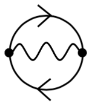
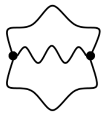
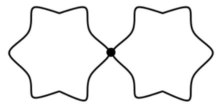
Finite Temperature Field Theory №3

Bryan Compiled @ 2020/06/04

QCD Partition Function at $\mathcal{O}(g^2)$

$\ln \mathcal{Z}_I^{(2)} = -\frac{1}{2}$  $-\frac{1}{2}$  $+\frac{1}{12}$  $+\frac{1}{8}$ 

(a) *(b)* *(c)* *(d)*

$$\ln \mathcal{Z}_I^{(2)} = \ln \mathcal{Z}^{(a)} + \ln \mathcal{Z}^{(b)} + \ln \mathcal{Z}^{(c)} + \ln \mathcal{Z}^{(d)} \tag{1}$$

The contribution of (a) is given by:

$$\ln \mathcal{Z}^{(a)} = \frac{1}{2!}\,(-1)^1 \frac{T}{V}\sum_k \frac{T}{V}\sum_p \mathrm{Tr}\left(S(k)\,(g\gamma^\nu T^b)\,S(p)\,(g\gamma^\mu T^a)\right)\left(\frac{V}{T}\right)\delta_{ab}\,\Delta_{\mu\nu}(p-k) \tag{2}$$

Here the trace goes over spinor, color and flavor indices. $S(k)$ is the quark propagator, with suppressed spinor, color and flavor indices, while $\delta_{ab}\,\Delta_{\mu\nu}$ is the gluon propagator, where a, b are adjoint indices; each vertex contributes a $(g\gamma^\mu T^a)$ factor.

In our convention, $k = (\omega_n, \mathbf{k})$ stands for the Euclidean 4-momentum with ω_n: the discrete Matsubara frequency. Each $\sum_k$ comes with a factor $\frac{T}{V}$ while each spacetime delta function comes with an inverse factor: $\frac{V}{T}$; this is due to the fact that:

$$1 = \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\,(2\pi)^4 \delta^4(k-k_0) \sim \frac{1}{\beta V}\sum_k \beta V \delta_{k,k_0} \tag{3}$$

Following the same recipe from QED, we can write down:

$$\begin{aligned}\ln \mathcal{Z}^{(a)} &= -\left(\mathrm{Tr}\left(T^a T^b\right)\delta_{ab}\right)\frac{1}{2}\,g^2 \frac{V}{T}\cdot\frac{T}{V}\sum_k \frac{T}{V}\sum_p \mathrm{Tr}\left(S(k)\,\gamma^\nu\,S(p)\,\gamma^\mu\right)\Delta_{\mu\nu}(p-k)\\ &= -\left(\frac{N_c^2-1}{2}\,N_f\right)\frac{g^2}{288}\frac{V}{T}\left(5T^4 + \frac{18}{\pi^2}T^2\mu^2 + \frac{9}{\pi^4}\mu^4\right)\end{aligned} \tag{4}$$

The (b) term is structually similar to the (a) term; now the amplitude can be written down simply by replacing the propagator $S(k) \mapsto W(k)$ of the ghost, while the vertex is $(g\gamma^\mu T^a) \mapsto (-igk^\nu T^b)$ instead:

$$\ln \mathcal{Z}^{(b)} = \frac{1}{2!}\,(-1)^1 \frac{T}{V}\sum_k \frac{T}{V}\sum_p \mathrm{Tr}\left(W(k)\,(-igp^\nu T^b)\,W(p)\,(-igk^\mu T^a)\right)\left(\frac{V}{T}\right)\delta_{ab}\,\Delta_{\mu\nu}(p-k) \tag{5}$$

