$f = \frac{c}{\lambda}$ $\lambda = \frac{c}{f}$

Assignment 3

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1 - The EDFA design is split into 2 stages, the first stage of the amplifier is the pre-amplifier like a setup for the second stage. The second stage is a power booster amplifier that configuration used to boost/amplify power. In the second stage it uses a mirror so that the signals make a double-pass through the stage to increase efficiency and gain.

The conditions on length for the design is different for the stages. L_1 could/should be relatively short since it only needs a small amount of gain. L_2 would be longer b/c it needs more gain but the mirror with double pass configuration helps limit the needed length. The total noise figure of the amplifier is mostly in the first stage. To get a low noise figure we use 980 nm pumping in the stage. As for the second stage we want high power conversion efficiency so we need 1480 nm of pumping.

2-a) $\lambda = 650\,nm$ $N_{in} = 4\times10^{16}$ photon/s $N_e = 6\times10^{16}$ electron/s

$E_{ph} = \frac{hc}{\lambda} = 3,058\times10^{-18}\,eV$

$\frac{P}{E_{ph}} = N_{in} \Rightarrow P = E_{ph} N_{in} = (3,058\times10^{-18}\,eV)(4\times10^{16}\,\text{photon/s})$

$\boxed{P_{in} = 12,23\,mW}$

b) $N_e = \frac{I_p}{q} \Rightarrow I_p = N_e q = (6\times10^{16}\,\text{electr/s})(1,602\times10^{-19}\,C/\text{electron})$

$\boxed{I_p = 9,612\,mA}$

c) $\eta = \frac{\text{electron generation}}{\text{photon incident}} = \frac{Ih\nu}{Pq}$

$\eta = \frac{6\times10^{16}\,\text{electr/s}}{4\times10^{16}\,\text{photon/s}} \Rightarrow \boxed{\eta = 1,5}$

d) $R_d = \frac{\eta\lambda}{1,24}$ or $I_p = R_d P_{in} \Rightarrow$

$\boxed{R_d = 0,786}$

e) There is photocurrent gain because $\eta > 1$ meaning that there was added current between the input and output.

$$f = \frac{c}{\lambda} \quad \lambda = \frac{c}{f}$$

3-a) $v = 2{,}25 \times 10^5\, m/s$ $\quad \eta = 1{,}0 \quad E_g = 1{,}43\, eV$

λ_{cutoff}

$$h\nu < E_g \Rightarrow \frac{hc}{\lambda} < E_g$$

$$\frac{hc}{E_g} < \lambda_{cutoff}$$

$$\frac{(4{,}1357 \times 10^{-15}\, eV/s)(3 \times 10^8\, m/s)}{1{,}43\, eV} < \lambda_{cutoff}$$

$$\boxed{\lambda_c > 867\, nm}$$

b) $$\eta = \frac{P_{abs}}{P_{in}} = 1 - e^{-\alpha W} = 1 - e^{-(10^4\, cm^{-1})\left(\frac{100\, cm}{1\, m}\right)(2 \times 10^{-6}\, m)}$$

$$\boxed{\eta = 0{,}865}$$

$$\tau_{tr} = \frac{W}{v_d} = \frac{2 \times 10^{-6}\, m}{2{,}25 \times 10^5\, m/s} = \boxed{8{,}89\, ps}$$

RC Time : $\tau_{RC} = (R_L + R_s) C_p$

$$C_p = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{(3{,}6)(8{,}85 \times 10^{-12}\, F/m)(400 \times (10^{-6})^2\, m)}{2 \times 10^{-6}} = 2{,}29 \times 10^{-14}\, F$$

$$\tau_{RC} = (1000\, \Omega + 0)\, 2{,}29 \times 10^{-14}\, F = 22{,}9\, ps$$

$$BW = \frac{1}{2\pi(\tau_{tr} + \tau_{RC})} = \frac{1}{2\pi(22{,}9\, ps + 8{,}89\, ps)}$$

$$\boxed{BW = 5{,}0064 \times 10^9\, Hz = 5{,}0064\, GHz}$$

4 - a) Please refer to the attached Matlab code and graph to see the plot.

