Robotics Lab

Assignment 1

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Contents

1 Task A

1.1 Calculations

Compute the gravity vector G(q_1, q_2, q_3)= $[?\ ?\ ?]^T$ which estimates the torque ($\frac{kg \cdot m^2}{s^2}$) caused by gravity at each joint.
The generally known formula for the calculation of the gravitational torque is defined as $\tau = \vec{r} \times \vec{F}$. In this example, the torque caused by gravitation on all three different joints should be calculated/estimated. Therefore the force in the formula can be replaced by the gravitational force.

$$\vec{F_G} = m_i \cdot g \cdot \vec{e_{y,0}} \tag{1.1.1}$$

The gravitational force is pulling in the y direction of the joint two. Therefore the cross-product in the torque formula only incorporates the perpendicular distance from the reference joint to the center of the mass. Consequently, the $\vec{r}$ can be described through trigonometric sin and cos relations of the different angles. As shown in the Robotics lecture the torque caused by m_1 on joint two can be estimated as

$$\tau_{1,m_1} = r_1 \cdot cos(q_1) \cdot m_1 \cdot g \tag{1.1.2}$$

In the following, the abbreviations will be used according to the task sheet ($s_i = sin(q_i)$, $c_i = cos(q_i)$, ...).
Before calculating the torque vectors caused by each center of mass on each joint, it is important to say that in the resting angle of $[0° \ 0° \ 0°]^T$ the coordinates of joints two and three are rotated by +90°. Following from that the c12 and c123 terms change to s12 and s123 ($cos(q_1 + q_2 + 90) = sin(q_1 + q_2)$ and $cos(q_1 + q_2 + q_3 + 90) = sin(q_1 + q_2 + q_3)$).
The complete gravitational torque on each joint caused by each mass can be calculated by adding up the vectors $\tau_G = \tau_{m1} + \tau_{m2} + \tau_{m3}$. In each of the τ_{mi}'s the row corresponds to the joint. So the first row describes the torque caused by mass m_i on joint two, and the second row on joint three and the third row on joint five.
Gravitational torque τ_{m1} caused by mass m_1 can be estimated as.

$$\tau_{m1} = \begin{pmatrix} r_1 \cdot c1 \cdot m_1 \cdot g \\ 0 \\ 0 \end{pmatrix} \tag{1.1.3}$$

As m_1 has no impact on joints two and three, the torque in the corresponding rows is zero. The same applies for row three in the following τ_{m2}.

$$\tau_{m2} = \begin{pmatrix} (l_1 \cdot c1 + r_2 \cdot s12) \cdot m_2 \cdot g \\ r_2 \cdot s12 \cdot m_2 \cdot g \\ 0 \end{pmatrix} \tag{1.1.4}$$

Following from that τ_{m3} can be calculated analogously as

$$\tau_{m3} = \begin{pmatrix} (l_1 \cdot c1 + l_2 \cdot s12 + r_3 \cdot s123) \cdot m_3 \cdot g \\ (l_2 \cdot s12 + r_3 \cdot s123) \cdot m_3 \cdot g \\ r_3 \cdot s123 \cdot m_3 \cdot g \end{pmatrix} \tag{1.1.5}$$

The complete gravitational torque cause by all masses on all joints is therefore.

$$\tau_G = \tau_{m1} + \tau_{m1} + \tau_{m3} \tag{1.1.6}$$

$$\tau_G = g \cdot \begin{pmatrix} m_3 \cdot (l_1 \cdot c1 + l_2 \cdot s12 + r_3 \cdot s123) + m_2 \cdot (l_1 \cdot c1 + r_2 \cdot s12) + m_1 \cdot r_1 \cdot c1 \\ m_3 \cdot (l_2 \cdot s12 + r_3 \cdot s123) + m_2 \cdot r_2 \cdot s12 \\ m_3 \cdot r_3 \cdot s123 \end{pmatrix} \tag{1.1.7}$$

In order to counteract this torque in the following tasks $-\tau_G$ must be applied to the joints.

