Maximizing Revenue of Golf Courses

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Motivating Example

A golf course wants to improve the profitability of the company

- Golf courses & tee times
- 0-4 players per tee time
- Price to charge is usually a “gut” decision

How can we maximize revenue?

What Affects Price?

$$Revenue = Price * Quantity$$

With $\pi(\theta|R)$

- What revenue can we expect at certain times?
- What tee times are “good” and which aren’t?
- How does changing price affect Quantity and Revenue?

Data

- 48765 rounds
- 2056 days
- 86 tee times
- Not all tee times are played every day
- 6:20am till 8:30pm

Data Matrix

n_i	TTime
15.00	6.20
124.00	6.30
⋮	⋮
638.00	9.20
763.00	9.30
⋮	⋮
693.00	15.10
604.00	15.20
660.00	15.30
⋮	⋮
63.00	19.40
45.00	19.50
16.00	20.00

Factors in Model

- θ_i mean of each tee time
- σ^2 between-tee-time variance, variance of pop'n of tee times
- τ^2 within-tee-time variance, variance of true revenue per time

Model

$$Y_{ij} \sim N(\theta_i, \sigma^2) \tag{1}$$
$$\theta_i \sim N(\theta_{i-1}, \tau^2) \tag{2}$$
$$\theta_0 \sim N(m, s^2) \tag{3}$$
$$\sigma^2 \sim IG(a_\sigma, b_\sigma) \tag{4}$$
$$\tau^2 \sim IG(a_\tau, b_\tau) \tag{5}$$

With posterior distribution:

$$\pi(\underline{\theta}, \sigma^2, \tau^2 | \underline{y}) \propto f(\underline{y}|\underline{\theta}, \sigma^2)\pi(\theta_i|\theta_{i-1}, \tau^2)\pi(\tau^2)\pi(\sigma^2)\pi(\theta_0) \tag{6}$$

Complete Conditionals

Complete conditional of θ_i:

$$[\theta_i] \propto f(\underline{y}|\underline{\theta}, \sigma^2)\pi(\theta_i|\theta_{i-1}, \tau^2) \tag{7}$$

$$\propto \prod_{i=1}^{k} \prod_{n=1}^{n_i} N_{y_{ij}}(\theta_i, \sigma^2) \prod_{i=1}^{k} N_{\theta_i}(\theta_{i-1}, \tau^2) \tag{8}$$

$$\propto \prod_{j=1}^{n_i} exp \frac{(y_{ij} - \theta_i)^2}{-2\sigma^2} \exp \frac{(\theta_i - \theta_{i-1})^2}{-2\tau^2} \exp \frac{(\theta_{i+1} - \theta_i)^2}{-2\tau^2} \dots \tag{9}$$

$$[\theta_i] \propto N(\frac{\sigma^2(\theta_{i-1} + \theta_{i+1}) + \bar{y} n_i \tau^2}{2\sigma^2 + \tau^2 n_i}, \frac{\tau^2\sigma^2}{2\sigma^2 + \tau^2 n_i}) \tag{10}$$

Complete Conditionals (Cont.)

Complete conditional of σ^2,τ^2:

$$[\sigma^2] \propto IG(a_\sigma + \frac{1}{2}\sum_{i=1}^{k} n_i, [\frac{1}{b_\sigma} + \frac{1}{2}\sum_{i=1}^{k}\sum_{j=1}^{n_i}(y_{ij} - \theta_i)^2]^{-1}) \tag{11}$$

$$[\tau^2] \propto IG(a_\tau + \frac{k}{2}, [\frac{1}{b_\tau} + \frac{1}{2}\sum_{i=1}^{k}(\theta_i - \theta_{i-1})^2]^{-1}) \tag{12}$$

$$\tag{13}$$

Assumptions in Model

Assumptions in this model:

- $y_{ij} \sim N(\theta_i, \sigma^2)$ implies tee times are similar (no other effects such as d.o.w,month,weather,etc.)
- τ^2 and σ^2 are independent.
- θ_i and θ_{i-1} are correlated.

