Math 102 · Lab 3 · Dot Product & Roots of Unity

- For vectors $\vec{x} = (x_1, x_2, \dots, x_n)$, $\vec{y} = (y_1, y_2, \dots, y_n) \in \mathbb{R}^n$, the **dot product of $\vec{x}$ and $\vec{y}$** is the number

$$\vec{x} \cdot \vec{y} = \sum_{i=1}^{n} x_i y_i = x_1 y_1 + x_2 y_2 + \cdots + x_n y_n.$$

- The **norm of $\vec{x}$** $= (x_1, x_2, \dots, x_n)$ is the number

$$\|\vec{x}\| = \sqrt{\vec{x} \cdot \vec{x}} = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$$

- An **n^{th} root of unity** is a complex number $z \in \mathbb{C}$ satisfying the equation $z^n = 1$.

- There are **"n" many** n^{th} roots of unity. Setting $\omega_n = e^{2\pi i/n}$, these are given by $\omega_n^0 = 1$, $\omega_n^1 = e^{2\pi i/n}$, $\dots$, $\omega_n^{n-1} = e^{2\pi(n-1)i/n}$.

So: An n^{th} root of unity has the form $\omega_n^k = e^{2\pi k i/n}$ for some $k = 0, 1, \dots, n-1$.

__Example:__ The "4" 4^{th} roots of unity are

$$\omega_4^0 = 1 \qquad \omega_4^1 = \omega_4 = e^{2\pi i/4} = e^{\frac{\pi}{2} i} = i$$

$$\omega_4^2 = e^{2\pi 2 i/4} = e^{4\pi i/4} = e^{\pi i} = -1$$

$$\omega_4^3 = e^{2\pi 3 i/4} = e^{6\pi i/4} = e^{\frac{3}{2}\pi i} = -i$$

Example: Find the "8" 8th roots of unity and plot them.

$$\omega_8^0 = 1$$

$$\omega_8^1 = \omega_8 = e^{2\pi i/8} = e^{\frac{\pi}{4} i}$$

$$\omega_8^2 = e^{2\pi 2 i/8} = e^{4\pi i/8} = e^{\frac{\pi}{2} i}$$

$$\omega_8^3 = e^{2\pi 3 i/8} = e^{6\pi i/8} = e^{\frac{3}{4}\pi i}$$

$\vdots$