The trace now goes over suppressed *adjoint* indices of $W(k)_{ab} = -\delta_{ab}\Delta(k)$ and $(T_a)_{bc} = f_{abc}$, where f_{abc} is the structure constant of $\mathrm{SU}(N_c)$. Therefore[1],

$$\begin{aligned}\ln \mathcal{Z}^{(b)} &= -\left(\mathrm{Tr}\left(T^a T^b\right)\delta_{ab}\right)\frac{1}{2}\,g^2 \frac{V}{T}\cdot\frac{T}{V}\sum_k \frac{T}{V}\sum_p \Delta(k)\,\Delta(p)\,(-k^\mu p^\nu)\,\Delta_{\mu\nu}(p-k)\\ &= -\left(\frac{N_c^2-1}{2}\,2N_c\right)\frac{1}{2}\,g^2 \frac{V}{T}\cdot\frac{T}{V}\sum_k \frac{T}{V}\sum_p \Delta(k)\,\Delta(p)\,(-k^\mu p^\nu)\,(-g_{\mu\nu})\,\Delta(p-k)\end{aligned} \tag{6}$$

[1] $\mathrm{Tr}\left(T^a T^b\right)_{\mathrm{ad}}$ in the adjoint representation is precisely the *Killing form* of the $\mathfrak{su}(N_c)$ algebra, which is $2N_c$ times the $\mathrm{Tr}\left(T^a T^b\right)_0$ in the fundamental representation; see `Wikipedia:` *Killing form*.

Now we compute the remaining $\sum_{k,p}$. We have:

$$\Delta(k)\,\Delta(p)\,(k\cdot p)\,\Delta(p-k)=\frac{k\cdot p}{k^2p^2\,(p-k)^2} \tag{7}$$

The generic method to carry out such summation is by using the mixed representation of the propagator; for some propagator $D(k)$, we have:

$$\begin{aligned} D(k)=D(w_n,\mathbf{k}) &= \int_0^\beta \mathrm{d}\tau\, e^{-i\omega_n\tau}\, T\sum_m e^{i\omega_m\tau} D(w_n,\mathbf{k}) \\ &= \int_0^\beta \mathrm{d}\tau\, e^{-i\omega_n\tau}\, \tilde{D}(\tau,\mathbf{k}), \end{aligned} \tag{8}$$

$$\begin{aligned} \tilde{D}(\tau,\mathbf{k}) &= T\sum_m e^{i\omega_m\tau} D(w_n,\mathbf{k}) \\ &= T\sum_m e^{i\omega_m\tau}\int\frac{\mathrm{d}\omega}{2\pi}\frac{\rho(\omega,\mathbf{k})}{\omega+i\omega_0} \\ &= \int\frac{\mathrm{d}\omega}{2\pi}\,\rho(\omega,\mathbf{k})\,T\sum_m\frac{e^{i\omega_m\tau}}{\omega+i\omega_0} \\ &= \int\frac{\mathrm{d}\omega}{2\pi}\,\rho(\omega,\mathbf{k})\,e^{-\omega\tau}\big(1\pm n_\pm(\omega)\big), \end{aligned} \tag{9}$$

$$\rho(\omega,\mathbf{k})=\frac{1}{i}\big(D(\omega+i\epsilon)-D(\omega-i\epsilon)\big)=2\,\mathrm{Im}\,D(\omega+i\epsilon,\mathbf{k}),\qquad n_\pm=\frac{1}{e^{\beta\omega}\mp 1}, \tag{10}$$

Then the Matsubara sum $\sum_{\omega_n}$ becomes a sum over exponentials like $e^{-i\omega_n\tau}$, which is easier to deal with. However, for this particular problem, there is a shortcut[2]; notice that the denominator of (7) is invariant under $p\mapsto k-p$, hence:

$$\begin{aligned} \sum_p\frac{k\cdot p}{k^2p^2\,(p-k)^2} &= \sum_{(k-p)}\frac{k\cdot(k-p)}{k^2\,(k-p)^2p^2} \\ &= \sum_p\frac{k\cdot(k-p)}{k^2p^2\,(p-k)^2} \\ &= \sum_p\frac{\frac{1}{2}\big(k\cdot p+k\cdot(k-p)\big)}{k^2p^2\,(p-k)^2} \\ &= \sum_p\frac{1}{2p^2\,(p-k)^2}, \end{aligned} \tag{11}$$

$$\begin{aligned} \frac{T}{V}\sum_k\frac{T}{V}\sum_p\Delta(k)\,\Delta(p)\,(k\cdot p)\,\Delta(p-k) &= \frac{1}{2}\,\frac{T}{V}\sum_p\frac{1}{p^2}\,\frac{T}{V}\sum_k\frac{1}{(p-k)^2} \\ &= \frac{1}{2}\left(\frac{T}{V}\sum_p\frac{1}{p^2}\right)^2=\frac{1}{2}\left(\frac{T^2}{12}\right)^2, \end{aligned} \tag{12}$$

