$$(x_1+\cdots+x_k)^n = \sum_{(n_1,\dots,n_k)\,:\,n_1+\cdots+n_k=n} \frac{n!}{n_1!\dots n_k!}\, x_1^{n_1}\dots x_k^{n_k}$$

$\begin{cases} P(S)=1 \\ P(E)\ge 0, \forall E\subseteq S \\ P\left(\bigcup_{i=1}^{\infty} E_i\right)=\sum_{i=1}^{\infty} P(E_i)\end{cases}$	$P(\cdot\,\lvert B)$ $P(B_k\lvert A)=\frac{P(A\lvert B_k)P(B_k)}{\sum_{i=1}^{n} P(A\lvert B_i)P(B_i)}$	

$$P\left(\cap_{i=1}^{k} A_i\right)=P(A_1)P(A_2|A_1)P(A_3|A_1\cap A_2)\dots P(A_k|\cap_{j}^{k-1} A_j)$$

$Discrete\ rv$	$Parameters$	$p_X(x)$	$Ran(X)$	$E(X)$	$Var(X)$
$X\sim Bin(n,p)$	$n=1,2,3,\dots$ $p\in(0,1)$	$\binom{n}{x}p^x(1-p)^{n-x}$	$x=0,1,2,\dots$	np	$np(1-p)$
$X\sim Bernoulli(p)=Bin(n=2,p)$					
$X\sim Possion(\lambda)$	$\lambda\in(0,\infty)$	$\frac{\lambda^x e^{-\lambda}}{x!}$	$x=0,1,2,\dots$	λ	λ
$Binomial(n,p)\approx Possion(np)$					
$X\sim N.Bin(k,p)$	$k=1,2,3,\dots$ $p\in(0,1)$	$\binom{x-1}{k-1}p^k(1-p)^{x-k}$	$x=k,k+1,k+2,\dots$	$\frac{k}{p}$	$\frac{k(1-p)}{p^2}$
$X\sim Geometric(p)=N.Bin(k=1,p)$ $f_X(x)=(1-p)^x p$ $P(X>s+t\lvert X>t)=P(X>s),\quad memorylessness\ (unique)$ $N.Bin=count(total\ trials\ until\ k\ successes)$					
$X\sim N.Bin(k,p), P(X\le s)\Leftrightarrow k\ successes, s-k\ failures, complete\ before\ s-k+1\ failures$ $P(E\ precedes\ F)=\frac{P(E)}{P(E)+P(F)}, disjoint\ competing\ events\ E,F$					
$P(X\le s)=P(Y\ge k),\quad connecting\ X\sim N.Bin(k,p), Y\sim Bin(s,p)$					
$X\sim Hypergeo(n,m,N)$	$\{m\cdot A,$ $(N-m)\cdot B\},$ $n\ draws$	$\frac{\binom{m}{x}\binom{N-m}{n-x}}{\binom{N}{n}}$	$x\in[\max(0,n+m-N),\min(m,n)]$	$\frac{nm}{N}$	$\frac{N-n}{N-1}np(1-p),$ $p=\frac{m}{N}$
$binomial\ without\ replacement, hence\ varying\ p$ $Hypergeo(n,m,N)\approx Bin\left(n,p=\frac{m}{N}\right)\ or\ Possion\left(\lambda=\frac{nm}{N}\right)\ as\ (n,)m,N\to\infty$					

$$\begin{aligned} E[g(X)] &\equiv E[Y] \\ &= \sum_{y\in\chi_Y} y p_Y(y) = \sum_{y\in\chi_Y} y\Big[\sum_{x\in\chi_{X(y)}} p_X(x)\Big] = \sum_{y\in\chi_Y}\sum_{x\in\chi_{X(y)}} g(x)p_X(x) \\ &= \sum_{x\in\cup_{y\in\chi_Y}\chi_{X(y)}=\chi_X} g(x)p_X(x)\end{aligned}$$

$$E[X]=\sum_{x\in\chi} P(X\ge x),\ \ tail\ sum$$

$$1=\sum_{x\in\chi} p_X(x)\,,\text{"}freedom\text{"}$$

$$Var[X]=E\left[\left(X-E(X)\right)^2\right]=\sum_{x\in\chi}\left(x-E(X)\right)^2 p_X(x)=E[X^2]-E[X]^2$$

