Assignment 1

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1- $n_3 = n_c = n_2$ β is an propagation constant.

TE => $E_z = 0$, TM => $H_z = 0$ $\Gamma = \frac{P_{core}}{P}$

The confinement factor is described as the ratio of the power in the core and the total power. Therefore the mode confinement factor will decrease as the mode order increases. This is due to the higher amount of field 'distribution' extending into the cladding at higher-order modes so the field is less found in the core. At higher order modes have ~~to~~ smaller β so γ is also smaller and it controls the rate of decay in the cladding. Therefore, as γ is smaller than the rate of decay is slower so more field in cladding leading to smaller confinement factor.

2- $n_1 = 3.6$ $n_2 = 3.57$ λ_0 800 nm.

$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$

$k_0 = \frac{2\pi}{\lambda_0}$ $n_{eff} = \frac{\beta}{k_0}$

$k_0 = \frac{2\pi}{800\times10^{-9}} = 2.5\pi\times10^{6}$

d = thickness
a = radius

$a = \frac{V\lambda}{2\pi}\cdot\frac{1}{\sqrt{n_1^2 - n_2^2}}$

Since that m = # of modes
$m = \frac{2V}{\pi}$

$V = \frac{m\pi}{2}$ => $V = \frac{5}{2}\pi$

$a = \frac{\frac{5\cancel{\pi}}{2}\lambda}{2\cancel{\pi}}\cdot\frac{1}{\sqrt{3.6^2 - 3.57^2}} = \frac{5}{4}(800\times10^{-9}m)\cdot\frac{1}{\sqrt{3.6^2 - 3.57^2}}$

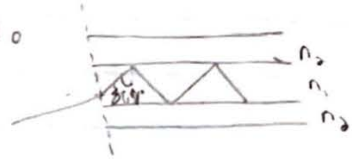
$a = 2.156\times10^{-6}m$

$\boxed{a = 2.156\ \mu m}$ => $\boxed{d = 4.312\ \mu m}$

radius diameter.

3 - $n_1 = 1{,}55$ $n_2 = 1{,}545$ n_0

TE : $\lambda_0 = 1550$ nm



a)

$$\boxed{\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = 1{,}49 \text{ rad} = 85{,}4°}$$

$$\boxed{\text{NA (numerical Aperture)} = n_0 \sin\theta_a = \sqrt{n_1^2 - n_2^2} = 0{,}124}$$

$\theta_{2\,\text{incident}} = 90 - 85{,}4° = 4{,}6°$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$1{,}0 \sin\theta_1 = 1{,}55 \sin(4{,}6°)$ $\Rightarrow$ Acceptance half-angle $\boxed{\theta_1 = \sin^{-1}(1{,}55 \sin(4{,}6°)) = 7{,}14°}$

b) $a_{max} = ?$, $m = 1$ $\Rightarrow$ $V = 2{,}405$ from notes.

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

$$\Rightarrow a = \frac{V\lambda}{2\pi} \frac{1}{\sqrt{n_1^2 - n_2^2}}$$

$$\Rightarrow a = \frac{V\lambda}{2\pi} \frac{1}{\sqrt{n_1^2 - n_2^2}}$$

$$a = \frac{1550 \times 10^{-9}\,\text{m}}{2\pi}(2{,}405)\left(\frac{1}{\sqrt{1{,}55^2 - 1{,}545^2}}\right)$$

$$\boxed{a_{max} = 4{,}769 \times 10^{-6}\,\text{m} = 4{,}769\ \mu\text{m}}$$ $\rightarrow$ radius

$$\boxed{d_{max} = 2a = 9{,}538\ \mu\text{m}}$$ $\rightarrow$ diameter

4. $$\nabla^2 \vec{E} - \varepsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

Faraday's Law $\rightarrow$ $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\vec{B} = \mu \vec{H}$ $\quad \nabla \cdot \vec{B} = 0$ $\quad \vec{D} = \epsilon \vec{E}$

Ampere's Law $\Rightarrow$ $\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

Proof: $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\Rightarrow$ Faraday's Law

$$\therefore \nabla \times \nabla \times \vec{E} = -\frac{\partial (\nabla \times \vec{B})}{\partial t}$$

Ampere's Law : $\nabla \times \vec{H} = \cancelto{0}{\vec{J}} + \frac{\partial \vec{D}}{\partial t}$

$$\Rightarrow \nabla \times \frac{\vec{B}}{\mu} = \frac{\partial \varepsilon \vec{E}}{\partial t}$$

$$\Rightarrow \nabla \times \vec{B} = \mu\varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \nabla \times \vec{E} = -\frac{\partial}{\partial t}\left(\mu\varepsilon \frac{\partial \vec{E}}{\partial t}\right)$$

$$= -\mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

using given property $\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot A) - \nabla^2 A$