[2]Reference: Laine & Vuorinen, *Basics of Thermal Field Theory*.

$$\boxed{\ln \mathcal{Z}^{(b)} = -\left(\frac{N_c^2-1}{2}\,2N_c\right)\frac{1}{2}\,g^2\frac{V}{T}\cdot\frac{1}{2}\left(\frac{T^2}{12}\right)^2 = -\frac{V}{T}\,N_c\big(N_c^2-1\big)\,\frac{1}{4}\,g^2\,\frac{T^4}{144}} \tag{13}$$

The (c) term is structually similar to the (b) term, but with a symmetrized 3-gluon vertex:

$$\left(\frac{1}{3!}\right) igf_{abc}\left(g_{\mu\nu}(k-p)_\rho + g_{\nu\rho}(p-q)_\mu + g_{\rho\mu}(q-k)_\nu\right) = \left(\frac{1}{3!}\right) igf_{abc}\,D_{\mu\nu\rho}(k,p,q) \tag{14}$$

To link the legs of two 3-gluon vertices as shown in (c), there are 3! possibilities. Therefore, we have:

$$\ln \mathcal{Z}^{(c)} = \frac{1}{2!}\cdot 3!\cdot\left(\frac{1}{3!}\right)^2 \frac{T}{V}\sum_k \frac{T}{V}\sum_p \operatorname{Tr}\Big(W(k)\left(igf_{abc}D_{\mu\nu\rho}(k,-p,p-k)\right)W(k)\left(igf^{bac}D^{\mu\nu\rho}(p,-k,k-p)\right)\Big)\left(\frac{V}{T}\right)\delta_{ab}\,\Delta_{\mu} \tag{15}$$

PAST WORK, AS TEMPLATE

With the metric convention: $g \sim (-+++)$, we have:

$$\mathcal{Z} = \int \mathcal{D}A^\mu \, e^{-S} \, \delta\big[\partial_\mu A^\mu - f\big] \det\big[\partial^2 \delta^4(x-y)\big] \tag{16}$$

Here S is the Euclidean action:

$$(-S) = \int \mathrm{d}^4 x \, \mathcal{L}_{t=-i\tau}, \qquad \int \mathrm{d}^4 x = \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3\mathbf{x} \tag{17}$$

For pure QED, we have:

$$\mathcal{L}_t = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} \tag{18}$$

Setting $t = i\tau$ is equivalent to carrying out a Wick rotation: $x^0 \mapsto -ix^0$, $A^0 \mapsto -iA^0$, while:

$$g_{\mu\nu}A^\mu A^\nu = g'_{\mu\nu}A'^\mu A'^\nu \quad \Longrightarrow \quad g_{\mu\nu} \longmapsto g'_{\mu\nu} = \delta_{\mu\nu} \tag{19}$$

Under this convention, the Euclidean action is formally unchanged; same applies for the gauge-fixing and the ghost term:

$$\mathcal{L}_{t=-i\tau} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu}, \quad g_{\mu\nu} \longmapsto \delta_{\mu\nu}, \tag{20}$$

$$\delta\big[\partial_\mu A^\mu - f\big] \quad \Longrightarrow \quad \mathcal{L}_{gf} = -\frac{1}{2\rho}\,(\partial_\mu A^\mu)^2, \tag{21}$$

$$\det\big[\partial^2\delta^4(x-y)\big] \quad \Longrightarrow \quad \mathcal{L}_{gh} \sim (\partial^2\bar\eta)\,\eta \sim -\partial_\mu\bar\eta\,\partial^\mu\eta, \tag{22}$$

Here we've dropped some total derivative terms in the ghost Lagrangian. The partition function is then reduced to:

$$\mathcal{Z} = \int \mathcal{D}A^\mu \, \mathcal{D}\bar\eta \, \mathcal{D}\eta \, e^{-S'}, \quad (-S') = \int \mathrm{d}^4x \left(\mathcal{L} + \mathcal{L}_{gf} + \mathcal{L}_{gh}\right)_{t=-i\tau} \tag{23}$$

