homework_1

October 4, 2023

[ ]:
```
import pyreadr
import pandas as pd
import numpy as np
from scipy.spatial.distance import pdist, squareform
import matplotlib.pyplot as plt
import seaborn as sns

from sklearn.cluster import AgglomerativeClustering
from scipy.cluster.hierarchy import (dendrogram, cut_tree, linkage)
```

[ ]:
```
def plot_dendrogram(model,filename, **kwargs):
    # Create linkage matrix and then plot the dendrogram

    # create the counts of samples under each node
    counts = np.zeros(model.children_.shape[0])
    n_samples = len(model.labels_)
    for i, merge in enumerate(model.children_):
        current_count = 0
        for child_idx in merge:
            if child_idx < n_samples:
                current_count += 1  # leaf node
            else:
                current_count += counts[child_idx - n_samples]
        counts[i] = current_count

    linkage_matrix = np.column_stack(
        [model.children_, model.distances_, counts]
    ).astype(float)

    # Plot the corresponding dendrogram
    dendrogram(linkage_matrix, **kwargs)
    plt.ylabel('Height')
    plt.xticks(rotation=45)
    plt.savefig('plots/{}.png'.format(filename),bbox_inches='tight')

def compute_linkage(hclust):
    """
```

```
    Create linkage matrix used to plot a dendrogram

    Follows [sklearn example](https://scikit-learn.org/stable/auto_examples/
↪cluster/plot_agglomerative_dendrogram.html)

    Parameters
    ----------

    hclust : `sklearn.cluster.AgglomerativeClustering`
        Fitted hierarchical clustering object.

    Returns
    -------

    linkage_matrix : np.ndarray
        Array to be passed to `dendrogram` from `scipy.cluster.hierarchy`.

    """

    counts = np.zeros(hclust.children_.shape[0])
    n_samples = len(hclust.labels_)
    for i, merge in enumerate(hclust.children_):
        current_count = 0
        for child_idx in merge:
            if child_idx < n_samples:
                current_count += 1  # leaf node
            else:
                current_count += counts[child_idx - n_samples]
        counts[i] = current_count

    linkage_matrix = np.column_stack([hclust.children_, hclust.distances_,
                                      counts]).astype(float)
    return linkage_matrix
```

1 Problem 1

[ ]:
```
problem_1_data = pyreadr.read_r('data/hw1prob1.Rdata')
```

[ ]:
```
for value in problem_1_data.values():
    problem_1_df = pd.DataFrame(value)
```

[ ]:
```
problem_1_df.columns
```

[ ]:
```
Index(['abandoned', 'abbeville', 'abbey', 'abbracciava', 'abilities',
       'ability', 'able', 'abode', 'about', 'above',
       …
```

```
       '  ', '   ', '  ', '    ', '  ', '  ', '  ',
       '     ', '  ', '  '],
      dtype='object', length=6820)
```

[ ]:
```
problem_1_df
```

[ ]:
```
           abandoned  abbeville  abbey  abbracciava  abilities  ability  able  \
tmnt leo         1.0        0.0    0.0          0.0        2.0      0.0   0.0
tmnt raph        0.0        0.0    1.0          0.0        1.0      0.0   0.0
tmnt mike        0.0        0.0    0.0          0.0        2.0      1.0   1.0
tmnt don         0.0        0.0    0.0          0.0        1.0      0.0   0.0
real leo         1.0        0.0    0.0          0.0        0.0      4.0   1.0
real raph        0.0        1.0    0.0          0.0        0.0      0.0   1.0
real mike        0.0        0.0    0.0          1.0        0.0      0.0   1.0
real don         0.0        0.0    0.0          0.0        0.0      0.0   0.0

           abode  about  above  …                          \
tmnt leo     0.0    5.0    1.0  …    0.0     0.0    0.0     0.0    0.0  0.0
tmnt raph    0.0    3.0    0.0  …    0.0     0.0    0.0     0.0    0.0  0.0
tmnt mike    1.0    4.0    0.0  …    0.0     0.0    0.0     0.0    0.0  0.0
tmnt don     0.0    3.0    0.0  …    0.0     0.0    0.0     0.0    0.0  0.0
real leo     0.0   15.0    2.0  …    1.0     1.0    1.0     1.0    1.0  1.0
real raph    0.0   12.0    1.0  …    1.0     0.0    0.0     1.0    0.0  1.0
real mike    0.0    3.0    1.0  …    1.0     1.0    1.0     1.0    1.0  1.0
real don     0.0    4.0    0.0  …    0.0     0.0    0.0     1.0    0.0  1.0


tmnt leo      0.0         0.0  1.0  0.0
tmnt raph     0.0         0.0  0.0  0.0
tmnt mike     0.0         0.0  1.0  0.0
tmnt don      0.0         0.0  0.0  0.0
real leo      1.0         1.0  1.0  1.0
real raph     1.0         0.0  1.0  1.0
real mike     1.0         1.0  1.0  1.0
real don      0.0         0.0  1.0  1.0

[8 rows x 6820 columns]
```

1.1 Problem 1 a

[ ]:
```
word_counts = problem_1_df.sum(axis=1)
```

[ ]:
```
word_counts
```

[ ]:
```
tmnt leo     3114.0
tmnt raph    1976.0
tmnt mike    3330.0
```

```
tmnt don     2143.0
real leo     8962.0
real raph    6524.0
real mike    4618.0
real don     1766.0
dtype: float64
```

1.1.1 Determine the number of documents that contain each word

This will allow me to keep track of number of documents that contain the given word, by making it 1 if it contains the word, 0 otherwise

[ ]:
```
documents_count_df = problem_1_df.applymap(lambda x: 1 if x != 0 and pd.
↪notna(x) else 0)
```

Sum each column to get n_w for each word

[ ]:
```
n_w = documents_count_df.sum()
n_w.head()
```

[ ]:
```
abandoned      2
abbeville      1
abbey          1
abbracciava    1
abilities      4
dtype: int64
```

[ ]:
```
dtm1 = pd.DataFrame(columns=problem_1_df.columns)
for k,v in word_counts.items():
    dtm1.loc[k] = problem_1_df.loc[k]/v
```

[ ]:
```
D = 8
```

[ ]:
```
dtm1 = dtm1 * np.log(D/dtm1.columns.map(n_w.to_dict()))
```

1.1.2 dtm2 - first scale by IDF and then normalize by word count

[ ]:
```
dtm2 = pd.DataFrame(columns=problem_1_df.columns)
```

[ ]:
```
dtm2 = problem_1_df * np.log(D/dtm2.columns.map(n_w.to_dict()))
```

[ ]:
```
for k,v in word_counts.items():
    dtm2.loc[k] = dtm2.loc[k]/v
```

1.1.3 dtm1 vs dtm2

They are identical. This makes since because you are only apply transformations in a different order. Both IDF and word count normalization depend on the original data and it does not matter which order you apply.

[ ]:
```
dtm1
```

[ ]:
```
           abandoned  abbeville     abbey  abbracciava  abilities   ability  \
tmnt leo    0.000445   0.000000  0.000000      0.00000   0.000445  0.000000
tmnt raph   0.000000   0.000000  0.001052      0.00000   0.000351  0.000000
tmnt mike   0.000000   0.000000  0.000000      0.00000   0.000416  0.000416
tmnt don    0.000000   0.000000  0.000000      0.00000   0.000323  0.000000
real leo    0.000155   0.000000  0.000000      0.00000   0.000000  0.000619
real raph   0.000000   0.000319  0.000000      0.00000   0.000000  0.000000
real mike   0.000000   0.000000  0.000000      0.00045   0.000000  0.000000
real don    0.000000   0.000000  0.000000      0.00000   0.000000  0.000000

               able     abode  about     above  …                \
tmnt leo   0.000000  0.000000    0.0  0.000223  …  0.000000  0.000000
tmnt raph  0.000000  0.000000    0.0  0.000000  …  0.000000  0.000000
tmnt mike  0.000208  0.000624    0.0  0.000000  …  0.000000  0.000000
tmnt don   0.000000  0.000000    0.0  0.000000  …  0.000000  0.000000
real leo   0.000077  0.000000    0.0  0.000155  …  0.000109  0.000155
real raph  0.000106  0.000000    0.0  0.000106  …  0.000150  0.000000
real mike  0.000150  0.000000    0.0  0.000150  …  0.000212  0.000300
real don   0.000000  0.000000    0.0  0.000000  …  0.000000  0.000000

                                                        \
tmnt leo   0.000000  0.000000  0.000000  0.000000  0.000000    0.000000
tmnt raph  0.000000  0.000000  0.000000  0.000000  0.000000    0.000000
tmnt mike  0.000000  0.000000  0.000000  0.000000  0.000000    0.000000
tmnt don   0.000000  0.000000  0.000000  0.000000  0.000000    0.000000
real leo   0.000155  0.000077  0.000155  0.000077  0.000109    0.000155
real raph  0.000000  0.000106  0.000000  0.000106  0.000150    0.000000
real mike  0.000300  0.000150  0.000300  0.000150  0.000212    0.000300
real don   0.000000  0.000392  0.000000  0.000392  0.000000    0.000000


tmnt leo   0.000092  0.000000
tmnt raph  0.000000  0.000000
tmnt mike  0.000086  0.000000
tmnt don   0.000000  0.000000
real leo   0.000032  0.000077
real raph  0.000044  0.000106
real mike  0.000062  0.000150
real don   0.000163  0.000392

[8 rows x 6820 columns]
```