<table>
<tr><th>$Continuous\ rv$</th><th>$Parameters$</th><th>$f_X(x)$</th><th>$Ran(X)$</th><th>$E(X)$</th><th>$Var(X)$</th></tr>
<tr><td>$X\sim N(\mu,\sigma^2)$</td><td>$\mu \in \mathbb{R}$
$\sigma \in (0,\infty)$</td><td>$\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$</td><td>$x = \mathbb{R}$</td><td>$\mu$</td><td>$\sigma^2$</td></tr>
<tr><td colspan="6">$Z = \frac{X-\mu}{\sigma}\sim N(0,1)$
$\Phi(a) := P(Z \le a) = 1 - \Phi(-a)$</td></tr>
<tr><td colspan="6">$normal\ approximation\ \ X \approx Z\sim N(\mu_X,\sigma_X^2)\ for\ X\sim distr, dynamically\ find\ \mu_X,\sigma_X^2$
$P(X=k) = P(X\le k) - P(X<k) \approx P\left(Z \le \frac{k+0.5-\mu}{\sigma}\right) - P(Z \le \frac{k-0.5-\mu}{\sigma})$
$P(X\ge k) \approx P\left(Z \ge \frac{k-0.5-\mu}{\sigma}\right), \qquad P(X>k) \approx P\left(Z \ge \frac{k+0.5-\mu}{\sigma}\right)$
$P(a\le X\le b), P(a<X<b), \dots$</td></tr>
<tr><td>$X\sim Gamma(\alpha,\beta)$</td><td>$\alpha,\beta \in (0,\infty)$</td><td>$\frac{\beta^\alpha x^{\alpha-1}}{\Gamma(\alpha)}e^{-\beta x}$</td><td>$x \ge 0$</td><td>$\frac{\alpha}{\beta}$</td><td>$\frac{\alpha}{\beta^2}$</td></tr>
<tr><td colspan="6">$\Gamma(t) := \int_0^\infty x^{t-1}e^{-x}dx$
$\Gamma(t) = (t-1)\Gamma(t-1);\ \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$
$Gamma = count(time\ interval\ between\ random\ events)$</td></tr>
<tr><td colspan="6">$X\sim Chisq(\theta) = Gamma\left(\frac{\theta}{2},\frac{1}{2}\right), as\ in \frac{(n-1)s_{n-1}^2}{\sigma^2}\sim Chisq(n-1)$</td></tr>
<tr><td colspan="6">$X\sim Exp(\lambda) = Gamma(1,\beta), \qquad Exponential$
$f_X(x) = \beta e^{-\beta x}, Ran(X) = \{x>0\}$
$P(X>s+t) = P(X>t)P(X>s), \qquad memorylessness(unique)$</td></tr>
<tr><td colspan="6">$Y\sim Exp(\lambda) \xrightarrow{X=\lfloor Y\rfloor+1} X\sim Geometric\left(p = 1-e^{-\lambda}\right)\ connecting\ Exp, Geometric$</td></tr>
<tr><td colspan="6">$P(Y\le t) = P(N(t)\ge\alpha), \qquad connecting\ Y\sim Gamma(\alpha,\lambda), N(t)\sim Possion(\lambda t)$
$count(arrivals\ in\ [0,t]) \Leftrightarrow duration(\alpha^{th} arrival)$</td></tr>
<tr><td>$X\sim Beta(a,b)$</td><td>$a,b \in (0,\infty)$</td><td>$\frac{x^{a-1}(1-x)^{b-1}}{B(a,b)}$</td><td>$x \in (0,1)$</td><td>$\frac{a}{a+b}$</td><td>$\frac{ab}{(a+b)^2(a+b+1)}$</td></tr>
<tr><td colspan="6">$B(a,b) := \int_0^1 x^{a-1}(1-x)^{b-1}dx$
$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} = \frac{a+b}{a}B(a+1,b) = \frac{a+b}{b}B(a,b+1)$
$Beta = \mathbb{P}(\mathbb{P}\ event)$</td></tr>
<tr><td colspan="6">$X\sim U_{[a,b]}, \qquad Uniform\ (a<b)$
$originates\ from\ Beta(a=1,b=1), f_X(x) = \frac{1}{Beta(1,1)} = 1, Ran(X) = (0,1)$
$f_X(x) = \frac{1}{k_2-k_1}, Ran(X) = (k_1,k_2)\ or\ [k_1,k_2]\ or\ \dots$</td></tr>
<tr><td colspan="6">$f_{Y=g(X)}(y) = f_X\left(g^{-1}(y)\right)\left|\frac{d}{dy}g^{-1}(y)\right|, \qquad distr\ of\ transformation$
$for\ strictly\ increasing\ and\ differentiatble\ g(\cdot)$</td></tr>
</table>