In the plot of exact vs. approximate we can see that approximately $10 < M < 60$ we are within 10% of the exact and approx. This is clear in the second graph which displays the % diff. between both curves

b)
$$SNR = \frac{(MRP_{in})^2}{2qM^2M^x(RP_{in}+I_d)\Delta f + 4(k_BT/R_L)F_n\Delta f}$$

$$SNR = \frac{(MRP_{in})^2}{BM^x + \sigma_T}$$

where $B = 2q(RP_{in}+I_d)\Delta f$

$\sigma_T = 4(k_BT/R_L)F_n\Delta f$

$$\frac{d\,SNR}{dx} = 0 = -\frac{(RP_{in})^2(BM^x + \frac{\sigma_T}{M^2})'}{(BM^x + \sigma_T/M^2)^2}$$

$$\Rightarrow \frac{dg}{dx} = \frac{-g'}{g^2}$$

$$\frac{d}{dx}\left(BM^x + \frac{\sigma_T}{M^2}\right) = 0$$

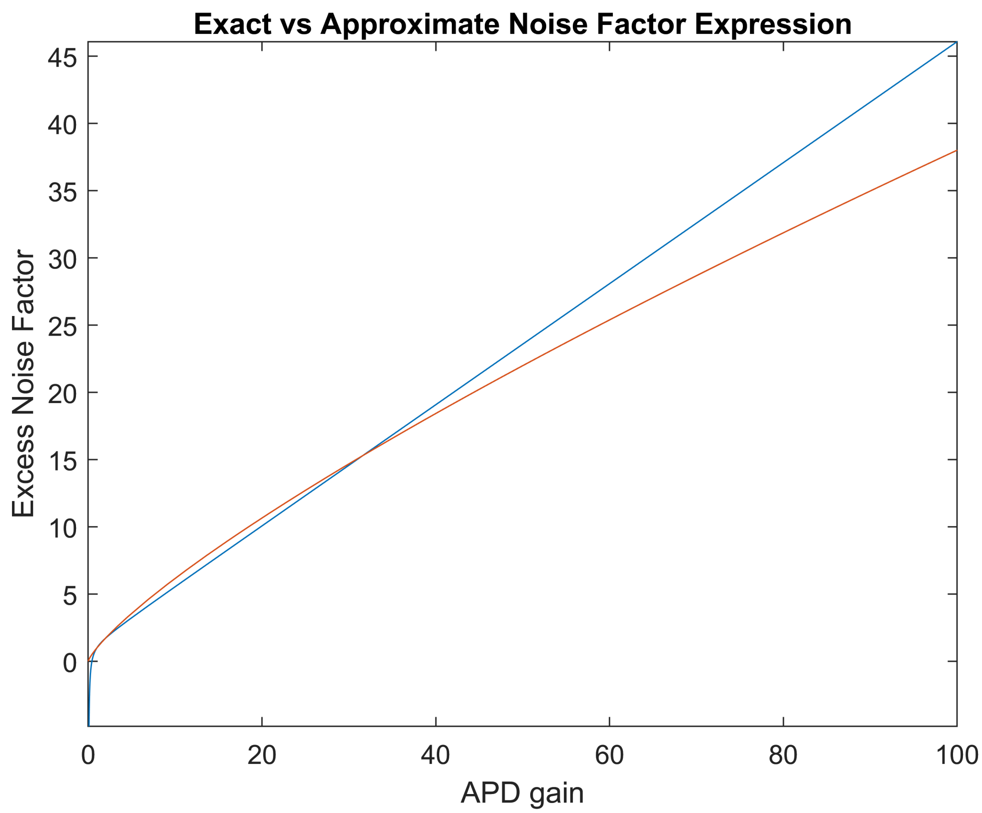
$$\Rightarrow M = \left(\frac{2\sigma_T}{Bx}\right)^{\frac{1}{x+2}}$$

$$M = \left(\frac{8(k_BT/R_L)F_n\Delta f}{2q(RP_{in}+I_d)\Delta f\, x}\right)^{\frac{1}{x+2}}$$

We find $M = 18.15$ for SNR_{max} with the given values. We find SNR is 23.73 dB which according to the graph is a very good approx. of the F_A expression. Since the max is at ≈ 17.

