Assignment 1

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1-a) $L_{kj} = \dfrac{A_{kj} - \sum_{m=1}^{j-1} (L_{km} U_{mj})}{u_{jj}} \quad k = (j+1) \dots n$

$U_{ik} = A_{ik} - \sum_{m=1}^{i-1} (L_{im} U_{mk}) \quad k = i \dots n$

$$U = \begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 2 & 2 & 8 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 2 & 2 & 2 & 1 & 0 \\ 4 & 1 & 2 & 2 & 1 \end{bmatrix}$$

Row 1

$u_{11} = a_{11}$

$u_{12} = a_{12}$

$u_{13} = a_{13}$

$u_{14} = a_{14}$

$a_{15} = a_{15}$

1, 2, 3, 4

Row 2

$u_{22} = a_{22} - (l_{21} u_{12}) = 9 - (2)(4) = 1$

$u_{23} = a_{23} - (l_{21} u_{13}) = 14 - (2)(6) = 2$

$u_{24} = a_{24} - (l_{21} u_{14}) = 8 - (2)(2) = 4$

$u_{25} = a_{25} - (l_{21} u_{15}) = 8 - (2)(1) = 6$

Row 3

$u_{33} = a_{33} - (l_{31} u_{13} + l_{32} u_{23})$

$= 12 - ((1)(6) + (2)(2)) = 12 - 6 - 4 = 2$

$u_{34} = a_{34} - (l_{31} u_{14} + l_{32} u_{24})$

$= 12 - ((1)(2) + (2)(4)) = 12 - 2 - 8 = 2$

$u_{35} = a_{35} - (l_{31} u_{15} + l_{32} u_{25})$

$= 21 - ((1)(1) - (2)(6)) = 21 - 1 - 12 = 8$

Column 1

$l_{21} = \dfrac{a_{21}}{u_{11}} = 2$

$l_{31} = \dfrac{a_{31}}{u_{11}} = \dfrac{2}{2} = 1$

$l_{41} = \dfrac{a_{41}}{u_{11}} = \dfrac{4}{2} = 2$

$l_{51} = \dfrac{a_{51}}{u_{11}} = \dfrac{8}{2} = 4$

Column 2

$l_{32} = \dfrac{a_{32} - l_{31} u_{12}}{u_{22}} = \dfrac{6 - (1)(4)}{1} = 2$

$l_{42} = \dfrac{a_{42} - l_{41} u_{12}}{u_{22}} = \dfrac{10 - (2)(4)}{1} = 2$

$l_{52} = \dfrac{a_{52} - l_{51} u_{12}}{u_{22}} = \dfrac{17 - (4)(4)}{1} = 1$

Column 3

$l_{43} = \dfrac{a_{43} - (l_{41} u_{13} + l_{42} u_{23})}{u_{33}}$

$= \dfrac{20 - (2)(6) - (2)(2)}{2} = \dfrac{20 - 12 - 4}{2} = 2$

$l_{53} = \dfrac{a_{53} - l_{51} u_{13} - l_{52} u_{23}}{u_{33}}$

$= \dfrac{30 - (4)(6) - (1)(2)}{2} = \dfrac{30 - 24 - 2}{2} = 2$

Row 4

$u_{44} = a_{44} - l_{41}u_{14} - l_{42}u_{24} - l_{43}u_{34}$

$= 17 - (2)(2) - (2)(4) - (2)(2)$

$= 17 - 4 - 8 - 4 = 17 - 16 = 1$

$u_{45} = a_{45} - l_{41}u_{15} - l_{42}u_{25} - l_{43}u_{35}$

$= 32 - (7)(1) - (2)(6) - (2)(8)$

$= 32 - 2 - 12 - 16 = 2$

Column 4

$u_{54} = \dfrac{a_{54} - (l_{51}u_{14} + l_{52}u_{24} + l_{53}u_{34})}{u_{44}}$

$= \dfrac{18 - (4)(2) - (1)(4) - (2)(2)}{1}$

$= 18 - 8 - 4 - 4 = 2$

Row 5

$u_{55} = a_{55} - l_{51}u_{15} - l_{52}u_{25} - l_{53}u_{35} - l_{54}u_{45}$

$= 34 - (4)(1) - (1)(6) - (2)(8) - (2)(2) = 34 - 4 - 6 - 16 - 4 = 4$

b)

$$A_2 = \begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 4 & 9 & 14 & 8 & 8 \\ 2 & 6 & 12 & 12 & 21 \\ 4 & 10 & 20 & 17 & 32 \\ 8 & 17 & 30 & 18 & 34 \end{bmatrix}$$

