ECSE 430: Assignment 2

Bryan Jay
260738764

1 - $n_1 = 1,47$ $\quad D_m = 30 \text{ ps}/(\text{nm} \cdot \text{km})$ $\quad \Delta = \frac{n_1 - n_2}{n_1} = 5,1 \times 10^{-3}$

$n_1 = 1,47 \Rightarrow (1-\Delta) n_1 = n_2 \Rightarrow n_2 = 1,462$

$$\Delta T_{modal} = \frac{L}{c_0} \frac{n_1^2}{n_2} \Delta = \frac{L}{c_0} \frac{n_1 \cancel{n_1}}{n_2} \frac{n_1 - n_2}{\cancel{n_1}} = \frac{L}{c_0} \frac{(n_1 - n_2) n_1}{n_2}$$

$$\Delta T_g = D L \Delta\lambda \qquad D = D_w + D_{mg} = D_m$$

$$= 30 \times 10^{-12} \times 10^9 \times 10^{-3} \text{ s/m}^2$$

$$= 30 \times 10^{-6}$$

$$\Delta T_{modal} = \Delta T_g$$

$$\frac{L}{c_0} \frac{(n_1 - n_2) n_1}{n_2} = D \cancel{L} \Delta\lambda \Rightarrow \Delta\lambda = \frac{(n_1 - n_2)}{D_m c_0}$$

$$\Delta\lambda = 8,988 \times 10^{-7} \text{ m} \Rightarrow \boxed{898,8 \text{ nm}}$$

2 - a) $n_1 = 1,48$, $\lambda_0 = 1310 \text{ nm}$ $\quad V = 2,405$ @ single-mode

$\Delta = 0,002$

$$V = \frac{\pi}{\lambda_0} d \sqrt{n_1^2 - n_2^2} \Rightarrow n_2 = n_1 (1 - \Delta) = 1,47704$$

$$\Rightarrow d = \frac{V \lambda_0}{\pi} \left(\sqrt{n_1^2 - n_2^2} \right)^{-1} = 10,719 \, \mu\text{m}$$

$\Delta = 0,02$ $\quad n_2 = 1,4504$

$$d = \frac{(2,405)(1310 \text{ nm})}{\pi} \left(\sqrt{1,48^2 - 1,45^2} \right)^{-1} = 3,4 \, \mu\text{m}$$

At a larger diameter the Δ is smaller which means it has a higher critical angle. Therefor, with a higher critical angle the diameter is greater to so that the TIR can be achieved in propogation.

$V = m \frac{\pi}{2}$

b) $V = \frac{d\pi}{\lambda_0} \sqrt{n_1^2 - n_2^2}$ $\quad d = 5\,\mu m$

$V = 2,405$ @ Mode 1

$\lambda_{cutoff} > \frac{d\pi}{V} \sqrt{n_1^2 - n_2^2}$

$n_2 = n_1(1-\Delta) = 1,48\,(1-0,004) = 1,47408$

$\boxed{\lambda_{cutoff} > 863,7\,nm}$

c) $V = \frac{d\pi}{\lambda_0} \sqrt{n_1^2 - n_2^2}$ $\quad \begin{cases} n_1 = 1,48 & d = 5\mu m \\ n_2 = 1,4726 & \lambda_0 = 1550nm \end{cases}$

$\boxed{V = 1,498}$

$b = \left(1,1428 - \frac{0,9960}{V}\right)^2 \Rightarrow \boxed{b = 0,2284}$

$n_{eff} = \sqrt{b\cdot(n_1^2 - n_2^2) + n_2^2} \Rightarrow \boxed{n_{eff} = 1,474}$

$v_p = f\lambda = \frac{c_0}{n_{eff}} = \boxed{2,035 \times 10^8\ m/s}$

d) It is intuitive because we only consider the range of propagation angle, $\theta < \pi/2$ and $\theta < \theta_c$ so we know with these limits that β must be contained by $k_0 n_2 < \beta < k_0 n_1$.