2 Task B

2.1 B1/B2 - P Controller and Tuning

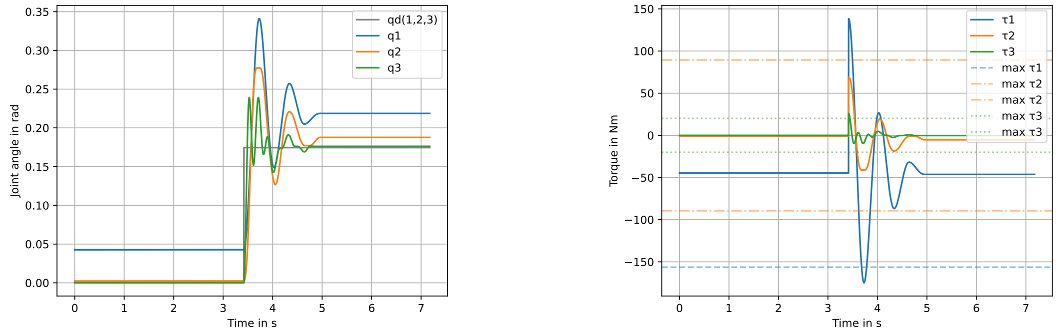
The controller was implemented according to the widely known formula of a simple proportional controller 2.1.1.

$$f = -k_p \cdot (x - x_d) \tag{2.1.1}$$

In our specific case, f is equal to the torque and the state x to the corresponding angle q_i. The process of tuning this controller is the adjustment of the three gains k_{p1}, k_{p2} and k_{p3} which determine how strong the controller will react to a deviation of the current angle (q_1, q_2, q_3) to the desired angles (q_{d1}, q_{d2}, q_{d3}).
Before presenting the tuned gains found by testing, the following two questions should be answered.

2.1.1 (a) What kind of behaviors do you observe with different gains?

The gains of the P-controller will determine different characteristics of our control system like rise-time, overshoot, oscillation, steady state error and control effort.
A higher k_p value leads to a more sensitive controller, meaning that it will start reacting to smaller deviations from the desired value. Therefore too large k_p values can lead to oscillation around the desired value. When comparing the reaction to the same specific deviation a controller with a smaller k_p will counteract slower and less "aggressive" than a controller with a bigger k_p. So a larger k_p lowers the rise time of the control system, decreases the steady state error, potentially increases oscillatory behavior and tends to increase the overshoot. Additionally the control effort increases with a larger k_p as the controller needs more power to react faster and stronger to the same deviation.
The listed behaviors are displayed in Figure 1, where a gainset above the tuned k_p was chosen to demonstrate the impact of a larger gain. The decreased rise-time alongside oscillatory behavior and decreased steady-state error are clearly visible in the graph. Also, the increased torque and therefore control effort can be seen in Figure 1b. As we have three joints the overshoot dynamics are more complicated. The increased gain leads to a larger overshoot of q_2 while decreasing the overshoot of q_1.

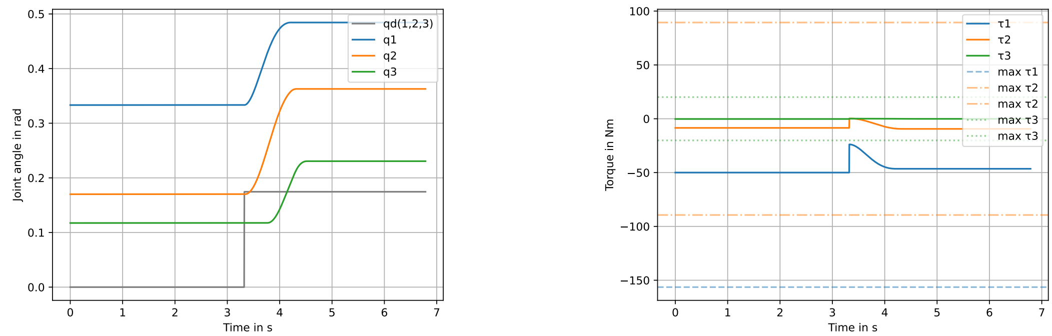


(a) Angular control repsonse.

(b) Torque control repsonse.

Figure 1: Reaction of a P-controller to a setpoint change of 0° to 10° with $k_{p1} = 1056$, $k_{p2} = 400$ and k_{p3}=150.

Choosing a k_p that is too small corresponds to the exact contrary reaction. It slows the controller down, increasing the rise time and steady-state error, while decreasing oscillatory behavior, control effort, and overshoot in simple systems. If k_p is too small a system is called overdamped. In this case, the controller is not able to sufficiently decrease steady-state error over time and has a slow reaction without any overshoot. This behavior can be observed in Figure 2.



(a) Angular control repsonse.

(b) Torque control repsonse.

Figure 2: Reaction of a P-controller to a setpoint change of 0° to 10° with $k_{p1} = 150$, $k_{p2} = 50$ and k_{p3}=2.

