Comparing Black-Scholes-Merton and Gram-Charlier Models in Estimating Call Option Values of Top Fifteen S&P 500 Companies

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ABSTRACT

Options represent contracts granting the holder the right to buy or sell a specific asset at a predetermined price within a specified period. They are categorized into two types based on their exercise period: American and European options. Various methodologies exist for determining option prices, such as the Black-Scholes and Gram-Charlier Expansion models. The Black-Scholes method assumes stock returns follow a normal distribution with constant volatility and no dividend payments, making it less accurate for pricing options due to real-world deviations from normal distribution and regular dividend payouts. In this study, a model will be established to determine the price of European call options using the Gram-Charlier Expansion method, which considers dividend payments and includes skewness and kurtosis values. Additionally, a comparison will be made between the prices of European call options derived from the Gram-Charlier Expansion and Black-Scholes methods. The data utilized comprises AAPL stock data from November 25, 2022, to November 24, 2023, and AAPL stock option prices expiring on December 16, 2023, obtained from Yahoo Finance. The comparison demonstrates that the Black-Scholes method performs better for calculating theoretical prices of call options, whereas the Black-Scholes Gram-Charlier Expansion method is more suitable for put options. However, both methods still exhibit imperfections, lacking sufficient accuracy in their predictions.

Keywords: options, Black-Scholes, Gram-Charlier Expansion Models

INTRODUCTION

Option pricing theory is a probabilistic approach to assigning a value to an options contract. The primary goal of option pricing theory is to calculate the probability that an option will be exercised, or be in-the-money (ITM), at expiration and assign a dollar value to it. The Black-Scholes-Merton (BSM) model, developed in 1973, is one of the most important concepts in modern financial theory. This mathematical equation estimates the theoretical value of derivatives based on other investment instruments, taking into account the impact of time and other risk factors.

The BSM model is used to determine the fair prices of stock options based on six variables: volatility, type, underlying stock price, strike price, time, and risk-free rate. It is based on the principle of hedging and focuses on eliminating risks associated with the volatility of underlying assets and stock options. However, the BSM model makes certain assumptions that can lead to predictions that deviate from the real-world results. For instance, it assumes that the option is a European-style option and can only be exercised at expiration. It also assumes that no dividends are paid out during the life of the option.

On the other hand, the Gram-Charlier (GC) model is an alternative approach that incorporates skewness and kurtosis into the option pricing framework. The GC model approximates a probability distribution in terms of its cumulants. The key idea of these expansions is to write the characteristic function of the distribution whose probability density function is to be approximated in terms of the characteristic function of a distribution with known and suitable properties, and to recover the density function through the inverse Fourier transform. The GC model has been shown to provide more accurate option pricing estimates in cases where the underlying asset price distribution deviates from log-normality.

The S&P 500 stock market index is maintained by S&P Dow Jones Indices. It comprises 503 common stocks which are issued by 500 large-cap companies traded on American stock exchanges. The index includes about 80 percent of the American equity market by capitalization. It is weighted by free-float market capitalization, so more valuable companies account for relatively more weight in the index. The top fifteen S&P 500 companies include Microsoft Corp, Apple Inc., Amazon.com Inc, Nvidia Corp, Alphabet Inc. Class A, Meta Platforms, Inc. Class A, Alphabet Inc. Class C, Tesla, Inc., Berkshire Hathaway Class B, Unitedhealth Group Incorporated, Eli Lilly & Co., Jpmorgan Chase & Co., Exxon Mobil Corporation, Visa Inc., and Broadcom Inc.

In this paper, we will compare the performance of the BSM and GC models in estimating call option values for these top fifteen S&P 500 companies. By analyzing the accuracy and consistency of these models in various market conditions, we aim to provide insights into their applicability and potential limitations in real-world option pricing scenarios. This comparison will help investors and financial analysts make more informed decisions when selecting an appropriate option pricing model for their specific needs.