The action can be further simplified by partial integration and dropping boundary terms:

$$\begin{aligned}
(-S') &= \int \mathrm{d}^4x \left(-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\rho}\,(\partial_\mu A^\mu)^2 - \partial_\mu\bar\eta\,\partial^\mu\eta\right) \\
&= \int \mathrm{d}^4x \left(-\frac{1}{4}\,(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{1}{2\rho}\,(\partial_\mu A^\mu)^2 - \partial_\mu\bar\eta\,\partial^\mu\eta\right) \\
&= \int \mathrm{d}^4x \left(-\frac{1}{2}\,(\partial_\mu A_\nu\,\partial^\mu A^\nu - \partial_\nu A_\mu\,\partial^\mu A^\nu) - \frac{1}{2\rho}\,(\partial_\mu A^\mu\,\partial_\nu A^\nu) - \partial_\mu\bar\eta\,\partial^\mu\eta\right) \\
&\sim \int \mathrm{d}^4x \left(-\frac{1}{2}\,\big(-A_\nu\,\partial^2 A^\nu + A_\mu\,\partial^\mu\partial_\nu A^\nu\big) + \frac{1}{2\rho}\,(A^\mu\,\partial_\mu\partial_\nu A^\nu) + \bar\eta\,\partial^2\eta\right) \\
&= \int \mathrm{d}^4x \left(-\frac{1}{2}\,A^\mu\Big(-\delta_{\mu\nu}\partial^2 + \partial_\mu\partial_\nu - \tfrac{1}{\rho}\,\partial_\mu\partial_\nu\Big)A^\nu + \bar\eta\,\partial^2\eta\right) \\
&= -\frac{1}{2}\int \mathrm{d}^4x \left(A^\mu\Big(-\delta_{\mu\nu}\partial^2 + \big(1-\tfrac{1}{\rho}\big)\,\partial_\mu\partial_\nu\Big)A^\nu - 2\bar\eta\,\partial^2\eta\right)
\end{aligned} \tag{24}$$

With $\beta = \frac{1}{T}$, expand A^μ, η into dimensionless Fourier modes, and we have:

$$A^\mu = \frac{1}{\sqrt{TV}} \sum_k e^{ik_\nu x^\nu} A^\mu_k, \qquad \sum_k e^{ik_\nu x^\nu} = V \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\, e^{i\mathbf{k}\cdot\mathbf{x}} \sum_{n\in\mathbb{Z}} e^{i\omega_n\tau} \tag{25}$$

$$
\begin{aligned}
\sum_{p,k}\int \mathrm{d}^4x\, e^{i\,(p+k)\cdot x} &= \sum_{p,k}(2\pi)^4\,\delta^4(p+k) \\
&= V^2\int \frac{\mathrm{d}^3\mathbf{p}\,\mathrm{d}^3\mathbf{k}}{(2\pi)^6}\sum_{m,n\in\mathbb{Z}}\beta\,\delta_{m,-n}\cdot(2\pi)^3\,\delta^3(\mathbf{p}+\mathbf{k}) \\
&= \beta V\cdot V\int\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\sum_{n\in\mathbb{Z}}\int \mathrm{d}^3\mathbf{p}\,\delta^3(\mathbf{p}+\mathbf{k})\sum_{n\in\mathbb{Z}}\delta_{m,-n} \\
&= \beta V\sum_k\int \mathrm{d}^3\mathbf{p}\,\delta^3(\mathbf{p}+\mathbf{k})\sum_{n\in\mathbb{Z}}\delta_{m,-n},
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
(-S') &= -\frac{1}{2TV}\sum_{p,k}\int \mathrm{d}^4x\, e^{i\,(p+k)\cdot x}\Big(A_p^\mu\Big(-\delta_{\mu\nu}(-k^2)+\big(1-\tfrac{1}{\rho}\big)\,(-k_\mu k_\nu)\Big)A_k^\nu - 2\bar{\eta}_p\,(-k^2)\eta_k\Big) \\
&= -\frac{\beta V}{2TV}\sum_k\Big(A_{-k}^\mu\Big(k^2\delta_{\mu\nu}-\big(1-\tfrac{1}{\rho}\big)\,k_\mu k_\nu\Big)A_k^\nu + 2\bar{\eta}_{-k}k^2\eta_k\Big) \\
&= -\frac{\beta^2}{2}\sum_k\Big(A_k^{\mu\dagger}D_{\mu\nu}^{-1}(k)A_k^\nu + 2\bar{\eta}_k^\dagger k^2\eta_k\Big)
\end{aligned}
\tag{27}
$$

