Lsn32

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A statistical hypothesis is a claim about the distribution of one or more random variables. When we test a statistical hypothesis we are using the data to make a claim about how plausible the hypothesis is. For example, we might claim that the probability of rolling a six is 1/6. In other words, if we let X be the number of 6s we roll out of ten trials we might say

$$X \sim \text{Binom}(10, p = 1/6)$$

Here our claim is that our parameter, p is 1/6. If we roll our die 10 times and we obtain 3 sixes, is our claim correct? Well... maybe? It's hard to judge the claim without an idea of other values that p could take on. For example, let's say we have one fair die and one unfair die where $p = 1/3$. If we roll our die 10 times and we obtain 3 sixes is it more likely that our die is fair or unfair? How can we tell?

Well, if $p = 1/6$ then the probability of rolling 3 sixes is .155, if $p = 1/3$ the probability of rolling 3 sixes is .26. So should we conclude $p = 1/3$?

Let's consider this another way. Let's say that if $p = 1/3$ and you guess $p = 1/3$ you win 10 dollars. However if $p = 1/6$ and you guess $p = 1/6$ you win 100 dollars. If you roll our die 10 times and get 3 sixes what would you guess?

What we went through above is the basics of statistical hypothesis tests. When we conduct a statistical hypothesis test we need to know what our null hypothesis is, what our alternative hypothesis is, what our test statistic is, and what our rejection region is.

Let's take the above example and go through it. In the absence of any data, would you guess that $p = 1/6$ or $p = 1/3$ given that you win 10 dollars on a bet of $p = 1/3$ and 100 dollars on a bet of $p = 1/6$?

Therefore, unless the data convince us, we should conclude that $p = 1/6$. Our test statistic is how we will test our data. What is our test statistic in this case?

Our rejection region is the realized values of our test statistic that will convince us to switch our bet from $p = 1/6$ to $p = 1/3$. For instance, if $X = 1$ would we want to switch our bet? What about $X = 5$? Think about what values of X would convince you to switch your bet from $p = 1/6$ to $p = 1/3$.

A type I error is made if we switch our bet when we shouldn't have. $\alpha$is the probability of obtaining our rejection region under H_0. For your rejection region, find α.

If α is too high or too low for your liking you can adjust your rejection region. Do so now if you want to.

Conversely, a Type II error is made if you fail to switch your bet but you should have. Meaning X wasn't in your rejection region but $p = 1/3$. Find the probabilty of committing a Type II error.

What we went through above is an example of a simple vs. simple hypothesis testing. This is a bit different then the hypothesis testing that is done in MA206/256, but is perhaps more intuitive. It is, perhaps, more common to use composite hypothesis. That is $p = 1/6$ vs $p > 1/6$. Now we aren't comparing $p = 1/6$ to $p = 1/3$, but rather comparing $p = 1/6$ to $p \in (1/6, 1]$. Let's say in this instance we decide to roll our die 10 times and reject if $x \geq 3$. Let's calculate α.

Now finding the probabilty of conducting a Type II error is a bit more difficult. Recall that a Type II error is made if H_a is true. So, if H_a is true we don't *know* p, but we know it's NOT 1/6. For instance, if $p = 1/3$ the probability of conducting a Type II error is:

However, if $p = 1/5$ it is:

If $p = 1/2$ it is:

So here we have to specify what our alternative is in order to find the probability of conducting a Type II error.

Let's work 10.6 from the text.

Now let's rework 10.6, but instead let's toss the coin 50 times. Find the rejection region matching the α value we found above.

For this rejection region find the value of β if $p = .7$.

What does this tell us about making a good test?