Lsn10

Clark

Admin/Questions

The kth moment (sometimes called the raw moment and denoted as μ_k') of a random variable is defined as $E[Y^k]$ and can always be found through either:

or in the case of discrete random variables

However, another method of finding the moments can be found by calculating $E[e^{ty}]$. Why this expectation? Recall that the Taylor series expansion about 0for e^x is:

So, assuming we have finite moments we can write:

The neat thing about this is if we consder the first derivative we end up with:

Now if we evaluate this function we have μ_1'.

Continuing on in this manner, the second derivative yields:

Which again, evaluating at $t = 0$ gives us the second moment.

Suppose that the waiting time for the first customer to enter a retail shop after 9:00 A.M. is a random variable $Y \sim \text{Exp}(\theta)$. Let's prove the result in the back of our book, that the MGF is $\frac{1}{1-\theta t}$ and use it to find $E[Y]$ and $E[Y^2]$

While this is all well and good, a more common use for MGFs is what's pointed out on pg 141 of our text. Specifically, if an MGF exists, it is *unique*. This gives us an alternative way to characterize a distribution outside of a pdf.

To really take advantage of this, we need a few additional facts proven about MGFs. Specifically, if Y is a random variable with MGF $m(t)$ and U is given by $U = aY + b$, the MGF of U is $e^{tb}m(at)$. To see this, let's start at:

$$
\begin{aligned}
m_u(t) =& \\
& \text{Next we substitute in } U = ay + b \\
& = \\
& \text{Now pull out and regroup to arrive at} \\
& =
\end{aligned}
$$

So, using this result, let $Y \sim N(\mu, \sigma)$ and, using the result in the back of the book that the mgf of a Normal random variable is $\exp\left(\mu t + \frac{t^2\sigma^2}{2}\right)$, find the mgf and hence the distribution of $X = -3Y + 4$

Let's practice a bit.

Keeping in mind that the formula for the binomial expansion is $(x + y)^n = \sum_{i=0}^{n} \binom{n}{i} x^{n-i} y^i$, argue that the mgf for a binomial random variable is $(pe^t + (1-p))^n$

Keeping the above result in mind, what random variable has the mgf $(.6e^t + .4)^3$?

Find the MGF for a Uniform(0,1) random variable.

Suppose Y is a Uniform (0,1) random variable. What is the mgf of $W = 3Y$? What is the distribution of W?

Extra Credit - 2 pts Use the MGF we found above to prove that the expected value of a Uniform (0,1) is 1/2. (Hint: L'Hopital's may help here)

Argue carefully that if $Y \sim N(0, 1)$ it must be that $E[Y^k] = 0$ for all odd k