[ ]:
```
dtm2
```

[ ]:

	abandoned	abbeville	abbey	abbracciava	abilities	ability	\
tmnt leo	0.000445	0.000000	0.000000	0.00000	0.000445	0.000000	
tmnt raph	0.000000	0.000000	0.001052	0.00000	0.000351	0.000000	
tmnt mike	0.000000	0.000000	0.000000	0.00000	0.000416	0.000416	
tmnt don	0.000000	0.000000	0.000000	0.00000	0.000323	0.000000	
real leo	0.000155	0.000000	0.000000	0.00000	0.000000	0.000619	
real raph	0.000000	0.000319	0.000000	0.00000	0.000000	0.000000	
real mike	0.000000	0.000000	0.000000	0.00045	0.000000	0.000000	
real don	0.000000	0.000000	0.000000	0.00000	0.000000	0.000000	

	able	abode	about	above	…		\
tmnt leo	0.000000	0.000000	0.0	0.000223	…	0.000000	0.000000
tmnt raph	0.000000	0.000000	0.0	0.000000	…	0.000000	0.000000
tmnt mike	0.000208	0.000624	0.0	0.000000	…	0.000000	0.000000
tmnt don	0.000000	0.000000	0.0	0.000000	…	0.000000	0.000000
real leo	0.000077	0.000000	0.0	0.000155	…	0.000109	0.000155
real raph	0.000106	0.000000	0.0	0.000106	…	0.000150	0.000000
real mike	0.000150	0.000000	0.0	0.000150	…	0.000212	0.000300
real don	0.000000	0.000000	0.0	0.000000	…	0.000000	0.000000

						\
tmnt leo	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
tmnt raph	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
tmnt mike	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
tmnt don	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
real leo	0.000155	0.000077	0.000155	0.000077	0.000109	0.000155
real raph	0.000000	0.000106	0.000000	0.000106	0.000150	0.000000
real mike	0.000300	0.000150	0.000300	0.000150	0.000212	0.000300
real don	0.000000	0.000392	0.000000	0.000392	0.000000	0.000000

tmnt leo	0.000092	0.000000
tmnt raph	0.000000	0.000000
tmnt mike	0.000086	0.000000
tmnt don	0.000000	0.000000
real leo	0.000032	0.000077
real raph	0.000044	0.000106
real mike	0.000062	0.000150
real don	0.000163	0.000392

[8 rows x 6820 columns]

1.2 Problem 1 b

[ ]:
```
dtm3 = pd.DataFrame(columns=problem_1_df.columns)
for k,v in word_counts.items():
    dtm3.loc[k] = problem_1_df.loc[k]/v
```

[ ]:
```
tmnt_mike = np.array(dtm3.loc['tmnt mike'])
```

Iterate through the data frame and calculate the euclidean norm for each vector compared to `tmnt_mike`

The closest document is `tmnt raph`

[ ]:
```
for row in dtm3.iterrows():
    euclidean_distance = np.linalg.norm(tmnt_mike - np.array(row[1]))
    print('{}: {}'.format(row[0], euclidean_distance))
```

```
tmnt leo: 0.04118467877954197
tmnt raph: 0.03782949128933655
tmnt mike: 0.0
tmnt don: 0.042006124349499935
real leo: 0.04887327167743683
real raph: 0.04487836641687292
real mike: 0.047500179860007326
real don: 0.06376091296934941
```

1.3 Problem 1c

Complete linkage does a better job when K=2 because it creates to groups, one of turtles and one of the artists.

[ ]:
```
euclidean_distances = pdist(dtm3, metric='euclidean')
distance_matrix = squareform(euclidean_distances)
```

[ ]:
```
distance_df = pd.DataFrame(distance_matrix, index=dtm3.index, columns=dtm3.
  ↪index)
distance_df
```

[ ]:

	tmnt leo	tmnt raph	tmnt mike	tmnt don	real leo	real raph	\
tmnt leo	0.000000	0.038599	0.041185	0.044445	0.042532	0.049604	
tmnt raph	0.038599	0.000000	0.037829	0.042870	0.054986	0.046176	
tmnt mike	0.041185	0.037829	0.000000	0.042006	0.048873	0.044878	
tmnt don	0.044445	0.042870	0.042006	0.000000	0.053655	0.050400	
real leo	0.042532	0.054986	0.048873	0.053655	0.000000	0.039363	
real raph	0.049604	0.046176	0.044878	0.050400	0.039363	0.000000	
real mike	0.064754	0.066762	0.047500	0.061904	0.053363	0.050380	
real don	0.067113	0.068849	0.063761	0.053722	0.055306	0.054376	

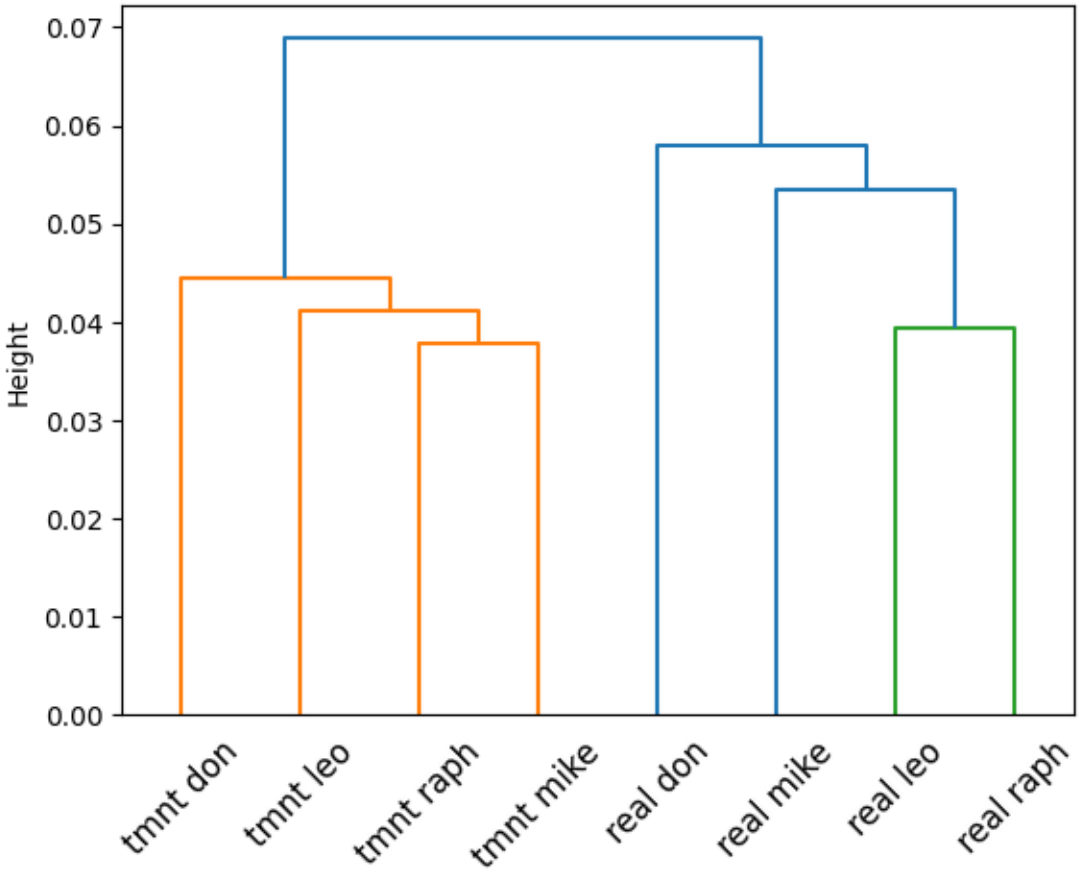
	real mike	real don
tmnt leo	0.064754	0.067113

```
tmnt raph   0.066762  0.068849
tmnt mike   0.047500  0.063761
tmnt don    0.061904  0.053722
real leo    0.053363  0.055306
real raph   0.050380  0.054376
real mike   0.000000  0.057994
real don    0.057994  0.000000
```

