CSE 564
Visualization & Visual Analytics

High-Dimensional Data

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Lecture	Topic	Projects
1	Intro, schedule, and logistics	
2	Applications of visual analytics, basic tasks, data types	
3	Introduction to D3, basic vis techniques for non-spatial data	Project #1 out
4	Data assimilation and preparation	
5	Bias in visualization	
6	Data reduction and dimension reduction	
7	Visual perception and cognition	Project #1 due
8	Visual design and aesthetics	Project #2 out
9	Python/Flask hands-on	
10	Cluster analysis: numerical data	
11	Cluster analysis: categorical data	
12	Foundations of scientific and medical visualization	
13	Computer graphics and volume rendering	Project #2 due / Project #3 out
14	Scientific and medical visualization	
15	Illustrative rendering	Project #3 due
16	High-dimensional data, dimensionality reduction	Final project proposal call out
17	Correlation visualization	
18	Principles of interaction	
19	Midterm #1	
20	Visual analytics and the visual sense making process	Final project proposal due
21	Evaluation and user studies	
22	Visualization of time-varying and time-series data	
23	Visualization of streaming data	
24	Visualization of graph data	Final Project preliminary report due
25	Visualization of text data	
26	Midterm #2	
27	Data journalism	
	Final project presentations	Final Project slides and final report due

Understanding High-D Objects

Feature vectors are typically high dimensional

- this means, they have many elements
- high dimensional space is tricky
- most people do not understand it
- why is that?

- well, because you don't learn to see high-D when your vision system develops

Object permanence (Jean Piaget)

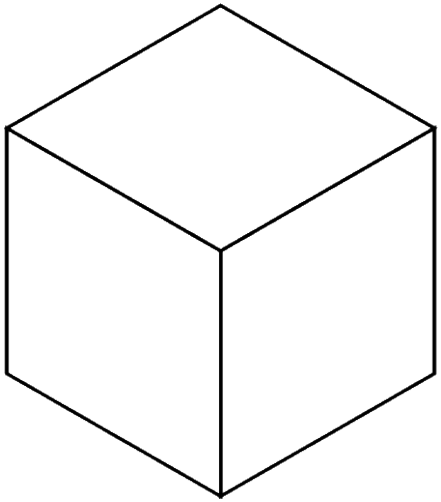
- the ability to create mental pictures or remember objects and people you have previously seen
- thought to be a vital precursor to creativity and abstract thinking

High-D Space is Tricky

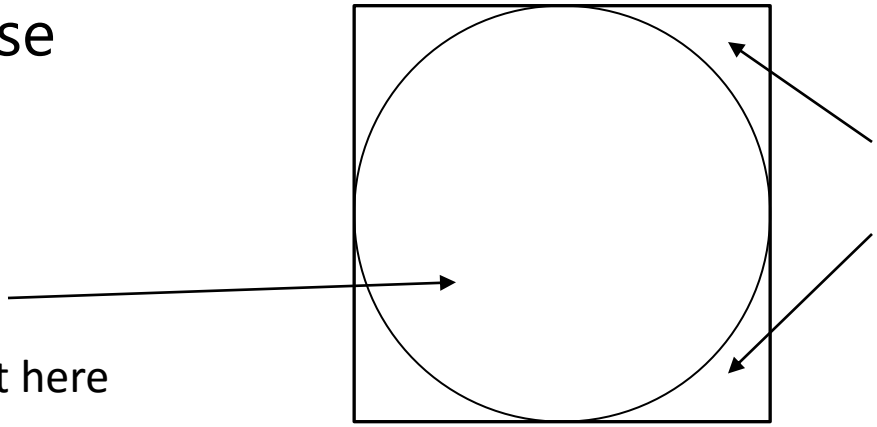
The curse of dimensionality

As $n \to \infty$

- Cube: side length l, diagonal d, volume V
- $V \to \infty$ for $l > 1$
- $V \to 0$ for $l < 1$
- $V = 1$ for $l = 1$
- $d \to \infty$

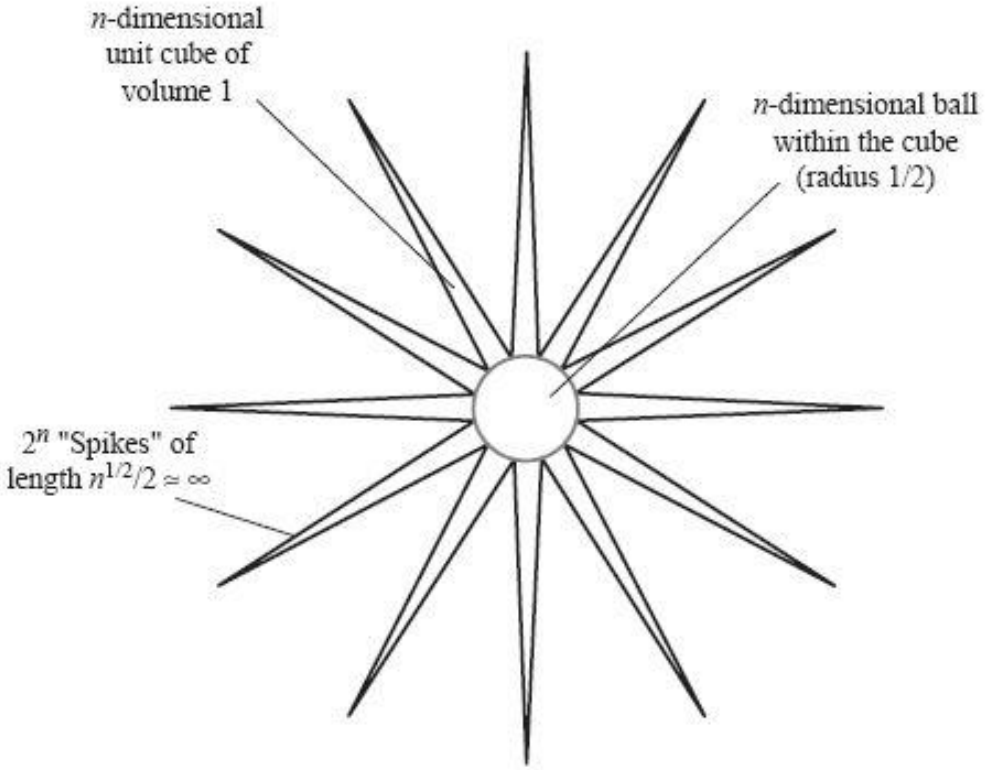


and very sparse



High-D Space is Tricky

Essentially hypercube is like a “hedgehog”



Curse of Dimensionality

Points are all at about the same distance from one another

- concentration of distances
- fundamental equation (Bellman, '61)

$$\lim_{n\to\infty} \frac{Dist_{\max} - Dist_{\min}}{Dist_{\min}} \to 0$$

- so as *n* increases, it is impossible to distinguish two points by (Euclidian) distance
  - unless these points are in the same cluster of points

Sparseness Demonstration

Space gets extremely sparse

- with every extra dimension points get pulled apart further
- distances become meaningless

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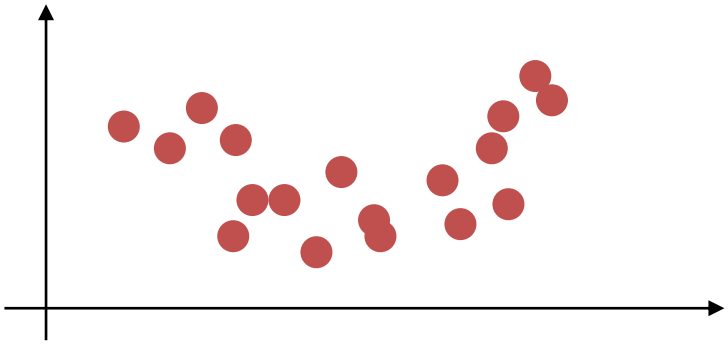
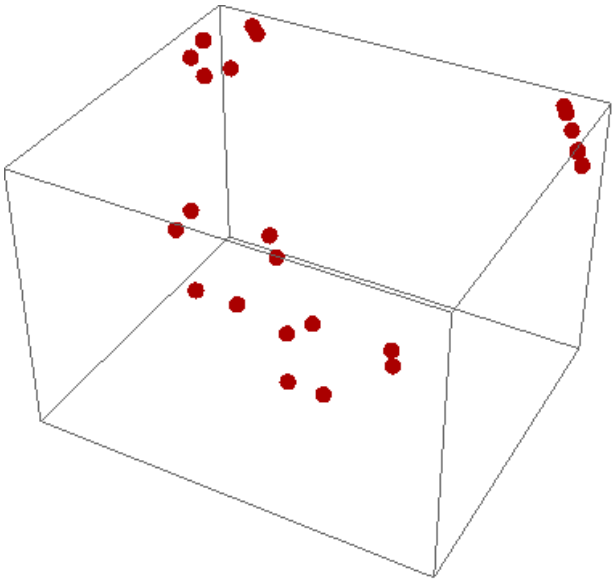
1D – points are very close



2D – points spread apart



3D – getting even sparser

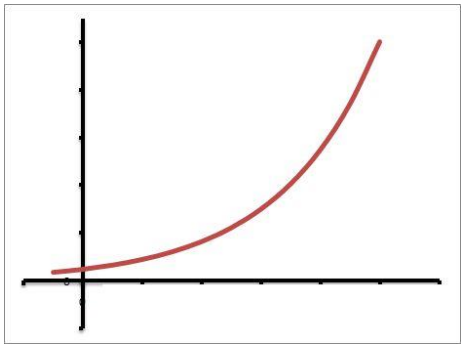
4D, 5D, … – sparseness grows further

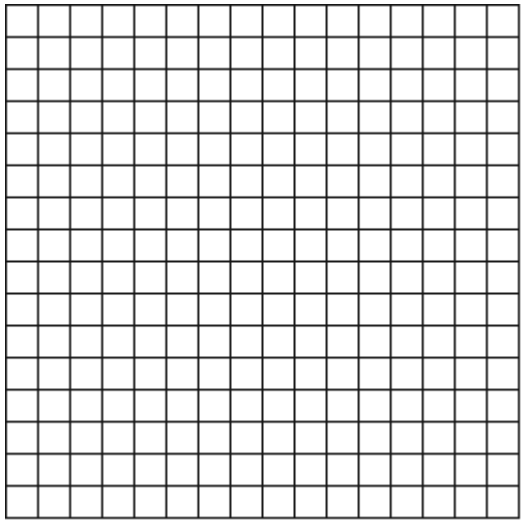
Space and Memory Management

Indexing (and storage) also gets very expensive

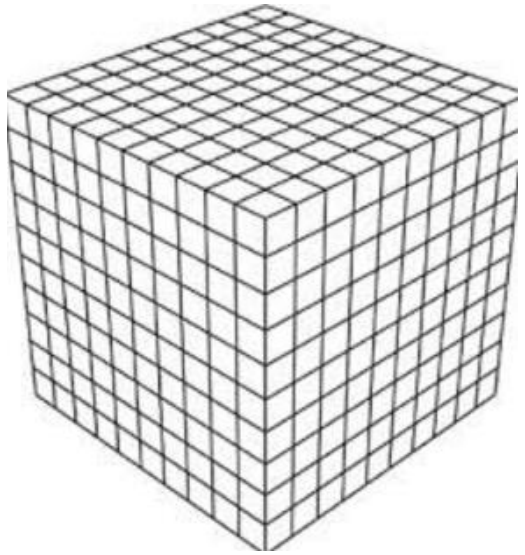
- exponential growth in the number of dimensions

16 cells





$16^2 = 256$ cells

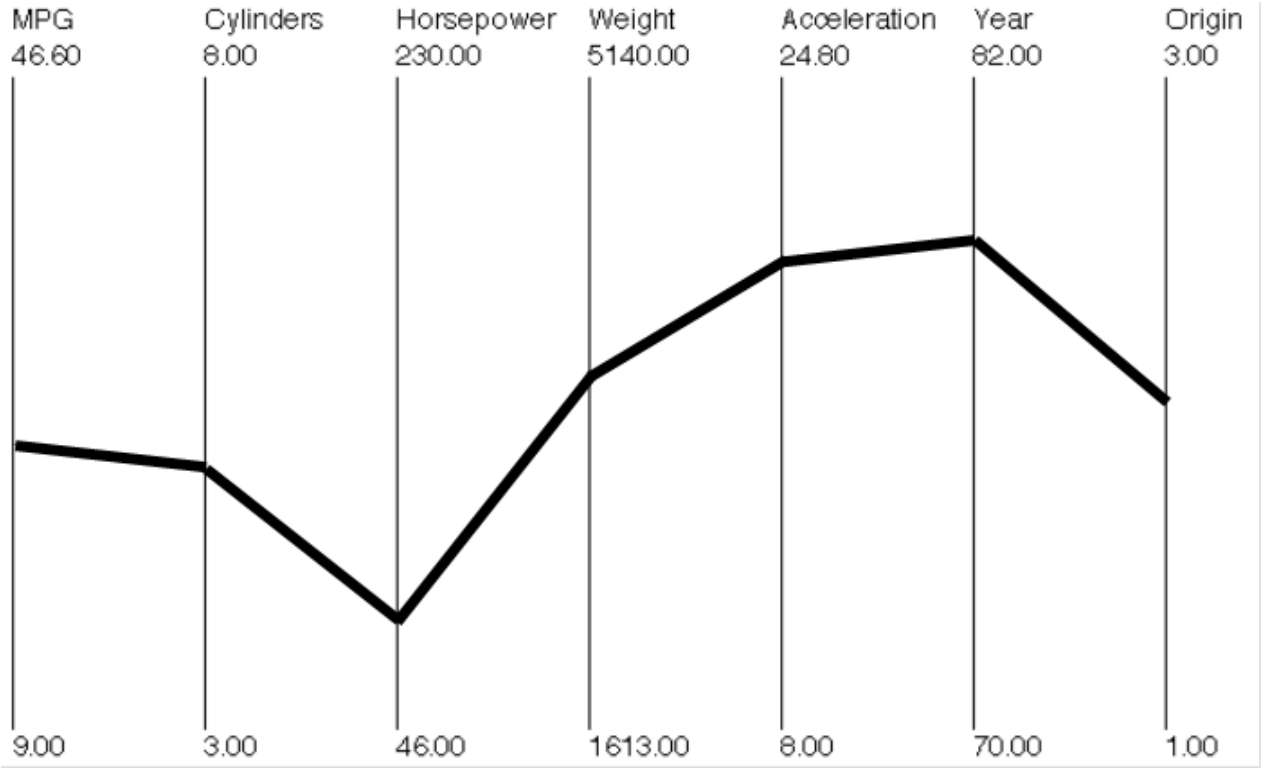


$16^3 = 4{,}096$ cells

- 4D: 65k cells 5D: 1M cells 6D: 16M cells 7D: 268M cells
- keep a keen eye on storage complexity

Recap: Parallel Coordinates

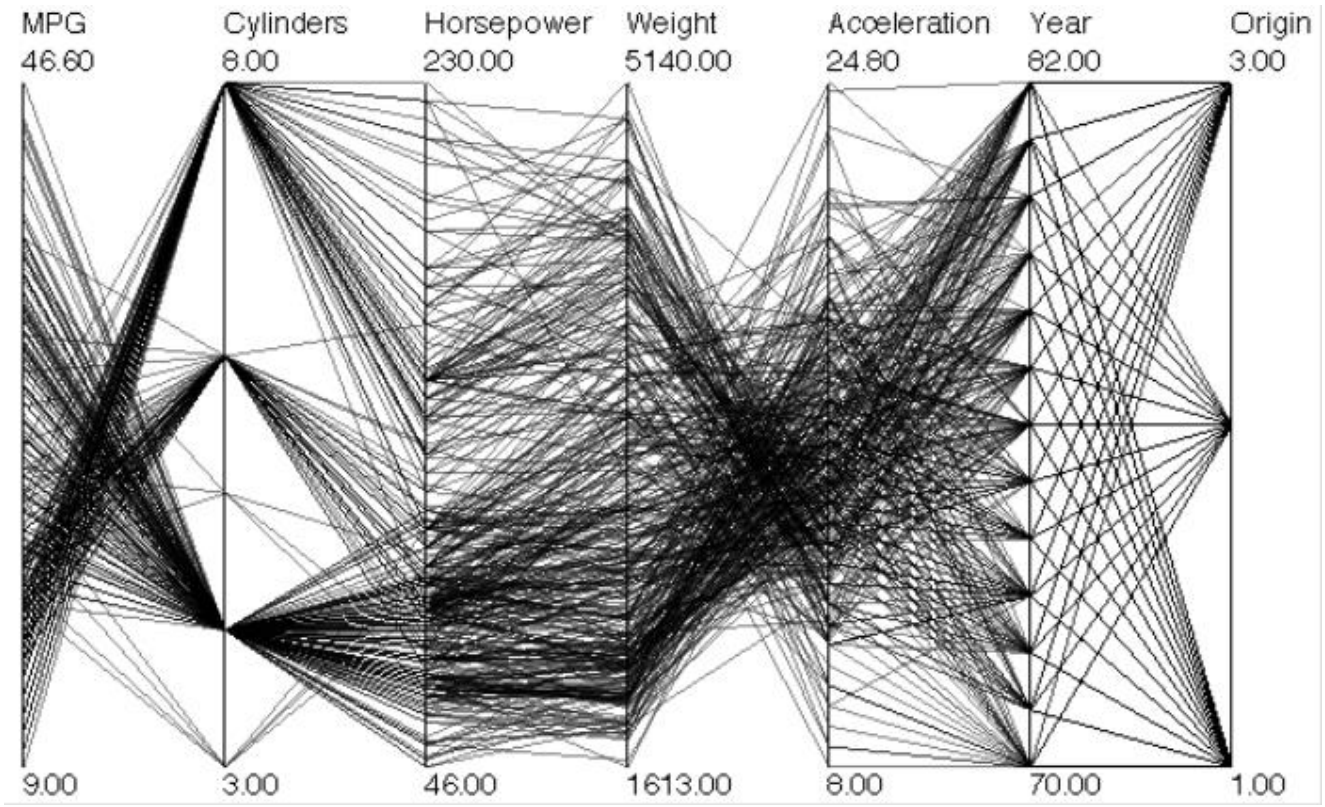
Parallel Coordinates – 1 Car



The N=7 data axes are arranged side by side

- in parallel

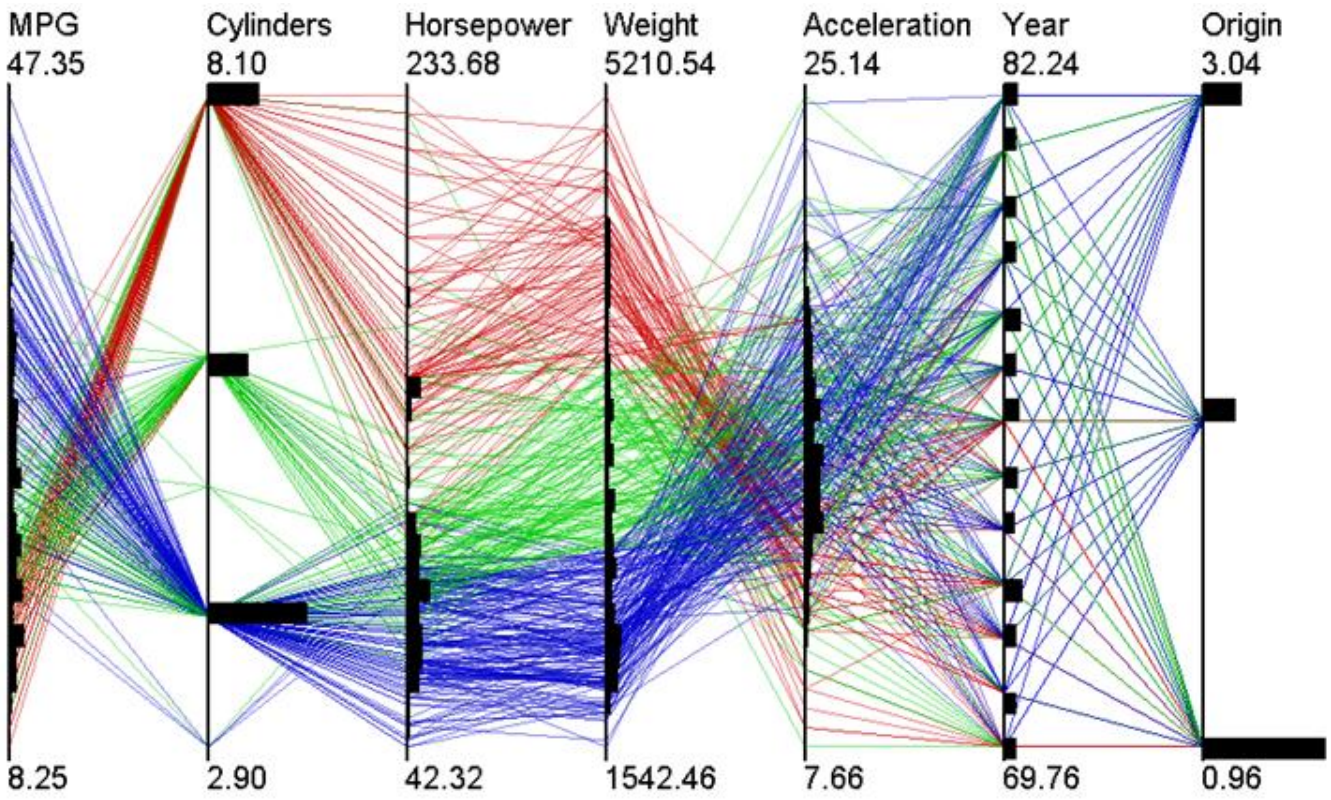
Parallel Coordinates – 100 Cars



Hard to see the individual cars?

