Lsn22

Clark

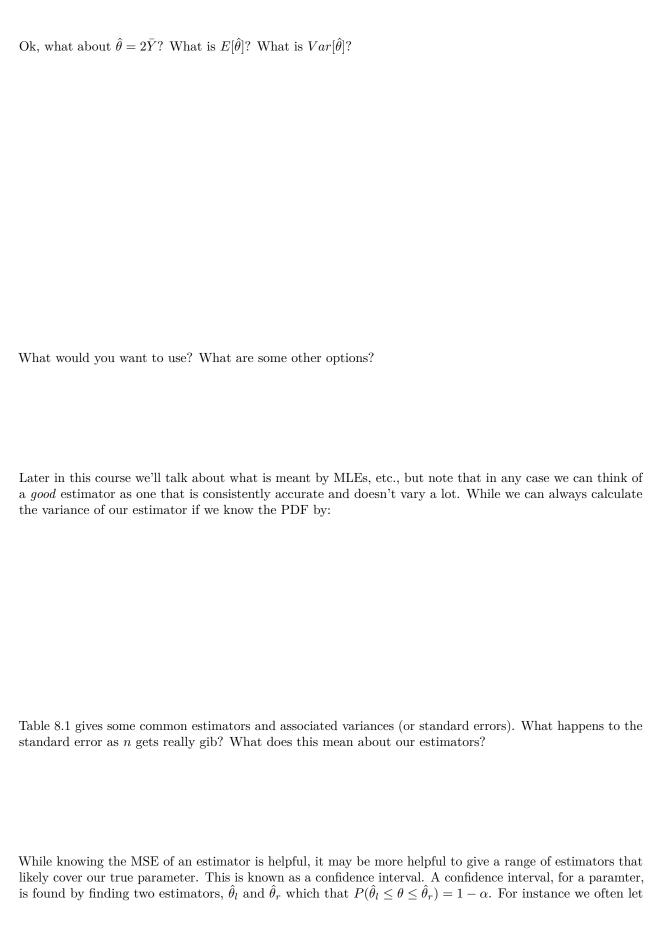
Admin

During WWII the Allies wanted to know how many Tanks the Germans were producing. Fortunately the Germans sequentially numbered their tanks. Each day the Allies would record the serial number of the tank they observed. Let's let Y be the number of tanks the Germans have and we'll assume that $Y \sim \text{Unif}(0, \theta)$ for some unknown (but desired to know!) θ .

How did you estimate θ ?

Let's look at $\hat{\theta}_{mle} = \max Y_1, Y_2, Y_3, \dots, Y_n$. What is $E[\hat{\theta}_{mle}]$?

What is $Var[\hat{\theta}_{mle}]$?



 $\alpha = .05$ and report a 95% confidence interval.

The key to finding confidence intervals is often finding what is known as a pivotal quantity. A pivotal quantity is not a statistic, but rather is a function of both our data and of our parameters of interest that we know the distribution of.

For example, recall that we know that

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi_{n-1}^2$$

In this case we have both a statistic, S^2 and our paramter of interest σ^2 that, together, we know the distribution of. Because we know the distribution we can find the points a and b such that

For instance, if n = 10, we can use R to find

qchisq(.025,9)

[1] 2.700389

qchisq(.975,9)

[1] 19.02277

So, with this in hand, we need to isolate σ^2 .

As you can guess, the difficulty in using this technique is finding a pivotal quantity that we can use to find a and b as in above. Though sometimes we can get lucky.

Let $X_1, \dots, X_n \sim Exp(\beta)$. Find the distribution of $\frac{\bar{X}}{\beta}$. Let's use MGF method here.

Now use this to find a 97.3% Confidence interval for $\beta.$