MA476 Lsn 4

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Admin

-HW Due

-Next HW is posted to website, due Lesson 7

Recall a random variable, $X(s)$ is a mapping from events in our sample space to $\mathbb{R}$. A random variable, $X(s)$ is discrete if the values $X(s) = x$ where $P(X(s) = x) > 0$ is contained within the set of integers. Or, in other words, the values of x where $P(X(s) = x) > 0$ are either finite or countably infinte. Note from here on out, we will usually drop the (s) and just write X while keeping in mind that a random variable is a function of points within our sample space.

However, note definition 3.2 in WMD. We could either calculate probabilities on our sample space directly or we can think of $P(Y = y)$ as the sum of the probabilities of all values of $s \in S$ such that $Y(s) \to y$.

Some important theorems/definitions/results

We let $p(y)$ be the probability distribution and we note that our probability distribution is a well defined probability. That is, $p(y) \in [0, 1]$ for all y and $\sum_y p(y) = 1$.

The probability distribution of a discrete random variable Y is given by $p(y) = c\lambda^y/y!$. Using the identity $e^\lambda = \sum_{y=0}^{\infty} \lambda^y/y!$ find c.

Find $P(Y > 2)$

Expected Value

We *define* the expected value of Y as $E[Y] = \sum_y yp(y)$. In teneral, we can caluate the expected value for any function of Y say $g(Y)$ as $E[g(Y)] = \sum_y g(y)p(y)$.

Some very important properties of expectation are given in Theorems 3.3, 3.4, and 3.5 in WMD.

Theorem 3.3 says that the expected value of a constant is a contant. That is, if c is any fixed number, $E[c] = c$. Constants can be multiplied by a variable. If we recall that $X \sim \text{Binomial}(n, p)$ means that $E[X] = np$. Then if we wanted to calculate $E[3X]$ we would have $E[3X] = 3np$. If we wanted $E[X + 3]$ we would have $E[X + 3] = 3 + np$.

Putting theorems 3.3-3.6 all together gives us a convenient way to find the Variance of a random variable. Recall that $V(Y) = E[(Y - \mu)^2]$. By 3.6 we can take a shortcut and say that $V(Y) = E(Y^2) - E(Y)^2$.

A single fair die is tossed once. Let Y be the number facing up. Find the expected value and variance of Y.

A person tosses a fair coin until a tail appears for the first time. If the tail appears on the nth flip, the person wins 2^n dollars. Let X denote the player's winnings. Show that $E[X] = \infty$.

Would you be willing to pay $1 million to play this game once? Does this contradict what you found above?