Here we've used the reality condition on A,η, namely $A_k^{\mu\dagger}=A_{-k}^\mu$, and defined the k–space inverse propagator $D_{\mu\nu}^{-1}(k)$. Similar result applies for η_k, except that we have to be careful about Grassmann variables. Carry out $\int \mathscr{D}A^\mu\,\mathscr{D}\bar{\eta}\,\mathscr{D}\eta$, and we have:

$$
\begin{aligned}
\mathcal{Z} &\sim \Big(\det_{\mu,\nu,k}\beta^2 D_{\mu\nu}^{-1}(k)\Big)^{-1/2}\Big(\det_k 2\beta^2k^2\Big)^{+1} \\
&= \prod_k\Big(\beta^{2\times4\times(-1/2)}\cdot\Big(\det_{\mu,\nu}D_{\mu\nu}^{-1}(k)\Big)^{-1/2}\cdot 2\beta^2k^2\Big) \\
&= \prod_k\Big(\beta^{-4}\Big(\det_{\mu,\nu}D_{\mu\nu}^{-1}(k)\Big)^{-1/2}\cdot 2\beta^2k^2\Big)
\end{aligned}
\tag{28}
$$

The determinant is evaluated as follows[3]:

$$
\begin{aligned}
\det_{\mu,\nu}D_{\mu\nu}^{-1} &= \det_{\mu,\nu}\Big(k^2\delta_{\mu\nu}-\big(1-\tfrac{1}{\rho}\big)\,k_\mu k_\nu\Big) \\
&= k^8\det_{\mu,\nu}\Big(\delta_{\mu\nu}-\big(1-\tfrac{1}{\rho}\big)\,\tfrac{k_\mu k_\nu}{k^2}\Big) \\
&= k^8\Big(1-\big(1-\tfrac{1}{\rho}\big)\,\tfrac{k^2}{k^2}\Big) \\
&= \tfrac{1}{\rho}k^8,
\end{aligned}
\tag{29}
$$

$$
\mathcal{Z}\sim\prod_k\tfrac{2}{\rho}\beta^{-4}k^{-4}\cdot\beta^2k^2\sim\prod_k\beta^{-2}k^{-2},
\tag{30}
$$

$$
\ln\mathcal{Z}\sim-\sum_k\ln\big(\beta^2k^2\big)
\tag{31}
$$

[3] Reference: `Wikipedia:` *Determinant # Sylvester's determinant theorem*.

We see that $\ln \mathcal{Z}$ is simply twice the result of a neutral scalar field, with mass $m \to 0$, i.e.

$$\ln \mathcal{Z} \sim -2 \times \frac{1}{2} \sum_k \ln\left(\beta^2 k^2\right) \sim -2 \sum_{\mathbf{k}} \left(\frac{1}{2}\,\beta E_k + \ln\left(1 - e^{-\beta E_k}\right)\right), \tag{32}$$

$$\mathcal{Z} \sim \prod_{\mathbf{k}} \left\{\exp\left(-\frac{1}{2}\,\beta E_k - \ln\left(1 - e^{-\beta E_k}\right)\right)\right\}^2, \tag{33}$$

$$\Omega = -T \ln \mathcal{Z} = 2 \sum_{\mathbf{k}} \left(\frac{1}{2}\,E_k + T \ln\left(1 - e^{-E_k/T}\right)\right), \tag{34}$$

$$p = -\frac{\Omega}{V} = -2 \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \left(\frac{1}{2}\,E_k + T \ln\left(1 - e^{-E_k/T}\right)\right), \tag{35}$$

Here $E_k = \|\mathbf{k}\|$. Ignore the vacuum contribution to p, and we have:

$$p = -2T \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \ln\left(1 - e^{-\|\mathbf{k}\|/T}\right) = 2\,\frac{\pi^2}{90}\,T^4 \tag{36}$$