- what can we do?

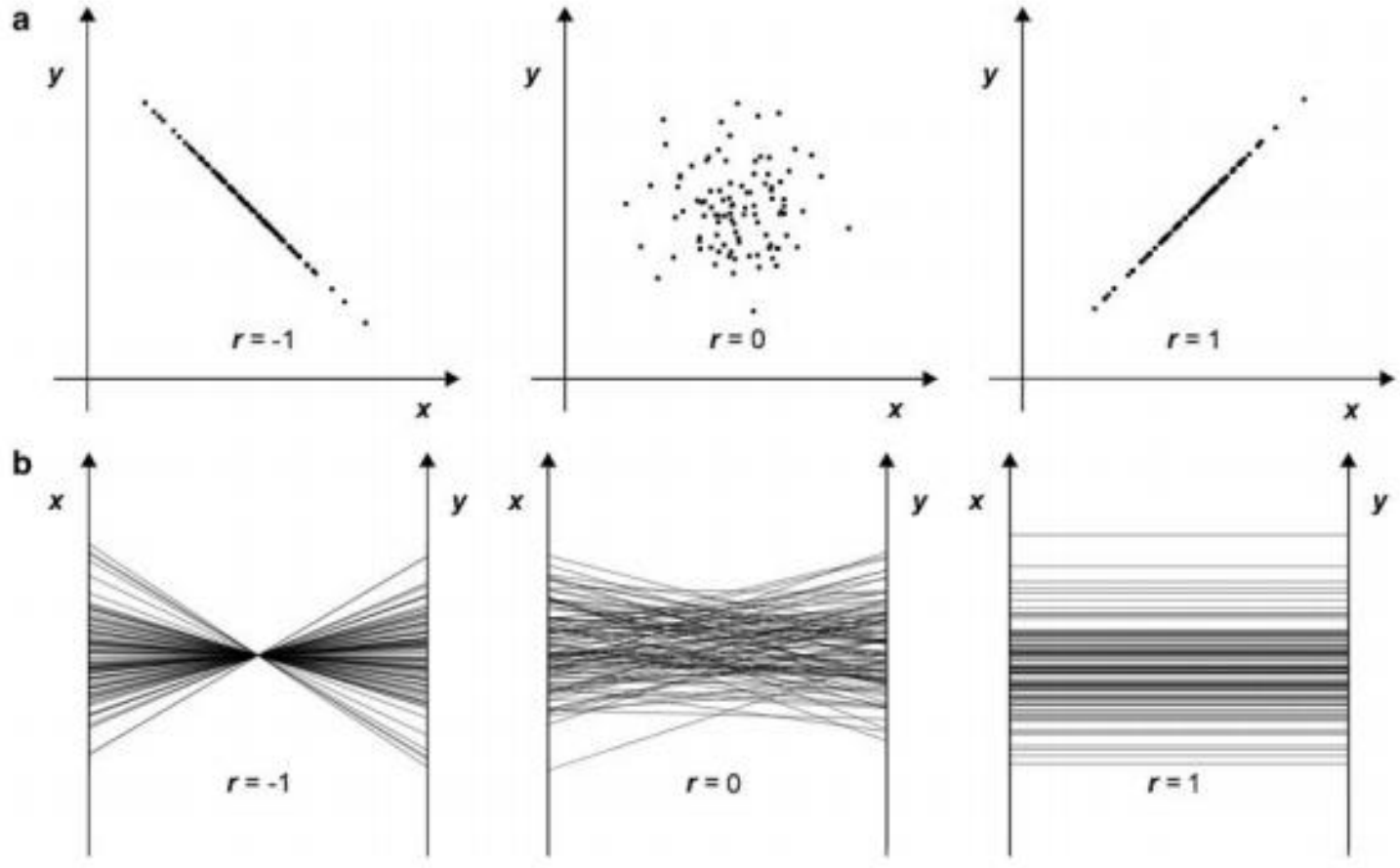
Parallel Coordinates – 100 Cars



Grouping the cars into sub-populations

- we perform clustering
- an be automated or interactive (put the user in charge)

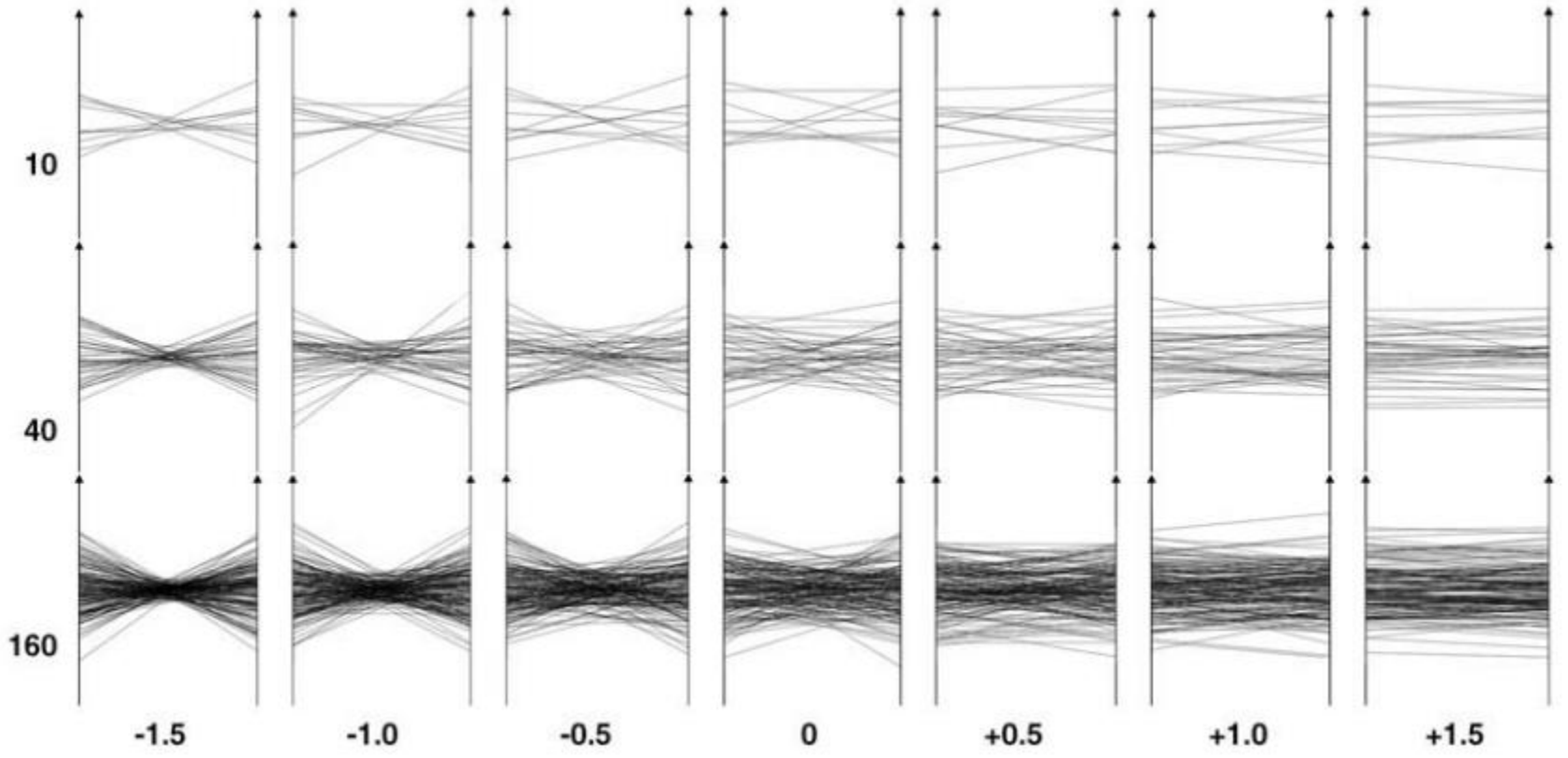
Patterns in Parallel Coordinates



correlation r=-1.0 r=0 r=1.0

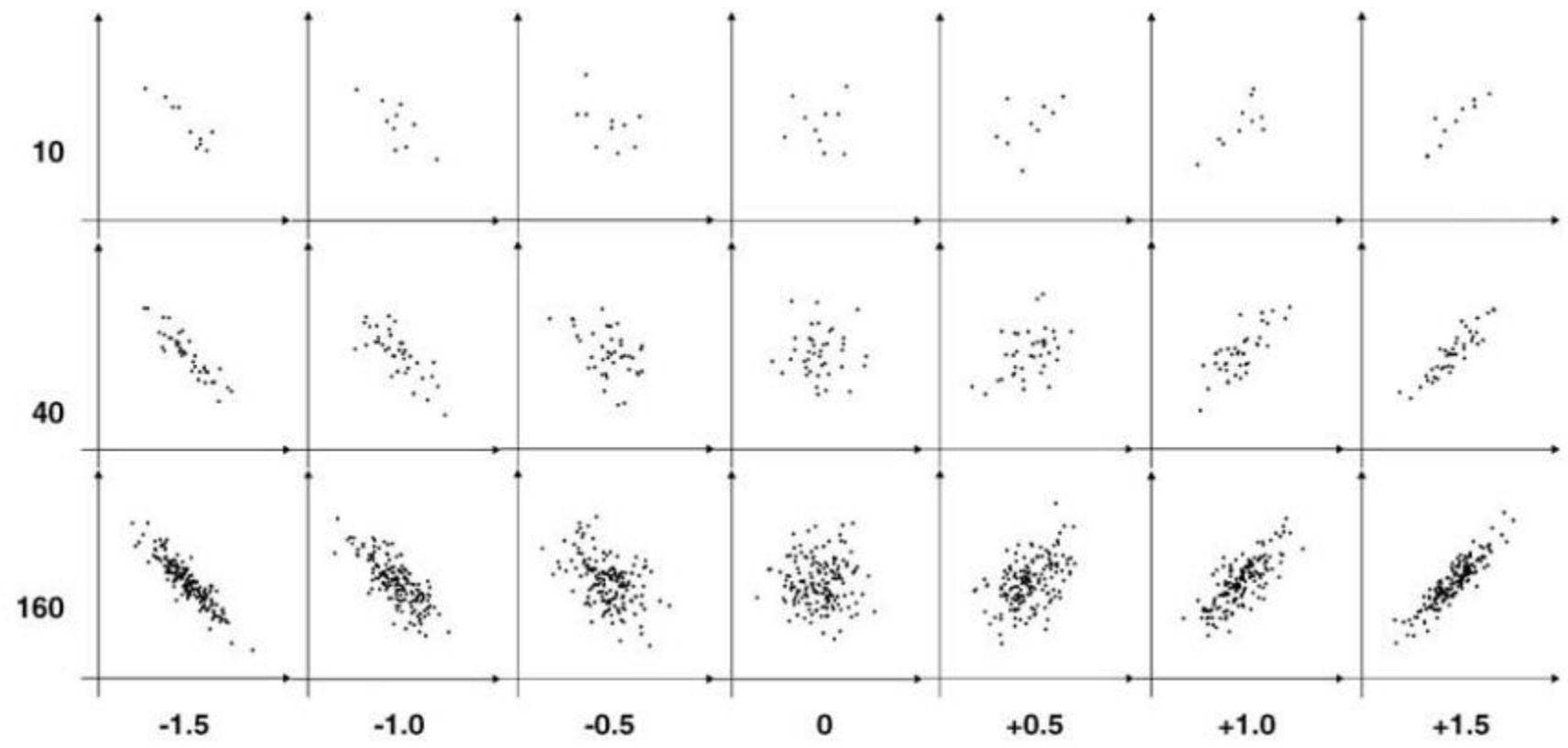
Patterns in Parallel Coordinates

points



Fisher-z (corresponding to $\rho = 0, \pm 0.462, \pm 0.762, \pm 0.905$)

Patterns in Scatterplots



Fisher-z (corresponding to $\rho = 0, \pm 0.462, \pm 0.762, \pm 0.905$)

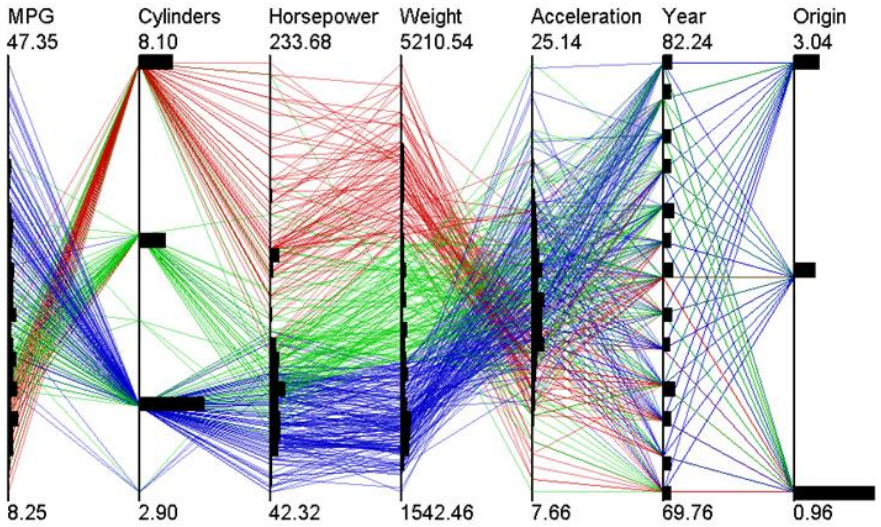
Li et al. found that twice as many correlation levels can be distinguished with scatterplots

Information Visualization Vol. 9, 1, 13 – 30

Axis Reordering Problem

There are n! ways to order the n dimensions

- how many orderings for 7 dimensions?
- 5,040
- but since can see relationships across 3 axes a better estimate is n!/((n-3)! 3!) = 35
- still a lot of axes orderings to try out → we need help

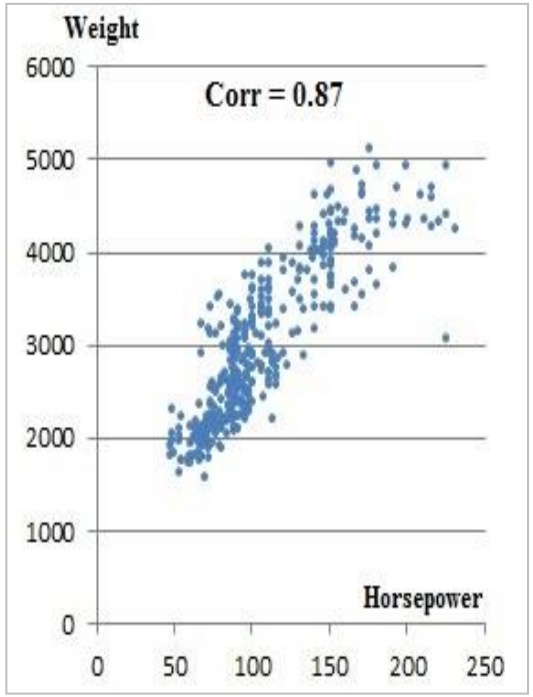


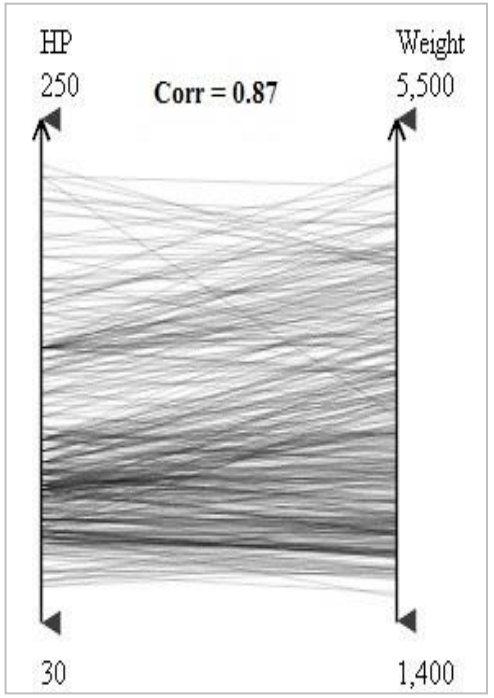
We Need a Measure for Relationships

Correlation

- a statistical measure that indicates the extent to which two or more variables fluctuate together