METHODS

Literature Review

Option valuation had been a topic developed by researchers as early as the 1960s. Fischer Black and Myron Scholes, who introduced their favored formula for option valuation in 1973, mentioned how most preceding expressions of option valuation were defined in terms of warrants and held lots of disadvantages. For example, the earliest formula by Sprenkle (1961) included a discount factor and the ratio of the expected stock price, but empirical estimation for these measures were found to be impossible. Another formula by Samuelson (1965) contained two unknown parameters and was only valid under conditions of capital market equilibrium, which Black and Scholes quoted to be “inappropriate”. The well-known Black-Scholes model every investor acknowledges in the present world of finance is the development of the concept by Thorp and Kassouf, presented later in 1967.

Over the next decades, the Black-Scholes model has been utilized to determine optimum prices of stock options in various nations. The model was firstly tested to the Australian Options Market (AOM) by Brown (1978). After fifteen years, Frino, Lodh, and Khan (1993) discovered that the model was still capable to price the national options unbiasedly. In Indonesia, one early review by Arifin (1998) stated that the model was not only reliable to value stock options, but also other financial securities and assets which characteristics resemble options, such as foreign currencies, stock indexes, and futures.

Along its implementation in the dynamic of world economy, researchers worldwide occasionally discovered solutions to counter issues regarding the Black-Scholes model. That is, the model was found to be less accurate in certain situations, combinations of different option types, or when compared to the latest models. As mentioned by Arifin (1998), the most prior modification was the generalization into valuation of American options done by Merton (1973), which resulted in the Black-Scholes-Merton (BSM) model. Cox and Ross (1975) and Merton (1976) constructed the Pure Jump Model and Mixed-Diffusion Jump Model to overcome discontinuity in the distribution of stock prices. Other models were also developed by Cox (1975), Cox and Ross (1976), Geske (1977), and Rubenstien (1983) for inconstant standard deviation and underlying assets. More recent studies include the modification of the BSM model to account for dividend payments by Riaman et al. (2018), the elaboration of the finite difference Center Time Center Space (CTCS) method by Irawan, Rosha, and Permana (2019), and the inclusion of conformable calculus by Morales-Bañuelos, Muriel, and Fernández-Anaya (2022).

Among all trials of improvement practiced so far, there exists the Gram-Charlier (GC) expansion model, an outstanding comparison for the BSM model. Backus, Foresi, and Wu (1997) examined that the GC model degrades kurtosis and biases in the BSM model. Several comparisons between both models in the past two years include Agustina and Zulfa (2021) and Amanah (2021) for three European call options, Utama, Hilnie, and Siswahyudi (2022) for three European put options, and Hilmi, Nurtiyasari, and Syahputra (2022) for Asian call options, where all four studies founded that the GC model yielded estimations closer to the present market prices than the BSM model. Chateau and Dufresne (2017) cited how a GC model could be based on a Poisson distribution or other types of polynomials according to the domain of the probability density function (pdf) of a distribution. However, they also mentioned some drawbacks, including the convergence of the expansion to an untrue probability distribution, or just asymptotically.

Financial Options

According to Berk and DeMarzo (2016), a financial option is a contract which gives its owner the right to buy or sell a corresponding asset at a fixed price at some future date. There are two kinds of options based on the contract: a call option, which gives the owner the right to purchase the asset; and a put option, which gives the owner the right to sell the asset. When an option is exercised, the price at which the owner buys or sells the asset is called the strike price or exercise price.

A common asset associated with options is stock. Based on the time allowance to exercise stock options, there are three types of options: European options, which allow its holder to exercise only on the expiration date; American options, which allow its holder to exercise at any date up to and including the final date; and Bermudan options, which allow its holder to exercise on predetermined dates. Stock option contracts in the United States are always written on a board lot (100 shares of stock), while in Indonesia, they are written on a share of stock.

