ECSE 430: Assignment 2

Bryan Jay
260738764

1 - $n_1 = 1{,}47$ $D_m = 30\ ps/(nm \cdot km)$ $\Delta = \frac{n_1 - n_2}{n_1} = 5{,}5 \times 10^{-3}$

$n_1 = 1{,}47 \Rightarrow (1-\Delta) n_1 = n_2 \Rightarrow n_2 = 1{,}462$

$$\Delta T_{modal} = \frac{L}{C_0} \frac{n_1^2}{n_2} \Delta = \frac{L}{C_0} \frac{n_1^{\cancel{2}}}{n_2} \frac{n_1 - n_2}{\cancel{n_1}} = \frac{L}{C_0} \frac{(n_1 - n_2) n_1}{n_2}$$

$\Delta T_g = D L \Delta\lambda$

$D = D_w + D_{M} = D_m$
$= 30 \times 10^{-12} \times 10^9 \times 10^{-3}\ s/m^2$
$= 30 \times 10^{-6}$

$\Delta T_{modal} = \Delta T_g$

$$\frac{\cancel{L}}{C_0} \frac{(n_1 - n_2) n_1}{n_2} = D_\lambda \cancel{L} \Delta\lambda \Rightarrow \Delta\lambda = \frac{(n_1 - n_2)}{D_m C_0}$$

$\Delta\lambda = 8{,}988 \times 10^{-7}\ m \Rightarrow \boxed{898{,}8\ nm}$

2 - a) $n_1 = 1{,}48$, $\lambda_0 = 1310\ nm$ $V = 2{,}405$ @ single-mode

$\underline{\Delta = 0{,}002}$

$$V = \frac{\pi}{\lambda_0} d \sqrt{n_1^2 - n_2^2} \Rightarrow n_2 = n_1(1-\Delta) = 1{,}47704$$

$$\Rightarrow d = \frac{V \lambda_0}{\pi} \left(\sqrt{n_1^2 - n_2^2}\right)^{-1} = 10{,}719\ \mu m$$

$\underline{\Delta = 0{,}02}$ $n_2 = 1{,}4504$

$$d = \frac{(2{,}405)(1310\ nm)}{\pi} \left(\sqrt{1{,}48^2 - 1{,}45^2}\right)^{-1} = 3{,}4\ \mu m$$

At a larger diameter the Δ is smaller which means it has a higher critical angle. Therefor, with a higher critical angle the diameter is ~~[illegible]~~ greater to so that the TIR can be achieved in propogation.

$V = m\frac{\pi}{2}$

b) $V = \frac{d\pi}{\lambda_0}\sqrt{n_1^2 - n_2^2}$

$d = 5\,\mu m$
$V = 2,405$ @ Mode 1

$\lambda_{cutoff} > \frac{d\pi}{V}\sqrt{n_1^2 - n_2^2}$

$n_2 = n_1(1-\Delta) = 1,48\,(1-0,004) = 1,47408$

$\boxed{\lambda_{cutoff} > 863,7\,nm}$

c) $V = \frac{d\pi}{\lambda_0}\sqrt{n_1^2 - n_2^2}$

$n_1 = 1,48$ $\quad d = 5\,\mu m$
$n_2 = 1,4726$ $\quad \lambda_0 = 1550\,nm$

$\boxed{V = 1,498}$

$b = \left(1,1428 - \frac{0,9960}{V}\right)^2 \Rightarrow \boxed{b = 0,2284}$

$n_{eff} = \sqrt{b\cdot(n_1^2 - n_2^2) + n_2^2} \Rightarrow \boxed{n_{eff} = 1,474}$

$V_p = f\lambda = \frac{c_0}{n_{eff}} = \boxed{2,035 \times 10^8\,m/s}$

d) It is intuitive because we only consider the range of propogation angle, $\theta < \pi/2$ and $\theta < \theta_c$ so we know with these limits that β must be contained by $k_0 n_2 < \beta < k_0 n_1$.

