APPENDIX

~~Proofs of~~ $q=1$, $q>r$, & backorder system

Backorder system (allowing $q \le r$)

* Zipkin p.188, 277 / H.W. p.183

Key observations

1. inv pos is uniformly dist in $\{r+1, \ldots, r+q\}$
2. demand in interval $[t, t+\tau]$ independent of position at epoch t.
3. Condition on inv pos $= s \in \{r+1, \ldots, r+q\}$
   (Also see p.183 for $(s, 1)$ base-stock w/ backorders).

   if pos $= s$ at time t

   then level at time $t+\tau = s-d$

   with prob $\dfrac{(\lambda\tau)^d}{d!} e^{-\lambda t}$

   so, prob <0 is $\displaystyle\sum_{k>s} \frac{x^k}{k!} e^{-x}$
4. averaging over $s \in \{r+1, \ldots, r+q\}$
   gives answer... EASY, but two ways of writing,
   Can use CCDF or a loss function...

APPENDIX
Proofs of $q=1$, $q>r$, & backorder systems, etc.

$q>r$ | H.W. p.199 / Zipkin p.278

- Is simple.
- Look at "cycles" which begin when $l=r$ and ~~end~~ end immediately before next time $l=r$.
- Obviously q units of demand served each cycle, since $q>r$.
- Demand lost each cycle is loss $= \sum_{k>r} (k-r) \frac{x^k}{k!} e^{-x}$

(really 211 for lost sales.)

$q=1$ | Erlang B loss / H.W. p.204* / Zipkin p.249–250

$(r,1)$ policy $=$ $(r+1)$ "base stock" policy.

Use Markov state transition probability analysis...

"Birth-death" process

States $(l, \underbrace{t_1, t_2, \ldots, t_{r+1}}_{\text{remaining lead times...}})$

$0 \le l \le r+1$

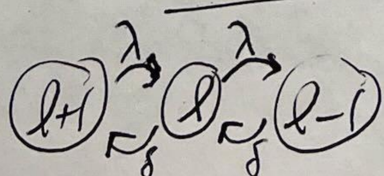
$0 < t_1 < t_2 < \ldots < t_{r+1} \le \tau$

Lead to Integro-diff EQns when constant lead time

But "balance equations" are easier to work with... if know are exponential...

Also, work for any lead-time distribution...

(~~Thakka~~ Lajos Takács, I think...)

Balance eqns: 

APPENDIX

* Poisson pdf: $\frac{(\lambda t)^k}{k!} e^{-\lambda t}$ prob get k

* exponential pdf: $\lambda e^{-\lambda t}$ density at t

  cdf: $1 - e^{-\lambda t}$ prob $< t$

* PASTA: Poisson arrivals see time averages,
  even when they affect the system --

* Little's Law: Queue in equilibrium

  (avg) length of time in queue
  = [(avg) length of queue] × [(avg) time between things exiting/entering]