Lsn34

Clark

Admin

Say we are testing, generically $H_0 : \theta = \theta_0$ vs $H_a : \theta > \theta_0$. Our first step is determining what our test statistic is. For large sample tests it may make sense to us:

When we use this as our test statistic it has a distribution of $Z \sim N(0, 1)$. From here we either fix or determine our rejection region so $P(Z > z) = \alpha$. Note here we can also solve for our rejection region in terms of $\hat{\theta}$

As we previously discussed, the probability of commiting a Type II error is:

In order to find this we need to stipulate a value for $\theta_a > \theta_0$. In terms of k we are searching for:

$$P(\hat{\theta} \leq k | \theta = \theta_a)$$

Note, for $\hat{\theta}$ we still have the CLT, so if $\theta = \theta_a$ our Z value is NOT $\frac{\hat{\theta}-\theta_0}{\sigma_{\hat{\theta}}}$ but rather since we are assuming $\theta = \theta_a$ we have:

$$Z_a = \frac{\hat{\theta} - \theta_a}{\sigma_{\hat{\theta}}}$$

Note the difference here. So, the probability of commiting a Type II error is the probability that

$$P(\hat{\theta} \leq k | \theta = \theta_a)$$

Or, $P(Z_a \leq \frac{k-\theta_a}{\sigma_{\hat{\theta}}})$

Again we have $Z_a \sim N(0, 1)$ so the calculations can be performed straight forwardly.

In Exercise 10.19 WMS state

The output voltage for an electric circuit is specified to be 130. A sample of 40 independent readings on the voltage for this circuit gave a sample mean of 128.6 and standard deviation 2.1. Test the hypothesis that the average coutput voltage is 130 against the alternative that it is less than 130. Use a test with level .05.

First we define our test statistic

In R we could write

```
Z<- (128.6-130)/(2.1/sqrt(40))
Z
```

```
## [1] -4.21637
```

```
RR<-qnorm(.05)    #Lower Tail test so .05 here
Z<RR
```

```
## [1] TRUE
```

Now, we note that if the voltage falls as low as 128, serious consequences may result. What is the probability of a type II error if $\mu = 128$?. Note here that

```
k<-2.1/sqrt(40)*RR+130
#In terms of Theta, our RR is
cat(-Inf,k,sep=",")
```

```
## -Inf,129.4538
```

So to find Power we need to find the probability that $\hat{\mu}$ is not in the Rejection if, indeed $\mu = 128$. If we are not in the rejection region than we have not observed $\hat{\mu} = 129.4538$ or lower, so we need the probability $\hat{\mu} > 129.45$ given $\mu = 128$, or in other words we need

$$P\left(\frac{\hat{\mu} - 128}{\frac{2.1}{\sqrt{40}}} > \frac{129.4538 - 128}{\frac{2.1}{\sqrt{40}}}\right)$$

Or, as we know the distribution of $\frac{\hat{\mu}-128}{\frac{2.1}{\sqrt{40}}}$ is $N(0,1)$ we want

```
1-pnorm((129.4538-128)/(2.1/sqrt(40)))
```

```
## [1] 5.977708e-06
```

So the probability of committing a Type II error is pretty low. What if we have serious consequences if the voltage falls below 129? Then we would have:

```
1-pnorm((129.4538-129)/(2.1/sqrt(40)))
```

```
## [1] 0.08585869
```

Graphically what is going on is:

As we previously talked about, there's no such thing as a free lunch. If you want a lower α you are going to get a higher β (see picture above). One way we can correct for this though is to change n. If we change n what happens to Z?

Graphically if we change n we get:

So, if we want a fixed value of α and a fixed value of β the one lever we have to pull is n.

Recall that α is

And β is

So if we fix μ_0, μ_a, α and β we can solve for what our book calls z_α and z_β

So solving for k yields

Letting these things be equal, we get a formula for n

$$n = \frac{(z_\alpha - z_\beta)^2 \sigma^2}{(\mu_\alpha - \mu_0)^2}$$

Let's put all this together with exercises 10.42 and 10.43

1.) Write out Null/Alternative

2.) Find Test Statistic

3.) Find Rejection Region if given α

4.) Determine Rejection Region in terms of $\hat{\theta}$, k

5.) Find $\frac{k-\theta_a}{\sigma_{\hat{\theta}}}$

6.) Calculate β

7.) Find z_α

```
qnorm(1-alpha)
```

8.) Find z_β

```
-qnorm(beta)
```