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Math 102 — Lab 4

Recall the formula $\text{proj}_{\mathbf{u}}\,\mathbf{v} = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|^2}\mathbf{u}$ for orthogonal projection of the vector $\mathbf{v}$ in the direction of the vector $\mathbf{u}$. (along the vector $\vec{u}$).

1. (a) Find the projection of the vector $(1, -2)$ in $\mathbb{R}^2$ along the vector $(2, 3)$. [1]

(b) Find the projection of the vector $(2, 1, -2)$ in $\mathbb{R}^3$ along the vector $(1, 1, 3)$. [1]

(c) Find the projection of the vector $(-2, 2, 4, -1)$ in $\mathbb{R}^4$ along the vector $(-1, 3, -3, 5)$. [1]

2. Consider the line $y - y_0 = m\,(x - x_0)$ in $\mathbb{R}^2$. The vector $\mathbf{v}$ from (x_0, y_0) to some other point (x_1, y_1) on the line is tangent to the line.

   (a) Find a vector $\mathbf{v}$ tangent to the line $y = 2x + 1$. [1]

   (b) Find a vector that goes from the point $(2, 2)$ to the line $y = 2x + 1$ and meets that line at a right angle. (Hint: First find any vector $\mathbf{w}$ that goes from the point $(2, 2)$ to the line. Then find the projection of $\mathbf{w}$ parallel and perpendicular to the line.] [2]

(or $\vec{w}$ could be a vector from the line to the point (2,2))

   (c) What is the distance from $(2, 2)$ to the line $y = 2x + 1$ (when mathematicians say distance, we usually mean what is commonly called the shortest distance)? [1]

3. What point on the line $y = 2x + 4$ is closest to the origin in $\mathbb{R}^2$? What is the distance from this line to the origin? [3]

Hint: Let $\vec{v}$ be a vector tangent to the line $y=2x+4$ and let P be some point on that line $y=2x+4$.

Finding $\text{proj}_{\vec{v}} \overrightarrow{OP}$ will help, where $\overrightarrow{OP} = P - (0,0)$.