# Lsn17

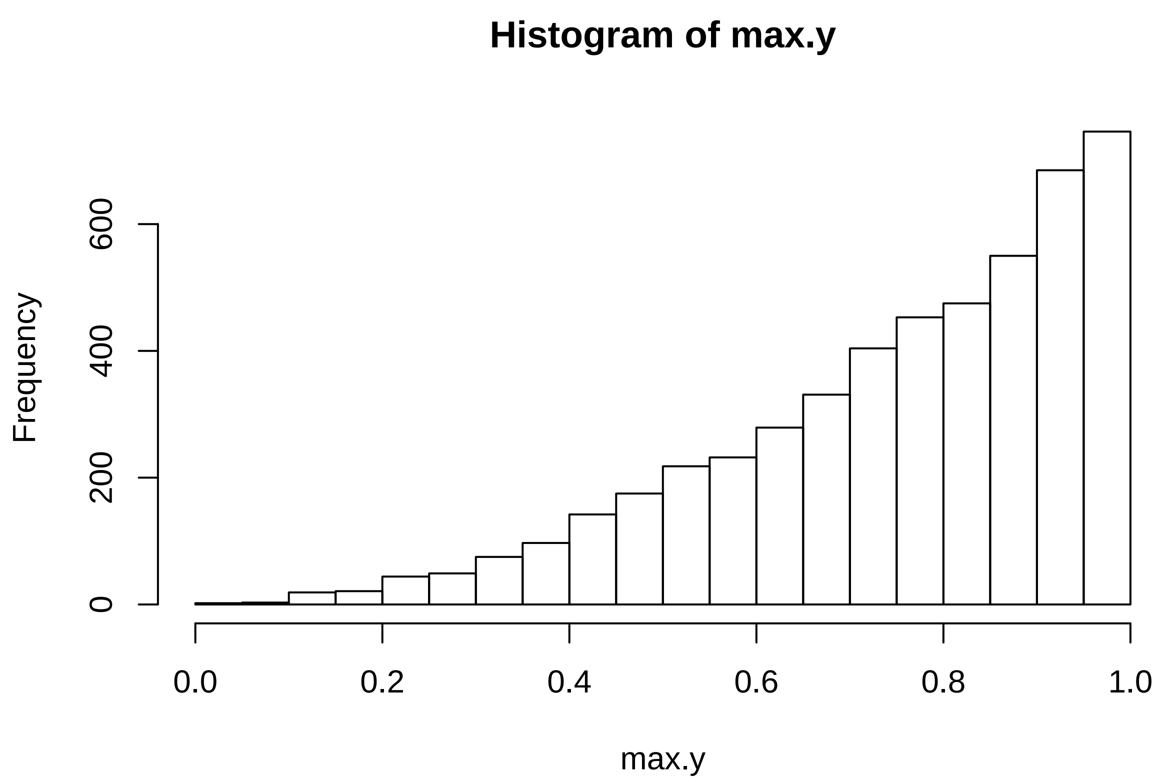
*Clark*

## Admin

Let $Y_1, Y_2, \cdots, Y_n$ be independent and identically distributed random variables all drawn from $F(y)$ with pdf $f(y)$. We *define* the order statistics as $Y_{(1)}, Y_{(2)}, \cdots, Y_{(n)}$ where $Y_{(1)} \leq Y_{(2)} \leq \cdots \leq Y_{(n)}$. Two very special order statistics are $Y_{(1)}$ and $Y_{(n)}$. The reason these are special is connsider an actual experiment. If we conduct an experiment $n$ times, a natural thing to ask is what is the smallest value we should expect to see and what is the largest value we should expect to see.

In order to answer that question for a given distribution we need the density function of $Y_{(1)}$ or $Y_{(n)}$. Note that this is not, necessarily, the same as the density function of $Y_i$. For example, let's say $Y_1, Y_2, Y_3$ are all drawn from a Uniform (0,1). In order to simulate the density of $Y_{(3)}$ we estimate the PDF we could do

```
max.y<-c()
for(i in 1:5000){
  samp<-runif(3)
  max.y[i]<-max(samp)
}
hist(max.y)
```




Here we see the density of the minimum is clearly NOT Uniform(0,1). In order to find the PDF of the maximum we can use the CDF method

Now if we take the derivative we get:

So for our example above we can calculate the PDF of $Y_{(3)}$ as:

Similarly for $Y_{(1)}$ we can use the CDF method:

Note above it was a lot easier to deal with the compliment. Let's find $Y_{(1)}$ for $Y_1, Y_2, Y_3 \sim \text{Exp}(1)$.

Verify that the above is a valid PDF.

The text goes on to talk about the joint density of the order statistics which is:

This isn't super useful except in some odd casees that we may or may not get to in this course. But intuitively it sort of makes sense.

Theorem 6.5. is the generalization of the PDF of the order statistic for any $f(y_{(k)})$. In this case, the density function can really be thought of as similar to the grade assigning problem that we had in HW 2. I "assign" $k-1$ of my Random Variables to be less than $y_k$, one value of my Random variable to be $y_k$, and $n-k$ random variables to begreater than $y_k$. Let's look at Theorem 6.5 and see where all those pieces come in to play.

If $X_1, X_2, \cdots, X_5$ are independent Exponential RVs with mean 1, what is the density of $X_{(3)}$?

Using Theorem 6.5, what is the Joint Density of $X_{(1)}$ and $X_{(3)}$

Let $Y_1$ and $Y_2$ be independent Uniform (0,1) random variables. Find $P(2(Y_{(1)}) < Y_{(2)})$