$(X_1,\dots,X_{r-1}) \sim Multinomial(n,p_1,\dots,p_{r-1})$ $X_r = n - \sum_{i=1}^{r-1} X_i\,,\ p = 1 - \sum_{i=1}^{r-1} p_i$	$\sum_{i=1}^{r} x_i = n$	$\frac{n!}{x_1!\dots x_r!} p_1^{x_1} \dots p_r^{x_r} = \frac{n!}{\prod_{i=1}^{r} x_i!} \prod_{i=1}^{r} p_i^{x_i}$
$(X_1,\dots,X_n) \sim N_n(\vec{\mu},\Sigma)$	$spd\ \Sigma \in \mathbb{R}^{n\times n}$ $\vec{\mu} \in \mathbb{R}^n$	$\frac{1}{(2\pi)^{\frac{n}{2}} \lvert\Sigma\rvert^{\frac{1}{2}}} e^{-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)}$

$Joint\ cdf, \quad F_{\vec{X}}(\vec{t})$	$\sum_{x_n \le t_n} \dots \sum_{x_1 \le t_1} p_{\vec{X}}(\vec{x})$	$\int_{-\infty}^{t_n} \dots \int_{-\infty}^{t_1} f_{\vec{X}}(\vec{x})\, d\vec{x}$
$Marginal\ cdf, \quad F_{X_i}(t)$	$\lim_{x_n\to\infty} (\backslash x_k) \dots \lim_{x_1\to\infty} F_{\vec{X}}(\vec{x}, x_i = t)$ $= \sum_{k\le t} p_{X_i}(k)$	$\int_{-\infty}^{t_1} f_{X_i}(x_i)\, dx_i$
$Joint$		$f_{\vec{X}}(\vec{x}) = \frac{\partial^n}{\partial t_n \dots \partial t_1} F_{\vec{X}}(\vec{t})$
$Marginal$ $(Joint\ Marginal)$	$p_{X_i}(x_i)$ $= \sum_{x_n} \dots (\backslash\, x_i) \sum_{x_1} p_{\vec{X}}(\vec{x}, x_i)$	$f_{X_i}(x_i)$ $= \int_{-\infty}^{\infty} \dots (\backslash x_i) \int_{-\infty}^{\infty} f_{\vec{X}}(\vec{x}, x_i)\, d\vec{x}$
$Conditional$ $(Conditional\ Joint)$	$p_{X\mid Y}(x\mid y) = \frac{p_{X,Y}(x,y)}{p_Y(y)}$ $= \frac{p_{X,Y}(x,y)}{\sum_x p_{X,Y}(x,y)}$	$f_{X\mid Y}(x\mid y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$ $= \frac{f_{X,Y}(x,y)}{\int_{-\infty}^{\infty} f_{X,Y}(x,y)dx}$
$\iint_R 1\, dA = \iint_{(x,y):graph} 1\, dxdy\ or\ dydx$		

$X \perp Y \iff p_{X,Y}(x,y) = p_X(x)p_Y(y)\ \ or\ \ f_{X,Y}(x,y) = f_X(x)f_Y(y)$
$f_X(x) = \int_{-\infty}^{\infty} f_{X\mid Y}(x\mid y)\, f_Y(y)dy, \qquad when\ X \sim distr'\ given\ fixed\ y$

