Contingent Valuation

Bryan S. Graham, UC - Berkeley & NBER

March 31, 2025

Imagine you have been hired by the City of Berkeley to evaluate the possible benefits associated with instituting a wildfire risk reduction program in the Berkeley Hills. This program would institute a regular schedule of removing excess wildfire fuel from the Berkeley Hills. The program would be financed by an annual per parcel property tax.

You decide to implement a double bounded dichotomous choice contingent valuation study. Specifically you randomly sample $i = 1, \ldots, N$ Berkeley households. For each household you *randomly* choose a hypothetical tax amount $B_{1i} \in \mathbb{B}^* = \{b_1^*, b_2^*, \ldots, b_K^*\}$ and ask them if they would approve the program at cost B_{1i}. For example you might set $\mathbb{B}^* = \{50, 100, 200\}$, in which case you would ask the household whether they support the project at a cost of either \$50, \$100 or \$200 (depending on which cost you randomly chose to pose).

If the respondent says "yes" they would support the program, you then ask if they would continue to support the program at the greater cost of $B_{2i} = 2B_{1i}$ (twice as much). For example if $B_{1i} = 50$ and the household responded yes, you would then ask them if they would continue to support the program at the higher cost $B_{2i} = 2B_{1i} = 2 \times 50 = 100$. If, instead, the respondent said "no" to the initial question, you would then ask them if they would be willing to support the project at the lower cost of $B_{2i} = \frac{1}{2}B_{1i}$.

You assume that (i) each household has an *unobserved* willingness-to-pay for the program, Y_i and (ii) they truthfully respond to the enumerator. That is they respond "yes" to the first question if $Y_i \geq B_{1i}$ and "no" otherwise (and similarly for the second question). Under these assumptions, the protocol, which is called a *double bounded dichotomous choice experiment*, reveals that a household's willingness-to-pay, Y_i, is in one of the four intervals

$$\left[0, \frac{1}{2}B_{1i}\right), \left[\frac{1}{2}B_{1i}, B_{1i}\right), [B_{1i}, 2B_{1i}), [2B_{1i}, \infty).$$

Observe that the four intervals will vary across households depending on the initial value of

B_{1i} which is selected. As an example, if $B_{1i} = 50$ we would get the intervals

$$[0, 25), [25, 50), [50, 100), [100, \infty),$$

whereas if $B_{1i} = 200$ were selected we would instead have the intervals

$$[0, 100), [100, 200), [200, 400), [400, \infty).$$

Also note that we assume that a household's willingness-to-pay for the project is bounded below by zero (e.g., that there is no household that would pay to *increase* fire risk).

An important implication of randomly choosing $B_{1i} \in \mathbb{B}^*$ is that unit i's actual valuation, Y_i, is independent of B_{1i} – the distribution of willingness-to-pay is independent of the initial costs posed to the respondent:

$$Y_i \perp B_{1i}. \tag{1}$$

Let $\mathbb{B} = \{b_0, b_1, \cdots, b_{L-1}, b_L\}$ denote the union of all the interval boundaries associated with all possible draws of $B_{1i} \in \mathbb{B}^*$. By construction $b_0 = 0$ and $b_L = \infty$. If $\mathbb{B}^* = \{50, 100, 200\}$, then, under the protocol described above we would have

$$\mathbb{B} = \{25, 50, 100, 200, 400\}.$$

Next consider the L intervals

$$[b_0, b_1), [b_1, b_2), \ldots, [b_{L-1}, b_L)$$

which partition the support of the willingness-to-pay distribution, $Y_i \in \mathbb{Y} = \mathbb{R}_+$. Our experiment doesn't necessarily reveal which of these (more refined) intervals a household's willingness-pay-lies in. For example if $B_{1i} = 100$ and the household responds "no" to both the first and second questions, then all we know is that their willingness to pay is less then \$50. This means it could lie in the interval $[b_0, b_1) = [0, 25)$ *or* the interval $[b_1, b_2) = [25, 50)$. In our example $L = 6$ with intervals of

$$[0, 25), [25, 50), \ldots, [400, \infty).$$

Let $D_{il} = 1$ if household i's willingness-to-pay *could logically fall into* interval the l^{th} interval $[b_{l-1}, b_l)$. In the example above we would have $D_{i1} = D_{i2} = 1$ and $D_{i3} = \cdots = D_{i6} = 0$, since the household's willingness-to-pay could be in the first $[0, 25)$ or second $[25, 50)$ intervals, but is definitely not in one of the intervals above 50. For each household in our sample we can determine the values of $\mathbf{D}_i = (D_{i1}, D_{i2}, \ldots, D_{iL})'$ on the basis of their survey responses. If $B_{i1} = 100$, and the household says "no" to the first question and "yes" to the second, then we would learn that their willingness to pay is at least 50 and less than 100. This gives

