Lsn22

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During WWII the Allies wanted to know how many Tanks the Germans were producing. Fortunately the Germans sequentially numbered their tanks. Each day the Allies would record the serial number of the tank they observed. Let's let Y be the number of tanks the Germans have and we'll assume that $Y \sim \text{Unif}(0, \theta)$ for some unknown (but desired to know!) θ.

How did you estimate θ?

Let's look at $\hat{\theta}_{mle} = \max Y_1, Y_2, Y_3, \cdots, Y_n$. What is $E[\hat{\theta}_{mle}]$?

What is $Var[\hat{\theta}_{mle}]$?

Ok, what about $\hat{\theta} = 2\bar{Y}$? What is $E[\hat{\theta}]$? What is $Var[\hat{\theta}]$?

What would you want to use? What are some other options?

Later in this course we'll talk about what is meant by MLEs, etc., but note that in any case we can think of a *good* estimator as one that is consistently accurate and doesn't vary a lot. While we can always calculate the variance of our estimator if we know the PDF by:

Table 8.1 gives some common estimators and associated variances (or standard errors). What happens to the standard error as n gets really gib? What does this mean about our estimators?

While knowing the MSE of an estimator is helpful, it may be more helpful to give a range of estimators that likely cover our true parameter. This is known as a confidence interval. A confidence interval, for a paramter, is found by finding two estimators, $\hat{\theta}_l$ and $\hat{\theta}_r$ which that $P(\hat{\theta}_l \leq \theta \leq \hat{\theta}_r) = 1 - \alpha$. For instance we often let

$\alpha = .05$ and report a 95% confidence interval.

The key to finding confidence intervals is often finding what is known as a pivotal quantity. A pivotal quantity is not a statistic, but rather is a function of both our data and of our parameters of interest that we know the distribution of.

For example, recall that we know that

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

In this case we have both a statistic, S^2 and our paramter of interest σ^2 that, together, we know the distribution of. Because we know the distribution we can find the points a and b such that

For instance, if $n = 10$, we can use R to find

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qchisq(.025,9)
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## [1] 2.700389
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qchisq(.975,9)
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## [1] 19.02277
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So, with this in hand, we need to isolate σ^2.

As you can guess, the difficulty in using this technique is finding a pivotal quantity that we can use to find a and b as in above. Though sometimes we can get lucky.

Let $X_1, \cdots, X_n \sim Exp(\beta)$. Find the distribution of $\frac{\bar{X}}{\beta}$. Let's use MGF method here.

Now use this to find a 97.3% Confidence interval for β.