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Math 102 — Lab 2

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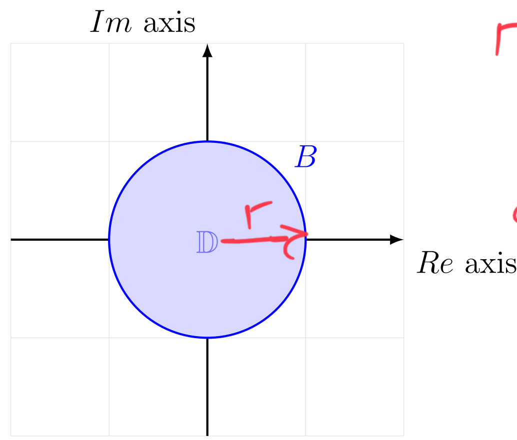
1. The unit disk $\mathbb{D}$ is one of the most important subsets in the complex plane and is sketched below. Note: the curve along the boundary of the unit disk is the unit circle in the complex plane, denoted by B.



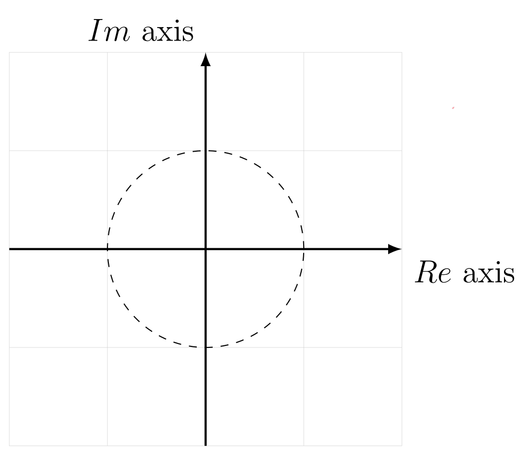
r = radius = 1

Clarification: $\mathbb{D}$ includes B

   (a) Describe the unit circle B in terms of complex numbers. [2]

   (b) Describe the region $\mathbb{D}$ in terms of complex numbers. [2]

   (c) Sketch the region outside of $\mathbb{D}$ and describe it in terms of complex numbers. [2]



2. Use properties of the modulus operation, $|z|$, to prove that $\left|\frac{1}{z}\right| = \frac{1}{|z|}$ for every nonzero $z \in \mathbb{C}$. [2]

Hint: Use the property $|ab| = |a||b|$, $a, b \in \mathbb{C}$, to conclude $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$, $z_1, z_2 \in \mathbb{C}$, $z_2 \neq 0$

(Set $a = z_2$, $b = \frac{z_1}{z_2}$)

3. Consider a transformation f defined by the equation $f(z) = \dfrac{1}{z}$. Note that as a function $f : \mathbb{C} \to \mathbb{C}$, f is not defined when $z = 0$.

   (a) Show that f sends nonzero numbers inside of $\mathbb{D}$ ("inside" means not on the boundary B) to numbers outside of $\mathbb{D}$. [1]

   (b) Show that f sends numbers outside of $\mathbb{D}$ to numbers inside of $\mathbb{D}$. [1]

Hint 1: $|f(z)| = \left|\frac{1}{z}\right| = \frac{1}{|z|}$

Hint 2: For real numbers $a, b > 0$: $a < b \Rightarrow \frac{1}{a} > \frac{1}{b}$