Hyperparameter Settings

- $a_\sigma = 50$
- $b_\tau = 5$
- $a_\sigma = 15$
- $b_\tau = 10$
- $\mathrm{m} = 60$
- $s^2 = 5$

Hyper Parameters

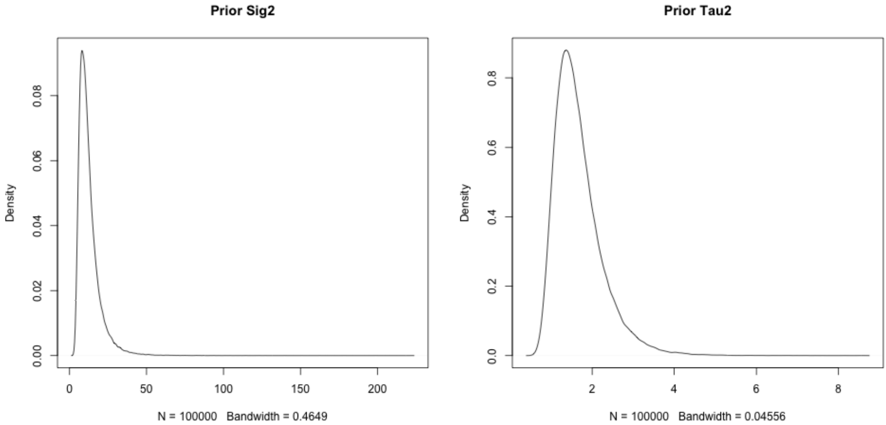


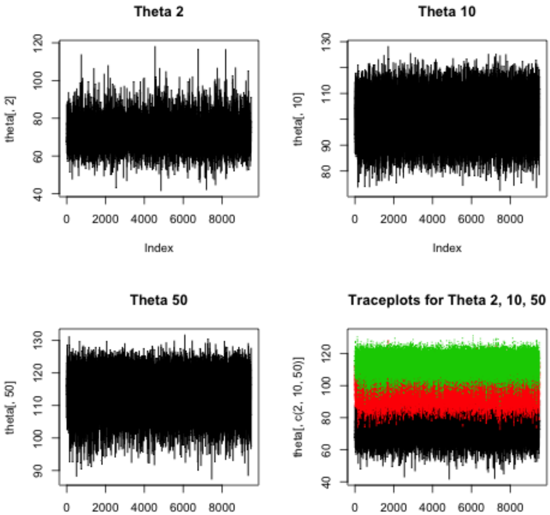
Figure: Determining prior values

MCMC Settings

- niter = 10000
- nburn = 500

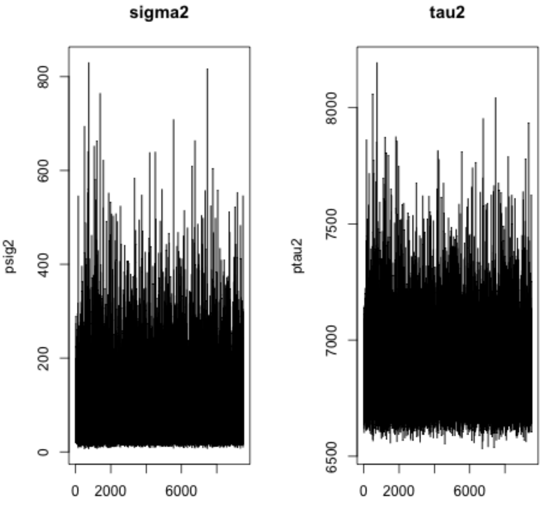
Trace Plots

Trace plots for a sample of θs



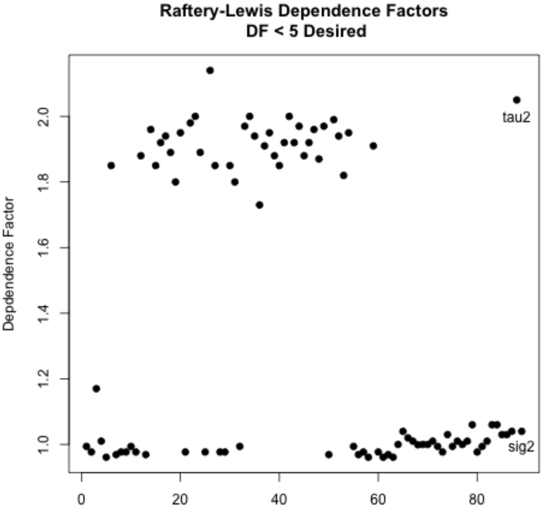
Trace Plots (Cont.)

Trace Plot for σ^2 and τ^2.



