Review Problems 1

Professor Bryan Graham

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In preparing for midterm exams review your lecture notes, assigned course readings, problem sets and prepare answers to the questions on the review sheet(s). You may bring to the exam a single 8.5 × 11 inch sheet of paper with notes on it. No calculation aides are allowed in the exam (e.g., calculators, phones, laptops etc.). You may work in pencil or pen, however I advise the use of pencil. Please be sure to bring sufficient blue books with you to the exam if possible. Scratch paper will be provided.

This review sheet is designed to assist you in your exam preparations. I suggest preparing written answers to each question. You may find it useful to study with your classmates. In the exam you may bring in a single 8.5 x 11 sheet of notes. No calculators or other aides will be permitted. Please bring blue books to the exam. The first midterm exam will occur in class on Thursday, March 5th.

[1] **Incoherence is costly**

Spanky is an avid player of Dungeon's and Dragons. When his character Ademaro, a 325 year old forest elf, is visiting the Yawning Portal he sees the following betting rates posted at the bar for an upcoming unicorn race. There are three unicorns racing: Sparklehoof, Shimmering Sea and Manificent Mountain Mare. The posted rates are pay α to win 1 electrum piece if Sparklehoof wins (event A), pay β to win 1 electrum piece if Shimmering Sea wins (event B) and pay γ to win 1 electrum piece if *either* Sparklehoof or Shimmering Sea win. Assuming that the Yawning Portal will accept any bet consistent with the advertised rate, construct three bets such that regardless of the outcome the Yawning Portal will lose money when $\gamma < \alpha + \beta$. You may also assume that tie races are not possible. Use the follow payoff table to describe the properties of your bets:

Yawning Portal Payoffs

	Payoff (Bet 1)	Payoff (Bet 2)	Payoff (Bet 3)	Total Payoff
$A \wedge \neg B$				
$\neg A \wedge B$				
$\neg A \wedge \neg B$				

What probability rule is the Yawning Portal violating?

[2] Life is better when you maximize expected utility

You are deciding whether to incorporate regular aerobic exercise into you life (specifically 45 minutes of vigorous training, three times a week). Let A denote the event "I exercise" and B the event "I die prior to my expected retirement at age 65". Your lifetime utility payoffs are

	B	$\neg B$
A	-10	90
$\neg A$	0	100

[a] Your friend, Not-a-Careful-Thinker says, "since your happier when you don't exercise regardless of how long your live you shouldn't exercise. It is a no-brainer." Your other friend, Blaise, says "dude, maximize expected utility". He says according to the best medical evidence

$$\Pr\left(B|\, A\right) = 0.10, \quad \Pr\left(B|\, \neg A\right) = 0.40.$$

Whose advice do you follow, why?

[b] Whose advice would you follow if, instead,

$$\Pr\left(B|\, A\right) = \Pr\left(B|\, \neg A\right) = 0.25.$$

If you answer to part [a] and [b] differ, then explain why.

[3] Linear regression

Let $X \sim \text{Uniform}\left[-1, 1\right]$ and assume that

$$Y = -\frac{2}{3} + X^2 + V, \;\; V|\, X \sim \mathcal{N}\left(0, \sigma^2\right) \tag{1}$$

[a] Calculate $\mathbb{E}\left[Y|\, X\right]$

[b] Calculate $\mathbb{E}\left[X^2\right]$ and $\mathbb{V}\left(X\right)$

[c] Calculate $\mathbb{E}\left[Y\right]$

[d] Calculate $\mathbb{C}\left(X, Y\right)$ and also the coefficient on X in $\mathbb{E}^*\left[Y|\, X\right]$.

[e] Let $U = Y - \mathbb{E}^*\left[Y|\, X\right]$. Show that $\mathbb{C}\left(U, X\right) = 0$. Give an intuitive explanation for this result.

[f] Find a function $g\left(X\right)$ such that that $\mathbb{C}\left(U, g\left(X\right)\right) = \mathbb{V}\left(X^2\right)$. Give an intuitive explanation for your answer.