$$\mathbf{D}_i = (0, 0, 1, 0, 0, 0)\,.$$

Because the value of B_{1i} is randomly assigned, the distribution of willingness-to-pay is independent of it. Hence the joint probability of probability the event $\mathbf{D}_i$ vector takes the configuration $\mathbf{D}_i = (d_1, d_2, \ldots, d_L)$ and $B_{1i} = b_l$ equals simply

$$\begin{aligned}\Pr\left(D_{i1} = d_1, D_{i2} = d_2, \ldots, D_{iL} = d_L, B_{1i} = b_l\right) &= \Pr\left(D_{i1} = d_1, D_{i2} = d_2, \ldots, D_{iL} = d_L \right| B_{1i} = b_l) \Pr\left(B_{1i} = \right. \\ &= \Pr\left(D_{i1} = d_1, D_{i2} = d_2, \ldots, D_{iL} = d_L\right) \Pr\left(B_{1i} = b_l\right) \\ &= \left\{d_1\left[F_1 - F_0\right] + d_2\left[F_2 - F_1\right] + \cdots + d_L\left[F_L - F_{L-1}\right]\right\} \Pr\left(B\right.\end{aligned}$$

where F_l denotes fraction of the population with a willingness-to-pay less than or equal to b_l (i.e., $F_l = \Pr\left(Y_i \leq b_l\right) = \mathbb{E}\left[1\left(Y_i \leq b_l\right)\right]$. The term $\Pr\left(B_{1i} = b_l\right)$ is determined by survey design and does not vary with any unknown parameter of interest.

An example:

$$\begin{aligned}\Pr\left(D_{i1} = 1, D_{i2} = 1, D_{i3} = 0, \ldots, D_{iL} = 0\right) =& 1 \cdot \left[F_1 - F_0\right] + 1 \cdot \left[F_2 - F_1\right] \\ &+ 0 \cdot \left[F_3 - F_2\right] + \cdots + 0 \cdot \left[F_L - F_{L-1}\right] \\ =& \left[F_1 - F_0\right] + \left[F_2 - F_1\right] \\ =& F_2 - F_0 \\ =& \Pr\left(Y_i \leq b_2\right) - \Pr\left(Y_i \leq b_0\right) \\ =& \Pr\left(Y_i \leq b_2\right).\end{aligned}$$

Let $\theta = (F_1, \ldots, F_{L-1})'$. Note that $F_0 = 0$ and $F_L = 1$ are known. However the remaining $L - 1$ points on the CDF of the willingness-to-pay distribution are unknown. Our goal is to estimate these CDF values. This is the best we can hope to learn from our data, since nothing in the data reveals anything about the distribution of willingness-to-pay *within* our L intervals.

The likelihood for the observed responses $\mathbf{D} = (\mathbf{D}_1, \ldots, \mathbf{D}_N)'$ is

$$L\left(\theta \middle| \mathbf{D}\right) = \prod_{i=1}^{N} \left(\sum_{l=1}^{L} D_{il}\left[F_l - F_{l-1}\right]\right) \tag{2}$$

with $0 = F_0 \leq F_1 \leq \cdots \leq F_{L-1} \leq F_L = 1$ and $\mathbf{D}_i = (D_{1i}, D_{2i}, \ldots, D_{iL})'$.

In principle we could choose $\hat{\theta}$ to maximize (4) directly subject to the inequality constraints (which guarantee that our estimated CDF is non-decreasing). This is certainly feasible, but in practice a simple iterative procedure – a specific instance of the EM-Algorithm – is convenient.

Turnbull estimator

Imagine we observed in which (of our refined) intervals each of our sampled households' willingness-to-pay lay. Specifically we observed

$$D_{il}^{*} = \mathbf{1}\left(b_{l-1} \leq Y_i \leq b_l\right)$$

for $l = 1, \ldots, L$. Note that, in this formulation one, and only one, element of $D_{i1}^{*}, \ldots, D_{iL}^{*}$ equals 1, with the balance equal to zero. With such data we can write down a *complete data likelihood* of

$$L^{c}\left(\theta \middle| \mathbf{D}^{*}\right) = \prod_{i=1}^{N}\left(\sum_{l=1}^{L} D_{il}^{*}\left[F_l - F_{l-1}\right]\right). \tag{3}$$

Using the fact that only one of the D_{il}^{*} indicators is non-zero for each household, it is possible to show that the maximum likelihood estimate (MLE) of F_l, for $l = 1, \ldots, L$ is

$$\hat{F}_l = \frac{1}{N}\sum_{i=1}^{N}\sum_{k=1}^{l} D_{ik}^{*}.$$

This is very intuitive (but good to verify directly). To estimate $F_l = \Pr\left(Y_i \leq b_l\right)$ we simply count the fraction of units in our sample whose willingness-to-pay is known to lie in an interval that is bounded above by b_l.