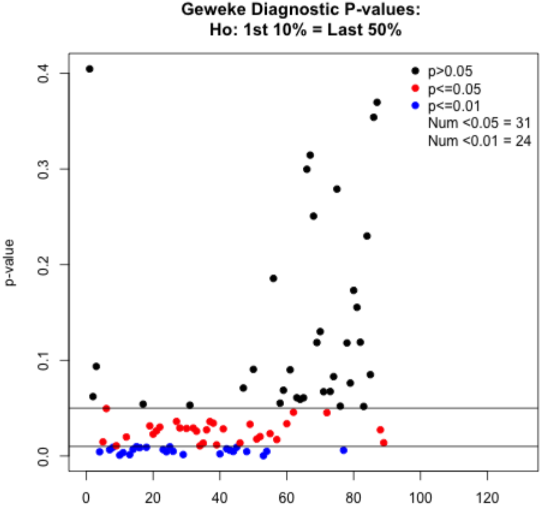
Raftery Lewis Dependence Factors

Seek dependence factors < 5.

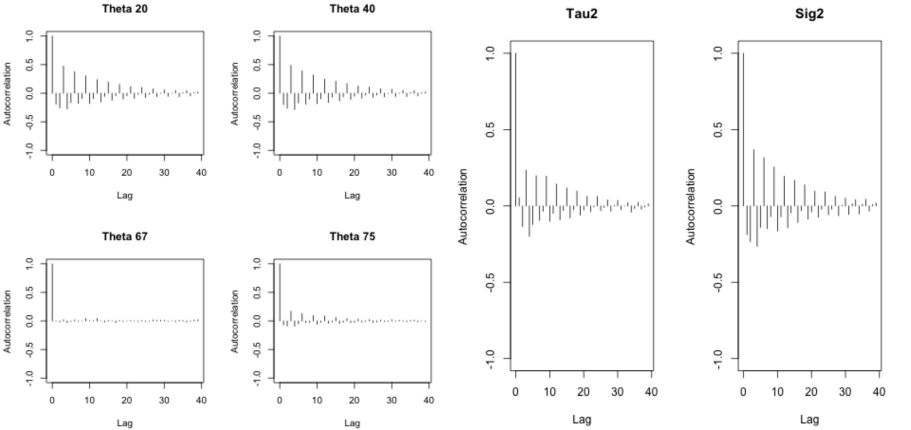


Geweke Diagnostic

H_0: Mean of first 10% = mean of last 50%



AutoCorrelation



GoF: Posterior Predictive Checking

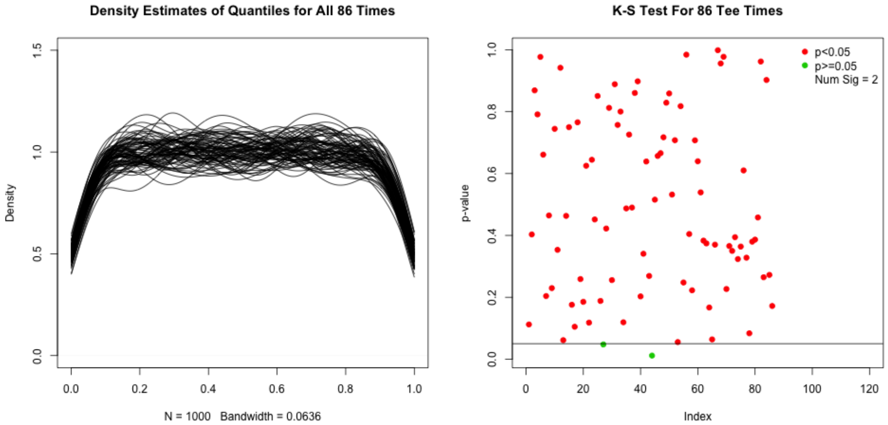


Figure: Posterior Predictive Checking

Covariance and Correlation Plots

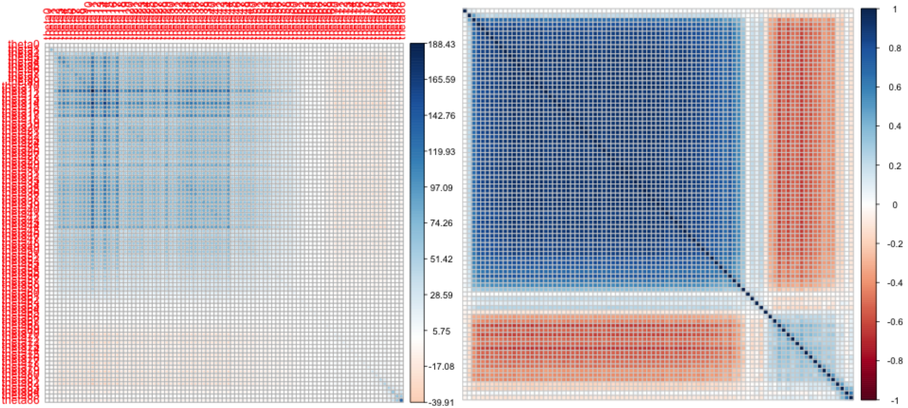


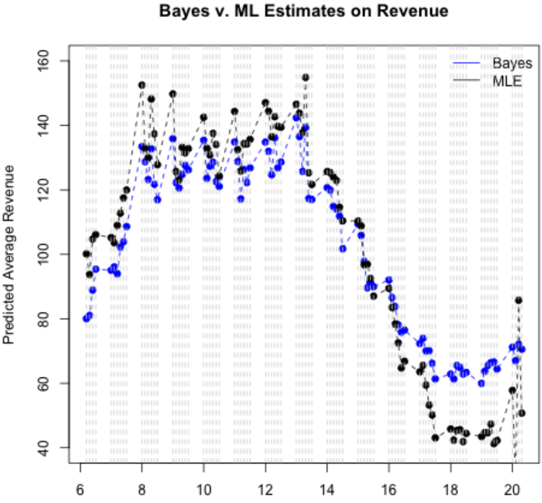
Figure: Covariance and Correlation Plots

Summary Stats

- $E\sigma^2 = 100.62$, $V\sigma^2 = 8488.427$
- $E\tau^2 = 6827.491$, $V\tau^2 = 38561.05$

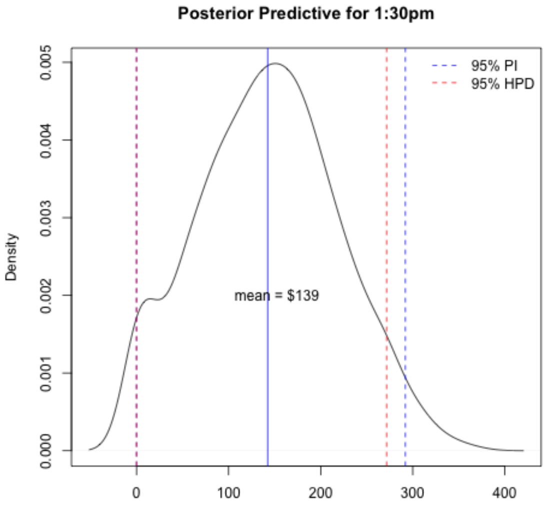