$$r_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{x})^2}}$$





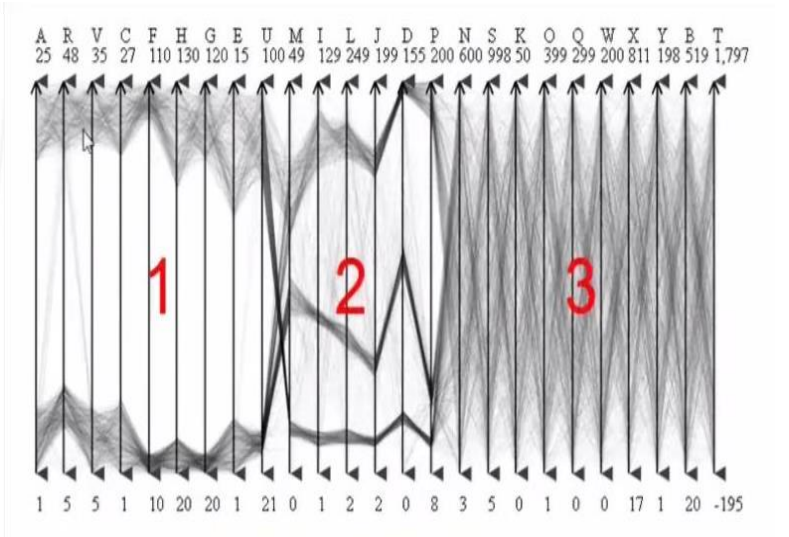
Building the Correlation Matrix

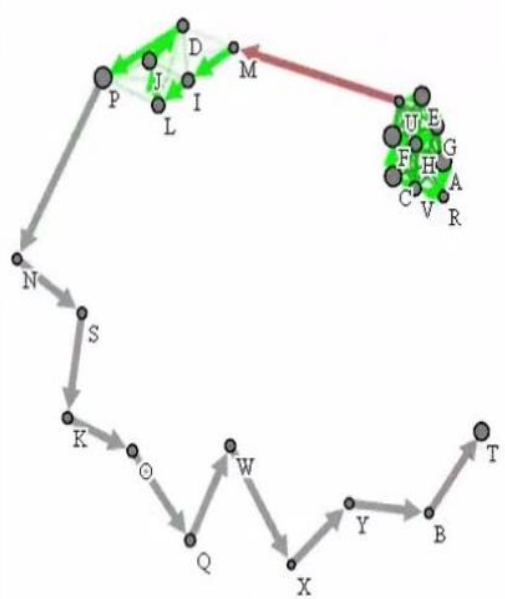
Create a correlation matrix

Run a mass-spring model

Run Traveling Salesman on the correlation nodes

Use it to order your parallel coordinate axes via TSP

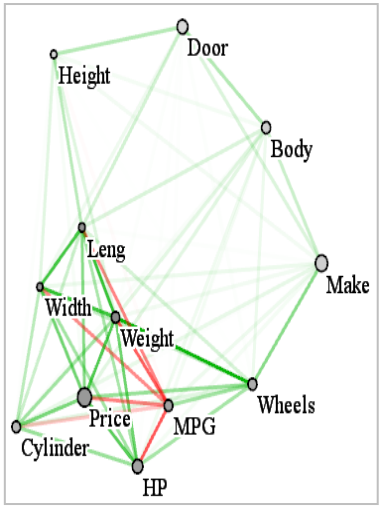




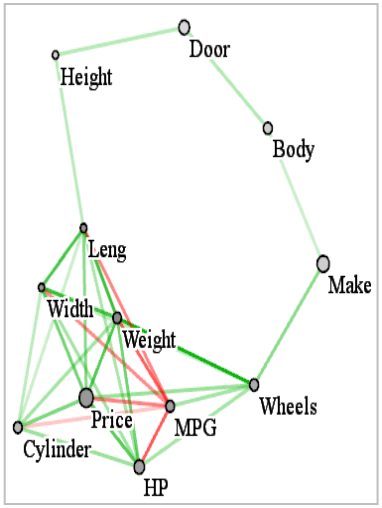
	MRK	MSFT	PFE	PG	T	TRV	UTX	VZ	WMT	XOM
MRK	1.	0.39	0.72	−0.43	0.57	0.031	−0.26	0.61	−0.11	−0.25
MSFT	0.39	1.	0.14	0.11	0.56	0.25	0.25	0.67	−0.074	0.24
PFE	0.72	0.14	1.	−0.77	0.08	−0.37	−0.65	0.19	−0.077	−0.72
PG	−0.43	0.11	−0.77	1.	0.25	0.68	0.92	0.086	0.072	0.9
T	0.57	0.56	0.08	0.25	1.	0.65	0.46	0.87	−0.059	0.54
TRV	0.031	0.25	−0.37	0.68	0.65	1.	0.83	0.43	−0.0067	0.81
UTX	−0.26	0.25	−0.65	0.92	0.46	0.83	1.	0.27	−0.033	0.93
VZ	0.61	0.67	0.19	0.086	0.87	0.43	0.27	1.	0.026	0.36
WMT	−0.11	−0.074	−0.077	0.072	−0.059	−0.0067	−0.033	0.026	1.	0.032
XOM	−0.25	0.24	−0.72	0.9	0.54	0.81	0.93	0.36	0.032	1.

Interaction with the Correlation Network

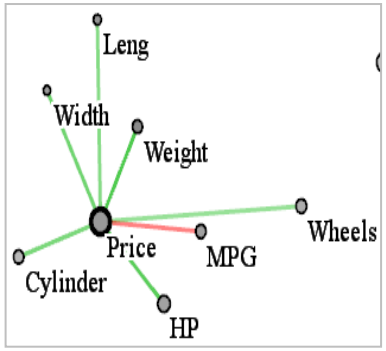
- Vertices are attributes, edges are correlations
  - vertex: size determined by $\sum_{j=0}^{D} \frac{|correlation(i,j)|}{D-1}$ $j \neq i$
  - edge: color/intensity → sign/strength of correlation



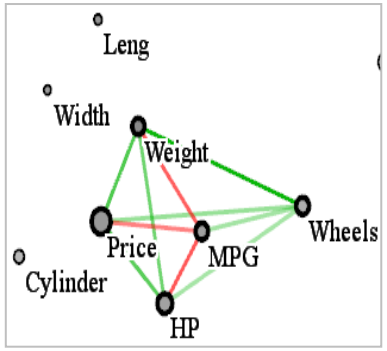
all edges



filtered by strength

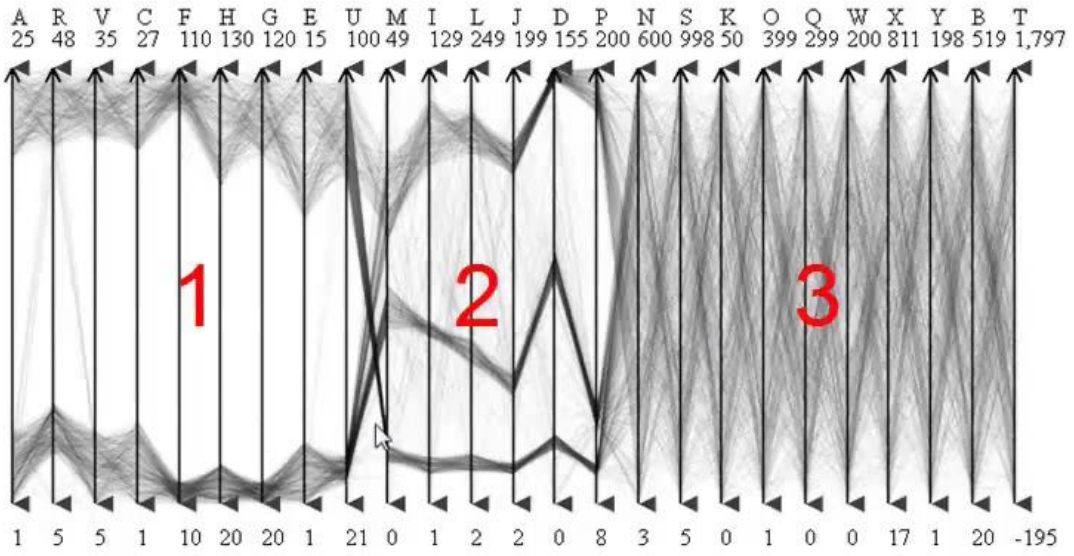


attribute centric



subset of attributes

Multiscale Zooming

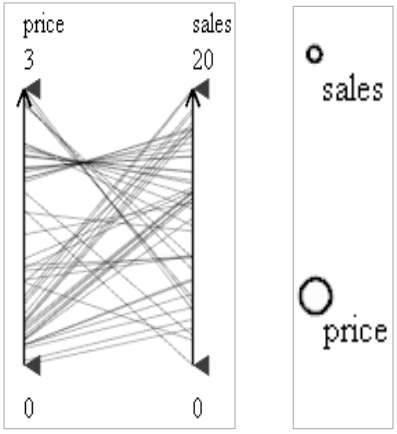


3 subspaces are well seperated.

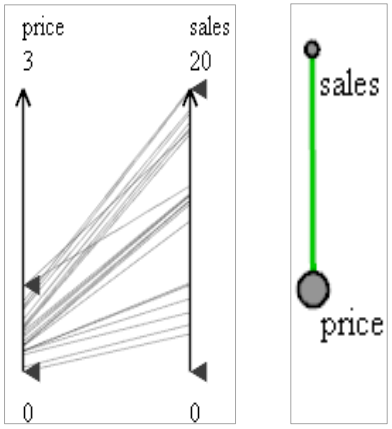
Bracketing and Conditioning

Correlation strength can often be improved by constraining a variable's value range

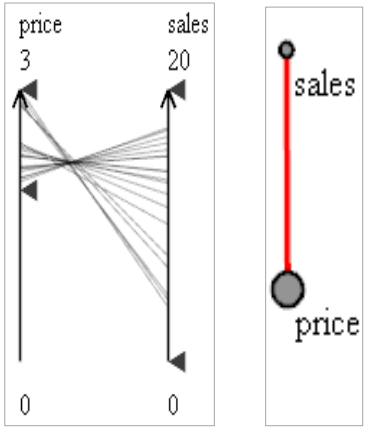
- this limits the derived relationships to this value range
- such limits are commonplace in targeted marketing, etc.



no bracketing



lower price range



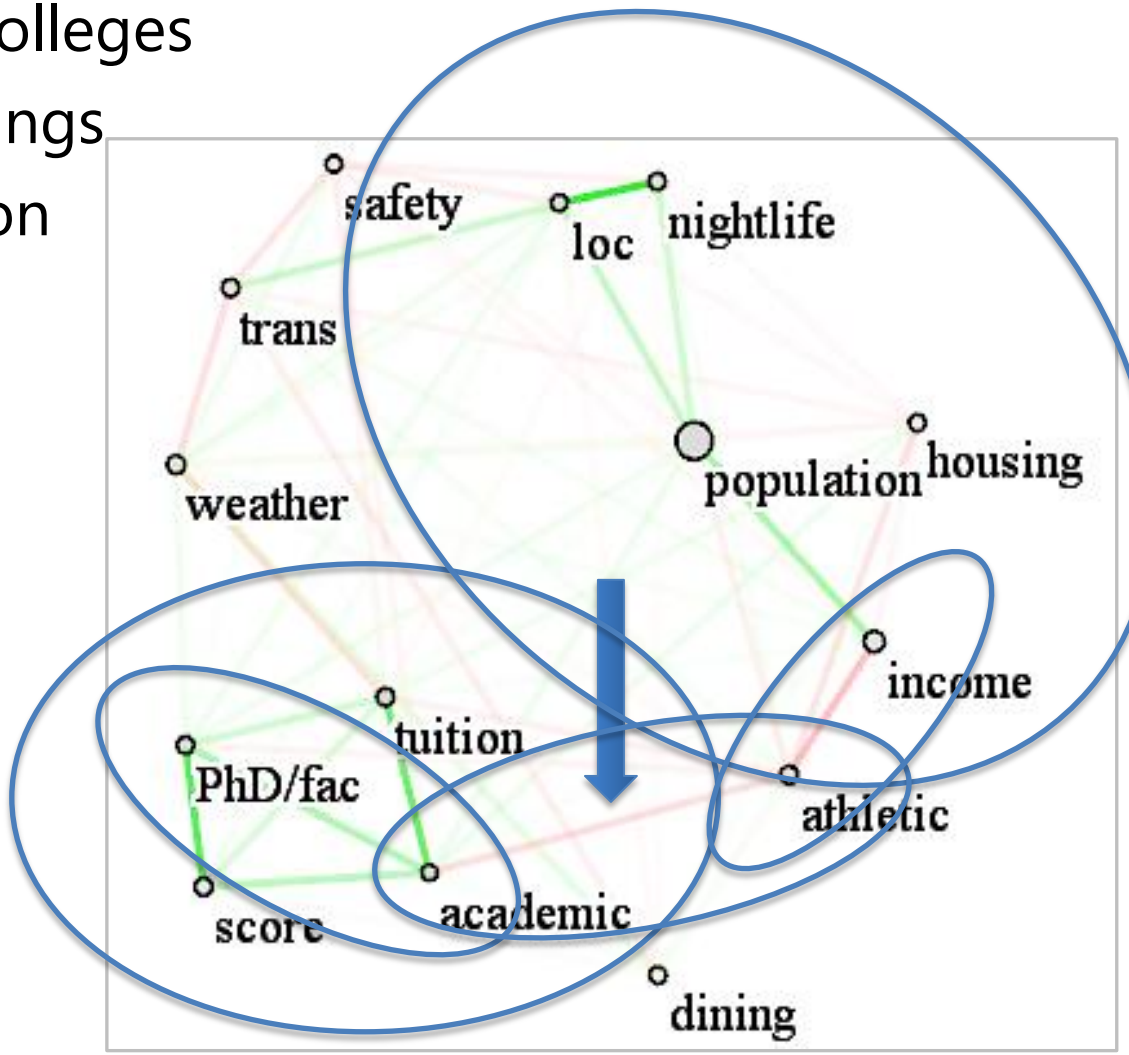
higher price range

Correlation Plots Are Powerful

Fused dataset of 50 US colleges

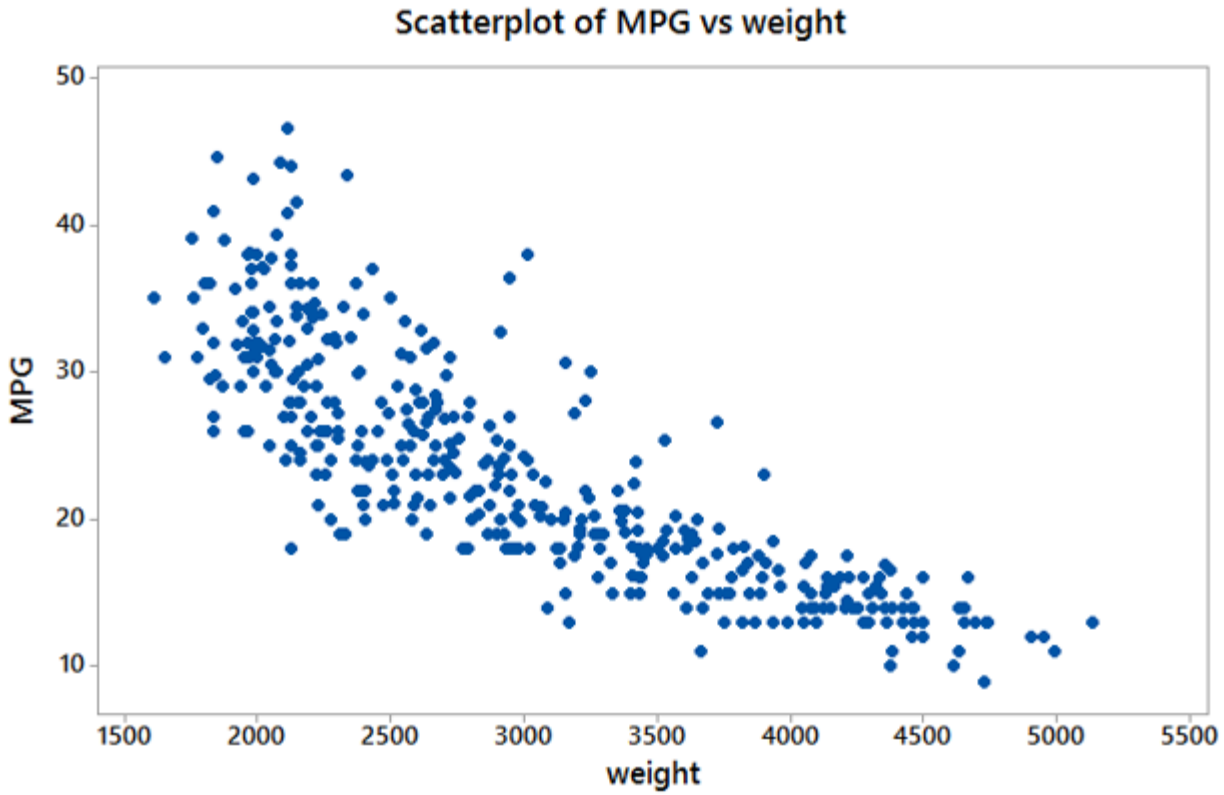
US News: academic rankings

College Prowler: survey on campus life attributes



Scatterplots

Projection of the data items into a bivariate basis of axes



Projection Operations

How does 2D projection work in practice?

- N-dimensional point $x = \{x_1, x_2, x_3, \ldots x_N\}$
- a basis of two orthogonal axis vectors defined in N-D space

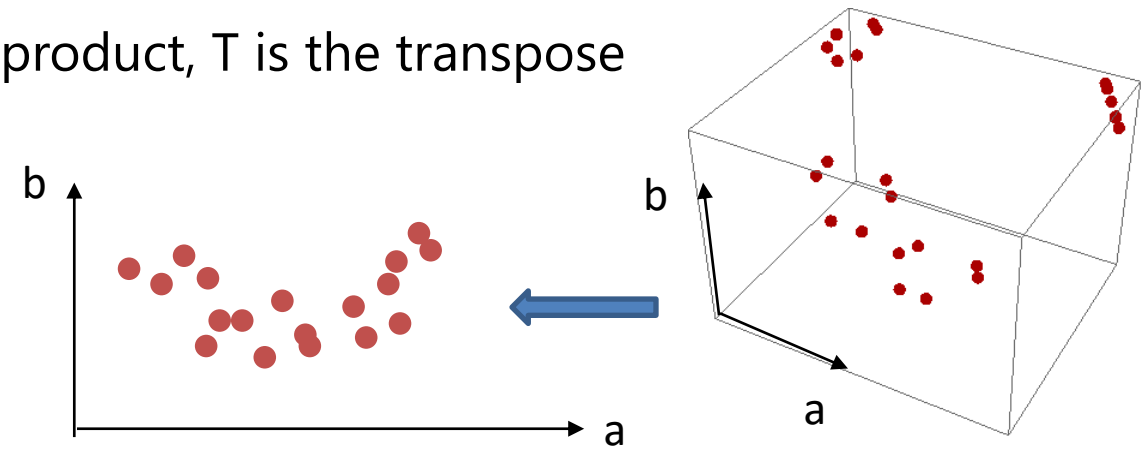
  $a = \{a_1, a_2, a_3, \ldots a_N\}$

  $b = \{b_1, b_2, b_3, \ldots b_N\}$
- a projection $\{x_a, x_b\}$ of x into the 2D basis spanned by {a, b} is:

  $x_a = a \cdot x^T$

  $x_b = b \cdot x^T$

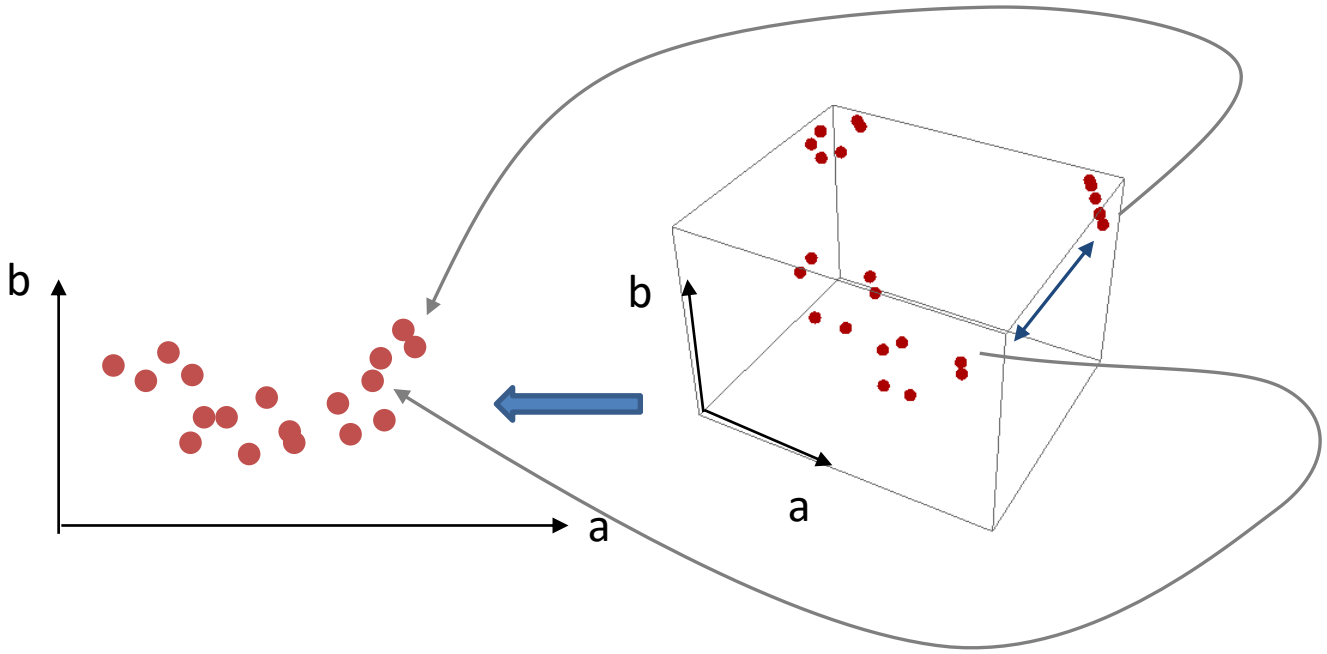
  where $\cdot$ is the dot product, T is the transpose



Projection Ambiguity

Projection causes inaccuracies

- close neighbors in the projections may not be close neighbors in the original higher-dimensional space
- this is called *projection ambiguity*



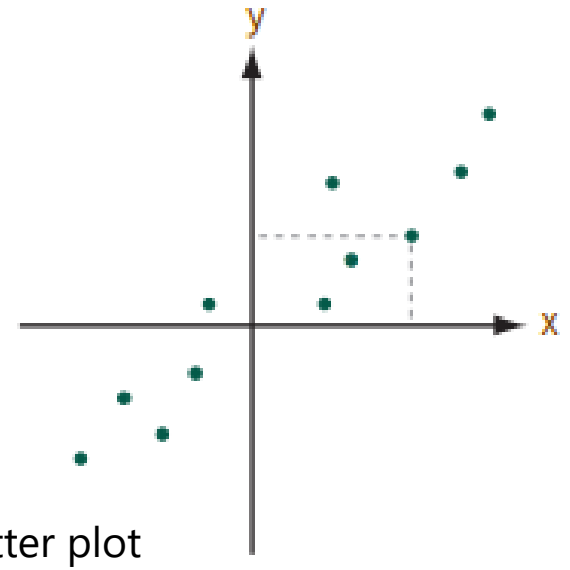
Biplots

Plots data points and dimension axes into a single visualization

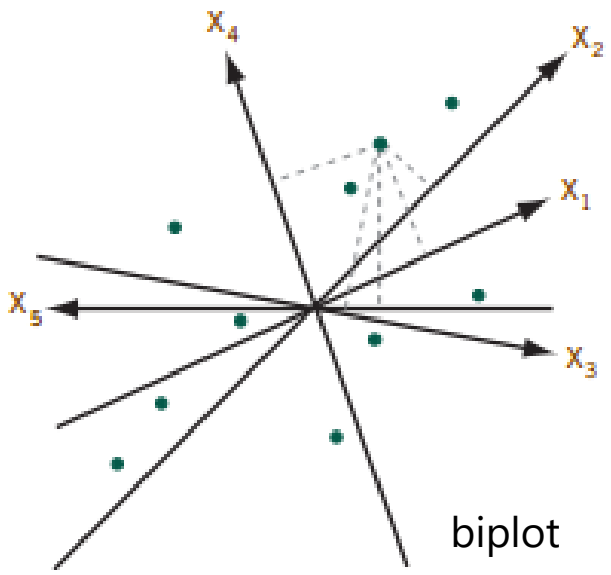
- uses first two PCA vectors as the basis to project into
- find plot coordinates [x] [y]

  for data points: [PCA_1 · data vector] [PCA_2 · data vector]

  for dimension axes: [PCA_1[dimension]] [PCA_2[dimension]]



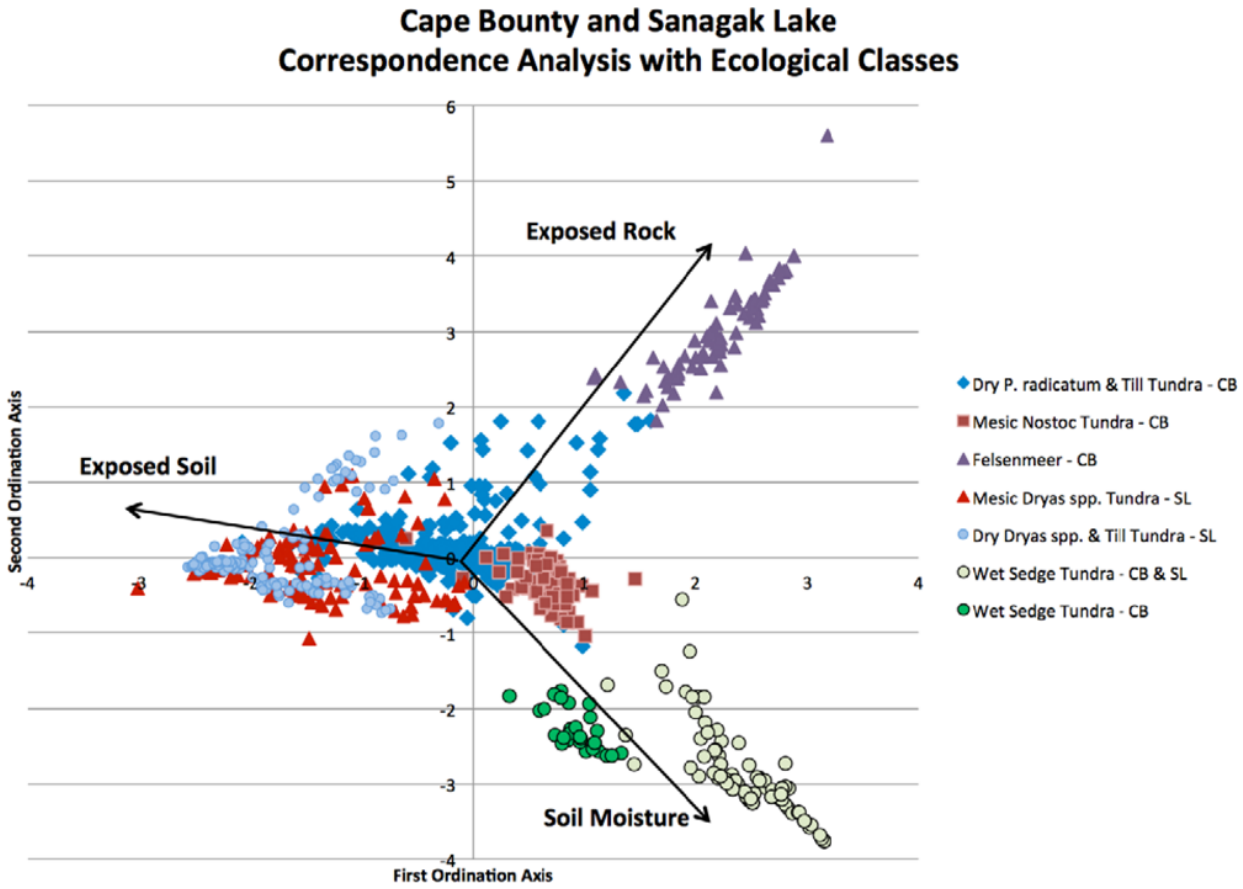
scatter plot



biplot

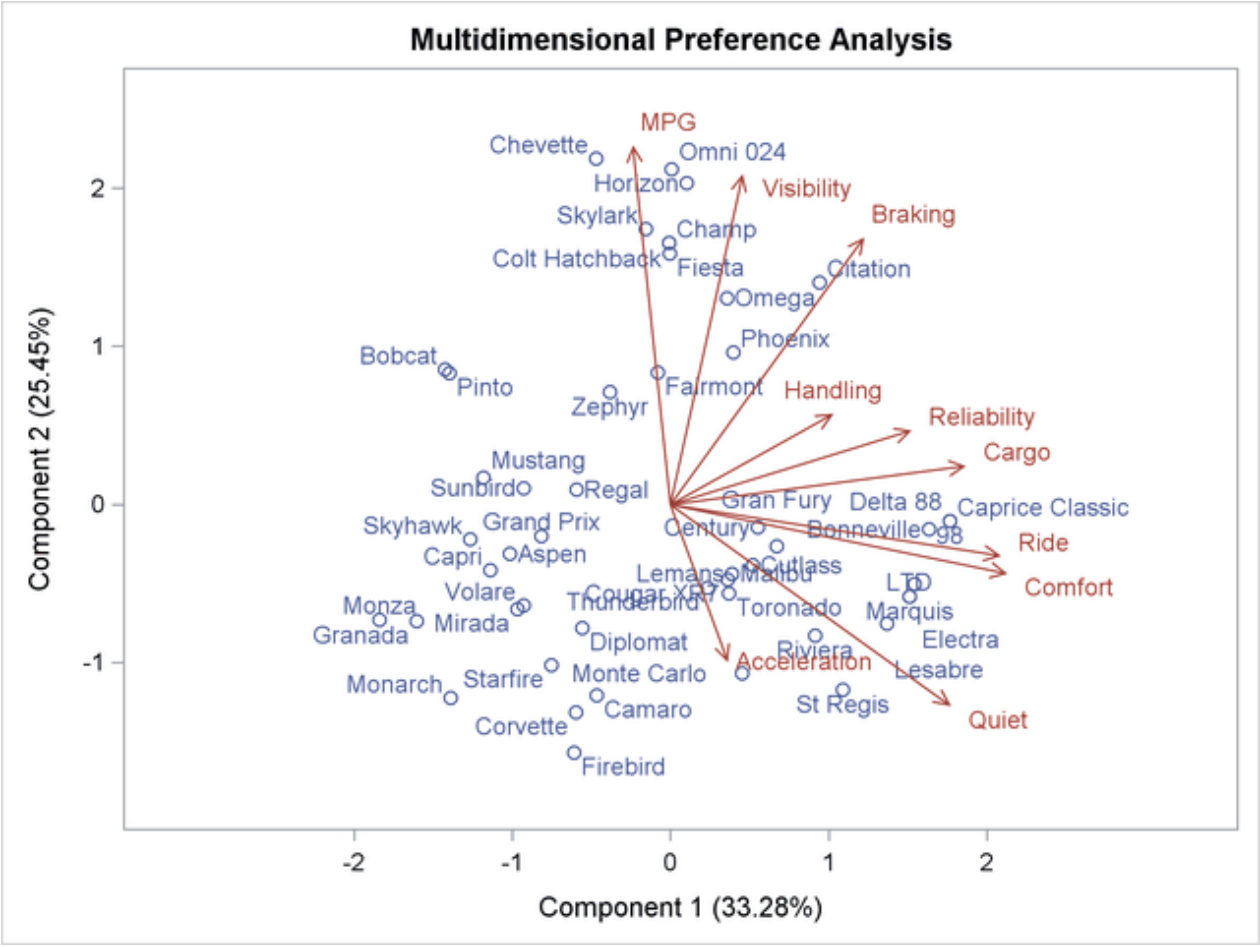
Biplots in Practice

See data distributions into the context of their attributes



Biplots in Practice

See data points into the context of their attributes



Biplots – A Word of caution

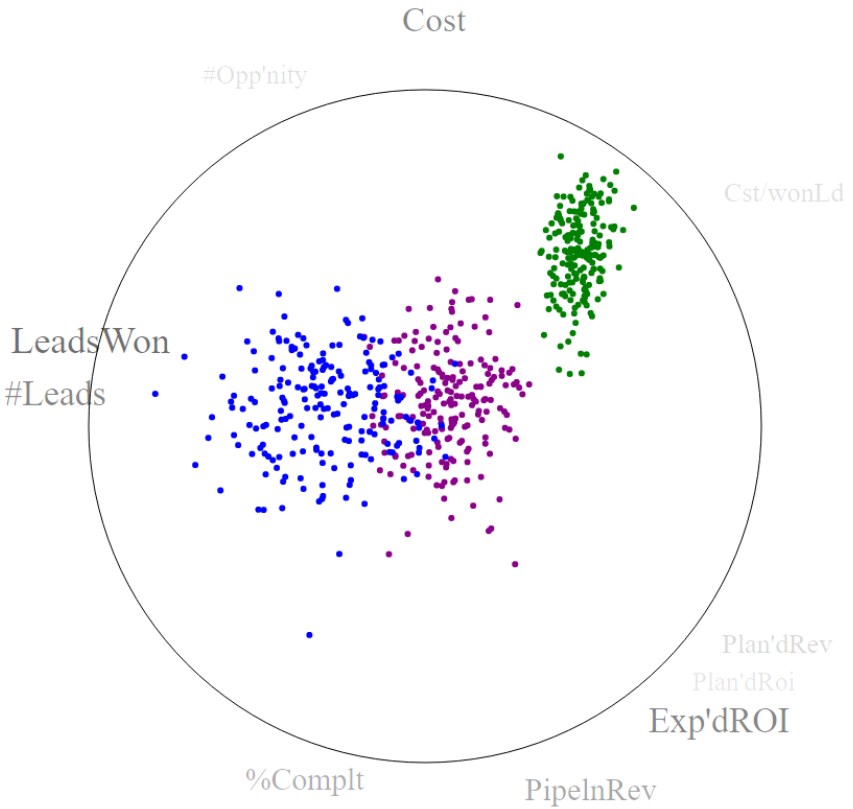
Do be aware that the projections may not be fully accurate

- you are projecting N-D into 2D by a linear transformation
- if there are more than 2 significant PCA vectors then some variability will be lost and won't be visualized
- remote data points might project into nearby plot locations suggesting false relationships → projection ambiguity
- always check out the PCA scree plot to gauge accuracy

Interactive Biplots

Also called multivariate scatterplot

- biplot-axes length vis replaced by graphical design
- less cluttered view
- but there's more to this .....



Meet the *Subspace Voyager*

Decomposes high-D data spaces into lower-D subspaces by

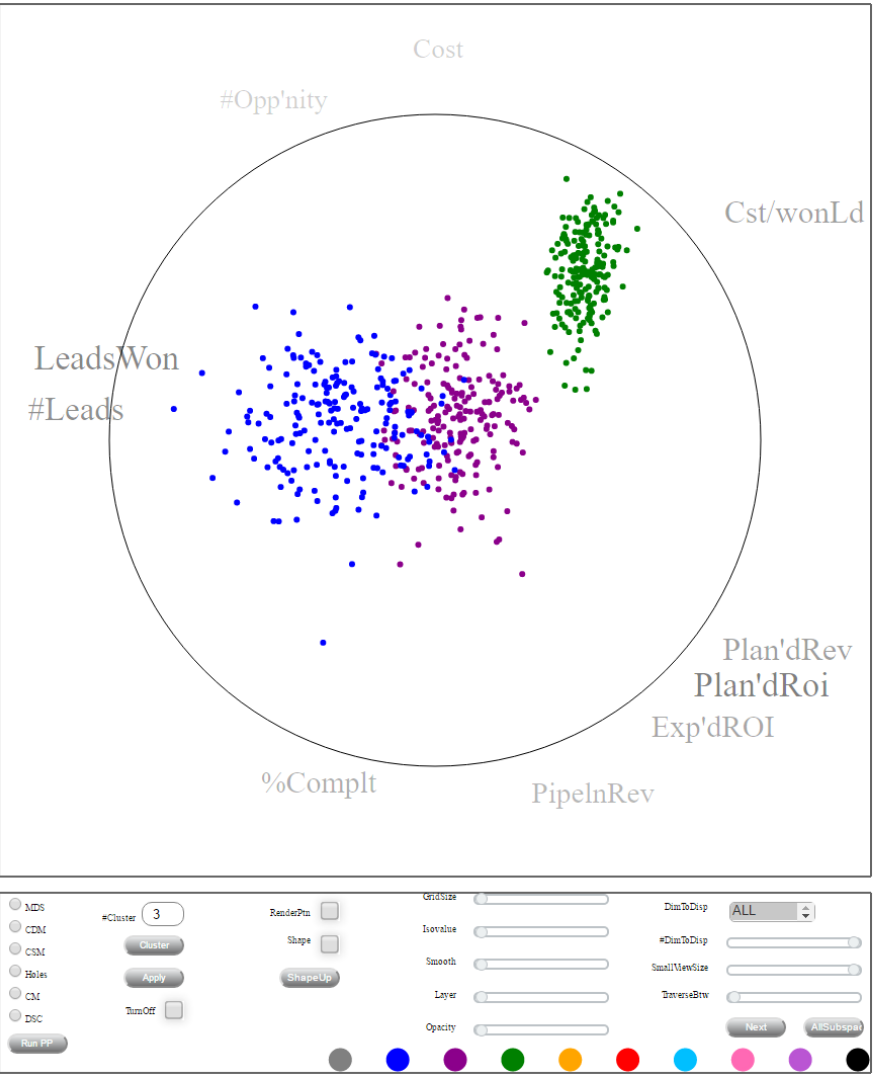
- clustering
- classification
- reducing clusters to intrinsic dimensionality via local PCA