3-a) $E_g = 1,30$ $\quad E_{f_c} - E_{f_v} = 1,35$ $\quad \alpha_{int} = 15\,cm^{-1}$ $\quad n_m = 3,5$
$L = 500\,\mu m$ $\quad \lambda_0 = 976\,nm$

$g = \alpha_{int} + \frac{1}{2L}\ln\left(\frac{1}{R_2 R_1}\right)$

$R_1 = R_2 = \left|\frac{n_m - 1}{n_m + 1}\right|^2 = \left|\frac{3,5-1}{3,5+1}\right|^2 = \frac{25}{81} = 0,308$

$g = \alpha_{int} + \frac{1}{2L}\ln\left(\frac{1}{R_2 R_1}\right) = 15\,cm^{-1} + \frac{1}{2(500\,\mu m)}\ln\left(\frac{1}{\left(\frac{25}{81}\right)\left(\frac{25}{81}\right)}\right)$

$= 15\,cm^{-1} + 23,5\,cm^{-1} =$

$\boxed{g = 38,5\,cm^{-1}}$

$$\Delta \nu_{FSR} = \frac{c_o}{2 n_m L}$$ ~~or~~ ~~use~~ $\boxed{q = \frac{2nL}{\lambda_0}}$

$$\frac{qc}{2nL} = \frac{c}{\lambda_0}$$

~~q = 3586~~

b) $\nu_q = q \frac{c}{2nL}$ $h = 6{,}626 \times 10^{-34}\ J \cdot s$

$$\frac{E_g}{h} < \nu < \frac{E_{fc} - E_{fv}}{h}$$

$$\frac{1{,}30\ eV}{4{,}1356 \times 10^{-15}\ eV \cdot s} < \nu < \frac{1{,}35\ eV}{4{,}1356 \times 10^{-15}\ eV \cdot s}$$

$$3{,}143 \times 10^{14}\ Hz < \nu < 3{,}2639 \times 10^{14}\ Hz$$

$$\nu = q \cdot \frac{3 \cdot 10^8}{2(3{,}5)(500 \times 10^{-6}\ m)}$$

$$3{,}143 \times 10^{14}\ Hz < 8{,}5714 \times 10^{10} q < 3{,}2639 \times 10^{14}\ Hz$$

$$3666{,}92 < q < 3807{,}9$$ $3807 - 3667 + 1 = \boxed{141 \text{ modes}}$

4- a) $n_1 = 3.3$ $\lambda = 850\ nm$ $\eta_{int} = 0{,}5$ $V_0 = 1V$
$n_2 = 1.0$ (air)

$$\eta_{tot} = \eta_{ext} \eta_{int} \frac{h\nu}{qV_0}$$

$$\alpha_{mir} = \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right) = \frac{1}{2(200 \mu m)} \ln\left(\frac{1}{(0{,}7)(0{,}7)}\right) = 1783{,}37\ m^{-1}$$

$$? \ \eta_{ext} = \frac{1}{n_1(n_1+1)^2} = \frac{1}{3.3(3.3+1)^2} = 0{,}01638$$

$$\eta_{tot} = \eta_{ext} \eta_{int} \frac{h\nu}{qV_0} = (0{,}01638)(0{,}5) \frac{\left(\frac{3 \times 10^8\ m/s}{850 \times 10^{-9}\ m}\right)(6{,}626 \times 10^{-34}\ J \cdot s)}{(1{,}602 \times 10^{-19}\ C)(1V)}$$

$$\boxed{\eta_{tot} = 0{,}01196 = 1{,}196\%}$$

Hilroy

b) $g = \alpha_{int} + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right) =$

$$200\, m^{-1} + \frac{1}{2(200\, \mu m)} \ln\left(\frac{1}{(0,7)(0,7)}\right)$$

$$= 200\, m^{-1} + 1783,37\, m^{-1}$$

$$g = 1983,37\, m^{-1}$$

$$g = \Gamma g_m \Rightarrow g_m = \frac{g}{\Gamma} = \frac{1983,37\, m^{-1}}{0,8}$$

$$\boxed{g_m = 2479,2125\, m^{-1}}$$