Math 102 · Lab 10 · Bases

- Let V be a vector space (over $\mathbb{R}$) and consider some subset $B = \{b_1, \ldots, b_n\} \subset V$. Set

$$W = \operatorname{span} B = \operatorname{span}\{b_1, \ldots, b_n\} = \{\lambda_1 \cdot b_1 + \cdots + \lambda_n \cdot b_n \mid \lambda_1, \ldots, \lambda_n \in \mathbb{R}\}$$

Facts: • W is a subspace of V

- If the vectors $b_1, \ldots, b_n$ are linearly independent, then $B = \{b_1, \ldots, b_n\} = (b_1, \ldots, b_n)$ is an ordered basis for W.

In this case, $\dim W = n$. Moreover, for any vector $w = w_1 \cdot b_1 + \cdots + w_n \cdot b_n \in W$ ($w_1, \ldots, w_n \in \mathbb{R}$), its coordinate vector (with respect to B) is

$$[w]_B = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} \in \mathbb{R}^n$$

Recall: • $b_1, \ldots, b_n$ are linearly independent if the only solution to the equation $\lambda_1 \cdot b_1 + \cdots + \lambda_n \cdot b_n = \vec{0}$ is $\lambda_1 = \cdots = \lambda_n = 0$.

- $B = \{b_1, \ldots, b_n\} = (b_1, \ldots, b_n)$ is an ordered basis for W if $b_1, \ldots, b_n$ are linearly independent and $W = \operatorname{span} B$.

- In general, if $\dim W = n$ then $W \neq \mathbb{R}^n$.

e.g.: $W = \operatorname{span}\left\{ \begin{bmatrix} 5 \\ 4 \\ -3 \end{bmatrix} \right\}$ is a 1-dimensional subspace of $\mathbb{R}^3$, but $W \neq \mathbb{R}^1 = \mathbb{R}$.