$p_{g(X,Y)}(k) = \sum_{(x,y):g(x,y)=k} \sum p_{X,Y}(x,y)\,, \quad for\ g = +,-,\times,\div$				
$f_{g(X,Y)}(k) = \frac{d}{dk} \iint_{(x,y):g(x,y)\le k} f_{X,Y}(x,y)dxdy\,, \quad for\ g = +,-,\times,\div$				
$f_{X+Y}(a) = \int_{-\infty}^{\infty} f_{X,Y}(x, a-x)dx, \qquad put\ y = a - x$ $f_{X-Y}(a) = \int_{-\infty}^{\infty} f_{X,Y}(x, x-a)dx, \qquad put\ y = x - a$ $f_{XY}(a) = \int_{-\infty}^{\infty} \frac{1}{\lvert x\rvert} f_{X,Y}\left(x, \frac{a}{x}\right) dx, \qquad put\ y = \frac{a}{x}$ $f_{\frac{X}{Y}}(a) = \int_{-\infty}^{\infty} \lvert y\rvert f_{X,Y}(ay, y)dy, \qquad put\ x = ay$				
$CLT\colon \{iid\ X_i\ \lvert finite\ common\ \mu, \sigma^2\}, \frac{\sum_{i=1}^{n} X_i - n\mu}{\sqrt{n}\sigma} = \frac{\sqrt{n}(\bar{X}-\mu)}{\sigma} \sim N(0,1),\ \ as\ n \to \infty$				
$Bin(\sum_{i=1}^{n} m_i\,, p)$	$Possion(\sum_{i=1}^{n} \lambda_i)$	$N.Bin(\sum_{i=1}^{n} 1\,, p)$	$N(\sum_{i=1}^{n} \mu_i\,, \sigma^2)$	$Gamma(\sum_{i=1}^{n} \alpha_i\,, \beta)$

$\mathcal{X} = \{(x_1,x_2)\colon p_{X_1,X_2}(x_1,x_2) > 0\ (noniid)\} = \cup_{(y_1,y_2)\in\mathcal{D}} \mathcal{X}_{y_1,y_2},$ $Define\ \mathcal{D} = \{y_1 = g_1(x_1,x_2),\ y_2 = g_2(x_1,x_2)\colon (x_1,x_2) \in \mathcal{X}\},$ $\mathcal{X}_{y_1,y_2} = \{(x_1,x_2) \in \mathcal{X}\colon y_1 = g_1(x_1,x_2),\ y_2 = g_2(x_1,x_2)\}$
$p_{Y_1=g_1(X_1,X_2),Y_2=g_2(X_1,X_2)}(y_1,y_2) = \sum_{(x_1,x_2)\in\mathcal{X}_{y_1,y_2}} p_{X_1,X_2}(x_1,x_2)\,, \qquad for\ (y_1,y_2) \in \mathcal{D}$
$Singleton\ counting\colon outer\ summation\ varies, inner\ summation\ \equiv for\ sub$

$\mathcal{X} = \{(x_1,x_2)\colon f_{X_1,X_2}(x_1,x_2) > 0\ (noniid)\}, \qquad Define\ \mathcal{D} = \{y_1 = g_1(x_1,x_2),\ y_2 = g_2(x_1,x_2)(aux)\}$
$(1)\ \mathcal{X} \overset{g_1,g_2\ bij}{\Longleftrightarrow} \mathcal{D}, \qquad (2)\ g_1^{-1}, g_2^{-1} \in C^1, \qquad (3)\ J = \left\lvert \frac{\partial(x_1 = g_1^{-1}(y_1,y_2), x_2 = g_2^{-1}(y_1,y_2))}{\partial(y_1,y_2)} \right\rvert \ne 0$
$f_{Y_1,Y_2}(y_1,y_2) = \lvert J\rvert f_{X_1,X_2}(x_1 = g_1^{-1}(y_1,y_2), x_2 = g_2^{-1}(y_1,y_2)), \qquad (y_1,y_2) \in \mathcal{D}$

$$E[g(X_1,\dots,X_n)] = \sum_{x_n}\dots\sum_{x_1} g(x_1,\dots x_n)p_{X_1,\dots X_n}(x_1,\dots,x_n)\,,or\ E[Y = g(X_1,\dots,X_n)] = \sum_{y\in\chi_Y} y p_Y(y)$$
$$E[g(X_1,\dots,X_n)] = \int_{-\infty}^{\infty}\dots\int_{-\infty}^{\infty} g(x_1,\dots x_n)f_{X_1,\dots X_n}(x_1,\dots,x_n)dx_1\dots dx_n\,,or\ E[Y] = \int_{-\infty}^{\infty} y f_Y(y)dy$$