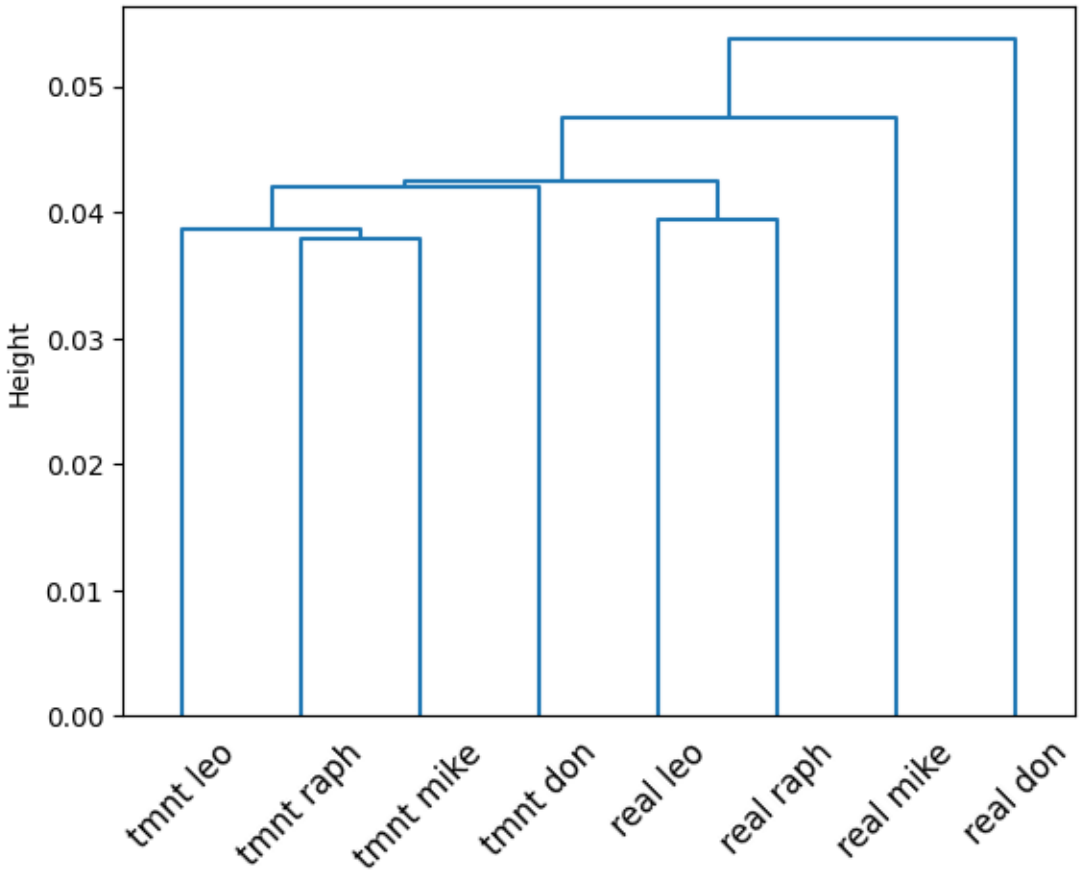
[ ]:
```
HClust = AgglomerativeClustering
hc_comp = HClust(distance_threshold=0,n_clusters=None ,linkage='complete')
hc_comp.fit(dtm3)
```

[ ]:
```
AgglomerativeClustering(distance_threshold=0, linkage='complete',
                        n_clusters=None)
```

[ ]:
```
plot_dendrogram(hc_comp, filename = 'tmnt_complete',labels = ["tmnt leo","tmnt␣
↪raph","tmnt mike","tmnt don","real leo","real raph","real mike","real don"])
```



[ ]:
```
hc_sing = HClust(distance_threshold=0,n_clusters=None ,linkage='single')
hc_sing.fit(dtm3)
plot_dendrogram(hc_sing, filename = "tmnt_single",labels = ["tmnt leo","tmnt␣
↪raph","tmnt mike","tmnt don","real leo","real raph","real mike","real don"])
```



1.4 Problem 1d

[ ]:
```
word_counts = problem_1_df.sum()
```

[ ]:
```
first_20 = pd.DataFrame(word_counts.sort_values(ascending=False),␣
↪columns=['count']).head(n=20)
first_20
```

[ ]:

	count
the	2664.0
and	1127.0
his	636.0
was	453.0
leonardo	342.0

```
that           297.0
for            282.0
with           279.0
michelangelo   277.0
raphael        212.0
from           209.0
this           189.0
which          129.0
turtles        127.0
donatello      117.0
him            116.0
series         115.0
were           111.0
who            110.0
one            103.0
```

[ ]:
```
first_20.sum()/word_counts.sum()
```

[ ]:
```
count    0.243425
dtype: float64
```

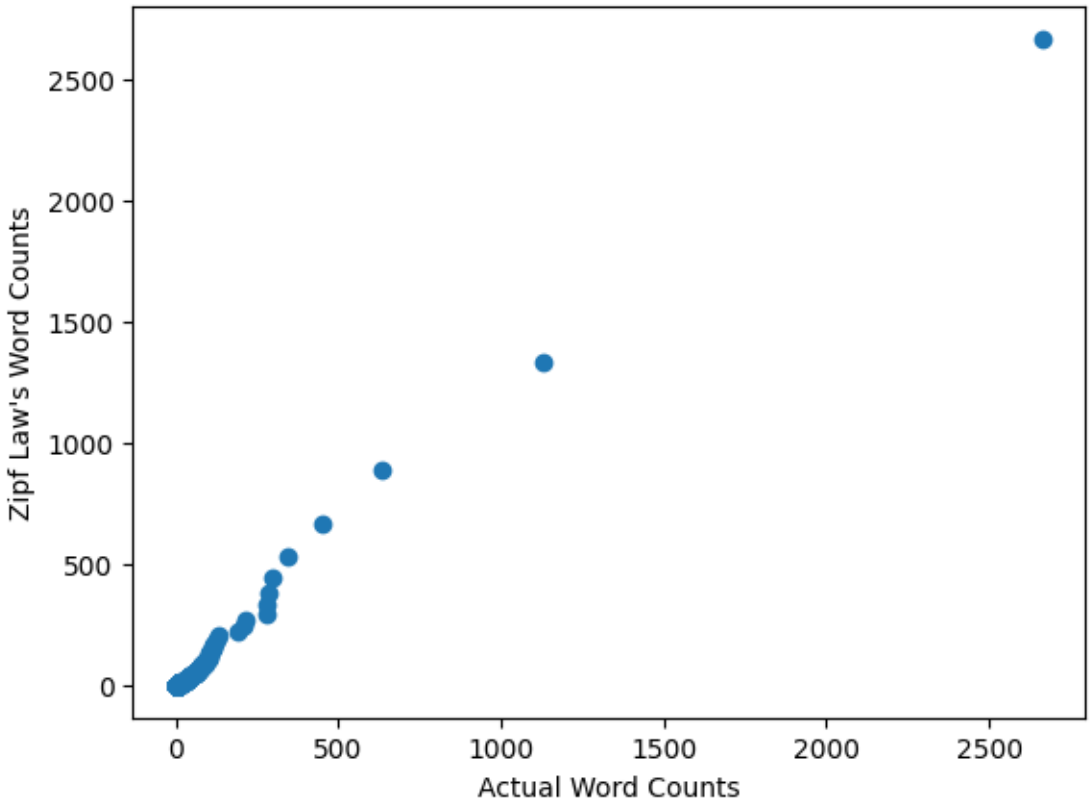
[ ]:
```
running_sum = 0
words = 0
for count in word_counts.sort_values(ascending=False):
    words+=1
    running_sum+=count
    if running_sum/word_counts.sum()>=0.5:
        print(words)
        print(running_sum)
        break
```

```
254
16231.0
```

1.5 Problem 1e

[ ]:
```
num_of_words = 6820
largest_count = 2664.0
zipf_law_values  = np.array([largest_count/(i+1) for i in range(num_of_words)])
actual_values = word_counts.sort_values(ascending=False).values
```

