ECSE 430: Assignment 2

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1 - $n_1 = 1{,}47$ $D_m = 30\ ps/(nm \cdot km)$ $\Delta = \frac{n_1 - n_2}{n_1} = 5.5 \times 10^{-3}$

$$n_1 = 1{,}47 \Rightarrow (1-\Delta)n_1 = n_2 \Rightarrow n_2 = 1{,}462$$

$$\Delta T_{modal} = \frac{L}{c_0}\frac{n_1^2}{n_2}\Delta = \frac{L}{c_0}\frac{n_1^{\cancel{2}}}{n_2}\frac{n_1 - n_2}{\cancel{n_1}} = \frac{L}{c_0}\frac{(n_1-n_2)n_1}{n_2}$$

$$\Delta T_g = D L \Delta\lambda \qquad D = \cancel{D_w} + D_{M} = D_m$$

$$= 30 \times 10^{-12} \times 10^{9} \times 10^{-3}\ s/m^2$$

$$= 30 \times 10^{-6}$$

$$\Delta T_{modal} = \Delta T_g$$

$$\frac{\cancel{L}}{c_0}\frac{(n_1-n_2)n_1}{n_2} = D\cancel{L}\Delta\lambda \Rightarrow \Delta\lambda = \frac{(n_1 - n_2)}{D_m c_0}$$

$$\Delta\lambda = 8{,}988 \times 10^{-7}\ m \Rightarrow \boxed{898.8\ nm}$$

2 - a) $n_1 = 1{,}48$, $\lambda_0 = 1310\ nm$ $V = 2{,}405$ @ single-mode

$\Delta = 0{,}002$

$$V = \frac{\pi}{\lambda_0} d \sqrt{n_1^2 - n_2^2} \Rightarrow n_2 = n_1(1-\Delta) = 1{,}47704$$

$$\Rightarrow d = \frac{V\lambda_0}{\pi}\left(\sqrt{n_1^2 - n_2^2}\right)^{-1} = 10{,}719\ \mu m$$

$\Delta = 0.02$ $n_2 = 1{,}4504$

$$d = \frac{(2{,}405)(1310\ nm)}{\pi}\left(\sqrt{1{,}48^2 - 1{,}45^2}\right)^{-1} = 3{,}4\ \mu m$$

At a larger diameter the Δ is smaller which means it has a higher critical angle. Therefor, with a higher critical angle the diameter is ~~[illegible]~~ greater to so that the TIR can be achieved in propogation.

$V = m\frac{\pi}{2}$

b) $V = \frac{d\pi}{\lambda_0}\sqrt{n_1^2 - n_2^2}$

$d = 5\,\mu m$
$V = 2{,}405$ @ Mode 1

$\lambda_{cutoff} > \frac{d\pi}{V}\sqrt{n_1^2 - n_2^2}$

$n_2 = n_1(1-\Delta) = 1{,}48(1-0{,}004) = 1{,}47408$

$\boxed{\lambda_{cutoff} > 863{,}7\,nm}$

c) $V = \frac{d\pi}{\lambda_0}\sqrt{n_1^2 - n_2^2}$

$n_1 = 1{,}48$ $d = 5\mu m$
$n_2 = 1{,}4726$ $\lambda_0 = 1550nm$

$\boxed{V = 1{,}498}$

$b = 1{,}1428 - \frac{0{,}9960^2}{V^2} \Rightarrow \boxed{b = 0{,}2284}$

$n_{eff} = \sqrt{b \cdot (n_1^2 - n_2^2) + n_2} \Rightarrow \boxed{n_{eff} = 1{,}2198}$

$V_p = f\lambda = \frac{c_0}{n_{eff}} = \boxed{2{,}459 \times 10^8\,m/s}$

d) It is intuitive because we only consider the range of propogation angle, $\theta < \pi/2$ and $\theta < \theta_c$ so we know with these limits that β must be contained by $k_0 n_2 < \beta < k_0 n_1$.

3-a) $E_g = 1{,}30$ $E_{fc} - E_{fv} = 1{,}35$ $\alpha_{int} = 15\,cm^{-1}$ $n_m = 3{,}5$
$L = 500\,\mu m$ $\lambda_0 = 976nm$

$g = \alpha_{int} + \frac{1}{2L}\ln\left(\frac{1}{R_2R_1}\right)$

$R_1 = R_2 = \left|\frac{n_m - 1}{n_m + 1}\right|^2 = \left|\frac{3{,}5 - 1}{3{,}5 + 1}\right|^2 = \frac{25}{81} = 0{,}308$

$g = \alpha_{int} + \frac{1}{2L}\ln\left(\frac{1}{R_2R_1}\right) = 15\,cm^{-1} + \frac{1}{2(500\mu m)}\ln\left(\frac{1}{\left(\frac{25}{81}\right)\left(\frac{25}{81}\right)}\right)$

$= 15\,cm^{-1} + 23{,}5\,cm^{-1} =$

$\boxed{g = 38{,}5\,cm^{-1}}$

b) $v_q = q \frac{c}{2nL}$ $h = 6,626 \times 10^{-34}$ J.s

$$\frac{E_g}{h} < v < \frac{E_{fc} - E_{fv}}{h}$$

$$\frac{1,30 eV \cdot 1,602\times10^{-19} J}{6,626\times10^{-34} J.s} < v < \frac{1,35 eV \cdot 1,602\times10^{-19} J}{6,626\times10^{-34} J.s}$$

$$3,143\times10^{14} Hz < v < 3,2639\times10^{14} Hz$$

$$v = q \cdot \frac{3\cdot10^{8}}{2(3,5)(500\times10^{-6} m)}$$

$$3,143\times10^{14} Hz < 4,39\times10^{13} q < 3,2639\times10^{14} Hz$$

$$3666,926 < q < 3807,9$$ $3807 - 3667 + 1 = \boxed{141 \text{ modes}}$

4- a) $n_1 = 3.3$ $\lambda = 850 nm$ $\eta_{int} = 0,5$ $V_0 = 1V$
$n_2 = 1.0$ (air)

$$\eta_{tot} = \eta_{ext} \eta_{int} \frac{hv}{qV_0}$$

$$\alpha_{mir} = \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right) = \frac{1}{2(200 \mu m)} \ln\left(\frac{1}{(0,7)(0,7)}\right) = 1783,37 m^{-1}$$

$$\eta_{ext} = \frac{1}{n_1(n_1+1)^2} = \frac{1}{3,3(3,3+1)^2} = 0,01638$$

$$\eta_{tot} = \eta_{ext} \eta_{int} \frac{hv}{qV_0} = (0,01638)(0,5) \frac{\left(\frac{3\times10^{8} m/s}{850\times10^{-9} m}\right)(6,626\times10^{-34} J.s)}{(1,602\times10^{-19} C)(1V)}$$

$$\boxed{\eta_{tot} = 0,01196 = 1,196\%}$$

b) $g = \alpha_{int} + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right) =$

$200\,m^{-1} + \frac{1}{2(200\,\mu m)} \ln\left(\frac{1}{(0,7)(0,7)}\right)$

$= 200\,m^{-1} + 1783,37\,m^{-1}$

$g = 1983,37\,m^{-1}$

$g = \Gamma g_m \Rightarrow g_m = \frac{g}{\Gamma} = \frac{1983,37\,m^{-1}}{0,8}$

$\boxed{g_m = 2479,2125\,m^{-1}}$