Allows users to interactively explore these lower-D subspaces

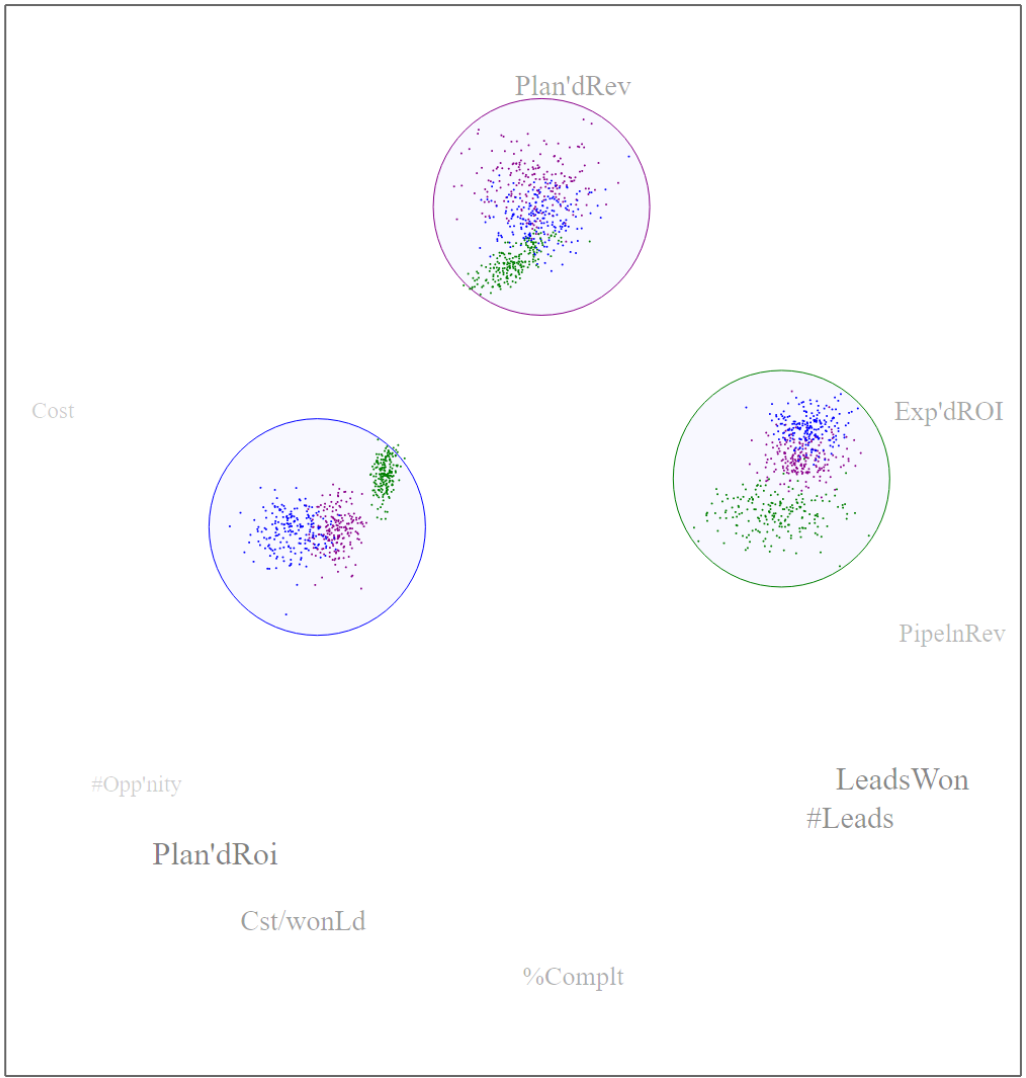
- explore them as a chain of 3D subspaces
- transition seamlessly to adjacent 3D subspaces on demand
- save observations as you go (and return to them just as well)

Visualize Raw Data w/the Subspace Voyager

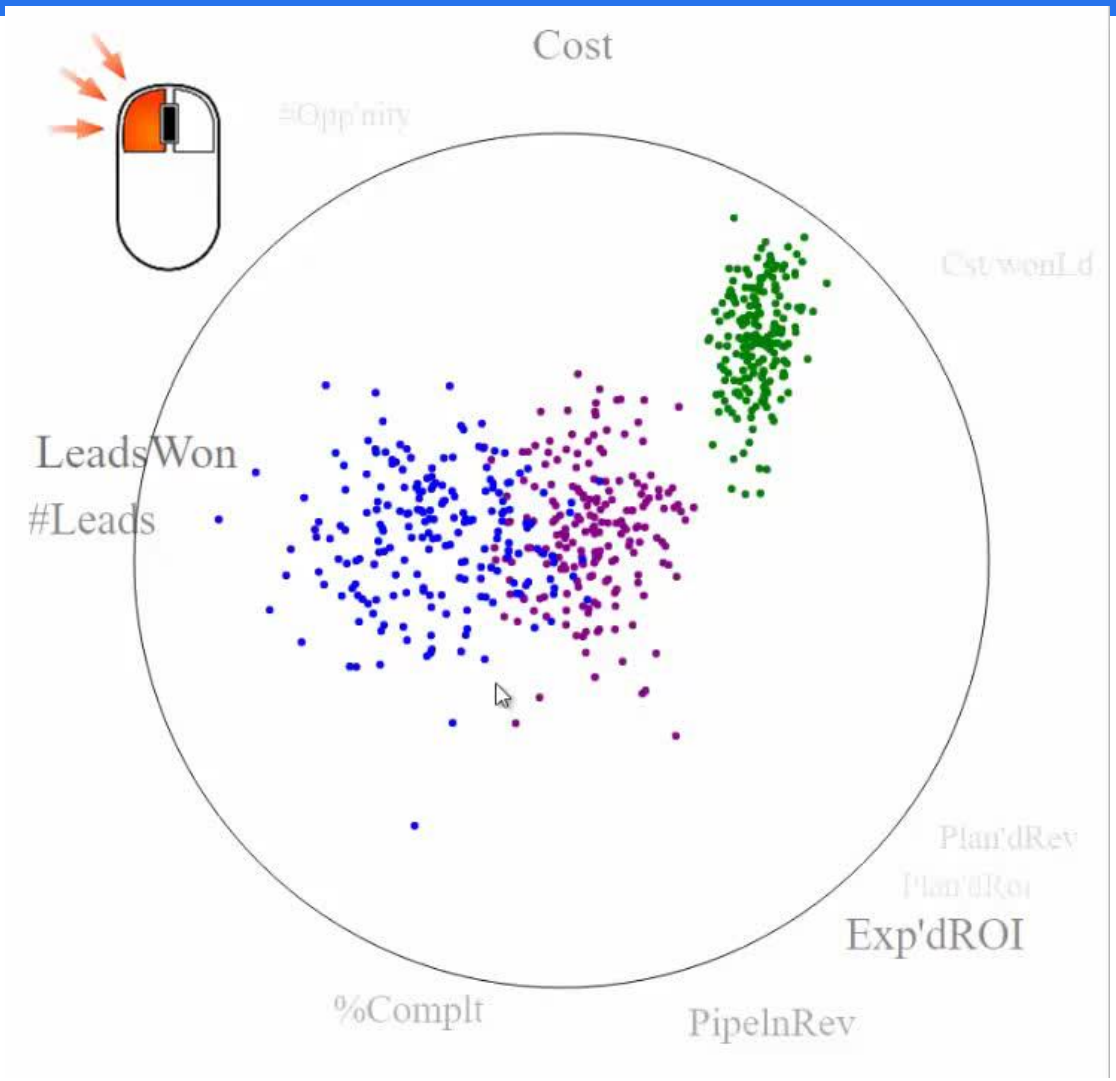
Interactive Scatterplot



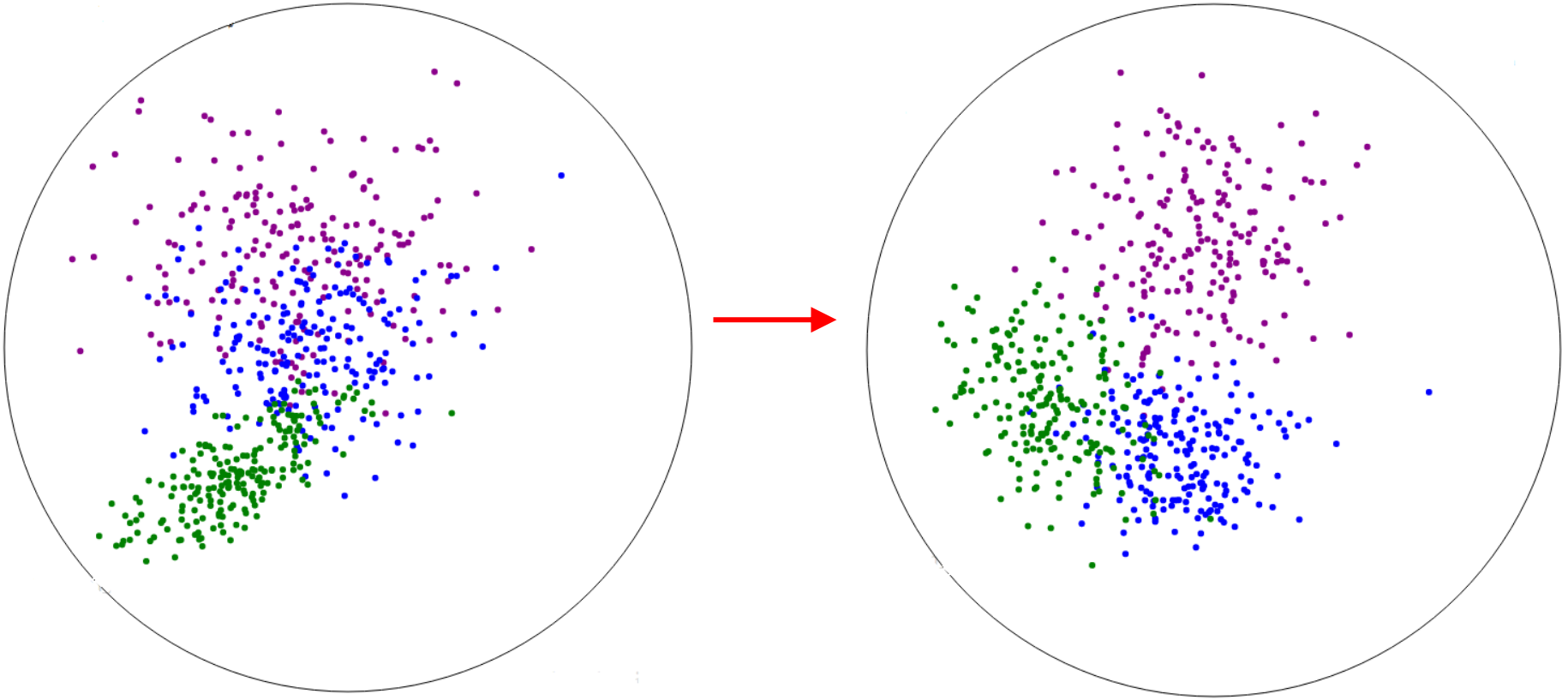
Subspace Trail Map



Trackball-Based Cluster Exploration



Interactive View Optimizer



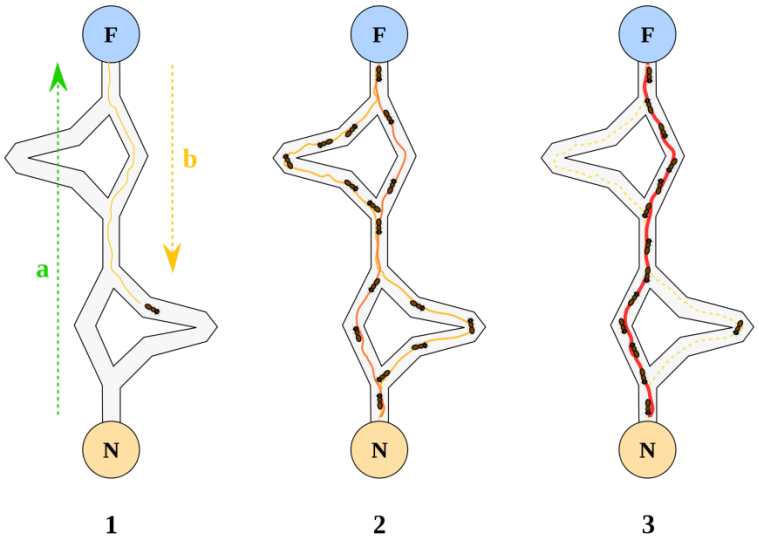
Uses genetic-algorithm driven projection pursuit

Several view quality metrics are available

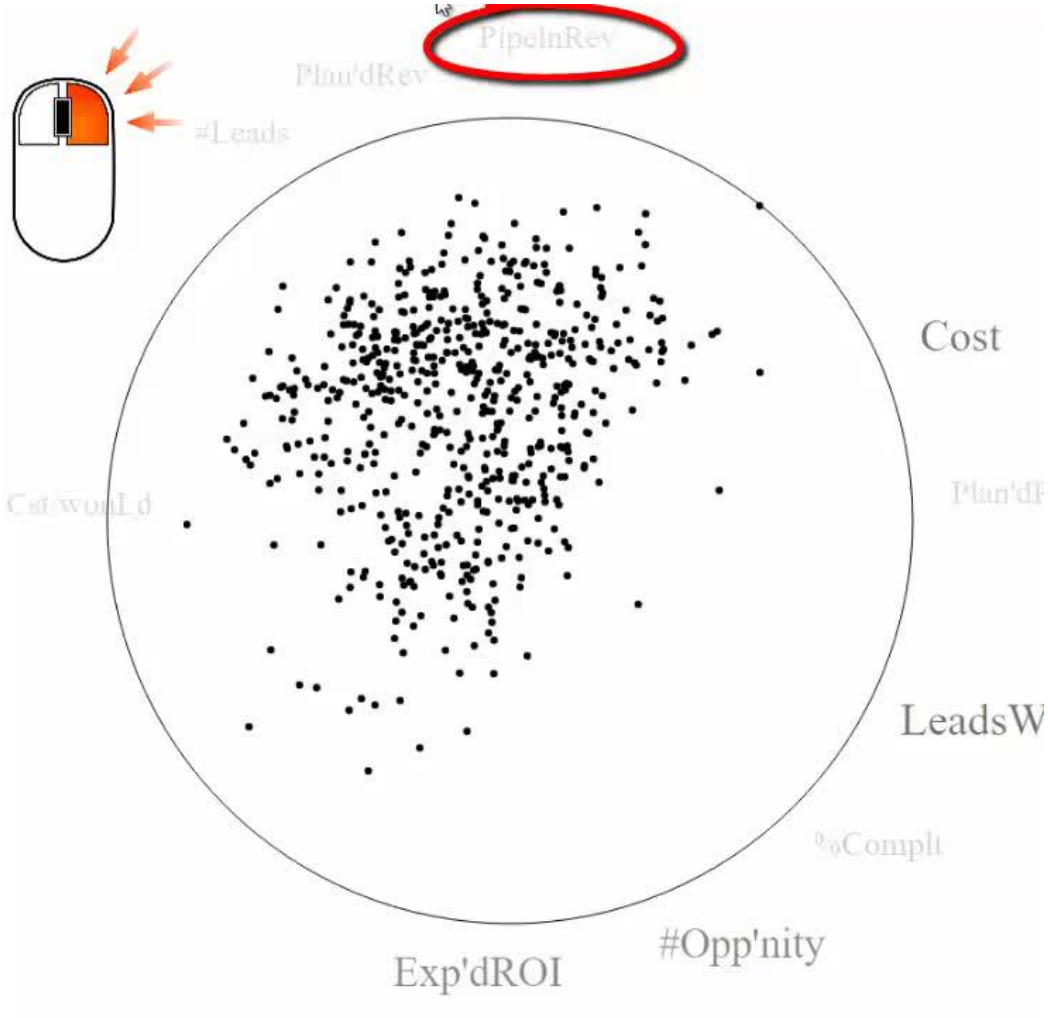
(Genetic) Ant Colony Algorithm

Generate many views and score them (one per ant)

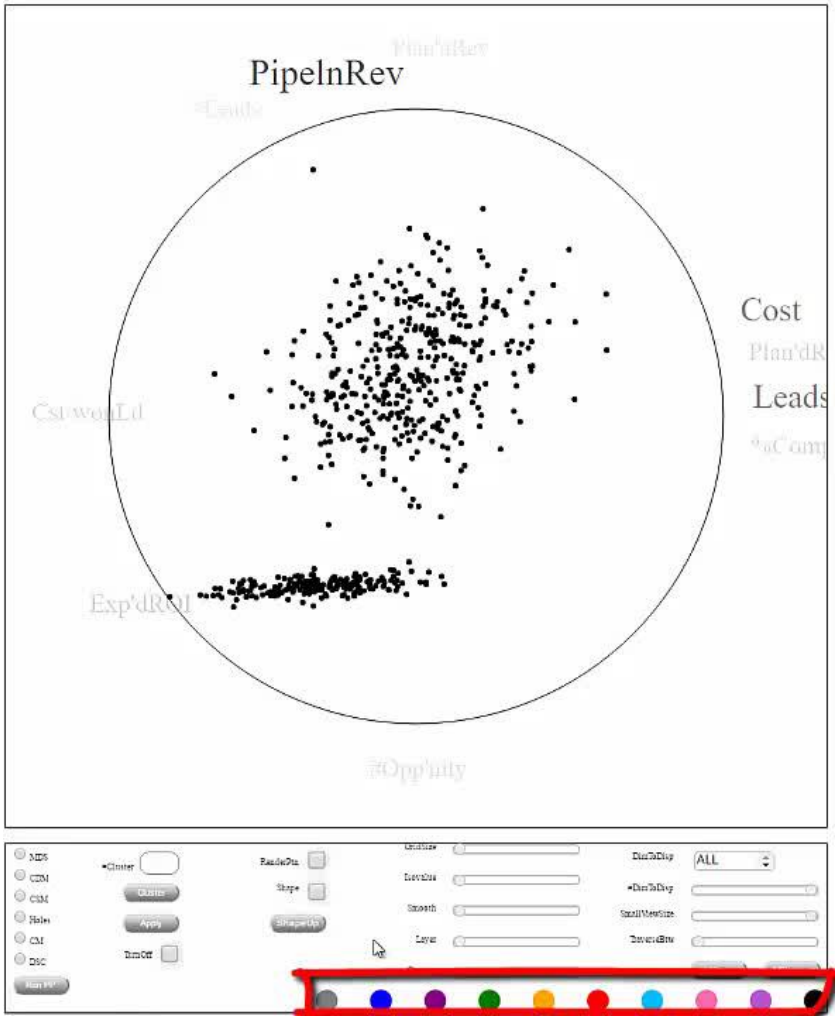
- poor scoring ants die and well-scoring ants survive
- sub paths of high scoring receive pheromone
- pheromone entices ants to take this path again
- each path variation is a parameter choice
- best view corresponds to the path that is converged on



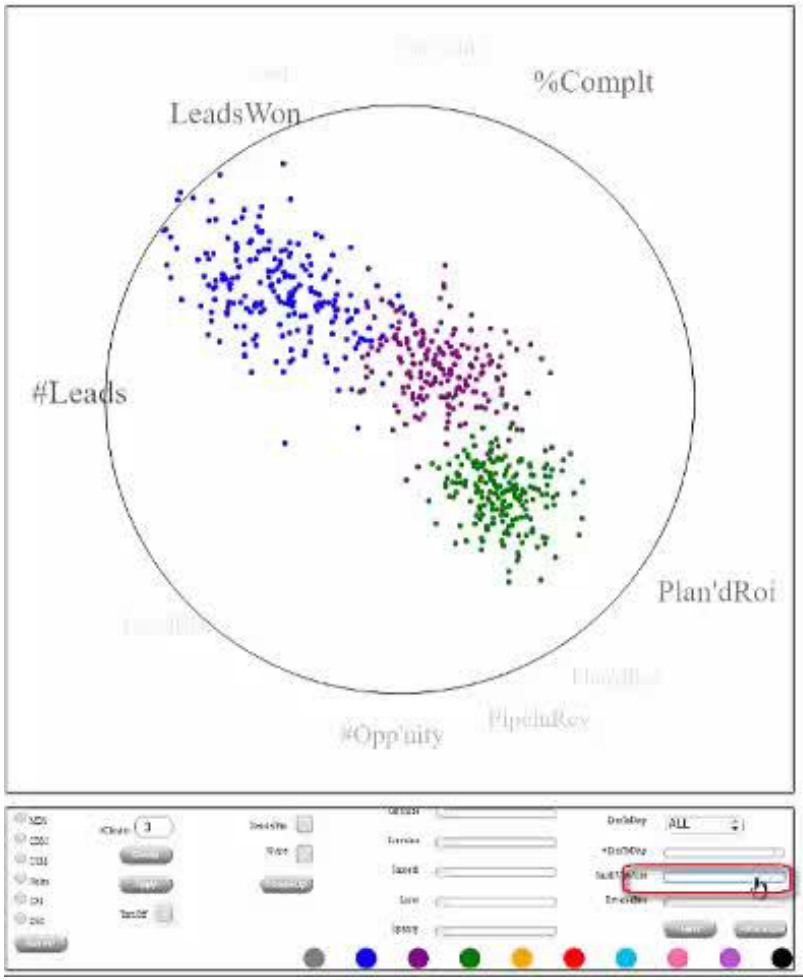
Chase Interesting Clusters – Transition to Adjacent 3D Subspaces