Summary

1. We've completed the calculation of pure QED partition function $\mathcal{Z}$ and thermodynamic potential Ω (energy density p) by introducing a "soft" gauge fix $\mathcal{L}_{gf}$. Alternatively, we can simply impose a "hard" Lorenz gauge fix $\delta\left[\partial_\mu A^\mu\right]$; this can be achieved by taking $\rho \to \infty$ in the Gaussian packet $-\frac{1}{2\rho}\left(\partial_\mu A^\mu\right)^2$, or by integrating out A^0 directly — similar to (27), we have:

$$\begin{aligned}
\mathcal{Z} &\sim \int \mathcal{D}A^\mu\, \mathcal{D}\bar\eta\, \mathcal{D}\eta\, \delta\left[\partial_\mu A^\mu\right] \exp\left(-\frac{\beta^2}{2} \sum_k \left(A_k^{\mu\dagger}\left(k^2 \delta_{\mu\nu} - k_\mu k_\nu\right) A_k^\nu + 2\bar\eta_k^\dagger k^2 \eta_k\right)\right) \\
&\sim \int \mathcal{D}A^i\, \mathcal{D}\bar\eta\, \mathcal{D}\eta\, \exp\left(-\frac{\beta^2}{2} \sum_k \left(A_k^{\mu\dagger}\left(k^2 \delta_{\mu\nu} - k_\mu k_\nu\right) A_k^\nu + 2\bar\eta_k^\dagger k^2 \eta_k\right)\right)_{A^0 = A^0[A^i]}
\end{aligned} \tag{37}$$

$$A^0[A^i] = -\int \partial_i A^i\, \mathrm{d}\tau\,, \quad k_\mu A_k^\mu = 0, \tag{38}$$

Here we've omitted a non-dynamical Jacobian $\det\left[\theta(\tau - \tau')\right] = \det \int^\tau \mathrm{d}\tau''\, \delta(\tau'' - \tau')$. The ghost integral gives the same contribution, while the A^i integral yields:

$$\mathcal{Z}_A = \int \mathcal{D}A^i\, \exp\left(-\frac{\beta^2}{2} \sum_k \left(A_k^{\mu\dagger}\left(k^2 \delta_{\mu\nu}\right) A_k^\nu\right)\right), \tag{39}$$

$$\begin{aligned}
A_k^{\mu\dagger}\left(k^2 \delta_{\mu\nu}\right) A_k^\nu &= A_k^{i\dagger}\left(k^2 \delta_{ij}\right) A_k^j + A_k^{0\dagger}\left(k^2\right) A_k^0 = A_k^{i\dagger}\left(k^2 \delta_{ij}\right) A_k^j + \frac{k^2}{\omega^2}\, A_k^{0\dagger}\left(\omega^2\right) A_k^0 \\
&\overset{(38)}{=} A_k^{i\dagger}\left(k^2 \delta_{ij}\right) A_k^j + \frac{k^2}{\omega^2}\, A_k^{i\dagger}(k_i k_j) A_k^0 = A_k^{i\dagger}\, k^2 \left(\delta_{ij} + \frac{k_i k_j}{\omega^2}\right) A_k^j \\
&= A_k^{i\dagger} D_{ij}^{-1}(k) A_k^j,
\end{aligned} \tag{40}$$

$$\begin{aligned}
\mathcal{Z}_A \sim \left(\det_{i,j,k} \beta^2 D_{ij}^{-1}(k)\right)^{-1/2} &= \prod_k \beta^{-3} k^{-3} \left(1 + \frac{\mathbf{k}^2}{\omega^2}\right)^{-1/2} = \prod_k \left(\beta^{-4} k^{-4} \cdot \beta\omega\right) \\
&= \prod_k \beta^{-4} k^{-4} \prod_{\mathbf{k}} \prod_n 2\pi n \sim \prod_k \beta^{-4} k^{-4}
\end{aligned} \tag{41}$$

We see that the result from a “hard” Lorenz gauge fixing is the same as before, up to a non-dynamical overall coefficient.

2. In our previous calculations, we notice that after functional integration, ρ is just a non-dynamical overall coefficient in $\mathcal{Z}$, hence it can be safely dropped from the final expression; see eq. (30). Therefore, the result is independent of parameter ρ.