2.1.2 (b) Why are well tuned gains different for each joint?

Well tuned gains are different for each joint due to physical structure of the system. The parts that connect the joints have different masses and lengths and therefore have different forces applied to them. As calculated in Task A joint two has to counteract the gravitational forces of all the accumulated masses m_1, m_2 and m_3 while joint five has only to counteract the force effects of m_3. This can be also seen in Figures 1, 2, 3, where despite the joint two having a magnitudes larger k_p, the steady state error of joint 5 ($q3$)

is significantly smaller. As the gravitational force on joint five is smaller, the controller needs to counteract sigifcantly less and is working sufficiently with a k_p that would not provide joint two with enough actuation power.
When looking for the ideal tuned P-controller, we quickly came to realize that the following criteria cannot all be fulfilled to the fullest extent, but have to be seen as a trade of.

- q_{des} is reached as fast as possible
- there is no oscillation/overshoot in any of the values of q,
- the robot's torque limits are not exceeded

In order to fulfill the first requirement, it is clear that the k_p values have to be drastically increased in order to decrease the rise-time and steady state error. Even though a P-controller on its own cannot remove the steady state error completely (one could use a PI controller to manage that), an increasing k_p would decrease the steady state error further and further. However the second requirement is exactly contrary to the first one. As no overshoot and oscillation can only be achieved by a smaller k_p which slows the controller down and makes it less accurate (increases the steady state error). The third requirements sets a hard maximum boundary for k_p, as an increasing k_p eventually requires more torque than the maximum, as can be seen in Figure 1b.
A well tuned P-controller trade of was found in the following parameters.

- k_{p1}=396
- k_{p2}=176
- k_{p3}=7

Those gains provide the controller a sufficiently high rise time, while avoiding extensive overshooting and oscillating (see Figure 3). At the same time it keeps the used torque in a safety margin away from the max allowed values as shown in the following Figure 4.

2.2 B3 - Plots of Robots behavior njmove

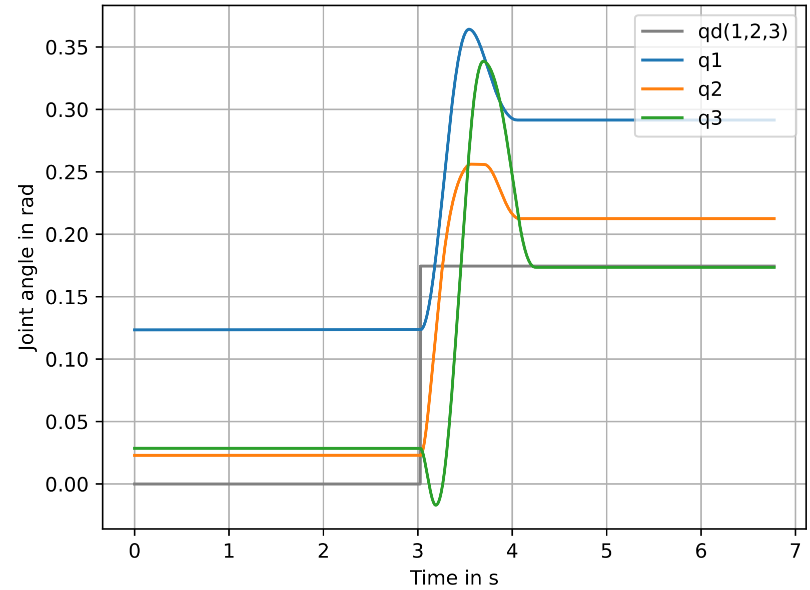


Figure 3: Angular control response of **tuned** P-controller to a setpoint change of 0° to 10° with $k_{p1} = 396$, $k_{p2} = 176$ and k_{p3}=7.