3-a) $E_g = 1,30$ $\quad E_{fc} - E_{fv} = 1,35$ $\quad \alpha_{int} = 15cm^{-1}$ $\quad n_m = 3,5$

$L = 500\,\mu m$ $\quad \lambda_0 = 976nm$

$g = \alpha_{int} + \frac{1}{2L} \ln\left(\frac{1}{R_2 R_1}\right)$

$R_1 = R_2 = \left|\frac{n_m - 1}{n_{m+1}}\right|^2 = \left|\frac{3,5-1}{3,5+1}\right|^2 = \frac{25}{81} = 0,308$

$g = \alpha_{int} + \frac{1}{2L} \ln\left(\frac{1}{R_2 R_1}\right) = 15cm^{-1} + \frac{1}{2(500\mu m)} \ln\left(\frac{1}{\left(\frac{25}{81}\right)\left(\frac{25}{81}\right)}\right)$

$= 15cm^{-1} + 23,5\,cm^{-1} =$

$\boxed{g = 38,5\,cm^{-1}}$

$$\Delta\nu_{FSR} = \frac{c_0}{2 n_m L} \quad \text{or use} \quad q = \frac{2nL}{\lambda_0}$$

b) $\nu_q = q \frac{c}{2nL}$ $\quad h = 6,626 \times 10^{-34}\ J \cdot s$

$$\frac{E_g}{h} < \nu < \frac{E_{fc} - E_{fv}}{h}$$

$$\frac{1,30\ eV}{4,1356 \times 10^{-15}\ eV \cdot s} < \nu < \frac{1,35\ eV}{4,1356 \times 10^{-15}\ eV \cdot s}$$

$$3,143 \times 10^{14}\ Hz < \nu < 3,2639 \times 10^{14}\ Hz$$

$$\nu = q \cdot \frac{3 \cdot 10^8}{2(3,5)(500 \times 10^{-6}\ m)}$$

$$3,143 \times 10^{14}\ Hz < 8,5714 \times 10^{10}\, q < 3,2634 \times 10^{14}\ Hz$$

$$3666,92 < q < 3807,9 \qquad 3807 - 3667 + 1 = \boxed{141 \text{ modes}}$$

4 - a) $n_1 = 3.3$ $\quad n_2 = 1.0$ (air) $\quad \lambda = 850nm$ $\quad \eta_{int} = 0,5$ $\quad V_0 = 1V$

$$\eta_{tot} = \eta_{ext}\, \eta_{int} \frac{h\nu}{qV_0}$$

$$\alpha_{mir} = \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right) = \frac{1}{2(200\mu m)} \ln\left(\frac{1}{(0,7)(0,7)}\right) = 1783,37\ m^{-1}$$

$$\eta_{ext} = \frac{1}{n_1(n_1+1)^2} = \frac{1}{3,3(3,3+1)^2} = 0,01638$$

$$\eta_{tot} = \eta_{ext}\, \eta_{int} \frac{h\nu}{qV_0} = (0,01638)(0,5) \frac{\left(\frac{3 \times 10^8\ m/s}{850 \times 10^{-9}\ m}\right)(6,626 \times 10^{-34}\ J \cdot s)}{(1,602 \times 10^{-19}\ C)(1\ V)}$$

$$\boxed{\eta_{tot} = 0,01196 = 1,196\%}$$

Hilroy

b) $g = \alpha_{int} + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right) =$

$200\,m^{-1} + \frac{1}{2(200\,\mu m)} \ln\left(\frac{1}{(0,7)(0,7)}\right)$

$= 200\,m^{-1} + 1783,37\,m^{-1}$

$g = 1983,37\,m^{-1}$

$g = \Gamma g_m \Rightarrow g_m = \frac{g}{\Gamma} = \frac{1983,37\,m^{-1}}{0,8}$