[ ]:
```
plt.scatter(actual_values,zipf_law_values)
plt.xlabel('Actual Word Counts')
plt.ylabel('Zipf Law\'s Word Counts')
plt.savefig('plots/zipf_law.png')
```



2 Problem 3

[ ]:
```
problem_3_data = pyreadr.read_r('data/hw1prob3.Rdata')
```

[ ]:
```
for k,v in problem_3_data.items():
    if k == 'x':
        x = pd.DataFrame(v)
    if k == 'd':
        distance = pd.DataFrame(v)
```

[ ]:
```
x.head()
```

[ ]:

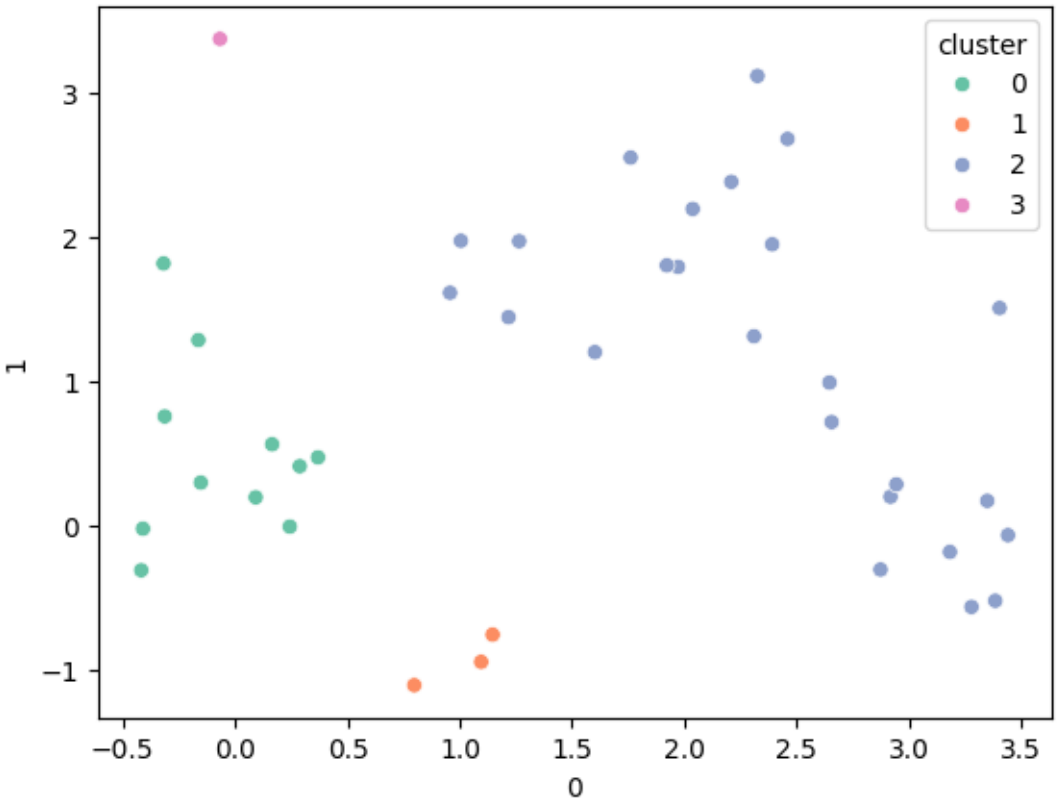
	0	1
0	-0.313227	0.755891
1	0.091822	0.194922
2	-0.417814	-0.310620
3	0.797640	-1.107350
4	0.164754	0.562465

[ ]:
```
distances = pd.read_csv('data/distances.csv')
distances = distances.drop(columns='Unnamed: 0', axis=1)
```

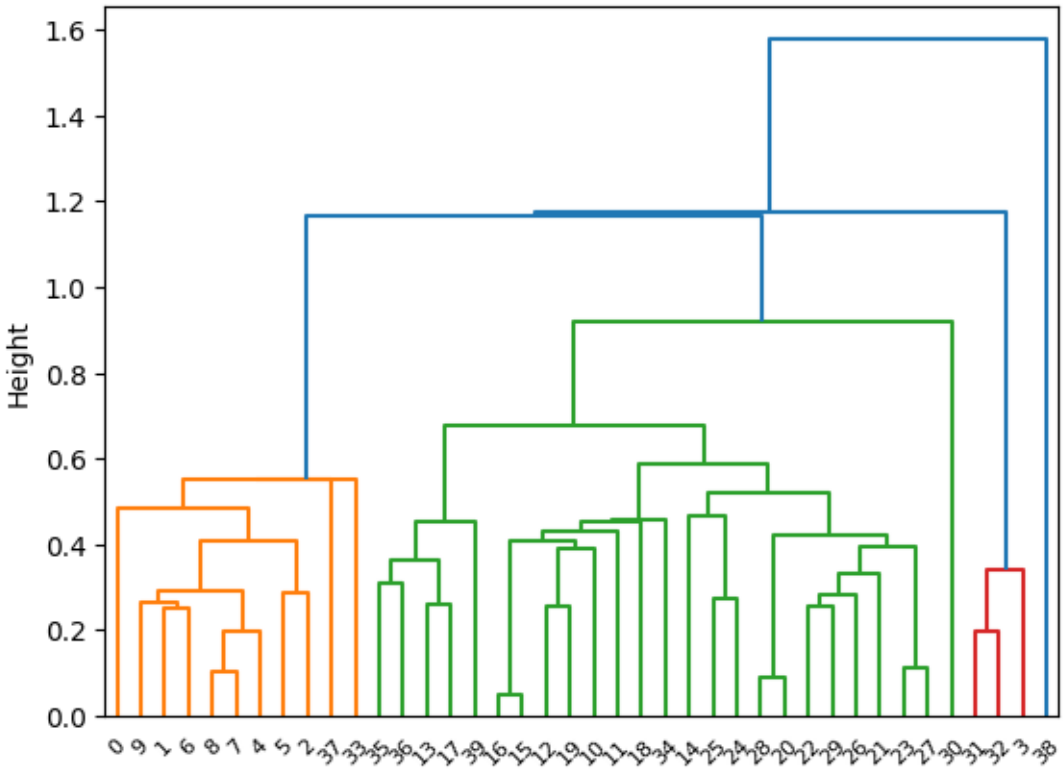
[ ]:
```
def h_cluster(data: pd.DataFrame,
              linkage: str,
              num_clusters: int,
              metric: str = 'euclidean',
              distances: pd.DataFrame = None,
              figure_name: str = 'plot')-> None:
    df = data.copy()
    hclust = HClust(distance_threshold=0,n_clusters=None,metric=metric,␣
 ↪linkage=linkage)
    if isinstance(distances, pd.DataFrame):
        hclust.fit(distances)
    else:
        hclust.fit(df)
    linkage_matrix = compute_linkage(hclust)
    data_labels = cut_tree(linkage_matrix,num_clusters)
    df['cluster'] = data_labels
    sns.scatterplot(x=df[0],y=df[1], hue=df['cluster'], palette = sns.
 ↪color_palette("Set2")[0:num_clusters])
    plt.savefig('plots/{}.png'.format(figure_name))

    return None
```

2.1 Problem 3a

[ ]:
```
h_cluster(data = x, linkage='single', num_clusters=4,␣
 ↪figure_name='single_linkage')
```