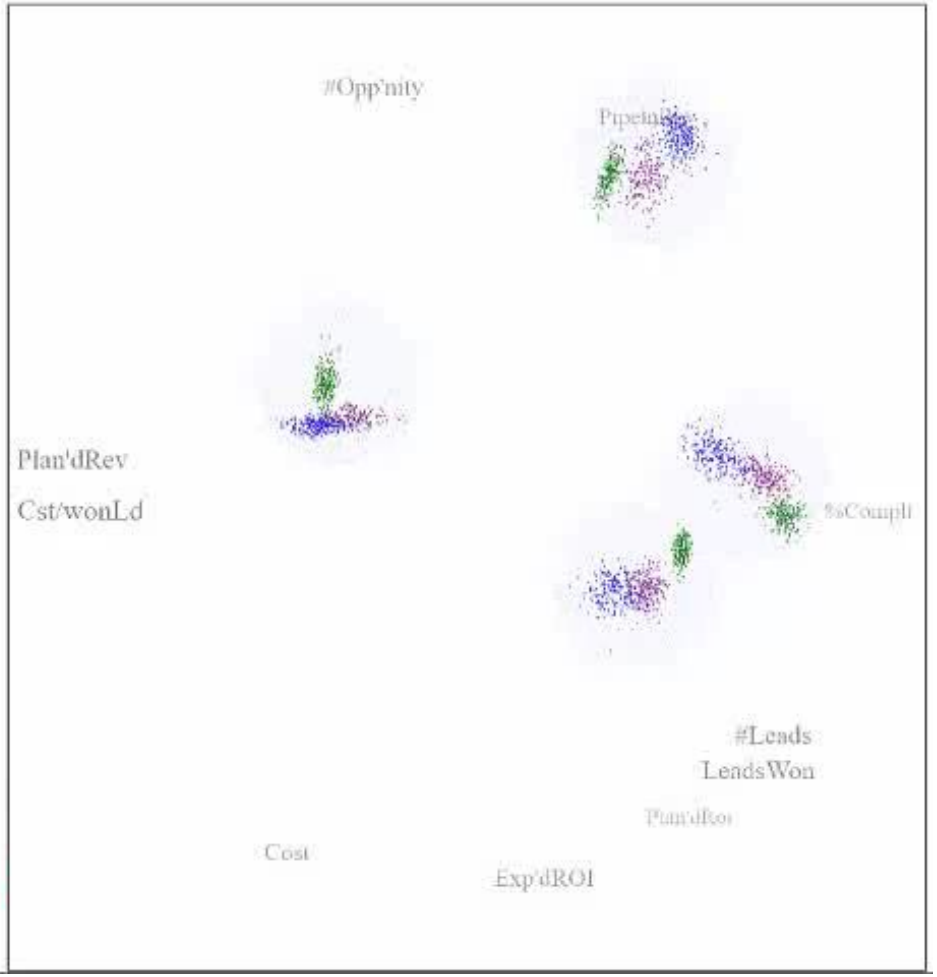


Edit and Annotate Clusters

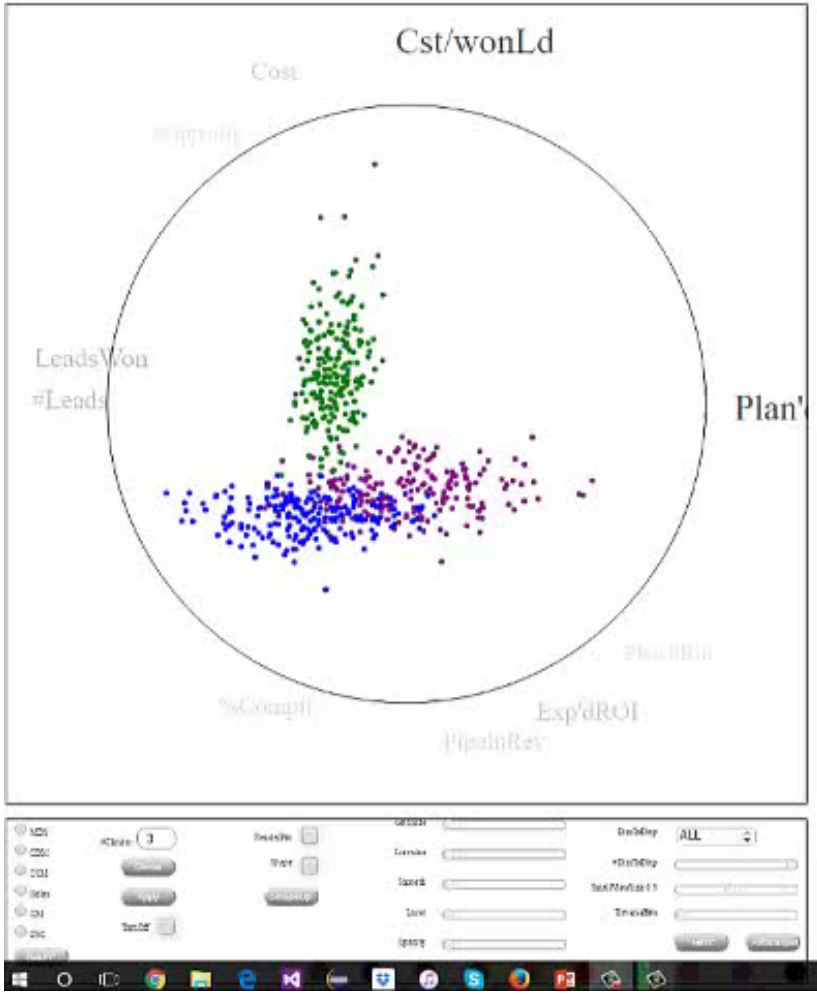


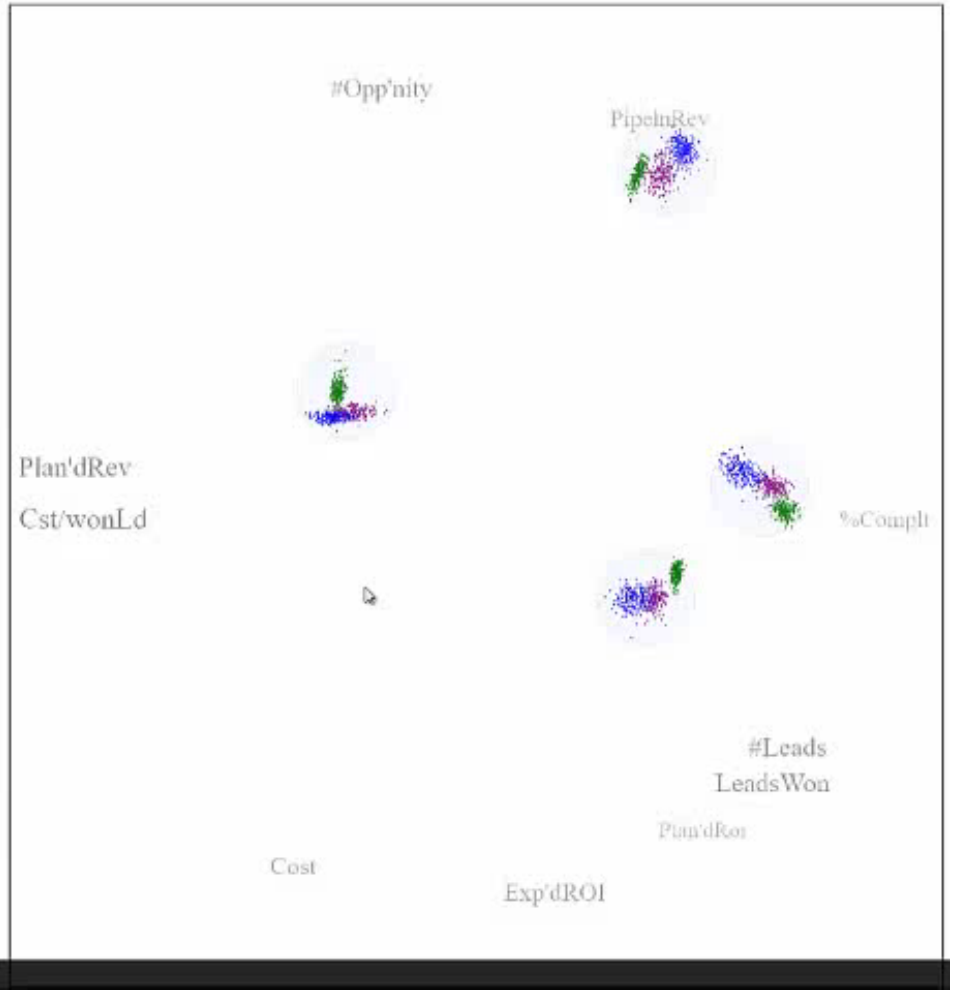
The Subspace Trail Map



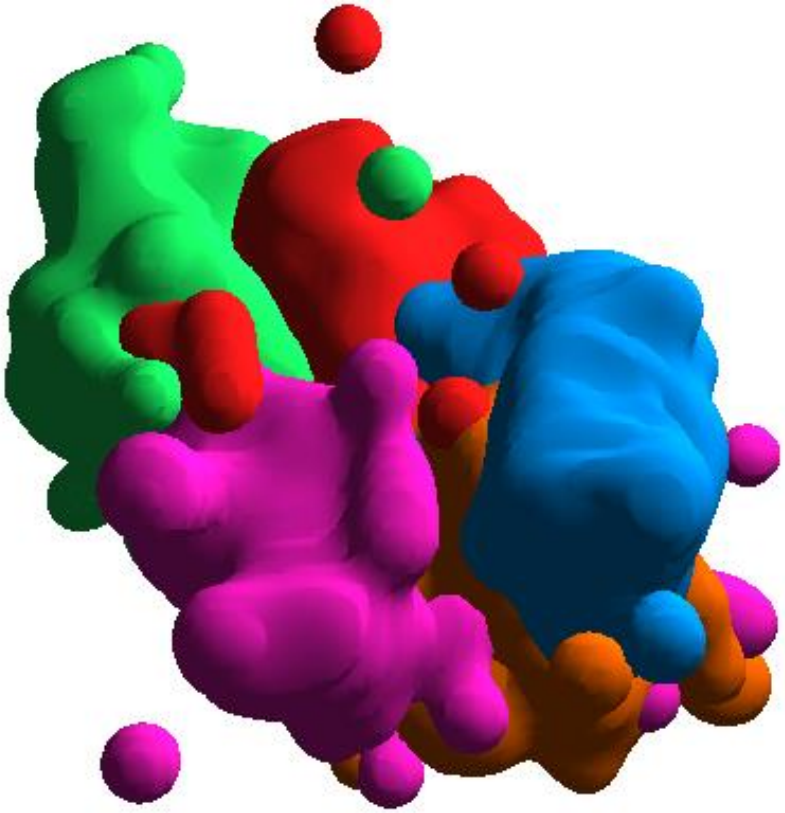


Walk The Subspace Trail Map

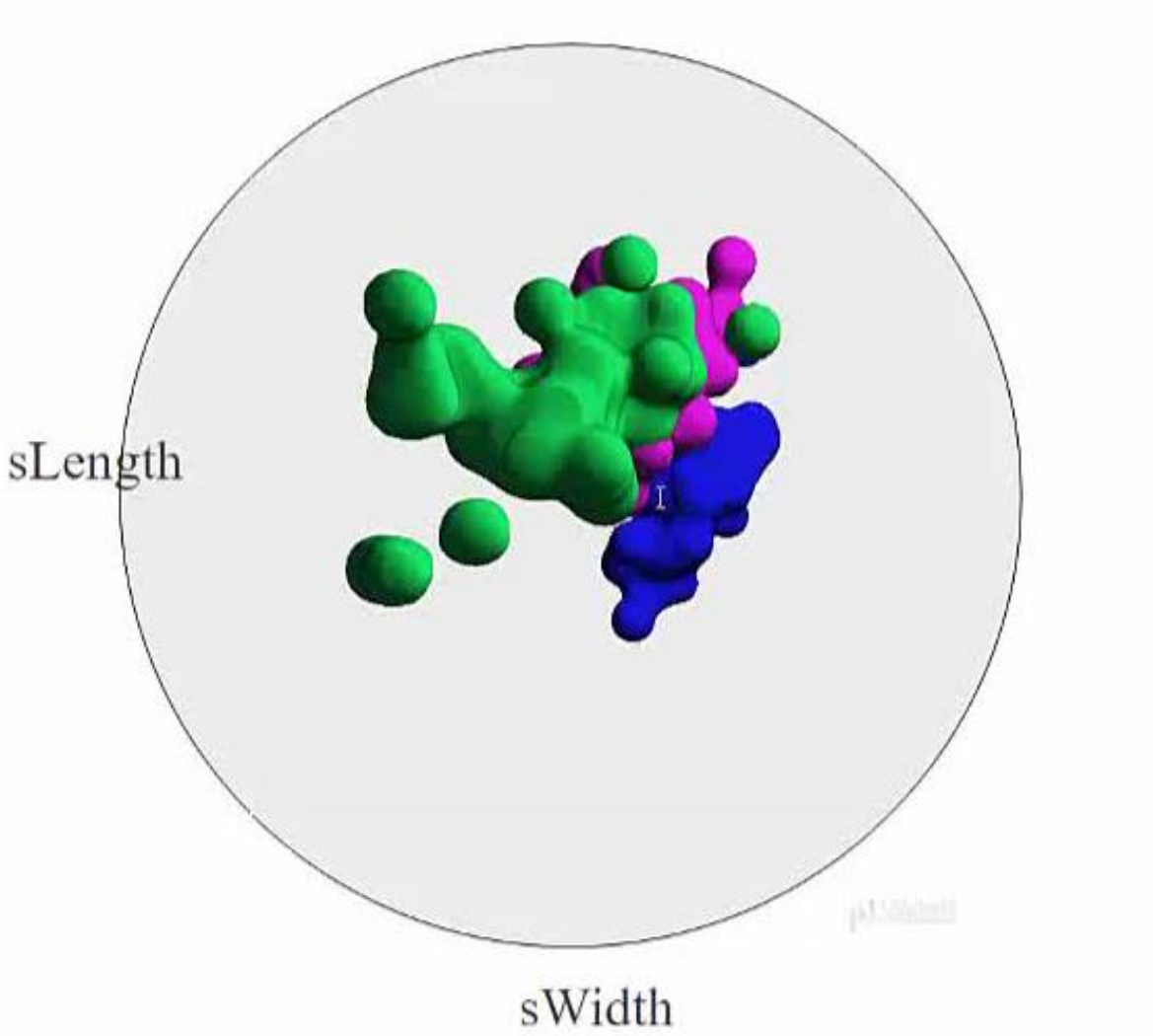




Clarify Spatial Relationships



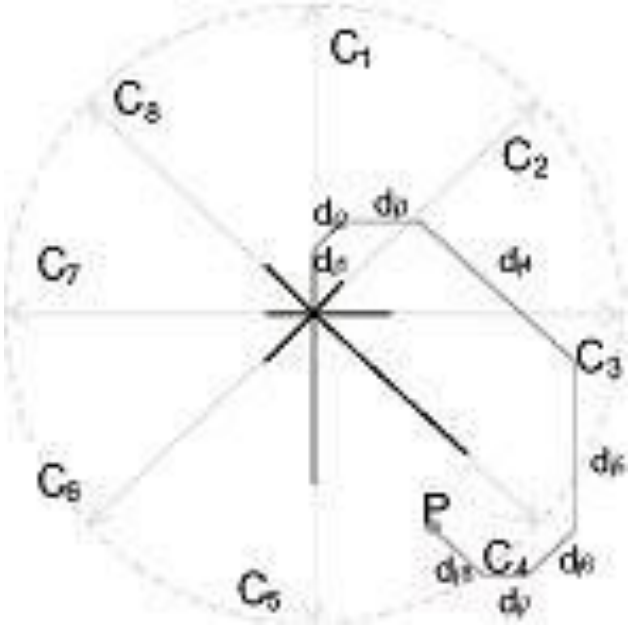
Clarify Spatial Relationships



Star Coordinates

Coordinate system based on axes positioned in a “star”, or circular pattern

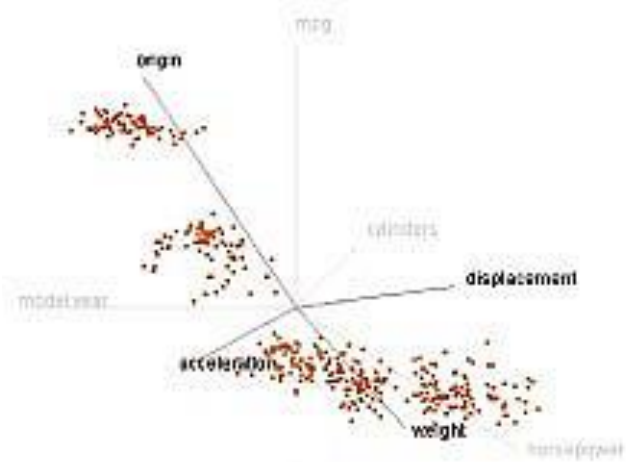
- a point P is plotted as a vector sum of all axis coordinates



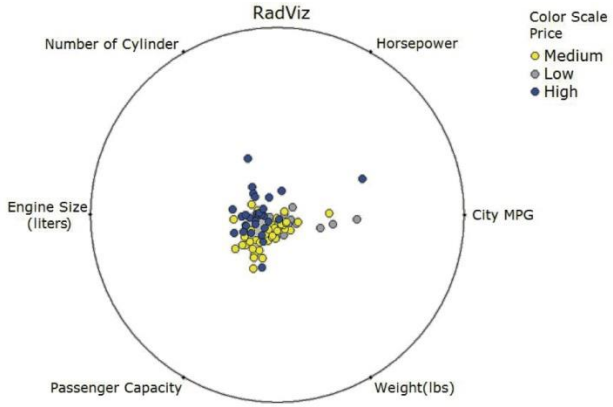
Star Coordinates

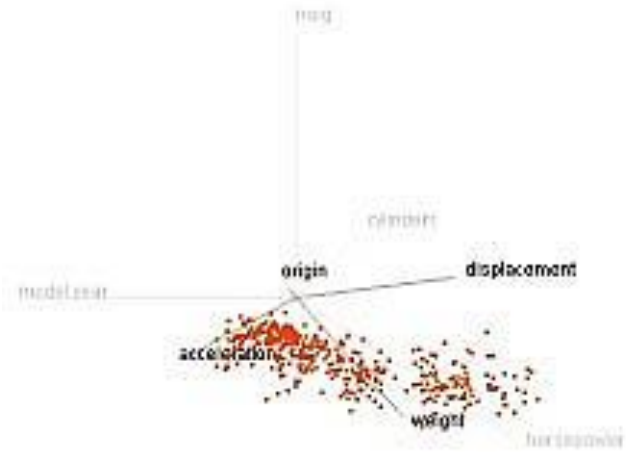
Operations defined on Star Coords

- scaling changes contribution to resulting visualization
- axis rotation can visualize correlations
- also used to reduce projection ambiguities