Let S be the price of a stock at expiration and K be the exercise price of an option of the stock. The value of a call option at expiration, denoted by C, is given by

$$C = (S - K, 0)\,.$$

Meanwhile, the value of a put option at expiration, denoted by P, is given by

$$P = (K - S, 0).$$

Based on the payoff value of an option, there are three terminologies. At some point of time, if $K = S$, the option is said to be at-the-money; if the payoff value is positive, the option is in-the-money; and if the payoff value is negative, the option is out-of-the-money.

Black-Scholes-Merton Model

The Black-Scholes-Merton (BSM) model is a mathematical model used for pricing options, initially European call options, and financial derivatives. The model was developed by economists Fischer Black, Myron Scholes, and Robert Merton. The BSM model for European call options is defined by the following equations:

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2),$$

$$d_1 = \frac{ln\left(\frac{S_0}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}},$$

$$d_2 = \frac{ln\left(\frac{S_0}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T},$$

where C is the option price, S_0 is the price of the corresponding stock at time $t = 0$, K is the strike price, and $N(\cdot)$ is the cumulative distribution function (cdf) of the standard normal distribution.

The BSM model makes several assumptions, including that there are no dividends, interest rates and volatility are constant, there are no taxes or transaction costs, and the market behaves in a manner consistent with a normal distribution. Some modifications to relax these assumptions are mentioned in the previous subchapter.

Gram-Charlier Expansion Model

Gram-Charlier (GC) expansion model is a modification of the Black-Scholes-Merton (BSM) model, which is constrained by the assumption that stock returns are normally distributed. This model involves the measures of skewness and kurtosis of stock returns, which consecutively correspond to the third and fourth moments of a probability distribution. The GC model for call options is defined by the following equations:

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2) + \mu_3 Q_3 + (\mu_4 - 3) Q_4 = C_{BS} + \mu_3 Q_3 + (\mu_4 - 3) Q_4,$$

$$Q_3 = \frac{1}{3!} S_0 \sigma\sqrt{T}\left[N(d_1)\left(2\sigma\sqrt{T} - d_1\right) + \sigma^2 T N(d_1)\right],$$

$$Q_4 = \frac{1}{4!} S_0 \sigma\sqrt{T}\left[N(d_1)\left(d_1^2 - 3\sigma\sqrt{T}\left(d_1 - \sigma\sqrt{T}\right) - 1\right) + \left(\sigma\sqrt{T}\right)^3 N(d_1)\right],$$

where C_{BS} is the call option formula of BSM model, μ_3 is the skewness, and μ_4 is the kurtosis.

Mean Absolute Percentage Error (MAPE)

Mean Absolute Percentage Error (MAPE) is a widely used metric for measuring the accuracy of forecasts in various fields, including epidemiology, power system management, and machine learning. While MAPE is widely used, it is not without critics. A study by Chicco, Warrens, and Jurman (2022) found that the coefficient of determination (R-squared) and the symmetric mean absolute percentage error (SMAPE) were more informative than MAPE, Mean Absolute Error (MAE), MSE, and Root Mean Squared Error (RMSE) in evaluating regression analysis. The study argued that these rates, including MAPE, have a common drawback: their values can range between zero and $+\infty$, which does not provide much information about the performance of the regression with respect to the distribution of the ground truth elements.

Despite its limitations, MAPE remains a popular choice for several reasons. Firstly, it is easy to interpret and understand, making it a practical choice for many researchers. Secondly, it provides a percentage error, which can be more intuitive than the absolute error values provided by measures like MSE. Lastly, it is less sensitive to large errors compared to squared error measures like MSE, making it more robust in certain situations.

