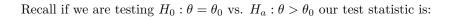
## Lsn35

## Clark

## Admin

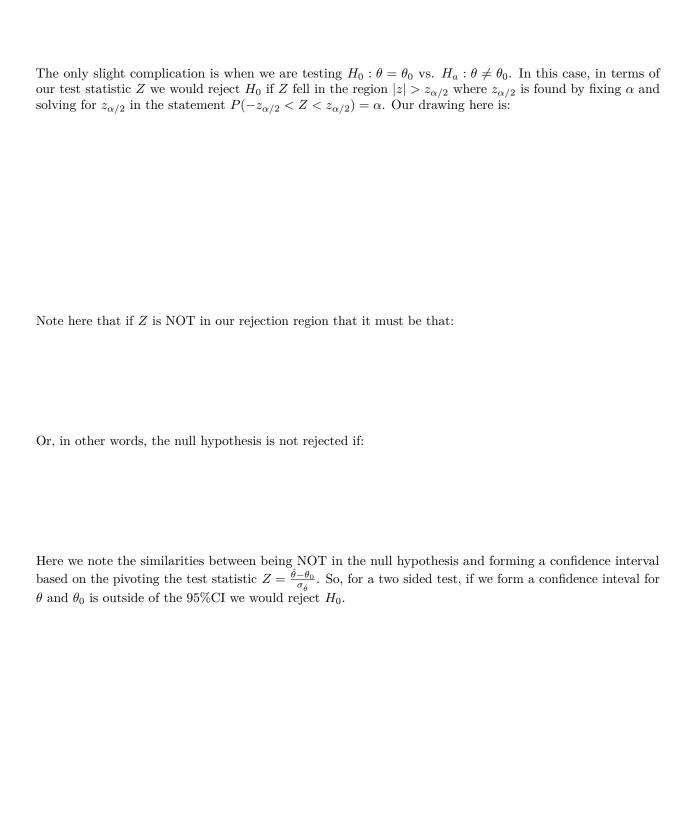


In this case, we also note that this implies a distribution of  $\hat{\theta}$  of:

Which we can draw as:

Even if  $H_0$  is true, there is a probabilty of  $\alpha$  that  $P(Z>z_\alpha)$  for any  $z_\alpha$  which corresponds to  $P(\hat{\theta}>k)=\alpha$ , for  $k=z_\alpha\sigma_{\hat{\theta}}+\theta_0$ . Which we can denote on our picture above.

Similarly, if we have  $H_0: \theta = \theta_0$  vs.  $H_a: \theta < \theta_0$ , for any fixed  $\alpha$  we can find k such that  $P(\hat{\theta} < k) = \alpha$ . In which case we have the drawing:



Similarly a large sample  $\alpha$ -level test of hypothesis for  $H_0: \theta = \theta_0$  vs  $H_a: \theta > \theta_0$  we would reject the null hypothesis if

$$\frac{\hat{\theta} - \theta_0}{\sigma_{\hat{\theta}}} > z_{\alpha}$$

Let's show that this is equivalent to rejecting  $H_0$  if  $\theta_0$  is less than the large-sample  $100(1-\alpha)\%$  lower confidence bound for  $\theta$ .

First, using the acceptance region, find, in terms of our parameters above, what values of  $\theta_0$  would cause us to accept  $H_0$  (I know. In MA206 they say, never say accept  $H_0$ ... But in practice it really doesn't matter.)

Now. Calculate a one sided lower confidence bound for  $\theta$ . Remember we need to use a Pivotal quantity. Probably makes sense to use  $\frac{\hat{\theta}-\theta}{\sigma_{\hat{\theta}}} \sim N(0,1)$  as our pivotal quantity. Then we need to find z such that  $P(\frac{\hat{\theta}-\theta}{\sigma_{\hat{\theta}}} \leq z) = 1 - \alpha$  and isolate  $\theta$  in the above inequality.