Lsn3

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Law of Total Probability

Start with a sample space, S, and define a *Partition* of S

Now note for any set $A \subset S$ we can write:

$$A = (A \cap B_1) \cup (A \cap B_2) \cup \cdots \cup (A \cap B_k)$$

What can we say about $(A \cap B_1) \cap (A \cap B_2)$?

Now, what can we say about $P(A)$?

As long as we assume $P(B_j) \neq 0$ we can write $P(A|B_j)P(B_j) = P(A \cap B_j)$ which, from above, leads us to the **Law of Total Probability**

At my high school, me and all the other cool kids used to play Risk some weekends. In the board game Risk sometimes the attacker gets to roll 2 dice and the defender only gets to roll 1 die. The attacker wins if *at least one* of his die is **higher** than the defenders. Who has the advantage in this situation?

$$A \equiv \text{Event the Attacker Wins}$$
$$D_i \equiv \text{Event the defenders roll is } i$$

Bayes Rule

Sometimes on the way to finding $P(A_i|B)$ its a lot easier to find $P(B|A_i)$

Claim: $P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_{j=1}^{k} P(B|A_j)P(A_j)}$

Proof:

Problem 2.134 in WMD

Random Variables

Formally, a random variable is a mapping from $S \to \mathbb{R}$ for sample points, $s \in S$. Random variables are not necessarily something that can be physically realized. They are things statisticians make up to help us in our probability calculations. We use random variables to conceptualize a real situation.

A pictures:

Flip two coins, what is our sample space?

Let $X(s) \equiv$ Number of heads. What outcomes map to $X(s) = 1$? What other values can X take on?

Can we come up with $P(X = 1)$? What about if we flip 10 coins? 100 coins? 1000 coins?