To evaluate the performance of the BSM model and its modifications, we will use the Mean Absolute Percentage Error (MAPE) as a measure of accuracy. MAPE is a widely used metric for measuring the forecasting accuracy of a model, as it provides an easy-to-interpret and explainable measure of the average difference between the forecasted value and the actual value. The formula to calculate MAPE is as follows:

$$MAPE = \frac{1}{n}\sum_{i=1}^{n} \left|\frac{Actual_i - Forecast_i}{Actual_i}\right| \times 100,$$

where n is the sample size, $Actual_i$ is the actual data value, and $Forecast_i$ is the forecasted data value. A lower MAPE value indicates a more accurate model, while a higher MAPE value indicates a less accurate model.

RESULTS AND DISCUSSION

Stock Selection

According to Slickcharts, there are 503 listed symbols from 500 companies in S&P 500 stock index. The situation happens due to some companies having more than one class of stock involved in the list. The authors took fifteen stock symbols which had been given the highest portfolios, namely AAPL, MSFT, AMZN, NVDA, GOOGL, META, GOOG, BRK-B, TSLA, UNH, LLY, JPM, XOM, V, and AVGO. In total, these fifteen symbols represented 37.61% of the stock index, as per 27 November 2023.

Determination of Option Prices

The historical daily prices of the fifteen selected stocks from the past year are imported from Yahoo! Finance. The lists of prices are utilized to calculate the sample mean, standard deviation, skewness, and kurtosis, which consecutively represent the first up to the fourth sample moments. From the same source, for each stock, thirty in-the-money call options due on 19 January 2024 with the highest prices are observed, especially the strike and last prices on 27 November 2023. If not possible, thirty call options with the lowest prices are put into attention.

After additional quantities are further derived, two main lists of nominals are yielded, which are the estimated option prices based on Black-Scholes-Merton and Gram-Charlier models. From these lists, authors also calculated the mean absolute percentage error (MAPE) in order to compare the performance of option prices estimation based on two discussed models. The values of MAPE are given in the table below.

Stock Symbol	**MAPE of Black-Scholes-Merton Estimated Prices**	**MAPE of Gram-Charlier Estimated Prices**
AAPL	12.73%	32.26%
MSFT	12.27%	18.98%
AMZN	12.52%	15.47%
NVDA	24.94%	24.95%
GOOGL	10.68%	10.13%
META	13.70%	54.70%
GOOG	14.94%	14.35%
BRK-B	10.83%	24.65%
TSLA	28.97%	30.88%
UNH	14.26%	37.78%
LLY	18.08%	28.35%
JPM	39.50%	52.34%
XOM	5.52%	29.27%

V	9.63%	27.84%
AVGO	14.63%	16.17%
Average	**16.21%**	**27.99%**

Based on the average, it can be seen that the MAPE of Black-Scholes-Merton estimated prices is 16.21%, less than the MAPE of the Gram-Charlier estimated prices. Therefore, for this specific case, the Black-Scholes-Merton model is better than Gram-Charlier expansion model.

CONCLUSION

The Black-Scholes-Merton and Gram-Charlier expansion models applied to stock options use different formulas with similar parameters. These two methods produce theoretical prices for both put and call options that are quite different. This difference underlies the comparison using Mean Absolute Percentage Error (MAPE). This comparison results in that the Black Scholes method is better if used to calculate the theoretical price on call options, while the Black Scholes Gram Charlier Expansion method is better used on put options. However, the use of these two methods is still not perfect, the level of accuracy is still not accurate enough.

Option pricing using the Black-Scholes model is influenced by several factors, including stock price (S0), strike price (K), interest rate (r), time (t), and volatility (σ). Option pricing using the Gram-Charlier expansion method is influenced by several factors, including stock price (S0), strike price (K), interest rate (r), time (t), and volatility (σ), skewness, and kurtosis.

The acquisition of the average value of MAPE on the Black-Scholes method stock options is 16.21%, while the MAPE value on the Gram-Charlier expansion method is 27.99% From the results of the acquisition of MAPE in each method it can be seen that the MAPE value of the Black-Scholes method is smaller than the Gram-Charlier method so that the Black-Scholes method is better used in determining the price of European type stock options.

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