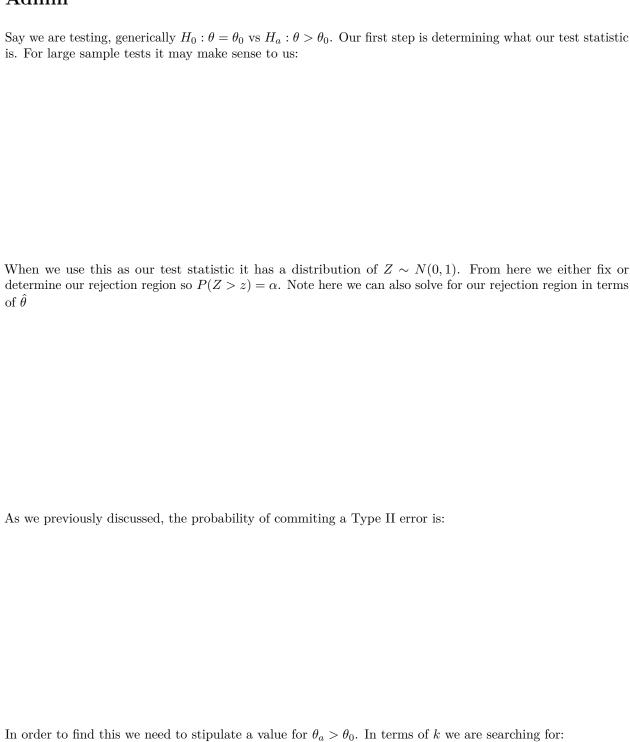
Lsn34

Clark

Admin



$$P(\hat{\theta} < k | \theta = \theta_a)$$

Note, for $\hat{\theta}$ we still have the CLT, so if $\theta = \theta_a$ our Z value is NOT $\frac{\hat{\theta} - \theta_0}{\sigma_{\hat{\theta}}}$ but rather since we are assuming $\theta = \theta_a$ we have:

$$Z_a = \frac{\hat{\theta} - \theta_a}{\sigma_{\hat{\theta}}}$$

Note the difference here. So, the probability of committing a Type II error is the probability that

$$P(\hat{\theta} \le k | \theta = \theta_a)$$

Or,
$$P(Z_a \leq \frac{k-\theta_a}{\sigma_{\hat{\theta}}})$$

Again we have $Z_a \sim N(0,1)$ so the calculations can be performed straight forwardly.

In Exercise 10.19 WMS state

The output voltage for an electric circuit is specified to be 130. A sample of 40 independent readings on the voltage for this circuit gave a sample mean of 128.6 and standard deviation 2.1. Test the hypothesis that the average coutput voltage is 130 against the alternative that it is less than 130. Use a test with level .05.

First we define our test statistic

In R we could write

```
Z<- (128.6-130)/(2.1/sqrt(40))
Z
```

```
## [1] -4.21637
```

```
RR<-qnorm(.05) #Lower Tail test so .05 here
Z<RR
```

```
## [1] TRUE
```

Now, we note that if the voltage falls as low as 128, serious consequences may result. What is the probability of a type II error if $\mu = 128$?. Note here that

```
k<-2.1/sqrt(40)*RR+130
#In terms of Theta, our RR is
cat(-Inf,k,sep=",")</pre>
```

-Inf,129.4538

So to find Power we need to find the probability that $\hat{\mu}$ is not in the Rejection if, indeed $\mu = 128$. If we are not in the rejection region than we have not observed $\hat{\mu} = 129.4538$ or lower, so we need the probability $\hat{\mu} > 129.45$ given $\mu = 128$, or in other words we need

$$P\left(\frac{\hat{\mu} - 128}{\frac{2.1}{\sqrt{40}}} > \frac{129.4538 - 128}{\frac{2.1}{\sqrt{40}}}\right)$$

Or, as we know the distribution of $\frac{\hat{\mu}-128}{\frac{2.1}{\sqrt{40}}}$ is N(0,1) we want

1-pnorm((129.4538-128)/(2.1/sqrt(40)))

[1] 5.977708e-06

So the probability of committing a Type II error is pretty low. What if we have serious consequences if the voltage falls below 129? Then we would have:

1-pnorm((129.4538-129)/(2.1/sqrt(40)))

[1] 0.08585869

Graphically what is going on is:

As we previously talked about, there's no such thing as a free lunch. If you want a lower α you are going to get a higher β (see picture above). One way we can correct for this though is to change n. If we change n what happens to Z?

Graphically if we change n we get:

So, if we want a fixed value of α and a fixed value of β the one lever we have to pull is n. Recall that α is

And β is

So if we fix μ_0 , μ_a , α and β we can solve for what our book calls z_{α} and z_{β}

So solving for k yields

Letting these things be equal, we get a formula for n

$$n = \frac{(z_{\alpha} - z_{\beta})^2 \sigma^2}{(\mu_{\alpha} - \mu_0)^2}$$

Let's put all this together with exercises 10.42 and 10.43

- 1.) Write out Null/Alternative
- 2.) Find Test Statistic
- 3.) Find Rejection Region if given α
- 4.) Determine Rejection Region in terms of $\hat{\theta}$, k
- 5.) Find $\frac{k-\theta_a}{\sigma_{\hat{\theta}}}$
- 6.) Calculate β
- 7.) Find z_{α}

qnorm(1-alpha)

8.) Find z_{β}

-qnorm(beta)