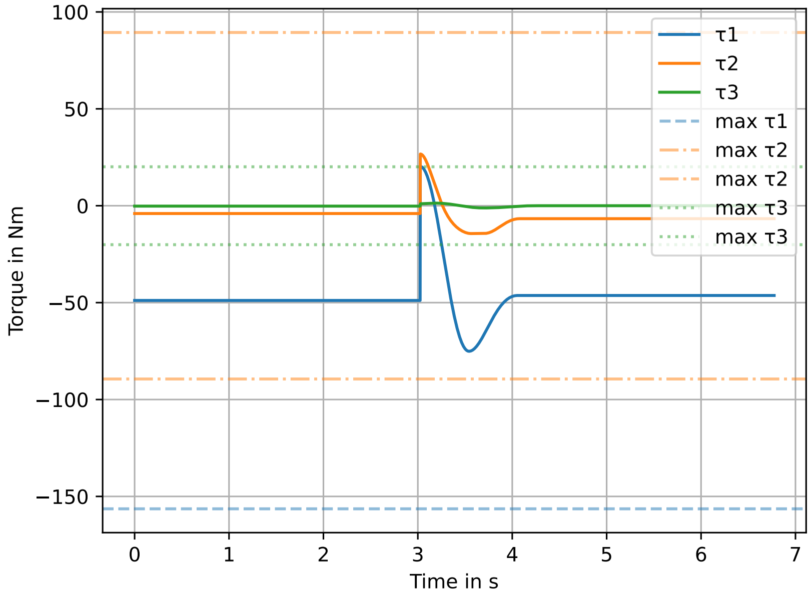


Figure 4: Torque control response of **tuned** P-controller to a setpoint change of 0° to 10° with $k_{p1} = 396$, $k_{p2} = 176$ and k_{p3}=7.

2.3 B6 - njgoto control

Tuned gains for P-controller with gravitational compensation: $k_{p1} = 350$, $k_{p2} = 150$ and k_{p3}=50.

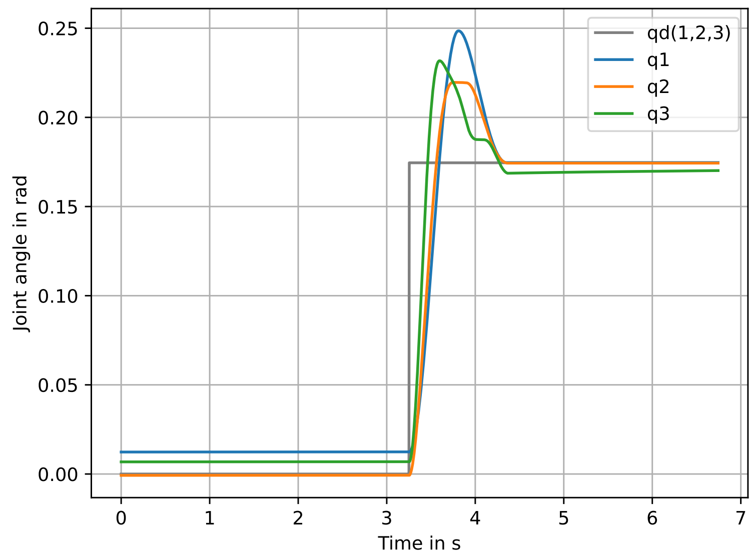


Figure 5: Angular control response of **tuned** P-controller with **gravitation force compensation** to a setpoint change of 0° to 10° with $k_{p1} = 350$, $k_{p2} = 150$ and k_{p3}=50.

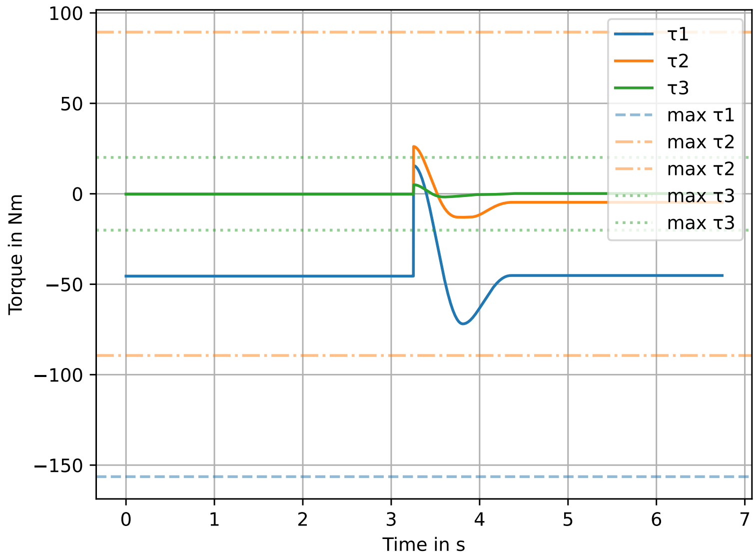


Figure 6: Torque control response of **tuned** P-controller with **gravitation force compensation** to a setpoint change of 0° to 10° with $k_{p1} = 350$, $k_{p2} = 150$ and k_{p3}=50.