Similar paradigm: RadViz





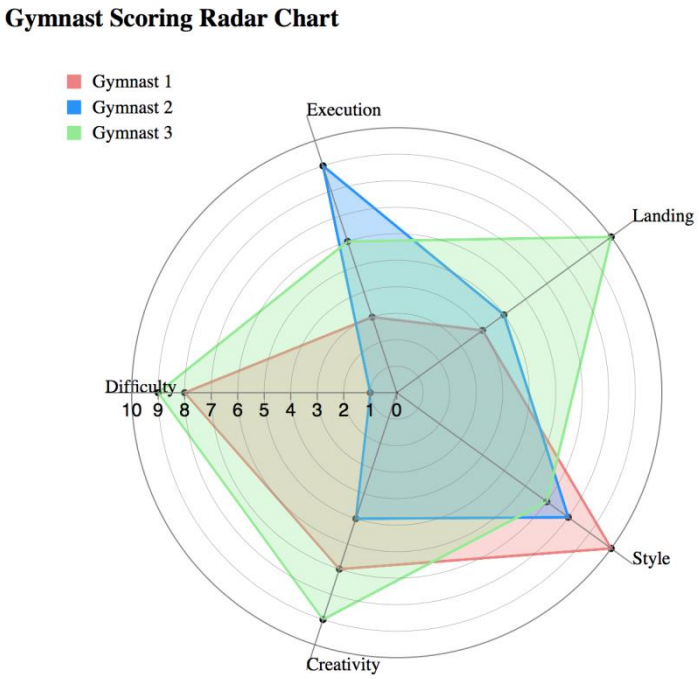
Radar Chart

Equivalent to a parallel coordinates plot, with the axes arranged radially

- each star represents a single observation
- can show outliers an commonalities nicely

Disadvantages

- hard to make trade-off decisions
- distorts data to some extents when lines are filled in



Commonalities

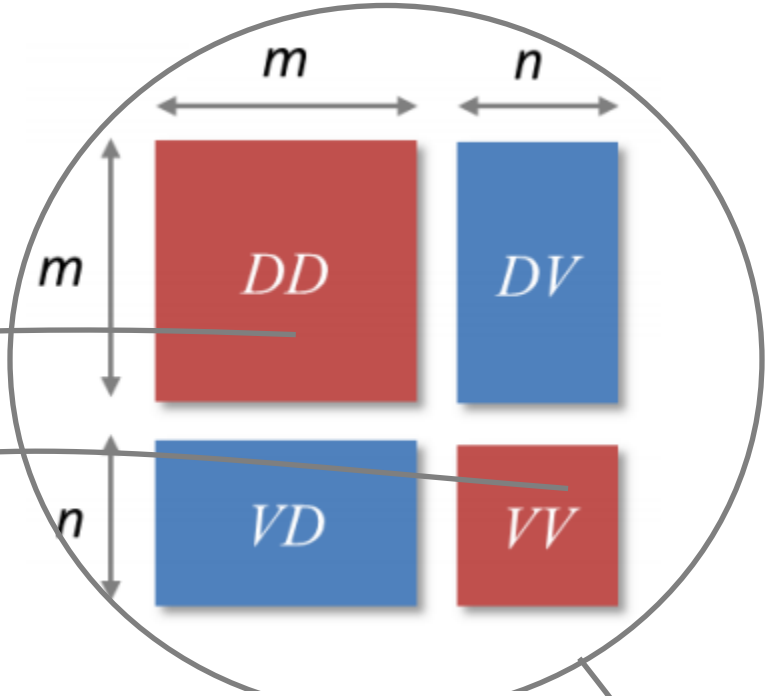
All of these scatterplot displays share the following characteristics

- allow users to see the data points in the context of the variables
- but can suffer from projection ambiguity
- some offer interaction to resolve some of these shortcomings
- but interaction can be tedious

Are there visualization paradigms that can overcome these problems?

- yes, algorithms that optimize the layout to preserve distances or similarities in high-dimensional space
- what is this algorithm?
- yes, MDS (Multi-Dimensional Scaling)
- we have discussed MDS before (so we will skip further discussion)

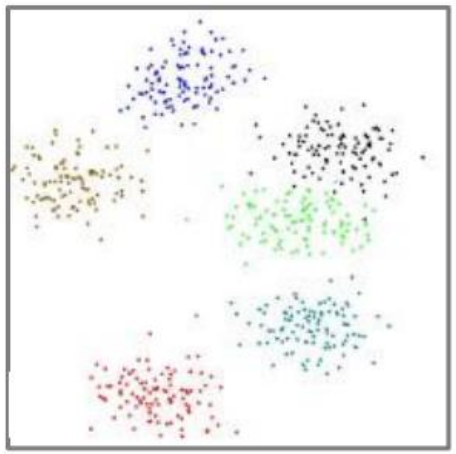
Uses of MDS



data similarity matrix (DD)

attribute similarity matrix (VV)

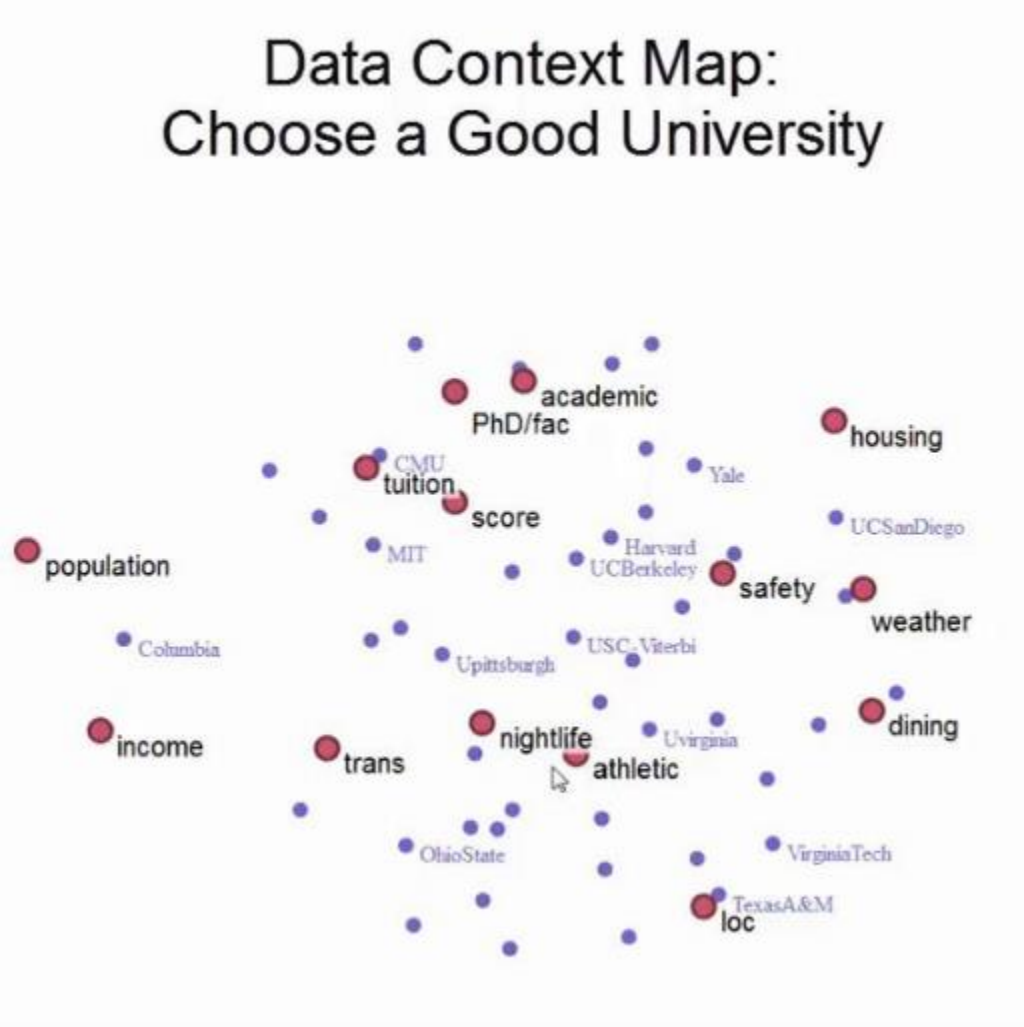
Data layout



+ data-attribute similarity matrix (VD, DV)

Yields the Data Context Map

Data visualized in the context of the attributes

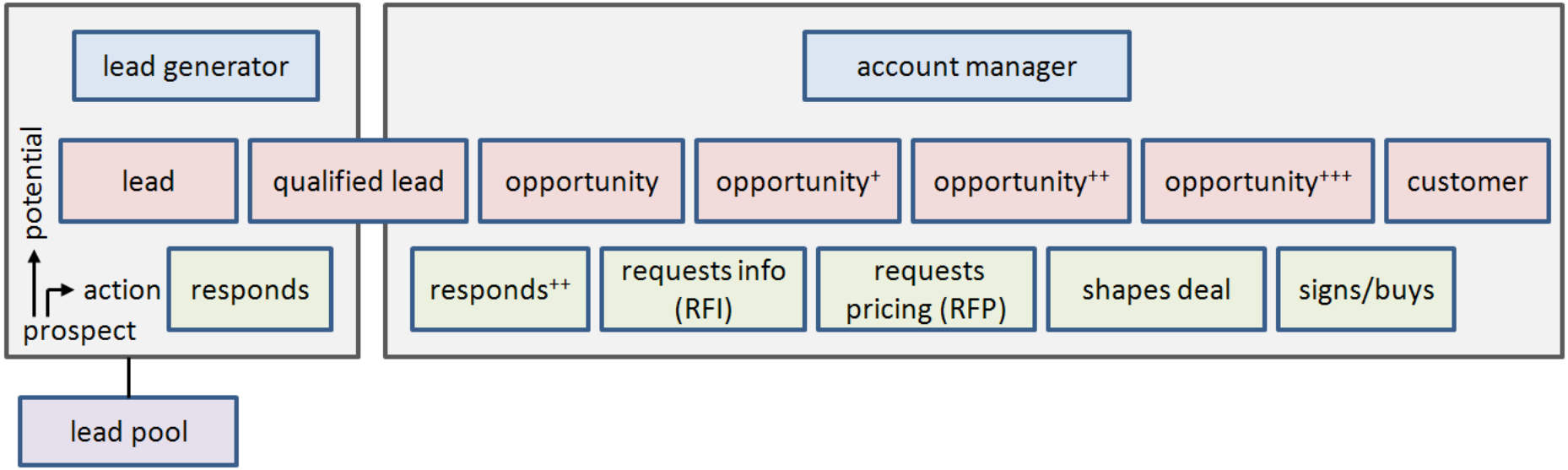


S. Cheng, K. Mueller, "The Data Context Map: Fusing Data and Attributes into a Unified Display," *IEEE Trans. on Visualization and Computer Graphics*, 22(1): 121-130, 2016.

Telling Stories with Parallel Coordinates

Example: Sales Strategy Analysis

Anatomy of a Sales Pipeline



The Setup

Scene:

- a meeting of sales executives of a large corporation, Vandelay Industries

Mission:

- review the strategies of their various sales teams

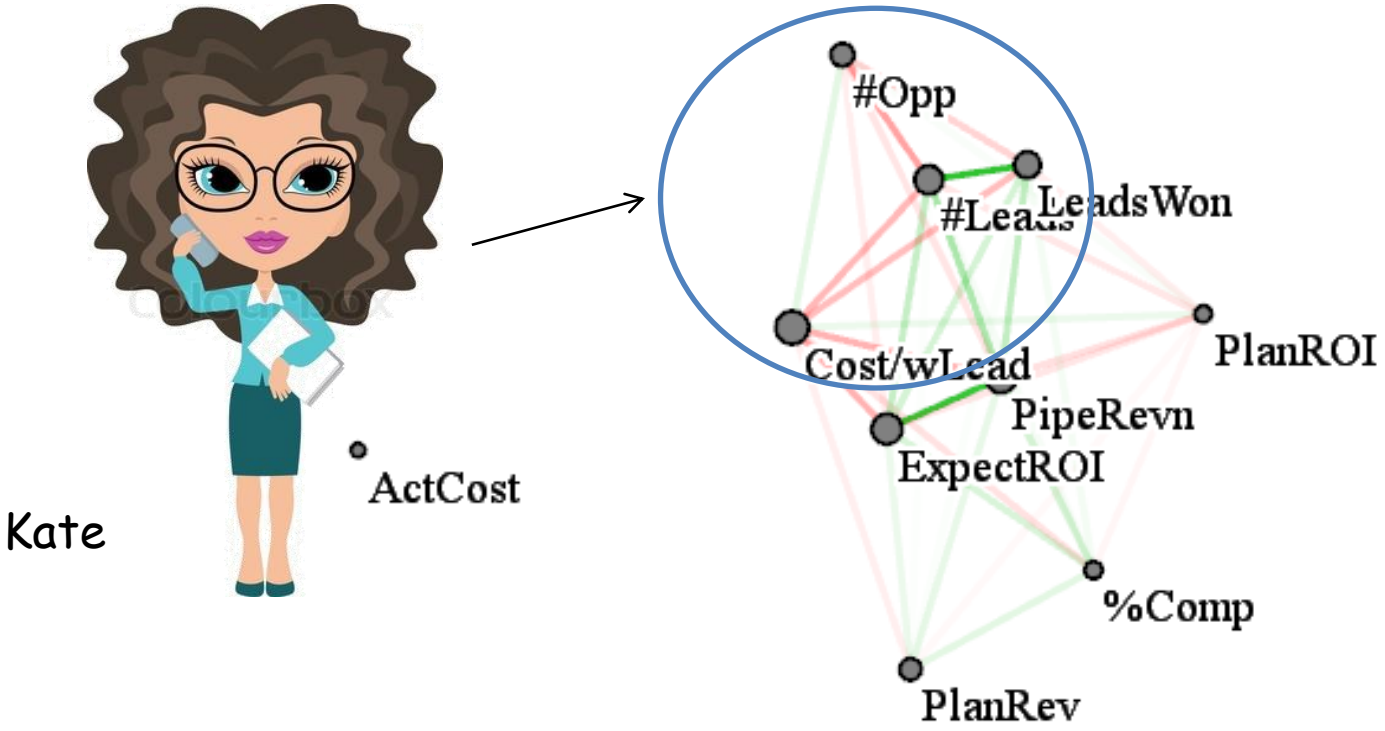
Evidence:

- data of three sales teams with a couple of hundred sales people in each team

Kate Explains It All

Meet Kate, a sales analyst in the meeting room:

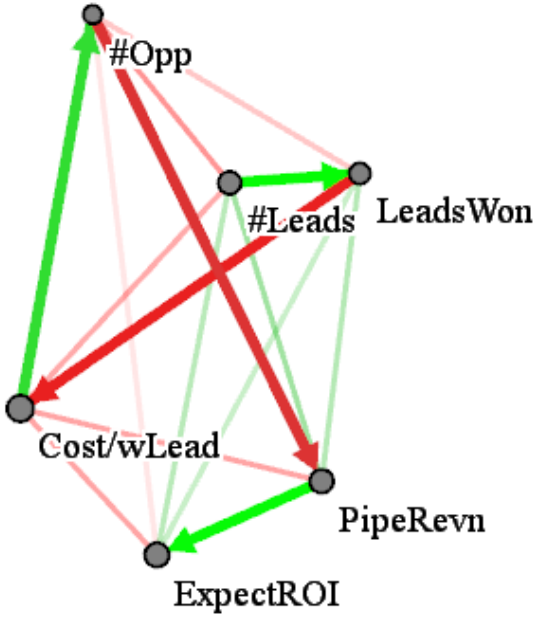
"OK…let's see, cost/won lead is nearby and it has a positive correlation with #opportunities but also a negative correlation with #won leads"

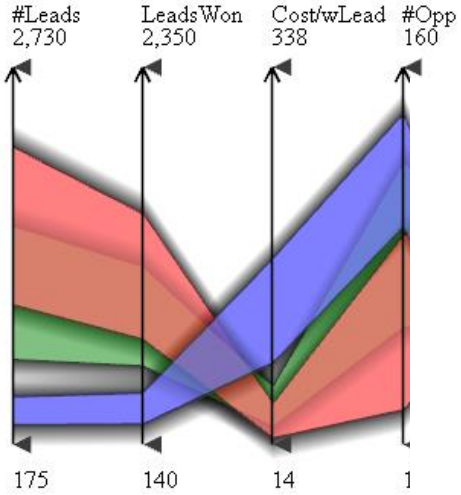


Kate Designs the Narration

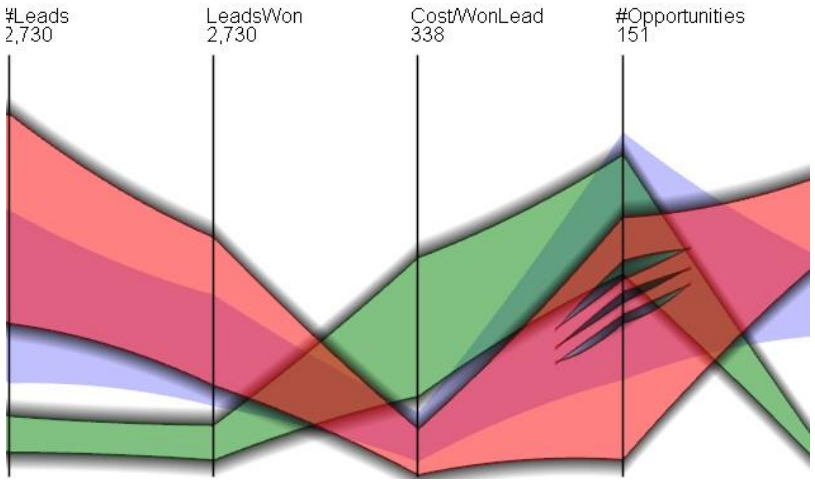
"Let's go and make a revealing route!"

- she uses the mouse and designs the route shown
- she starts explaining the data like a story …





Further Insight



Kate notices something else:

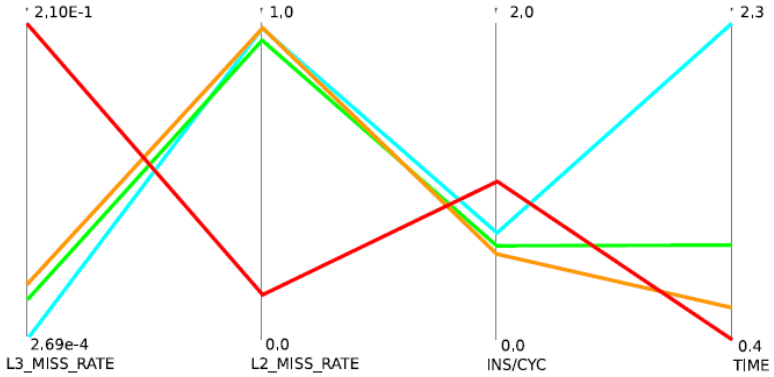
- now looking at the red team
- there seems to be a spread in effectiveness among the team
- the team splits into three distinct groups

She recommends: “Maybe fire the least effective group or at least retrain them”

Recent Reviewer Comment

From a paper sent to a software visualization conference:

Figure 8

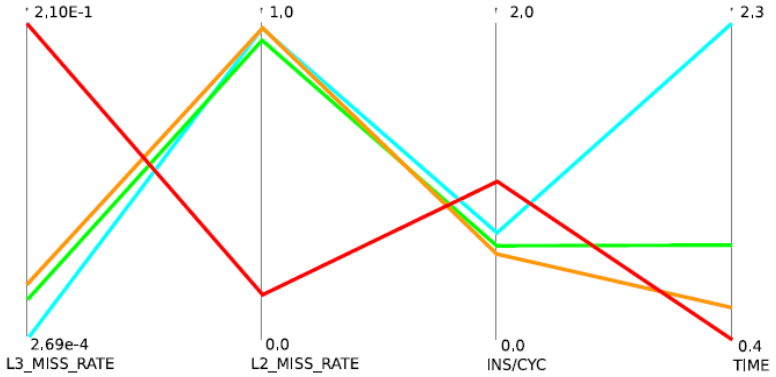


- Multiple visualizations appear to present categorical data as line graphs, which seems a strange choice.

