Math 102 · Lab 1 · Complex Numbers & Linear Combinations

Definition: ① A linear combination of vectors $\vec{u}_1, \vec{u}_2, \vec{u}_3$ and $\vec{u}_4$ in $\mathbb{R}^3$ is any vector of the form

$$\lambda_1 \cdot \vec{u}_1 + \lambda_2 \cdot \vec{u}_2 + \lambda_3 \cdot \vec{u}_3 + \lambda_4 \cdot \vec{u}_4$$

$\lambda_i \cdot \vec{u}_i$ — scalar multiplication

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4 \in \mathbb{R}$ (i.e., $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are numbers)

② The span of the vectors $\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{u}_4 \in \mathbb{R}^3$ is the set

$$\operatorname{span}\{\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{u}_4\} := \{\text{All linear combinations of } \vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{u}_4\}$$
$$:= \{\lambda_1 \cdot \vec{u}_1 + \lambda_2 \cdot \vec{u}_2 + \lambda_3 \cdot \vec{u}_3 + \lambda_4 \cdot \vec{u}_4 \mid \lambda_1, \lambda_2, \lambda_3, \lambda_4 \in \mathbb{R}\}$$

Example 1: Let $\vec{x}, \vec{u}_1, \vec{u}_2, \vec{u}_3$, and $\vec{u}_4$ be vectors in $\mathbb{R}^3$ and suppose that:

$$\boxed{3 \cdot \vec{u}_1 - \tfrac{4}{5} \cdot \vec{u}_4 + 6 \cdot \vec{x} = -\vec{u}_2 + 0 \cdot \vec{u}_3} \quad (*)$$

Write $\vec{x}$ as a linear combination of $\vec{u}_1, \vec{u}_2, \vec{u}_3$, and $\vec{u}_4$.
Is $\vec{x} \in \operatorname{span}\{\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{u}_4\}$?

Solution: Rearrange $(*)$: $6\vec{x} = -3 \cdot \vec{u}_1 - \vec{u}_2 + 0 \cdot \vec{u}_3 + \frac{4}{5} \cdot \vec{u}_4$

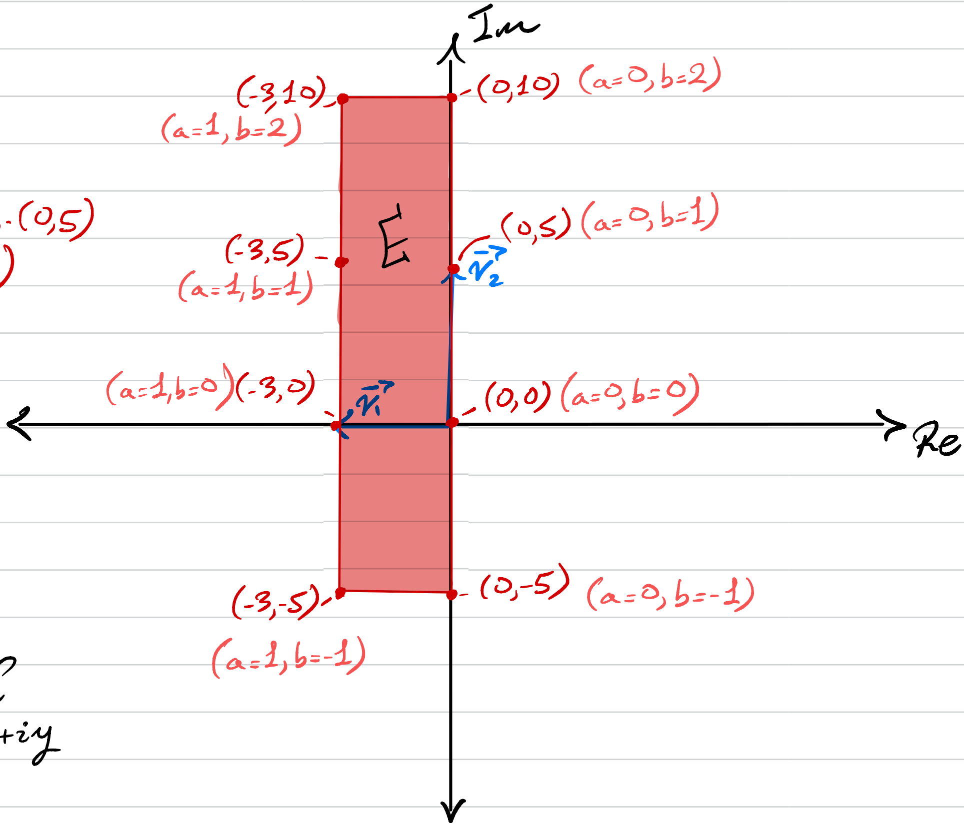
$$\Rightarrow \quad \vec{x} = -\frac{3}{6} \cdot \vec{u}_1 - \frac{1}{6} \vec{u}_2 + 0 \cdot \vec{u}_3 + \frac{4}{30} \cdot \vec{u}_4$$
$$= \lambda_1 \cdot \vec{u}_1 + \lambda_2 \cdot \vec{u}_2 + \lambda_3 \cdot \vec{u}_3 + \lambda_4 \cdot \vec{u}_4, \quad \text{where } \lambda_1 = -\frac{3}{6}, \lambda_2 = -\frac{1}{6}, \lambda_3 = 0, \lambda_4 = \frac{4}{30}$$

Is $\vec{x} \in \operatorname{span}\{\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{u}_4\}$? YES!

Example 2: Sketch the set of all vectors of the form $a\cdot\vec{v_1}+b\cdot\vec{v_2}$, where $0\le a\le 1$, $-1\le b\le 2$, and $\vec{v_1}=(-3,0)$, $\vec{v_2}=(0,5)$ are vectors in $\mathbb{R}^2$.

Solution:

$$a\cdot\vec{v_1}+b\cdot\vec{v_2}$$
$$=a\cdot(-3,0)+b\cdot(0,5)$$
$$=(-3a,5b)$$



$$\mathbb{R}^2 \text{``}=\text{''} \mathbb{C}$$
$$(x,y) \longleftrightarrow x+iy$$

$$E=\{(x,y)\in\mathbb{R}^2 \mid -3\le x\le 0,\ -5\le y\le 10\}$$
$$=\{x+iy\in\mathbb{C} \mid -3\le x\le 0,\ -5\le y\le 10\}$$