Posterior Predictive Means

What we expect the average tee times per day to be:



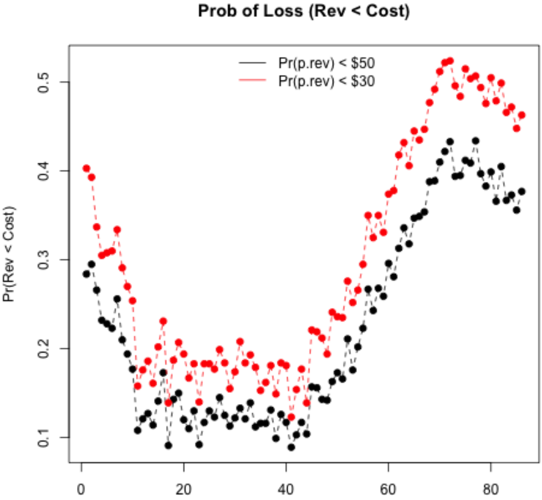
Posterior Predictive for a Tee Time

Given the process:



Optimizing Revenue:

Probability of Losing Money given a certain cost:



Optimizing Revenue:

What's the best revenue for 11:10am? \$120

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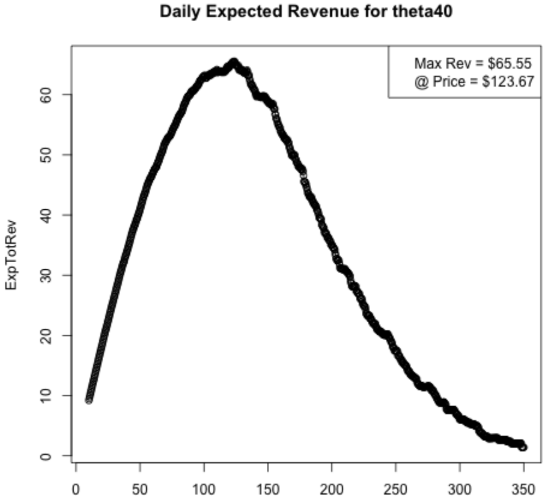

Conclusions

- Successful model
- Certain tee times are more stable than others
- Some tee times are not profitable

Thank You

Sources

- http://www.people.fas.harvard.edu/~plam/teaching/methods/convergence/convergence_print.pdf