# Lsn35

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Recall if we are testing $H_0 : \theta = \theta_0$ vs. $H_a : \theta > \theta_0$ our test statistic is:

In this case, we also note that this implies a distribution of $\hat{\theta}$ of:

Which we can draw as:

Even if $H_0$ is true, there is a probabilty of $\alpha$ that $P(Z > z_\alpha)$ for any $z_\alpha$ which corresponds to $P(\hat{\theta} > k) = \alpha$, for $k = z_\alpha \sigma_{\hat{\theta}} + \theta_0$. Which we can denote on our picture above.

Similarly, if we have $H_0 : \theta = \theta_0$ vs. $H_a : \theta < \theta_0$, for any fixed $\alpha$ we can find $k$ such that $P(\hat{\theta} < k) = \alpha$. In which case we have the drawing:

The only slight complication is when we are testing $H_0 : \theta = \theta_0$ vs. $H_a : \theta \neq \theta_0$. In this case, in terms of our test statistic $Z$ we would reject $H_0$ if $Z$ fell in the region $|z| > z_{\alpha/2}$ where $z_{\alpha/2}$ is found by fixing $\alpha$ and solving for $z_{\alpha/2}$ in the statement $P(-z_{\alpha/2} < Z < z_{\alpha/2}) = \alpha$. Our drawing here is:

Note here that if $Z$ is NOT in our rejection region that it must be that:

Or, in other words, the null hypothesis is not rejected if:

Here we note the similarities between being NOT in the null hypothesis and forming a confidence interval based on the pivoting the test statistic $Z = \frac{\hat{\theta}-\theta_0}{\sigma_{\hat{\theta}}}$. So, for a two sided test, if we form a confidence inteval for $\theta$ and $\theta_0$ is outside of the 95%CI we would reject $H_0$.

Similarly a large sample $\alpha$-level test of hypothesis for $H_0 : \theta = \theta_0$ vs $H_a : \theta > \theta_0$ we would reject the null hypothesis if

$$\frac{\hat{\theta} - \theta_0}{\sigma_{\hat{\theta}}} > z_\alpha$$

Let's show that this is equivalent to rejecting $H_0$ if $\theta_0$ is less than the large-sample $100(1-\alpha)\%$ lower confidence bound for $\theta$.

First, using the acceptance region, find, in terms of our parameters above, what values of $\theta_0$ would cause us to accept $H_0$ (I know. In MA206 they say, never say accept $H_0 \ldots$ But in practice it really doesn't matter.)

Now. Calculate a one sided lower confidence bound for $\theta$. Remember we need to use a Pivotal quantity. Probably makes sense to use $\frac{\hat{\theta}-\theta}{\sigma_{\hat{\theta}}} \sim N(0,1)$ as our pivotal quantity. Then we need to find $z$ such that $P(\frac{\hat{\theta}-\theta}{\sigma_{\hat{\theta}}} \leq z) = 1 - \alpha$ and isolate $\theta$ in the above inequality.