$$E[\textstyle\sum_{i=1}^{n} X_i] = \sum_{i=1}^{n} E[X_i],\ \ further\ put\ X_i = c_i g_i(X_i), for\ generic\ \{X_i\}$$
$$E[\textstyle\sum_{i=1}^{n} X_i] = \sum_{i=1}^{n} E[X_i],\ \ further\ put\ X_i = c_i g_i(X_i)\ \ for\ independent\ \{X_i\}$$

$$Cov(X,Y) := E[(X - E(X)(Y - E(Y)],\qquad Covariance$$
$$Cov(X,Y) = E(XY) - E(X)E(Y)$$
$$X \perp Y \xRightarrow{E(XY)=E(X)E(Y)} Cov(X,Y) = 0$$
$$X \perp Y \xLeftarrow{(X,Y)\sim N_2(\vec{\mu},\Sigma)} Cov(X,Y) = 0,\qquad counter\ examples$$

$$Cov(X,Y) = Cov(Y,X);\ \ Cov(X,X) = Var(X);\ \ Cov(aX,bY) = abCov(X,Y)$$
$$Cov\left(\textstyle\sum_{i=1}^{n} X_i, \sum_{j=1}^{m} Y_j\right) = \sum_{i=1}^{n}\ \sum_{j=1}^{m} Cov(X_i,Y_j)$$

$$Var(\textstyle\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} Var(X_i) + 2\sum_{1\le i<j\le n}\ \sum\ Cov(X_i,X_j) \xrightarrow{ind,Cov(X_i,Y_j)=0} \sum_{i=1}^{n} Var(X_i)$$

$$-1 \le \rho(X,Y) := \frac{Cov(X,Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}} \le 1, if\ Var(X),Var(Y) > 0,\qquad Correlation\ (linearity)$$
$$Var\left(W = \frac{X}{\sigma_X} - \frac{Y}{\sigma_Y}\right) \ge 0\ \Leftrightarrow Cov(X,Y) \le \sqrt{Var(X)}\sqrt{Var(Y)},$$
$$\rho(X,Y) = 1, Var(W) = 0, W = E(W)\ with\ p = 1, Y = [-\sigma_Y E(W)] + (\frac{\sigma_Y}{\sigma_X})X$$

$$E(X|Y=y) = \sum_k k\ p_{X|Y}(k|y)\ or\ \int_{-\infty}^{\infty} k\ f_{X|Y}(k|y)dk\,,\qquad Conditional\ Expectation$$
$$is\ rv\ function\ of\ fixed\ Y,\qquad E(X|Y)\ has\ distribution$$
$$E[E(X|Y)] = E(X),\qquad Double\ Expectation\ Formula$$

$M_X(t) = E(e^{tX})$	$\sum_{g(x)}[e^{tg(x)}p_X(x)]\ \ or\ \ \int_{-\infty}^{\infty}[e^{tg(x)}f_X(x)]$
$M_{aX+b}(t) = e^{tb}M_X(at),\qquad rv\ transform$	$M_{X+Y}(t) = M_X(t)M_Y(t)\ \ for\ X \perp Y$
$E(X) = M_X^{(1)}(0),\qquad E(X^k) = M_X^{(k)}(0)$	$Var(X) = E(X^2) - E(X)^2$
$E(X^3),\qquad Skewness$	$E(X^4),\qquad Kurtosis$

$X\sim Possion(\lambda)$	$M_X(t) = e^{\lambda(e^t-1)}$
$X\sim Bin(n,p)$	$M_X(t) = (pe^t + 1 - p)^m$
$X\sim N.Bin(k,p)$	$M_X(t) = \left[\frac{pe^t}{1-(1-p)e^t}\right]^k$

$Z\sim N(0,1)$ $X\sim N(a,b)$	$M_Z(t) = e^{\frac{t^2}{2}},\qquad hence\ M_{X=a+\sqrt{b}Z}(t) = e^{at+\frac{bt^2}{2}}$
$X\sim Exp(\lambda)$	$M_X(t) = \frac{\lambda}{\lambda - t},\qquad for\ t < \lambda$
$X\sim Gamma(\alpha,\beta)$	$M_X(t) = \left(\frac{\beta}{\beta - t}\right)^{\alpha},\qquad for\ t < \beta$
$X\sim U_{[a,b]}$	$M_X(t) = \frac{e^{bt} - e^{at}}{(b-1)t}$

$$mgf\ uniquely\ determines\ distribution\ \Rightarrow exploit\ change\ of\ parameters$$