Unfortunately this complete data MLE is not available since we do not observe D_{il}^{*}. However it is possible to mimic this estimator.

Let $\theta^{(s)}$ be some value of θ – the reason for the specific choice of notation will be come clear shortly. Assume that $\theta^{(s)}$ is, in fact, the true population value of θ. Under this assumption we can use Bayes' rule to estimate

$$\mathbb{E}\left[D_{il}^{*} \middle| \mathbf{D}_i; \theta^{(s)}\right] = \Pr\left(D_{il}^{*} = 1 \middle| \mathbf{D}_i; \theta^{(s)}\right) \stackrel{def}{\equiv} \tilde{D}_{il}^{*}\left(\theta^{(s)}\right) = \frac{D_{il}\left[F_l^{(s)} - F_{l-1}^{(s)}\right]}{\sum_{k=1}^{L} D_{ik}\left[F_k^{(s)} - F_{k-1}^{(s)}\right]}. \tag{4}$$

The $\Pr\left(\cdot \middle| \cdot; \theta^{(s)}\right)$ notation emphasizes that we are calculating the conditional probability under the law where $\theta = \theta^{(s)}$. Equation (4) gives the posterior probability of the event $D_{il}^* = 1$ given the observed information $\mathbf{D}_i$ under the assumption that the distribution of willingness-to-pay is such that $\theta = \theta^{(s)}$. Using equation (4) we can compute the posterior expectation of (3):

$$\mathbb{E}\left[L^c\left(\theta | \mathbf{D}^*\right) | \mathbf{D}_i; \theta^{(s)}\right] = \prod_{i=1}^{N} \left(\sum_{l=1}^{L} \tilde{D}_{il}^* \left(\theta^{(s)}\right) \left[F_l - F_{l-1}\right] \right).$$

With this "estimate" of D_{il}^* we can then construct an estimate of $F_l = \Pr\left(Y_i \leq b_l\right)$ along the lines of our complete data MLE:

$$F_l^{(s+1)} = \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{l} \tilde{D}_{ik}^* \left(\theta^{(s)}\right). \tag{5}$$

Equation (5) equals the proportion of individuals with an *expected* WTP less than or equal to b_l under the assumption that $\theta = \theta^{(s)}$.

Note that (5) gives use a new "estimate" of θ equal to $\theta^{(s+1)} = \left(F_1^{(s+1)}, \ldots, F_{L-1}^{(s+1)}\right)'$. We can use this estimate to compute new posterior expectations for each household's willingness-to-pay interval (using equation (4) above with $\theta^{(s+1)}$ replacing $\theta^{(s)}$). With these new estimates, $\tilde{D}_{il}^* \left(\theta^{(s+1)}\right)$ for $l = 1, \ldots, L$, we can then revaluate equation (5), constructing the update $\theta^{(s+2)}$. Eventually we will find that $\theta^{(s)} \approx \theta^{(s+1)}$. When this occurs our estimate of θ is "self-consistent", in the sense that the share of observations in our sample which we believe fall into each of the L WTP intervals coincides with our beliefs about the corresponding population shares.

With some work it is possible to formally show that if we start with some $\theta^{(0)}$ and iterate between (4) and (5) as described above, that $\theta^{(s)} \approx \theta^{(s+1)}$will occur at a local maxima of our likelihood function (2). By choosing different starting values we can find the global maximum, and hence the MLE of θ, with high probability.

Further reading

The Turnbull Estimator was introduced by Turnbull (1976). Interval censored data arises frequently in empirical applications (e.g., in survival analysis). Consequently variants of Turnbull's estimator are used in many different settings. Carson (2012) provides a sympathetic introduction to contingent valuation (which is a widely used, but also controversial, approach to valuing amenities for which no market transaction data exist).

References

Carson, R. T. (2012). Contingent valuation: a practical alternative when prices aren't available. *Journal of Economic Perspectives*, 26(4), 27 – 42.

Turnbull, B. W. (1976). The empirical distribution function with arbitrarily grouped, censored and truncated data. *Journal of the Royal Statistical Society: Series B*, 38(3), 290 – 295.