$a_{22} - a_{21} \times a_{12}$

$\Rightarrow$ Pivot

$$\begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 2 & 9 & 14 & 8 & 8 \\ 1 & 6 & 12 & 12 & 21 \\ 2 & 10 & 20 & 17 & 32 \\ 4 & 17 & 30 & 18 & 34 \end{bmatrix}$$

$\Rightarrow a_{ij} - a_{i1} \times a_{1j}$

$$\begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 2 & 1 & 2 & 4 & 6 \\ 1 & 2 & 6 & 10 & 20 \\ 2 & 2 & 8 & 13 & 30 \\ 4 & 1 & 6 & 10 & 30 \end{bmatrix}$$

$a_{22} - a_{21} \times a_{12} = 9 - 2 \times 4 = 1$

a_{32} $6 - 1 \times 4 = 2$

$10 - 2 \times 4 = 2$

$17 - 4 \times 4 = 1$

a_{23} $14 - 2 \times 6 = 2$

$12 - 1 \times 6 = 6$

$20 - 2 \times 6 = 8$

$30 - 4 \times 6 = 6$

a_{24} $8 - 2 \times 2 = 4$

$12 - 1 \times 2 = 10$

$17 - 2 \times 2 = 13$

$18 - 4 \times 2 = 10$

a_{25} $8 - 2 \times 1 = 6$

$21 - 1 = 20$

$32 - 2 = 30$

$34 - 4 = 30$

2) Pivot

$$\begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 2 & 1 & 2 & 4 & 6 \\ 1 & 2 & 6 & 10 & 20 \\ 2 & 2 & 8 & 13 & 30 \\ 4 & 1 & 6 & 10 & 30 \end{bmatrix}$$

a_{33}
$6 - 2 \times 2 = 2$
$8 - 2 \times 2 = 4$
$6 - 1 \times 2 = 4$
a_{34}
$10 - 2 \times 4 = 2$
$13 - 2 \times 4 = 5$
$10 - 1 \times 4 = 6$
a_{35}
$20 - 2 \times 6 = 8$
$30 - 2 \times 6 = 18$
$30 - 1 \times 6 = 24$

2) $a_{ij} - a_{i2} \times a_{2j}$

$$\begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 2 & 1 & 2 & 4 & 6 \\ 1 & 2 & (2) & 2 & 8 \\ 2 & 2 & 4 & 5 & 18 \\ 4 & 1 & 4 & 6 & 24 \end{bmatrix}$$

2) Pivot

$a_{ij} - a_{i3} \times a_{3j}$

$$\begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 2 & 1 & 2 & 4 & 6 \\ 1 & 2 & 2 & 2 & 8 \\ 2 & 2 & 2 & 1 & 2 \\ 4 & 1 & 2 & 2 & 8 \end{bmatrix}$$

a_{44}: $5 - 2 \times 2 = 1$
$6 - 2 \times 2 = 2$
a_{45}: $18 - 2 \times 8 = 2$
$24 - 2 \times 8 = 8$

2)
$$\begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 2 & 1 & 2 & 4 & 6 \\ 1 & 2 & 2 & 2 & 8 \\ 2 & 2 & 2 & 1 & 2 \\ 4 & 1 & 2 & 2 & -12 \end{bmatrix}$$

$a_{ij} - a_{iu} \times a_{uj}$
$8 - 2 \cdot 2 = 4$

2) $L =$
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 2 & 2 & 2 & 1 & 0 \\ 4 & 1 & 2 & 2 & 1 \end{bmatrix}$$

$U =$
$$\begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 2 & 2 & 8 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

c) The main similarities are that the diagonal in both L matrices are all 1 and that the first row in U is the same as each other and A matrix row 1.

The difference is that the gaussian method uses a pivot method on submatrices so simplify and it is quite recursive calculation. While the doolittle method is more iterative as we proceed down each row and column in the matrix