Ec143, Spring 2026

Professor Bryan Graham

Problem Set #1 (Pencil & Paper Part)

Due: Wednesday, February 4th

[1] **Bayesian Rulers and Sure Losers**

Reading: Vineberg (2022), Diaconis & Skyrms (2018, Ch. 2), Ramsey (1931), de Finetti (1992)

Imagine you are Misstra Know-It-All, a tenured professor at a prestigious university with an unhealthy ego. One morning a graduate student corners you in the hallway excited about her new research idea. The idea is that NBA G League basketball players will shoot better from outside the three point line if they view a highlight real of classic three point baskets in an undistracted environment just prior to competition. The student situates her idea within the wider academic literature on the connection between systematic approaches to getted psyched up and long run life outcomes. "If we can all get psyched up, we can vanquish inequality," she claims.

The student plans to temporarily relocate to Santa Cruz (she would like to work with the Santa Cruz Warriors/Surfers). Before each of the ~25 Surfer home games the student intends to randomly assign members of the Surfers' starting line up to either treatment (view highlight real) or control (normal pre-game routine) and then compare three point field goal percentages across the two groups. There are $25 \times 5 = 125$ player-by-game observations, each randomly assigned to either treatment or control.

Let E be the outcome of the student's hypothesis test; $E = 0$ if the student fails to reject the null hypothesis that basketball players watching highlight reels have the same three point shot efficiency as those that do not; $E = 1$ if the student rejects this null. Let $H \in \{0, 1\}$ according to whether the hypothesized treatment effect is actually present ($H = 1$) or not ($H = 0$).

Let $\Pr(E|\neg H) = \Pr(E = 1|H = 0) = \alpha$ denote the *size* of your students' test. Let $\Pr(E|H) = \Pr(E = 1|H = 1) = \beta$ denote its *power*. Let γ be the student's *a priori* belief that H is true: $\gamma = \Pr(H) = \Pr(H = 1)$.

[a] Use Baye's Rule to show that

$$\Pr(H|E) = \frac{\beta\gamma}{\beta\gamma + \alpha(1-\gamma)}, \ \Pr(H|\neg E) = \frac{(1-\beta)\gamma}{(1-\beta)\gamma + (1-\alpha)(1-\gamma)}.$$

[b] Your student admits to you that if their research idea "works", by which she means the test rejects ($E = 1$), all but assuring publication in a prestigious journal, she will use what she calls the "conservative posterior"

$$p^*\left(H|\,E\right) = \Pr\left(H|\,E\right) - \epsilon,\ 0 < \epsilon < \frac{\beta\gamma}{\beta\gamma + \alpha\left(1-\gamma\right)}.$$

That is she will use Bayes' rule to update her beliefs, but then adjust her resulting confidence in H downwards, so as to not be "overly-optimistic" about the truthiness of H. On the other hand she is also worried about prematurely discarding a good idea. If the test fails to reject she plans to instead use an "optimistic posterior" of

$$p^*\left(H|\,\neg E\right) = \Pr\left(H|\,\neg E\right) + \epsilon,\ 0 < \epsilon < \frac{\left(1-\alpha\right)\left(1-\gamma\right)}{\left(1-\beta\right)\gamma + \left(1-\alpha\right)\left(1-\gamma\right)}.$$

You suggest to the student that these adjustments are ill-advised and recommend that she read de Finetti (1992) and Savage (1972), but she is insistent. Sensing a teachable moment you making the following proposal:

Today, at the student's fair price, you will sell her the following wagers:

1. A payout of \$1000 if it turns out H and E are true and zero otherwise;
2. A payout of $\$1000 \times \Pr\left(H|\,E\right)$ if $\neg E$ (i.e., if her test fails to reject) and zero otherwise;
3. A payout of $\$1000\epsilon$ if E (if her test rejects) and zero otherwise.

What are the student's fair prices for the three wagers above? Hint: consider the following quote from Thomas Bayes:

> The *probability of any event* is the ratio between the value at which an expectation depending on the happening of the event ought to be computed, and the value of the thing expected upon it's happening Bayes (1763, p. 376).

[c] Suppose the test fails to reject, $\neg E$. Show that your students total winnings in that case equal $-\left[\beta\gamma + \alpha\left(1-\gamma\right)\right] \times \$1000\epsilon$. Why does she lose more money when her test is powerful and her *a priori* confidence in H is high?

[d] *After* she completes her data collection and analysis, if E (i.e., if her test rejects) you agree to buy back the first bet above at her "conservative" post-analysis fair price of

$p^* (H|E) \times \$1000$. Why is this the appropriate fair price? Show that if the test does reject E, her winnings also equals $-[\beta\gamma + \alpha(1-\gamma)] \times \$1000\epsilon$.

[e] After your student returns from Santa Cruz in debt, you assign her the following post-fieldwork reflection exercise: "You can believe whatever you what, but your beliefs must be coherent. Comment?" Write your own response to this prompt (2 - 3 paragraphs).

[2] Brave Bayesians

Reading: Bar-Hillel (1979)

Your doctor, as part of a routine physical, screens you for a rare disease (frequency one in one million). The power of the test he uses is $\beta = 0.95$, its size is $\alpha = 0.05$. Let H denote a positive test and E actually have the disease. Your doctor calls you saying," I am afraid I have some bad news, you test came back positive –you have the disease". You respond "that is very unlikely to be the case". Explain (Hint: compute $\Pr(E|H)$).

[3] Confident Bayesians

You work for an NGO which provides after school services for disadvantaged youth. A prominent academic evaluates the efficacy of your program via a randomized control trial (RCT). She uses one-sided test with power $\beta = 0.80$, and size is $\alpha = 0.05$. At the completion of her study she reports finding no evidence of a positive effect of your program on youth outcomes. She recommends defunding your program in order to redirect resources to a program that actually "works". Offer a counter-argument to your funding source.

[4] Overconfident social scientists

You are near retirement, after a long and illustrious career in the social sciences. You've done it all, Op-Eds, a best selling trade book, coverage of your research demonstrating counter-intuitive truths in the *New Yorker, Atlantic* and other outlets prized by the culturati, lucrative speaking engagements to corporate audiences, and a large social media presence. You are a "thought leader" *par excellence*. As you clean out your office you realize you've tested 1000 hypotheses over the course of your career. You failed to reject 925 of theses hypotheses. None of these research projects led to a publication. You did reject 75 hypotheses and managed to publish in all these cases. Throughout your career you've employed research designs with power $\beta = 0.80$, and size is $\alpha = 0.05$. What proportion on your published body of research is actually correct?

References

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