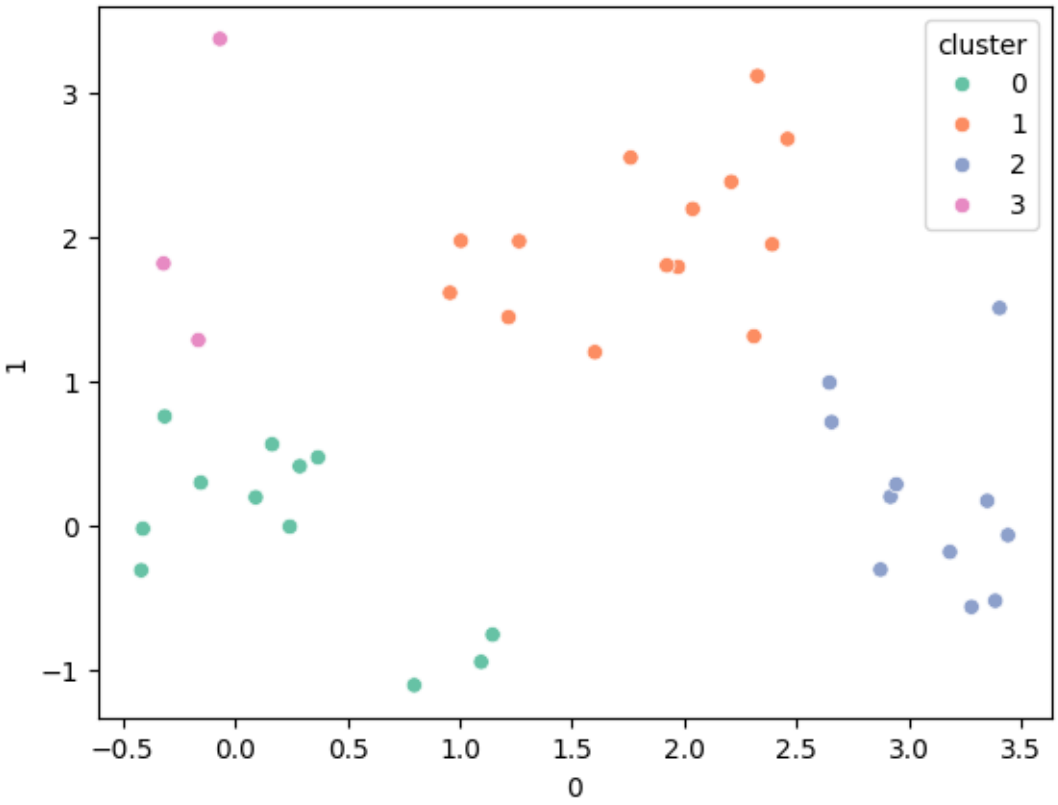


[ ]:
```
hc = HClust(distance_threshold=0,n_clusters=None ,linkage='single')
hc.fit(x)
plot_dendrogram(hc, filename="3a_single_dendrogram")
```

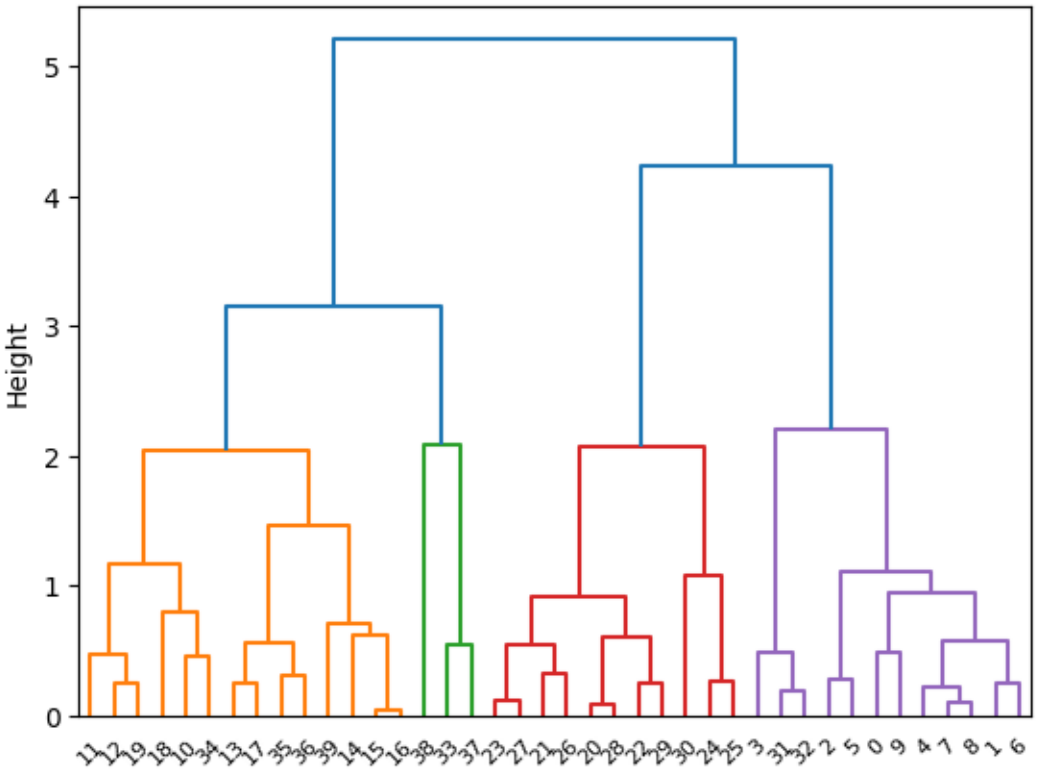


2.2 Problem 3b

[ ]:
```
h_cluster(data = x, linkage='complete', num_clusters=4,␣
↪figure_name='complete_linkage')
```

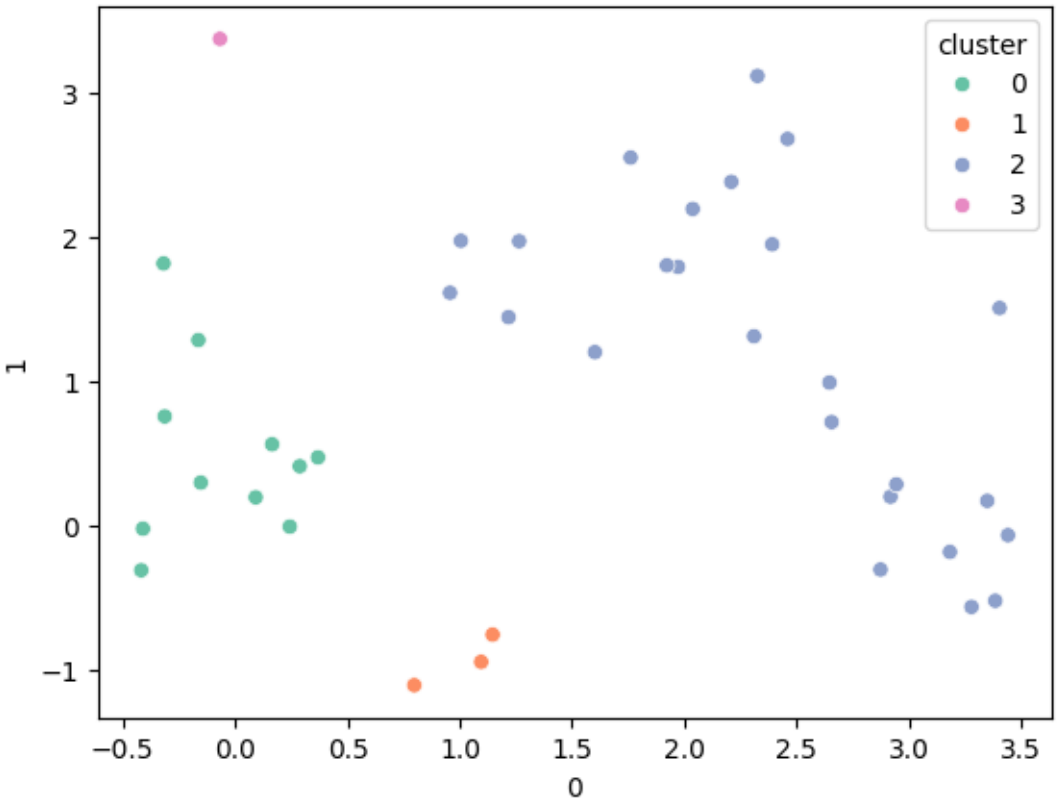


[ ]:
```
hc = HClust(distance_threshold=0,n_clusters=None ,linkage='complete')
hc.fit(x)
plot_dendrogram(hc, filename="3a_complete_dendrogram", color_threshold = 2.5)
```

