Lsn28

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Up to this point we've been talking about properties of point estimators and comparing/evaluating what makes something a good point estimator. Remember that a point estimator is

However, we have not give you a tool for actually finding a point estimator. One of the most straight forward ways to estimate a parameter, or a function of parameters, is the *method of moments*. Recall that the kth moment of a random variable is:

What the method of moments does is use the *sample moment* to estimate the *population moment*. The kth sample moment is:

For example, if $X_1, X_2, \cdots, X_n$ come from a Poisson distribution, the first moment is λ. Therefore, in order to esimate λ the MOM says to use $\bar{X}$ as an estimate. In this case, we might think this is a pretty good estimate.

It's unbiased:

It's consistent:

It is a function of the sufficient statistic:

So, I guess, yay? But herein lies the first of seeveral issues with the MOM. It does not necessarily yield a unique estimate. If $X \sim Po(\lambda)$ what is $E[X^2]$?

So, we could also use the second sample moment to estimate λ. If we do this, our esimate is no longer unbiased.

So while the MOM doesn't necessarily yield the best estimator, it does potentially serve as a start point. And if it turns out it yields an unbiased estimator that is a function of a sufficient statistic, then we likely have the MVUE.

Again, the steps of using the MOM estimator are: First find the population moment, equate the population moment with the sample moment, then simplify if necessary.

Let's work problem 9.69

Sometimes we need more than one moment to find an estimator for our parameters. Let's let $Y_1, \cdots, Y_n \sim \text{Gamma}(\alpha, \beta)$.

The MOM becomes:

Let $Y_1, \cdots, Y_n \sim N(\mu, \sigma^2)$ Find the MOM for μ and σ^2.