[g] Describe how your answers in (d) to (f) would change if (1) held but now $X \sim \text{Uniform}[0,2]$. You may find it helpful to sketch a figure. [4] **More linear regression**

You are given a random sample from South Africa in the late 1980s. Each record in this sample includes, Y, an individual's log income at age 40, X the log permanent income of their parents, and D a binary indicator equaling 1 if the respondent is White and zero if they are Black. Let the best linear predictor of own log income at age forty given parents' log permanent income and own race be

$$\mathbb{E}^*[Y|X,D] = \alpha_0 + \beta_0 X + \gamma_0 D.$$

[a] Let $Q = \Pr(D=1)$, assume that $\mathbb{V}(X|D=1) = \mathbb{V}(X|D=0) = \sigma^2$ and recall the analysis of variance formula $\mathbb{V}(X) = \mathbb{V}(\mathbb{E}[X|D]) + \mathbb{E}[\mathbb{V}(X|D)]$. Show that

$$\mathbb{V}(X) = Q(1-Q)\{\mathbb{E}[X|D=1] - \mathbb{E}[X|D=0]\}^2 + \sigma^2.$$

[b] Let $\mathbb{E}^*[D|X] = \kappa + \lambda X$. Show that

$$\lambda = \frac{Q(1-Q)\{\mathbb{E}[X|D=1] - \mathbb{E}[X|D=0]\}}{Q(1-Q)\{\mathbb{E}[X|D=1] - \mathbb{E}[X|D=0]\}^2 + \sigma^2}.$$

[c] Assume that $\beta_0 = 0$. Show that in this case $\gamma_0 = \mathbb{E}[Y|D=1] - \mathbb{E}[Y|D=0]$.

[d] Let $\mathbb{E}^*[Y|X] = a + bX$. Maintaining the assumption that $\beta_0 = 0$ show that

$$b = \frac{Q(1-Q)\{\mathbb{E}[Y|D=1] - \mathbb{E}[Y|D=0]\}\{\mathbb{E}[X|D=1] - \mathbb{E}[X|D=0]\}}{Q(1-Q)\{\mathbb{E}[X|D=1] - \mathbb{E}[X|D=0]\}^2 + \sigma^2}.$$

[e] Let $Q(1-Q) = 1/10$, $\sigma^2 = 3/10$ and $\mathbb{E}[Y|D=1] - \mathbb{E}[Y|D=0] = \mathbb{E}[X|D=1] - \mathbb{E}[X|D=0] = 3$. Provide a numerical value for $\mathbb{V}(X)$ and b.

[f] On the basis of β_0 a member of the National Party argues that South Africa is a highly mobile society. One the basis of b a member of the African National Congress argues that it is a highly immobile one. Comment on the relative merits of these two assertions.

[5] Frisch-Waugh Theorem Redux

Let Y be a scalar random variable, X a K vector of covariates (which includes a constant), and W a vector of additional covariates (which excludes a constant). Consider the long (linear) regression

$$\mathbb{E}^*[Y|W,X] = X'\beta_0 + W'\gamma_0. \tag{2}$$

Next define the short and auxiliary regressions

$$\mathbb{E}^* [Y | X] = X' b_0 \tag{3}$$

$$\mathbb{E}^* [W | X] = \Pi_0 X. \tag{4}$$

[a] Let $V = W - \mathbb{E}^* [W | X]$ be the projection error associated with the auxiliary regression. Show that

$$\begin{aligned} \mathbb{E}^* [Y | V, X] &= \mathbb{E}^* [Y | X] + \mathbb{E}^* [Y | 1, V] - \mathbb{E} [Y] \\ &= \mathbb{E}^* [Y | X] + \mathbb{E}^* [Y | V] \end{aligned}$$

where $\mathbb{E}^* [Y | 1, V]$ denotes the linear regression of Y onto a constant and V, while $\mathbb{E}^* [Y | V]$ denotes the corresponding regression without a constant (HINT: Observe that $\mathbb{C} (X, V) = 0$).

[b] Next show that $\mathbb{E}^* [Y | V, X] = \mathbb{E}^* [Y | W, X]$ and hence that the coefficient on V in $\mathbb{E}^* [Y | V, X]$ coincides with that on W in $\mathbb{E}^* [Y | W, X]$.

[c] Let $U = Y - \mathbb{E}^* [Y | X]$ be the projection error associated with the short regression. Derive the coefficient on V in the linear regression of U onto V (excluding a constant).

[d] Discuss the possible practical value of the results shown in [b] and [c] above.