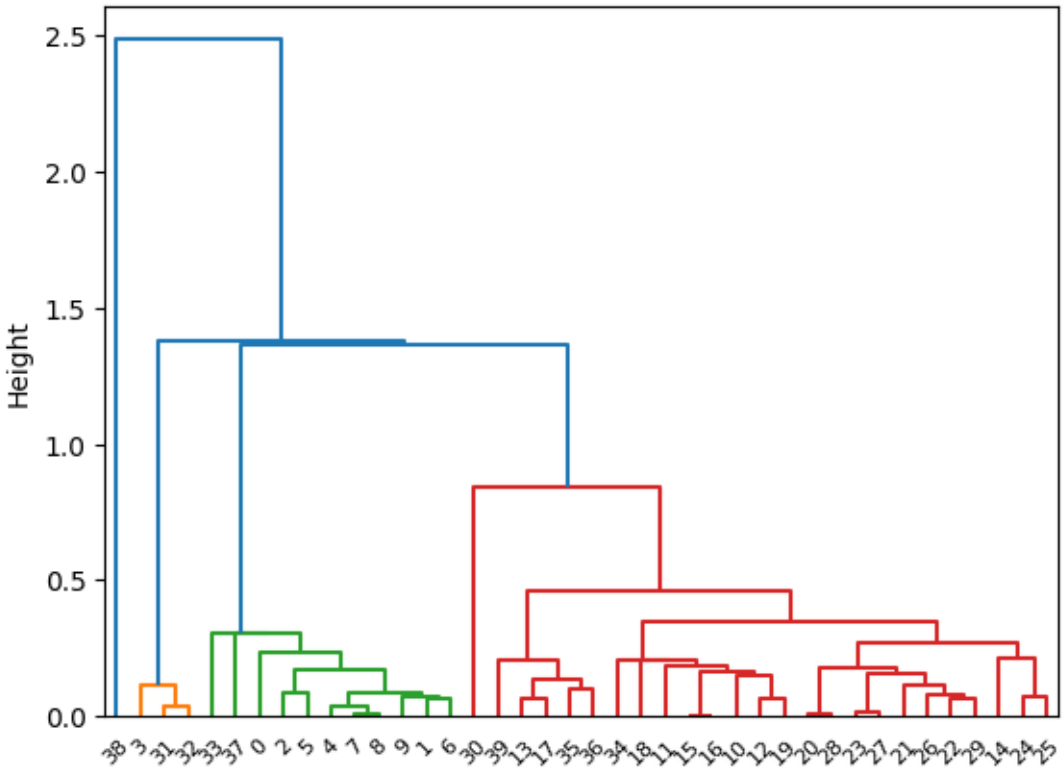


2.3 Problem 3c

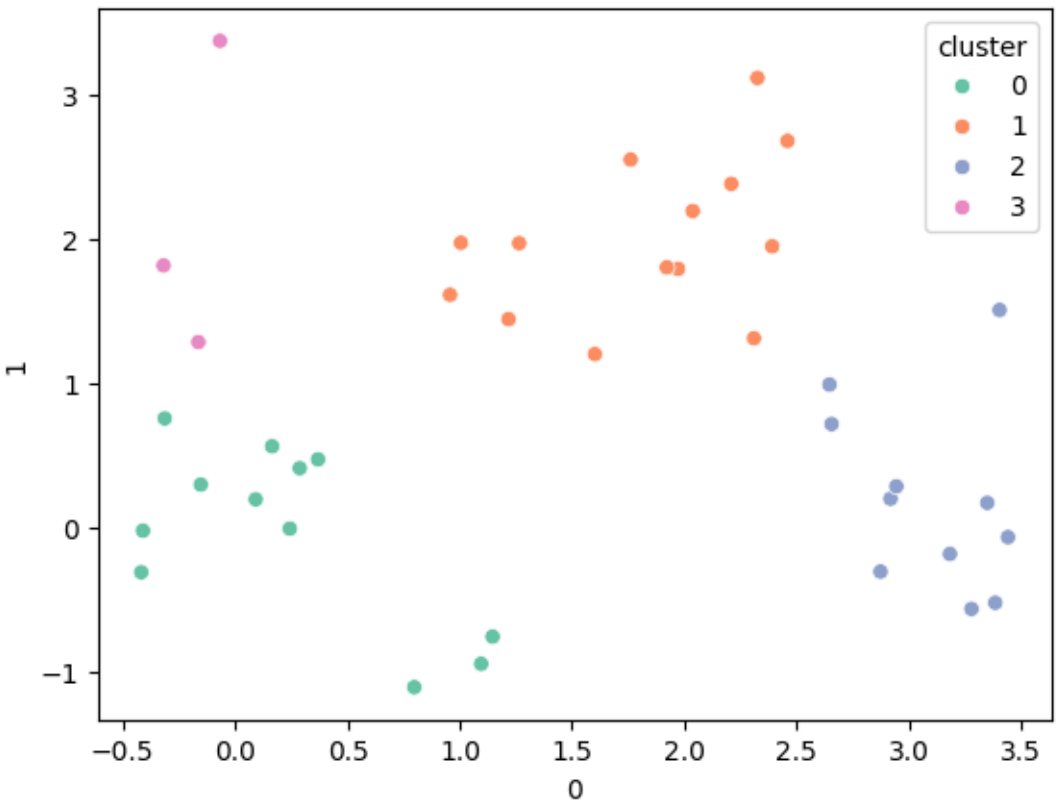
[ ]:
```
h_cluster(data = x, metric='precomputed', distances=distances**2,␣
↪linkage='single', num_clusters=4, figure_name='single_squared')
```



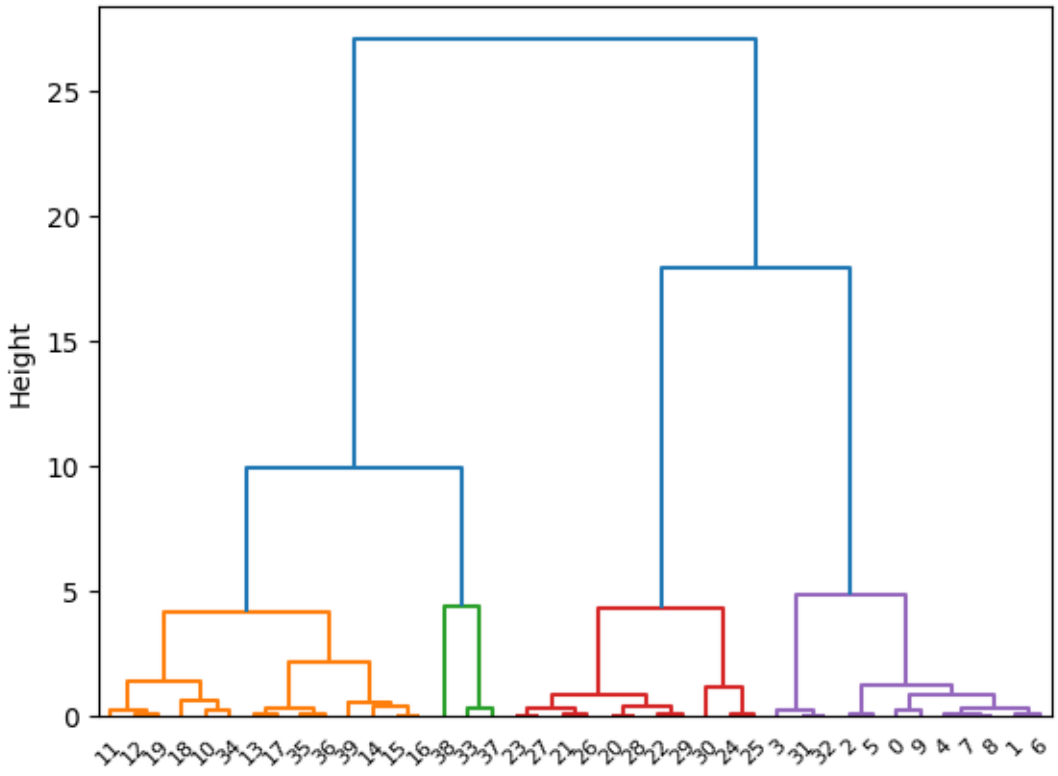
[ ]:
```
hc = HClust(distance_threshold=0,metric='precomputed',n_clusters=None␣
 ↪,linkage='single')
hc.fit(distances**2)
plot_dendrogram(hc, filename="3c_single_squared_dendrogram", color_threshold=1)
```



[ ]:
```
h_cluster(data = x, metric='precomputed', distances=distances**2,␣
↪linkage='complete', num_clusters=4, figure_name='complete_squared')
```