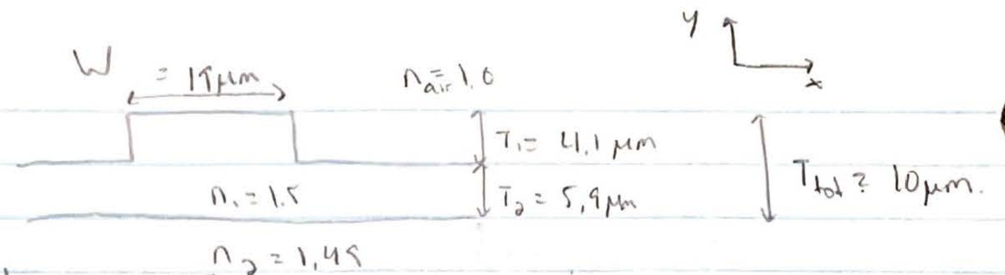
$$\Rightarrow \nabla(\nabla \cdot \vec{E}) - \nabla^2 \cdot \vec{E} = -\mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

We know there is no free electrical charge so

$\nabla \cdot \vec{D} = \nabla \cdot \varepsilon \vec{E} = 0$ so $\nabla \cdot \vec{E} = 0$

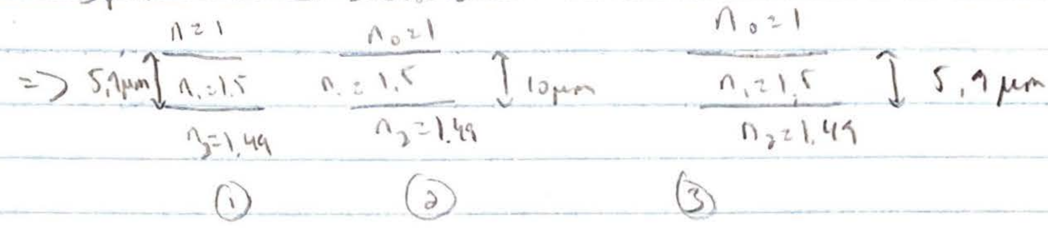
$$\therefore -\nabla^2 \cdot \vec{E} = -\mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} \Rightarrow \boxed{\nabla^2 \cdot \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}}$$

5-



$$V = \frac{2\pi d}{\lambda_0}\sqrt{n_1^2 - n_2^2} \qquad \lambda = 1.55\ \mu m.$$

Need to compose the waveguide in x, y - direction
-> split into 3 sections



(1) $V_1 = \frac{2\pi d_1}{\lambda_0}\sqrt{n_1^2 - n_2^2} = \frac{2\pi(5{,}9\ \mu m)}{1{,}55\ \mu m}\sqrt{1{,}5^2 - 1{,}49^2} = 4{,}136$

(2) $V_2 = \frac{2\pi d_2}{\lambda_0}\sqrt{n_1^2 - n_2^2} = \frac{2\pi(10\ \mu m)}{1{,}55\ \mu m}\sqrt{1{,}5^2 - 1{,}49^2} = 7{,}009$

(3) $V_3 = V_1 = 4{,}136$

assymmetry parameter

$$C = \frac{n_2^2 - n_0^2}{n_1^2 - n_2^2} = \frac{1{,}49^2 - 1{,}0^2}{1{,}5^2 - 1{,}49^2} = 40{,}808$$

find b $\quad V\sqrt{(1-b)} - m\frac{\pi}{2} = \tan^{-1}\sqrt{\frac{b}{1-b}}$

use $C = \frac{1{,}49^2 - 1^2}{1{,}5^2 - 1{,}49^2} = 40{,}81$

Used Matlab to solve eqn. , $n_{eff} = \frac{\beta}{k_0} = \sqrt{n_2^2 + b(n_1^2 - n_2^2)}$

* (1) $b_1 = 0{,}636$, $n_{eff} = 1{,}49$
(2) $b_2 = 0{,}847$ $\quad n_{eff} = 1{,}498$
(3) $b_3 = b_1 = 0{,}636$ $\quad n_{eff} = 1{,}49$

$$V = \frac{2\pi a}{\lambda_0}\sqrt{n_1^2 - n_2^2} = \frac{2\pi(19\times10^{-6})}{1{,}55\times10^{-6}}\sqrt{1{,}498^2 - 1{,}49^2} =$$

$\boxed{V = 5{,}96}$

β refer to *