2.4 B7 - jgoto Control

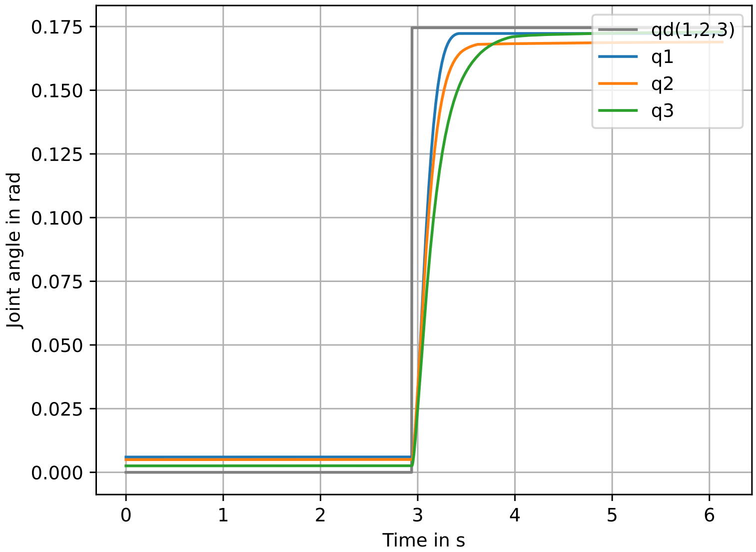


Figure 7: Angular control response of **tuned** PD-controller with **gravitation force compensation** to a setpoint change of 0° to 10° with
$k_{p1} = 1140$, $k_{p2} = 490$, $k_{p3} = 100$, $k_{d1} = 150$, $k_{d2} = 80$, $k_{d3} = 20$.

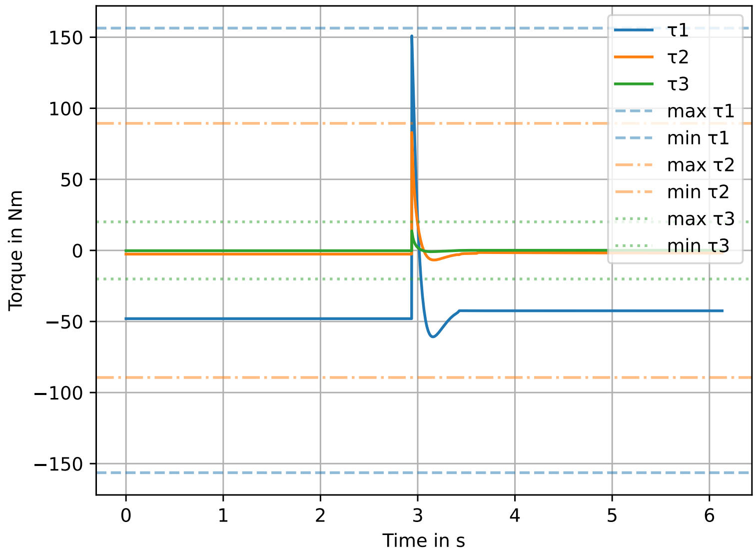


Figure 8: Torque control response of **tuned** PD-controller with **gravitation force compensation** to a setpoint change of 0° to 10° with
$k_{p1} = 1140$, $k_{p2} = 490$, $k_{p3} = 100$, $k_{d1} = 150$, $k_{d2} = 80$, $k_{d3} = 20$.

The gains k_{pi} and k_{di} were chosen as large as possible with respect to a 5Nm torque margin to the max value. Even though *as large as possible* could imply going as close to the limit as possible we considered a 5 Nm safety margin beneficial, considering the minor performance increase through larger kp and the major risk of reaching the max torque value.

2.4.1 Why can the k_p now be higher compared to the P-controller?

The k_p in the PD-controller can be higher than in the P-controller because the additional derivative part of the controller is damping the overshoot sufficiently. The derivative part is working against a rapidly changing state. In this case the higher the angular velocity dq the stronger the controller's derivative part is working against it. This effectively dampens rapid movements that the proportional part initiates due to a deviation of the state to the desired state.

Student Name	A1	B1	B2	B3	B4	B5	B6	B7	Documentation
Maxim Fenko	X	X	X	X	X	X	X	X	X
Abdelkarim Ben Salah	X	X	X	X	X	X	X	X	
Bryan Oppong-Boateng	X	X	X			X	X	X	