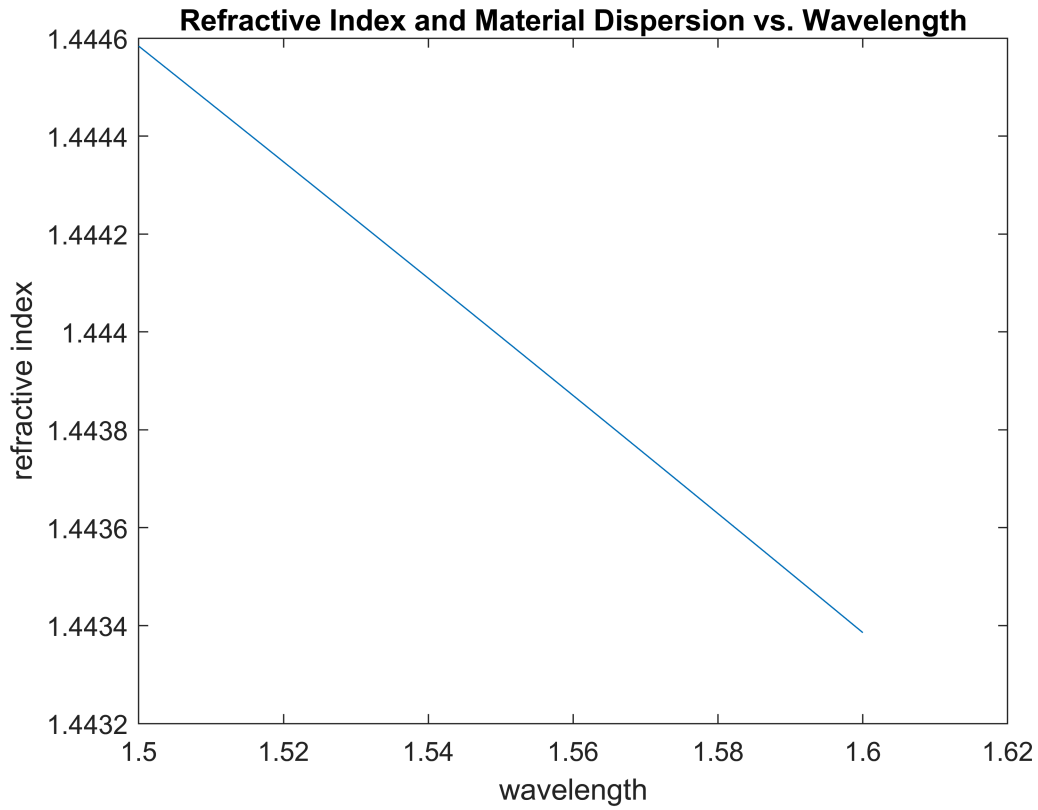
$g_m = 2479,2125\,m^{-1}$

Part 1 - Plot of refractive index vs wavelength

```
% declare the Sellmeier constants
B1 = 0.696;
B2 = 0.408;
B3 = 0.897;

C1 = 4.679e-3;
C2 = 1.351e-2;
C3 = 97.934;

% use the linspace to get the range based off of the wavelength
% need to put the lambda values in micro meters for Sellmeier eqn.
lambda_o = linspace(1500e-3, 1600e-3, 1000)
```

```
lambda_o = 1×1000
    1.5000    1.5001    1.5002    1.5003    1.5004    1.5005    1.5006    1.5007 ···
```

```
% declare matrix for refractive index
n = zeros(size(lambda_o));

% use the sellmeier equation to get n
for i = 1:length(lambda_o)
    n(i) = sqrt(1 + (B1*lambda_o(i)^2)/(lambda_o(i)^2 - C1) + (B2*lambda_o(i)^2)/(lambda_o(i)^2
end

% plot the refractive index and material dispersion as a funct of wavelength below
plot(lambda_o, n)
xlabel('wavelength')
ylabel('refractive index')
title('Refractive Index and Material Dispersion vs. Wavelength')
```



Part 2 - Plot of waveguide dispersion vs wavelength

```
% set constants
delta = 0.003;
d = 2e-6;

% using the delta eqn from Q1 we get n2
n2 = n - delta*n;

% get V using the eqn from Q2
V = (pi.*d.*n2.*sqrt(n.^2 - n2.^2))/lambda_o;

% get b from given eqn
b = (0.5 - 0.1/V)^2;

% Couldnt figure out the rest :(
```

5 - b) $\Delta T = DL \Delta\lambda$

given $\omega = \frac{2\pi c}{\lambda}$

$\Rightarrow \Delta\omega = \frac{2\pi c}{\lambda^2} \Delta\lambda$

$\Delta\lambda = \frac{\lambda^2 \Delta\omega}{2\pi c}$

$\Delta\lambda = \frac{\lambda^2 \Delta 2\pi f}{2\pi c}$

$\Delta\lambda = \frac{\lambda^2 \Delta f}{c}$

$\Rightarrow \Delta f = \frac{1}{T}$

so $\Delta\lambda = \frac{\lambda^2}{cT} = \frac{(1550 \times 10^{-9}\,m)^2}{(3 \times 10^8\,m/s)\left(\frac{1}{50 \times 10^9}\right)}$

$\Delta\lambda = 4,804 \times 10^{-10}\,m = 0,4804\,nm$

I need my graphs D

D = ?

$\Rightarrow$ then $\Delta T = DL \Delta\lambda$

* plug in results here.

c) $\lambda = 1550\,nm$

we need the graph for this but my guess is that we do have anomalous dispersion