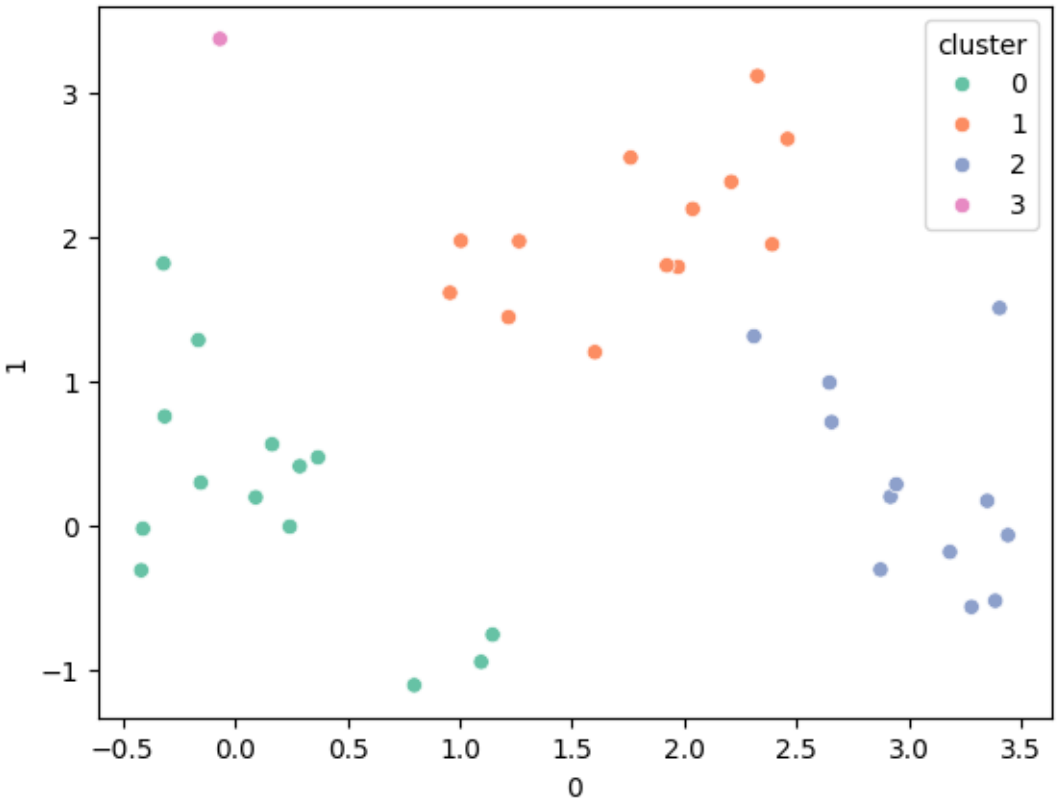


[ ]:
```
hc = HClust(distance_threshold=0,metric='precomputed',n_clusters=None␣
 ↪,linkage='complete')
hc.fit(distances**2)
plot_dendrogram(hc, filename="3c_complete_squared_dendrogram", color_threshold␣
 ↪= 7.5)
```

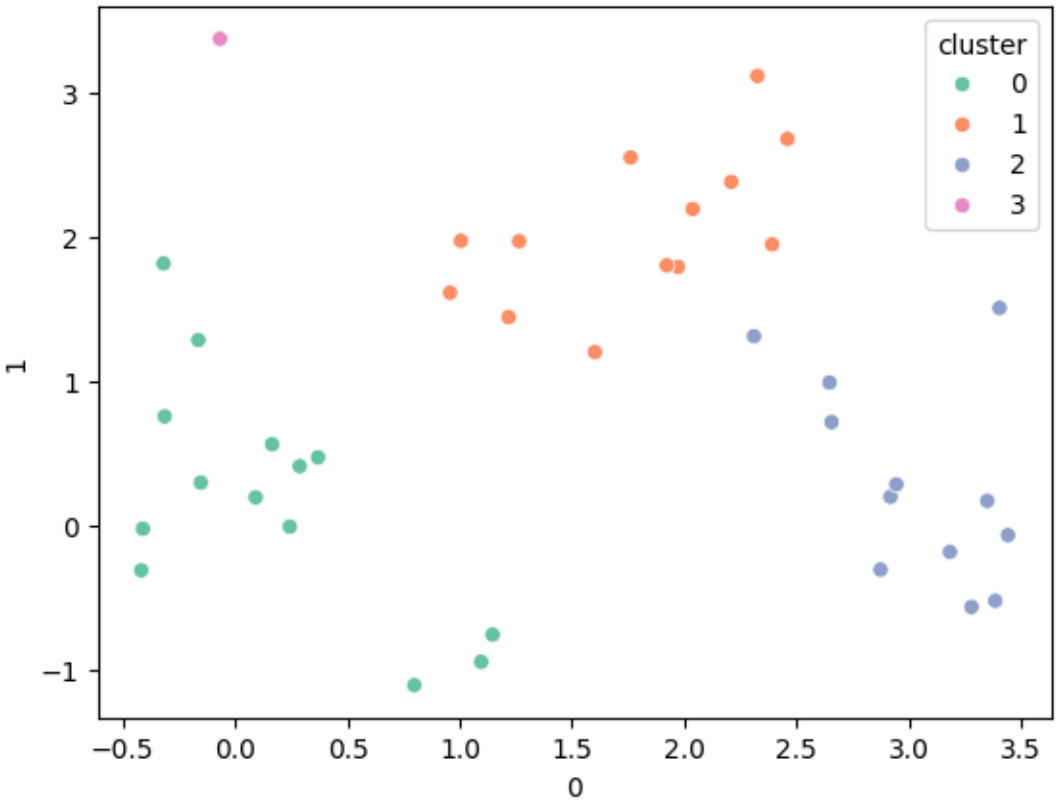


2.4 Problem 3e

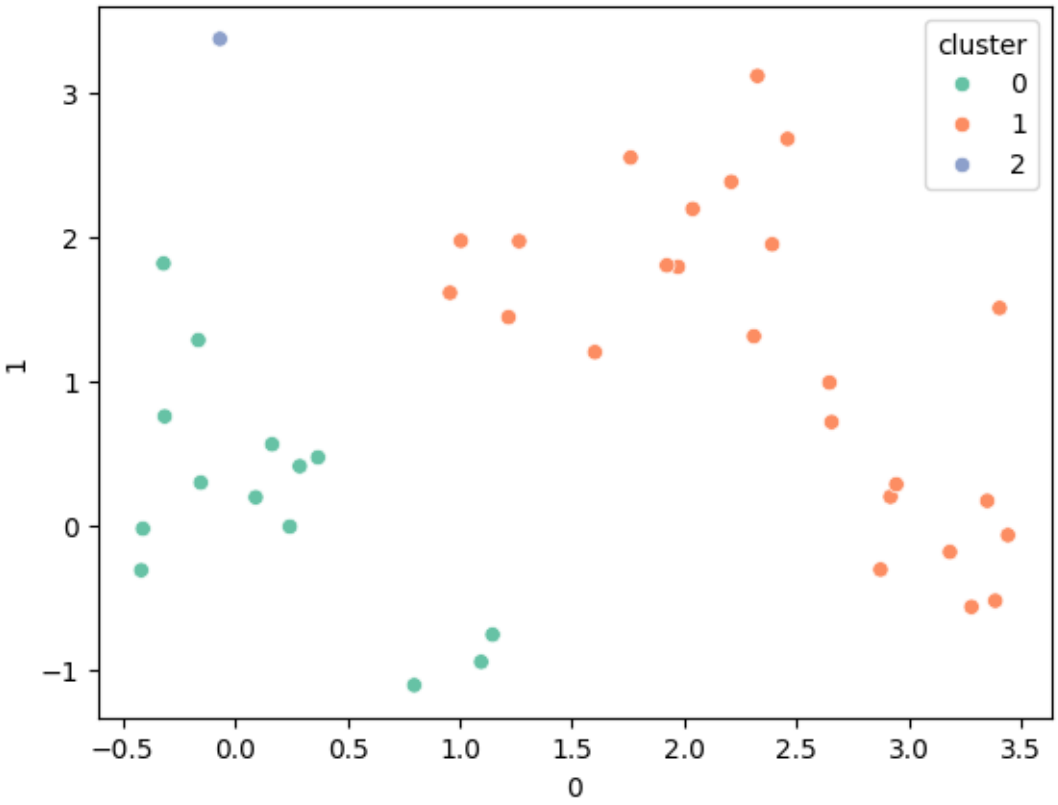
[ ]:
```
h_cluster(data = x, linkage='average', num_clusters=4,␣
↪figure_name='average_linkage_4')
```



