Problem Set #3

Math, Jorge Barro
Bryan Chia

Problem 6.1

The optimization problem can be written as:

$$\begin{aligned} \underset{w}{\text{maximize}} \quad & e^{w^T x} \\ \text{subject to} \quad & G(w) = w^T x + w^T A w - w^T A y \leq -a \\ & H(w) = y^T w - w^T x = b \end{aligned}$$

Problem 6.5

$$\begin{aligned} \underset{P,L}{\text{maximize}} \quad & \pi(M, K) = 0.07M + 0.05K \\ & \pi(P, L) = 0.07(4P + 2L) + 0.05(3P + L) \\ \text{subject to} \quad & P \leq 240,000 \\ & L \leq 6000 \end{aligned}$$

Problem 6.6

$$Df(x, y) = [6xy + 4y^2 + y, \quad 3x^2 + 8xy + x]$$

$$D^2 f(x, y) \begin{bmatrix} 6y & 6x + 8y + 1 \\ 6x + 8y + 1 & 8x \end{bmatrix}$$

There is only one point where the first derivative is 0 and that is the point $(0, 0)$. Taking A_k as the submatrices, we have that $det(A_1) = 0$ and $det(A_2) = -1$. Therefore, the point is a saddle point.

Problem 6.11

Proof.

$$\begin{aligned} f'(x_0) &= 2ax_0 + b \\ f(x_0) &= 2a \\ x_1 &= x_0 - \frac{2ax_0 + b}{2a} = -\frac{b}{2} \end{aligned}$$

Sub in x_0 for x_1 and iterate again:

$$x_1 = -\frac{b}{2} - \frac{2ax_0 + b}{2a} = -\frac{b}{2}$$

Since $x_0 = x_1$, our iteration stops after one iteration. □

Problem 6.14

In Jupyter Notebook in folder.