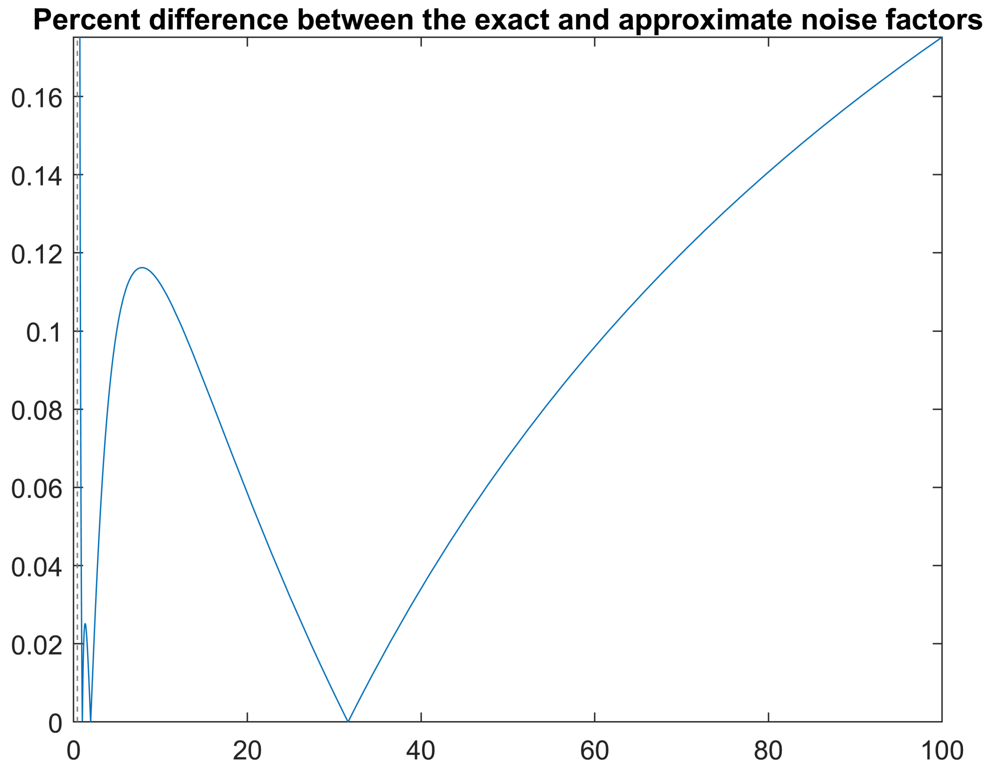
Question 4 - part a

```
syms F_A(M) F_A_apx(M) f_diff(M)

% set the given constants
k_A = 0.45;
x = 0.79;

% create the approx and exact functions
F_A(M) = k_A*M + (1 - k_A) * (2 - 1/M);
F_A_apx(M) = M^x;

% plots
fplot(F_A, [0, 100]);

hold on

fplot(F_A_apx,[0, 100]);
title('Exact vs Approximate Noise Factor Expression')
ylabel('Excess Noise Factor')
xlabel('APD gain')

hold off
```




```
% plot of the % difference between the functions
f_diff(M) = abs(F_A(M) - F_A_apx(M)) / F_A(M);

fplot(f_diff(M), [0,100])
```

```
title('Percent difference between the exact and approximate noise factors')
```



part B

```
syms M_g(x) SNR(M) F_A

% set the constants
R = 1; % A/W
I_d = 10e-9; % A
F_n = 2.5;
T = 300; % K
R_L = 50; % ohms
delta_f = 2e9; % Hz
P_in = 2e-6; % W
k_B = 1.38e-23;
q = 1.602e-19; % C

% create the M function derrived in the notes to find max SNR
M_g(x) = ((8 * F_n * delta_f * (k_B * T / R_L)) / (2 * q * x * delta_f * (R * P_in + I_d)))^(1

% M-value for max SNR
APD_gain = vpa(M_g(0.79))
```

APD_gain = 18.154177880867923023991642885102

```
% SNR function
SNR(M) = 10 * log10((M * R * P_in)^2 / ((2 * q * M^2 * (k_A*M + (1 - k_A) * (2 - 1/M)) * delta_
```

```
% SNR max value based on M-value
SNR_M = vpa(SNR(APD_gain))
```

SNR_M = 23.735395344992784343534522900102

```
% plot of SNR
fplot(SNR(M), [0,100])
title('SNR vs ADP gain')
ylabel('SNR')
xlabel('APD gain')
```