Recent Reviewer Comment

From a paper sent to a software visualization conference:

Figure 8



- Multiple visualizations appear to present categorical data as line graphs, which seems a strange choice. Figure 8, for example, at first sight appeared to be showing a change over time, but in fact further inspection shows that the different x-coordinates are almost entirely unrelated to one another and in no particular order.

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From a paper sent to a software visualization conference:

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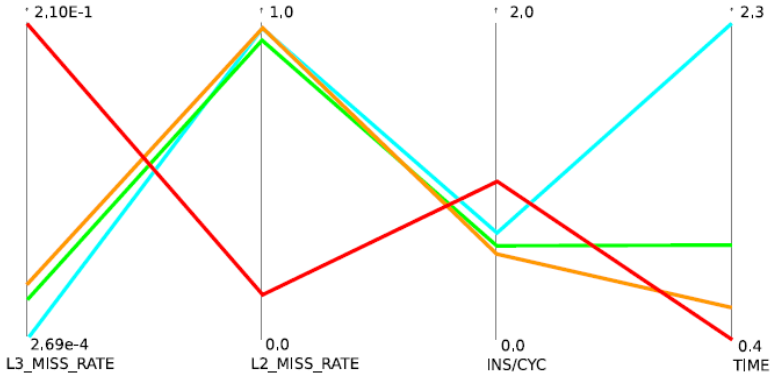


- Multiple visualizations appear to present categorical data as line graphs, which seems a strange choice. Figure 8, for example, at first sight appeared to be showing a change over time, but in fact further inspection shows that the different x-coordinates are almost entirely unrelated to one another and in no particular order. This is such an unusual choice that I'm not sure that I am understanding the role of the graphs correctly.

How to Teach Mainstream Users

Learning Visualizations by Analogy

Puripant Ruchikachorn and Klaus Mueller

Stony Brook
University

User Studies

Encode user responses based on task complexities

- none (0): cannot report any findings
- low (1): understand representation visual encoding
- medium (2): identify groups and outliers
- high (3): recognize correlations and trends

User Studies – Car Dataset

Visual understanding:

(1) The MPG of the orange-highlighted car is ~40% of its range

(2) There is just one line at the top of the acceleration scale

(3) Heavier cars are faster

Data Understanding:

(1) The number of cylinders of the orange-highlighted car is 4, one fifth between 3 and 8.

(2) Many cars have the same numbers of cylinders, mostly even numbers particularly 4 and 8.

(3) Heavier cars have more cylinders and hence more horsepower and speed.

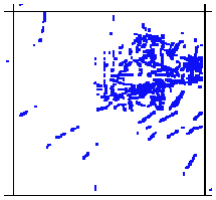
Results

Participants		V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11
Parallel Coordinates Plot	Before	3	0	0	0	1	0	2	1	0	3	3
	After	3	2	2	1	2	2	3	2	1	3	3
	Diff.	0	2	2	1	1	2	1	1	1	0	0

D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11
0	2	3	1	1	3	1	1	2	0	3
2	3	3	3	1	3	2	2	3	2	3
2	1	0	2	0	0	1	1	1	2	0

Scatterplot For Two Attributes

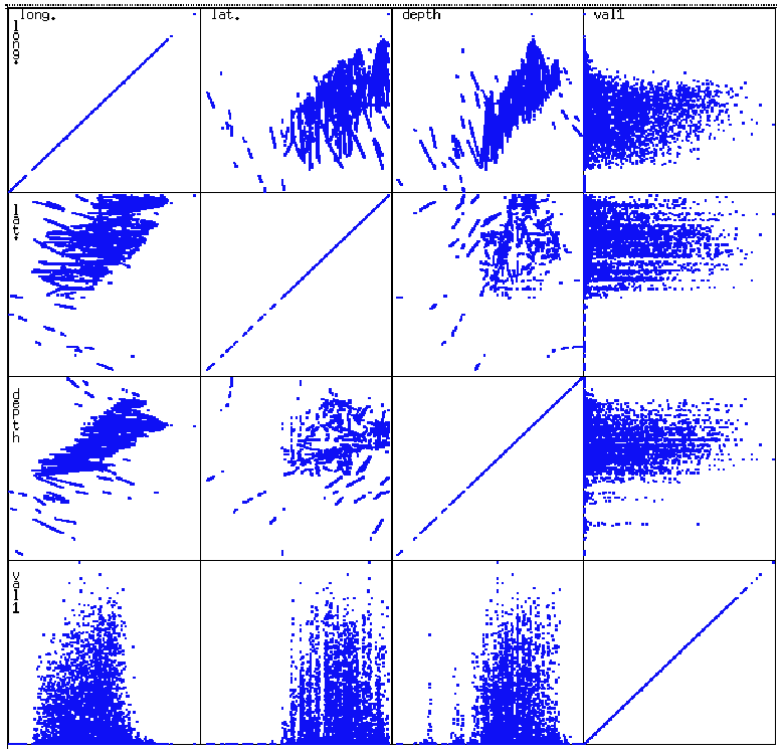
Appropriate for the display of bivariate relationships



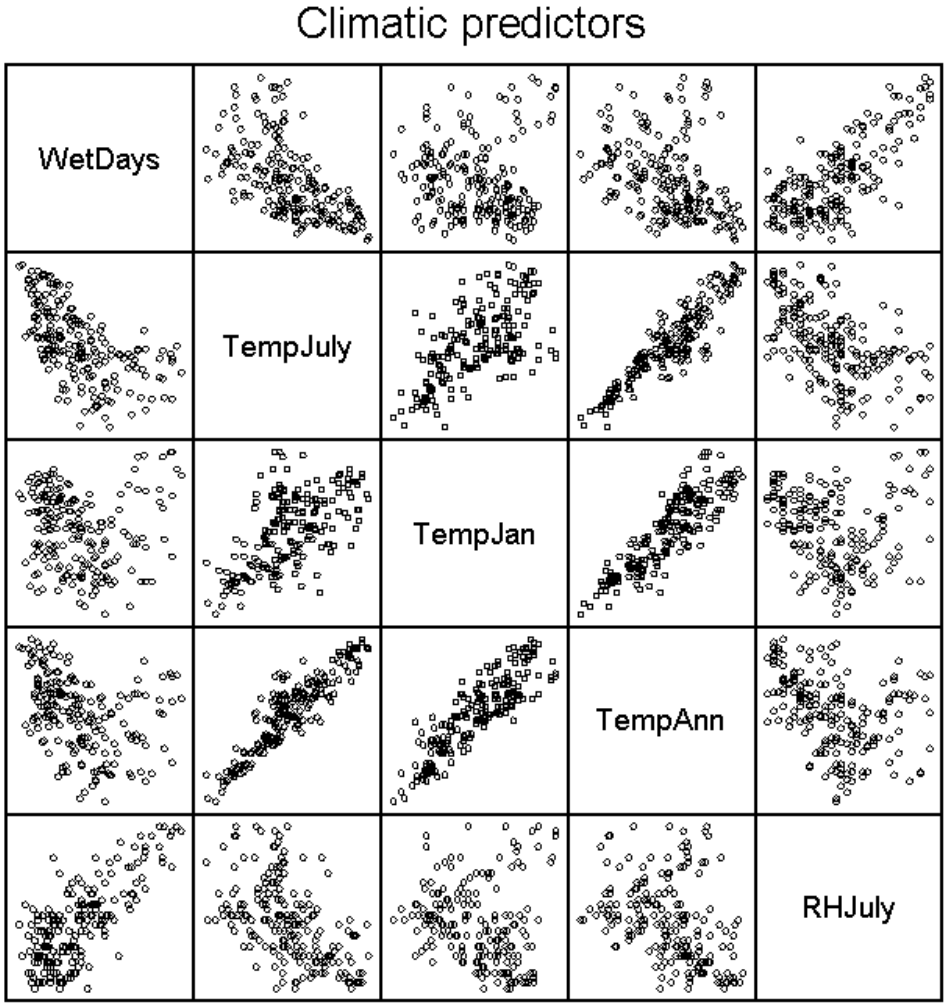
Scatterplot For many Attributes

What to do when there are more than two variables?

- can arrange multivariate relationships into scatterplot matrices
- not overly intuitive to perceive multivariate relationships



Scatterplot Matrix (SPLOM)



Scatterplot Matrix

Scatterplot version of parallel coordinates

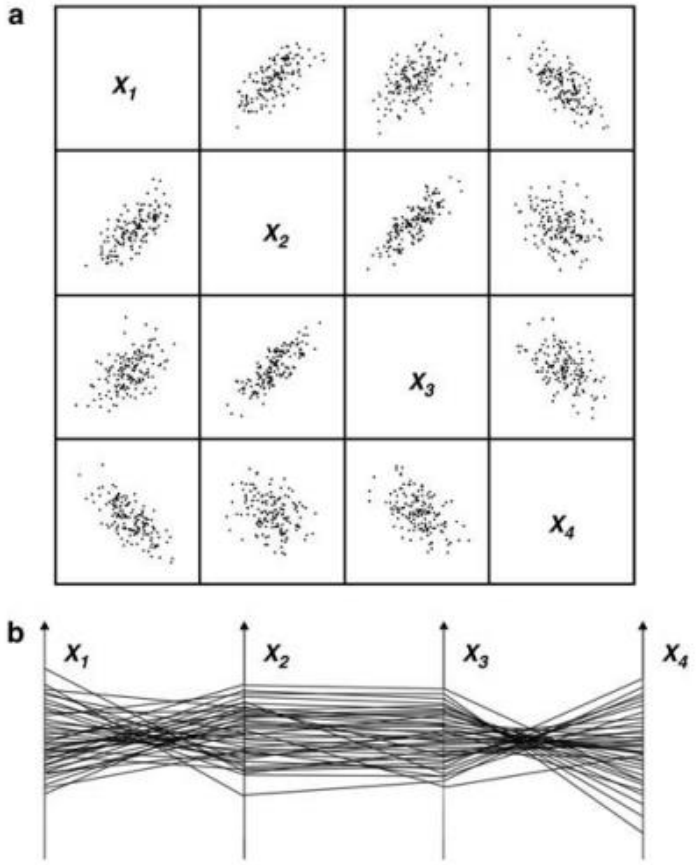
- distributes n(n-1) bivariate relationships over a set of tiles
- for n=4 get 16 tiles
- can use n(n-1)/2 tiles

For even moderately large n:

- there will be too many tiles

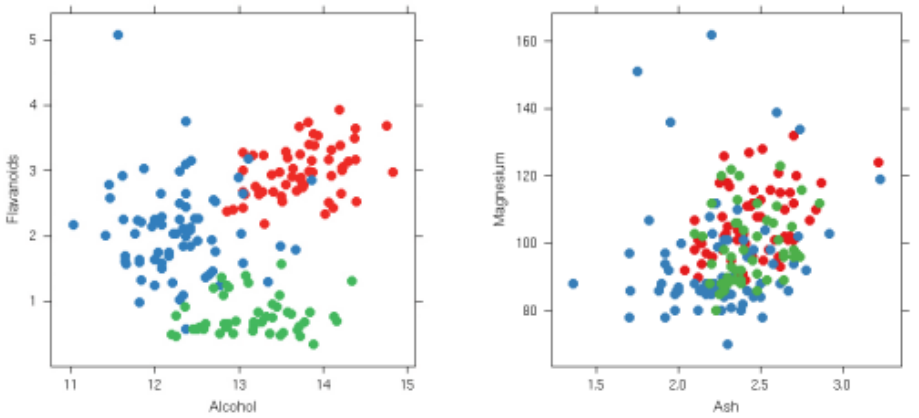
Which plots to select?

- plots that show correlations well
- plots that separate clusters well

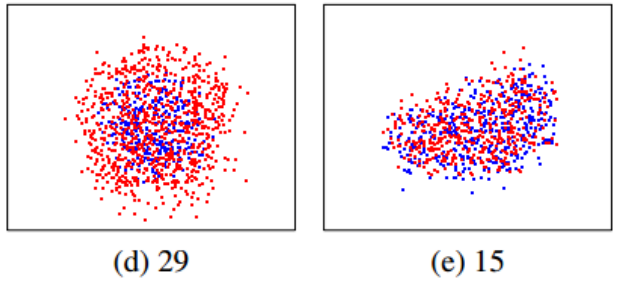


Automated Scatterplot Selection

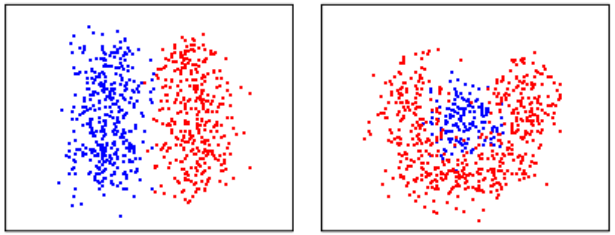
Several metrics, a good one is Distance Consistency (DSC)



(a) **DSC**=90 (b) **DSC**=49



(d) 29 (e) 15



(a) 99 (b) 74

bad

OK

$$\mathbf{DSC} = \frac{\left|x' \in v(X) : \mathbf{CD}(x', centr'(c_{clabel(x)}) \;=\; true\right|}{k}$$

- measures how "pure" a cluster is
- pick the views with highest normalized DSC

M. Sips et al., Computer Graphics Forum, 28(3): 831–838, 2009

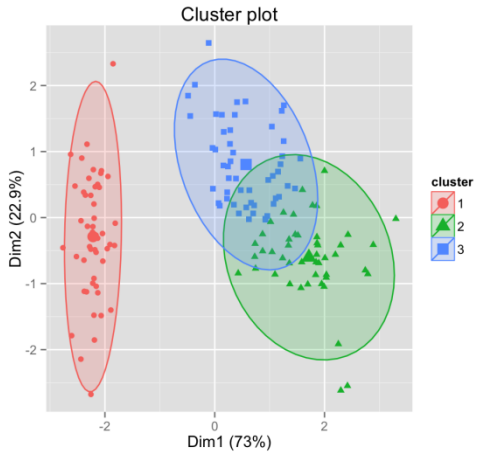
Dunn Index

Favors clusters that are compact and are well isolated

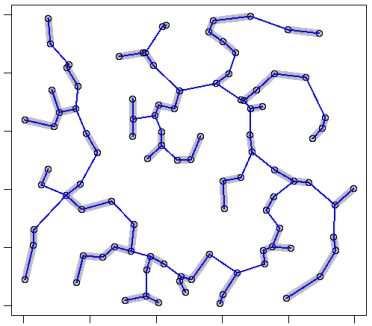
$$DI_m = \frac{\min\limits_{1 \leqslant i < j \leqslant m} \delta(C_i, C_j)}{\max\limits_{1 \leqslant k \leqslant m} \Delta_k}.$$

$\Delta_i = \frac{\sum_{x \in C_i} d(x, \mu)}{|C_i|}, \mu = \frac{\sum_{x \in C_i} x}{|C_i|}$, calculates distance of all the points from the mean.

$\delta(C_i, C_j)$ be this intercluster distance metric, between clusters C_i and C_j.

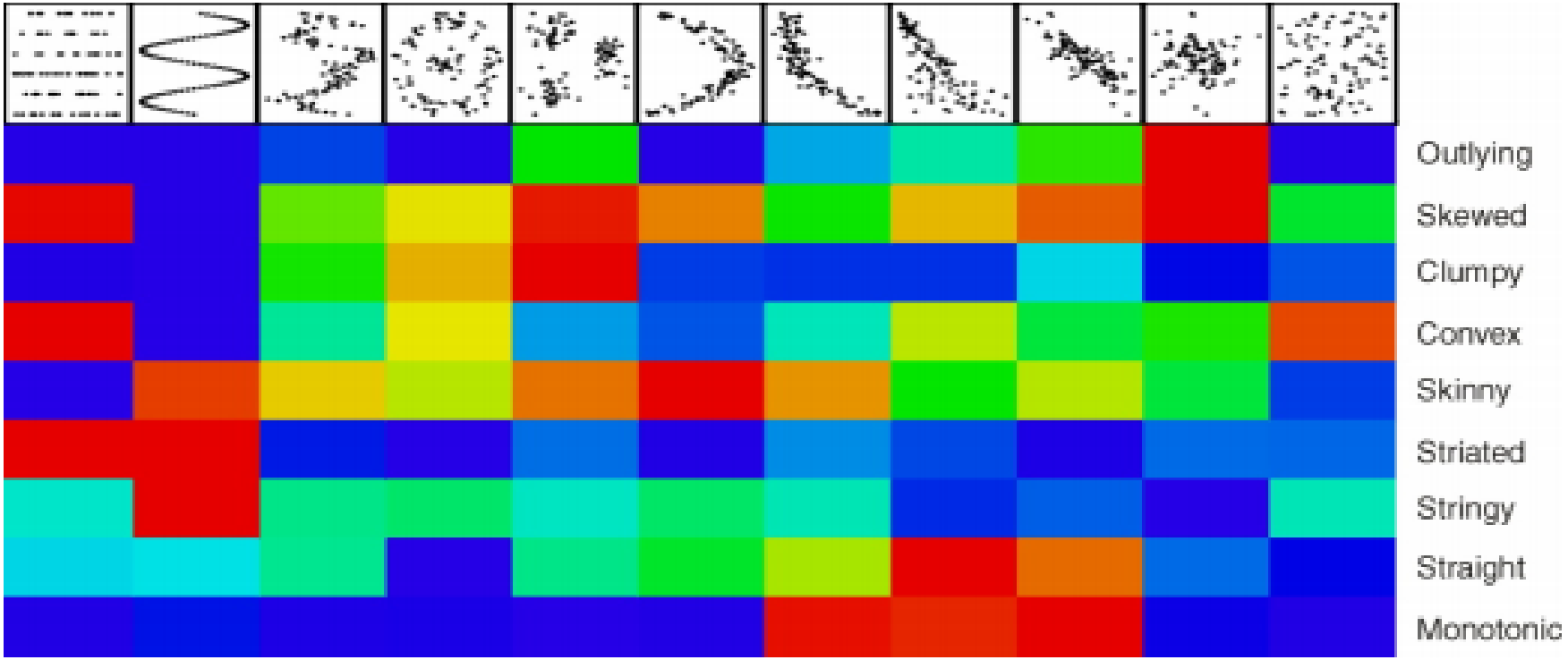


Scagnostics



Describe scatterplot features by graph theoretic measures

- mostly built on minimum spanning tree
- can be used to summarize large sets of scatterplots



Scatterplot of Scatterplots

Use scagnostics to quickly survey 1,000s of scatterplots

- compute scagnostics measures
- create scatterplot matrix of these measures
- each scatterplot is a point