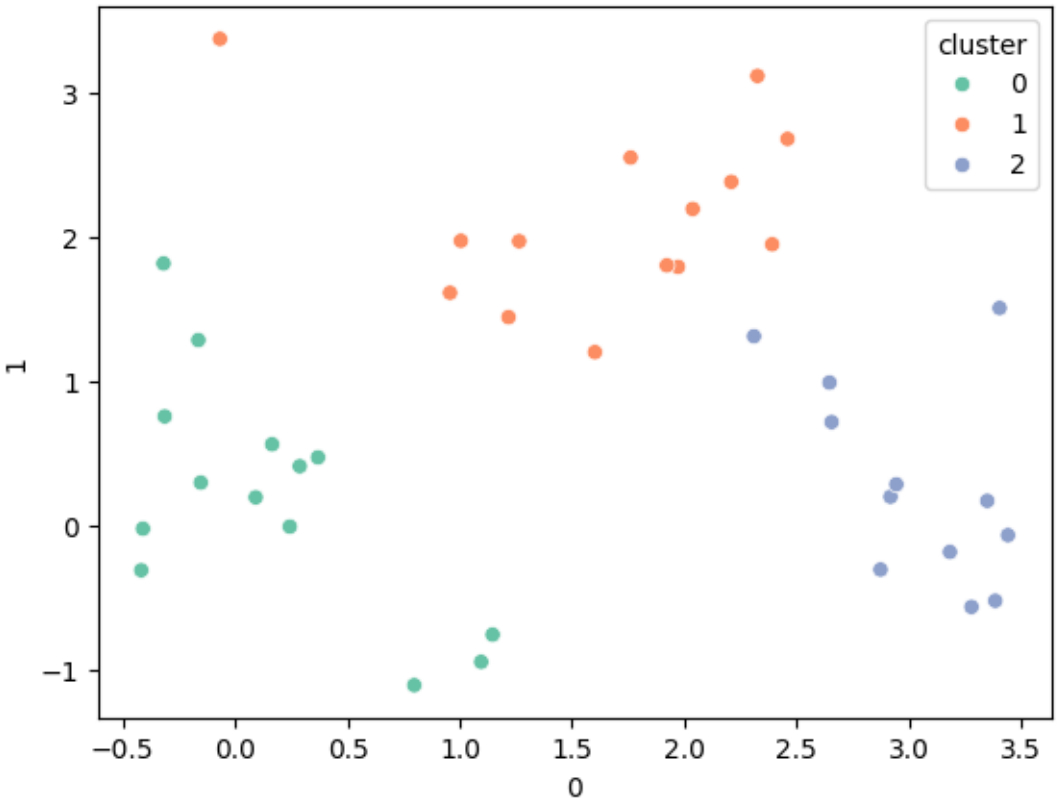
[ ]:
```
h_cluster(data = x, metric='precomputed', distances=distances**2,␣
↪linkage='average', num_clusters=4, figure_name='average_squared_4')
```



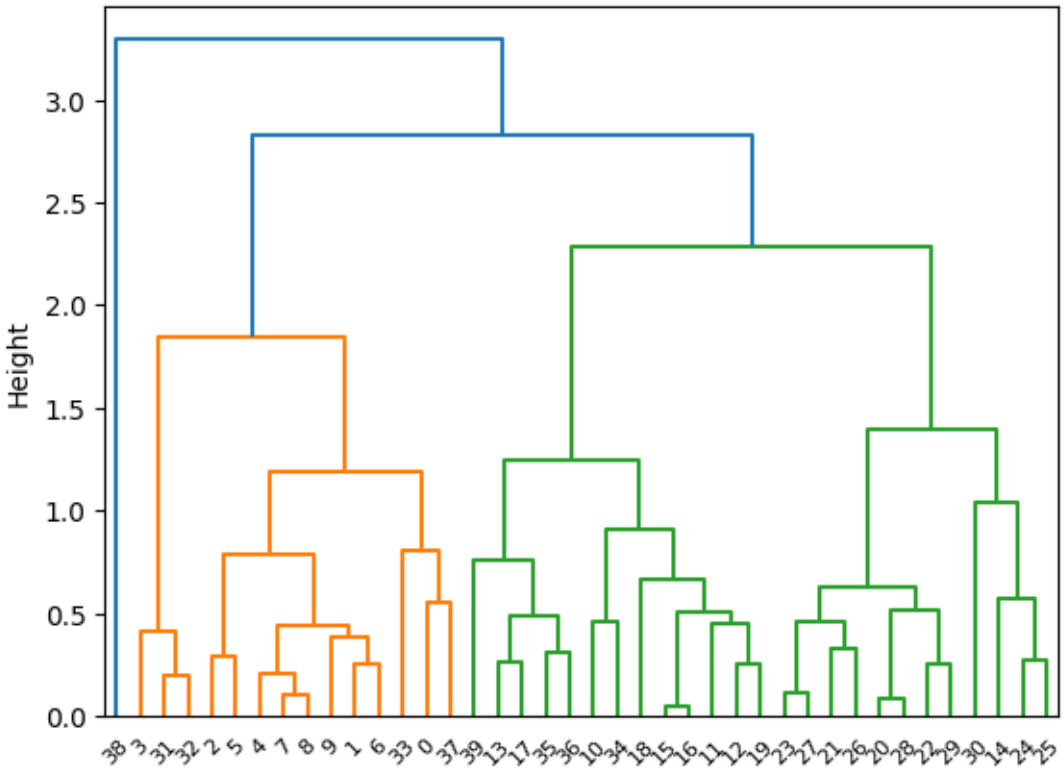
[ ]:
```
h_cluster(data = x, linkage='average', num_clusters=3,␣
↪figure_name='average_linkage_3')
```



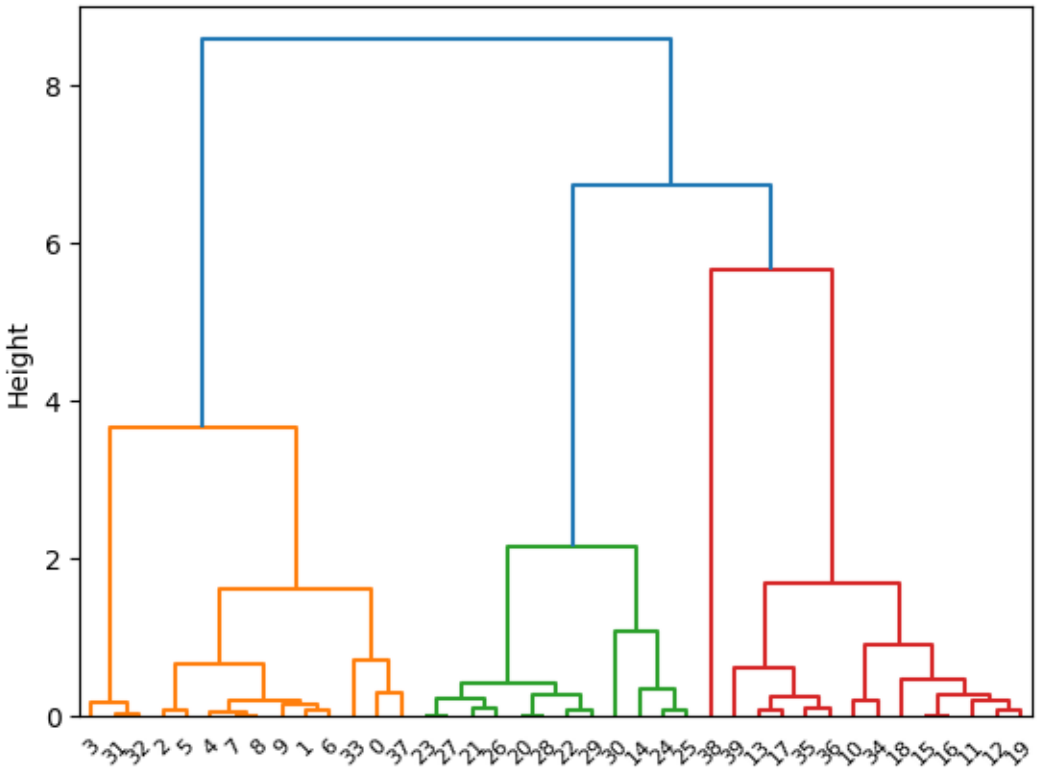
[ ]:
```
h_cluster(data = x, metric='precomputed', distances=distances**2,␣
↪linkage='average', num_clusters=3, figure_name='average_squared_3')
```



[ ]:
```
hc = HClust(distance_threshold=0,metric='precomputed',n_clusters=None␣
  ↪,linkage='average')
hc.fit(distances)
plot_dendrogram(hc, filename="3e_average_dendrogram")
```



[ ]:
```
hc = HClust(distance_threshold=0,metric='precomputed',n_clusters=None␣
 ↪,linkage='average')
hc.fit(distances**2)
plot_dendrogram(hc, filename="3e_average_squared_dendrogram")
```


Height
8
6
4
2
0
3 31 32 2 5 4 7 8 9 1 6 33 0 37 23 27 21 26 20 28 22 29 30 14 24 25 38 39 13 17 35 36 10 34 18 15 16 11 12 19