Assignment 5 part a

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1-a) $\dot{x}_n = \frac{x_{n+1} - x_n}{h}$ poles: ~~-5~~, ~~-3~~, -2, -7, -8

=> use test eqn.: $\dot{x} = \lambda x$

Apply Fwd Euler $x_{n+1} = x_n + h\dot{x}_n$

$$x_{n+1} = x_n + h\lambda x_n$$
$$= x_n(1 + h\lambda)$$

=> $t = nh$ -> $x_n = (1 + h\lambda)^n x_0$

$s = -5$

$$|1 + h\lambda| < 1$$
$$-1 < 1 + h\lambda < 1$$
$$-2 < h\lambda < 0$$
$$-2 < -5(h) < 0$$
$$0 < h < \frac{2}{5} = 0.4$$
$$h < 0.4$$

$s = -3$

$$|1 + h\lambda| < 1$$
$$-2 < h\lambda < 0$$
$$0 < h < \frac{2}{3}$$
$$h < 0.\overline{67}$$

$s = -2$

$$|1 + h\lambda| < 1$$
$$-2 < h\lambda < 0$$
$$0 < h < 1$$
$$h < 1$$

$s = -7$

$$|1 + h\lambda| < 1$$
$$-2 < h\lambda < 0$$
$$0 < h < \frac{2}{7}$$
$$h < 0.286$$

$s = -8$

$$|h\lambda + 1| < 1$$
$$-2 < h\lambda < 0$$
$$0 < h < \frac{2}{8} = \frac{1}{4} = 0.25$$

$$h < 0.25$$

$\boxed{h < 0.25 \text{ for stability to be reached.}}$

$s = \cancel{-5}, \cancel{-7}, -3, -8, -2$

b) Backward Euler : $x_{n+1} = x_n + h\dot{x}_{n+1}$

$$\Downarrow$$

$$x_{n+1} = x_n + h\lambda x_{n+1}$$

$$x_{n+1} - h\lambda x_{n+1} = x_n$$

$$x_{n+1} = (1-\lambda h)^{-1} x_n$$

$$|1 - h\lambda| > 1$$

$\Rightarrow$ $1 - h\lambda > 1$ or $\cancel{1 - h\lambda < -1}$ $\rightarrow h \not< 0$

$s = -5$

$1 - h(-5) > 1$ or $1 - h(-5) < -1$

$1 + 5h > 1$ or $h(5) < -2$

$5h > 0$

$h > 0$

$s = -7$

$1 + 7h > 1$

$7h > 0$

$h > 0$

$s = -3$

$h > 0$

$s = -8$

$h > 0$

$s = -2$

$h > 0$

For all poles we get $h > 0$, since all poles are < 0. If a pole is > 0 then the step size h will have a different condition

$$h > 0$$

$s = -5, -3, -2, -7, -8$ $\quad z = \frac{5}{2}h$

c) TR rule: $x_{n+1} = x_n + \frac{h}{2}(\dot{x}_n + \dot{x}_{n+1})$

$$x_{n+1} = x_n + \frac{h}{2}(\lambda x_n + \lambda x_{n+1})$$

$$\Rightarrow x_{n+1} = \left(1 - \frac{h\lambda}{2}\right)^{-1}\left(1 + \frac{h\lambda}{2}\right) x_n$$

$$\left| \frac{1 + \frac{h\lambda}{2}}{1 - \frac{h\lambda}{2}} \right| < 1$$

$s = -5$

$$-1 < \frac{1 + \frac{5h}{2}}{1 - \frac{5}{2}h} < 1 \Rightarrow \left|1 + \frac{5h}{2}\right|^2 < \left|1 - \frac{5}{2}h\right|^2$$

$$\cancel{1} + 5h + \cancel{\frac{25h}{4}} < \cancel{1} - 5h + \cancel{\frac{25h}{4}}$$

$$10h < 0$$

$$h < 0$$

Condition for stability is always: $Re[h\lambda] < 0$

- Since all poles are negative and real then we will always get $h < 0$. Since All poles are $Re(\lambda) < 0$ then we know that the TR rule is stable for all values of h

$h > 0$

2 - $x_n = x_{n-1} + \frac{5h}{12}\dot{x}_n + \frac{8h}{12}\dot{x}_{n-1} + \frac{h}{12}\dot{x}_{n-2}$

MNA: $Gx(t) + C\dot{x}(t) = b(t)$

$$\Rightarrow \quad Gx_n + C\dot{x}_n = b_n \quad \Rightarrow \quad \frac{Gx_n - b_n}{C} = \dot{x}_n$$

$$Gx_{n-1} + C\dot{x}_{n-1} = b_{n-1} \quad \Rightarrow \quad \frac{Gx_{n-1} - b_{n-1}}{C} = \dot{x}_{n-1}$$

$$Gx_{n-2} + C\dot{x}_{n-2} = b_{n-2} \quad \Rightarrow \quad \frac{Gx_{n-2} - b_{n-2}}{C} = \dot{x}_{n-2}$$

$$x_n = x_{n-1} + \frac{5h}{12}\dot{x}_n + \frac{8h}{12}\dot{x}_{n-1} + \frac{h}{12}\dot{x}_{n-2}$$

$$\Rightarrow \frac{12}{h}(x_n - x_{n-1}) = 5\dot{x}_n + 8\dot{x}_{n-1} + \dot{x}_{n-2}$$

$$\Rightarrow \frac{12}{h}(x_n - x_{n-1}) = 5\left(\frac{Gx_n - b_n}{C}\right) + 8\left(\frac{Gx_{n-1} - b_{n-1}}{C}\right) + \frac{Gx_{n-2} - b_{n-2}}{C}$$

$$\frac{12C}{h}(x_n - x_{n-1}) = 5x_nG - 5b_n + 8Gx_{n-1} - 8b_{n-1} + Gx_{n-2} - b_{n-2}$$

$$\therefore \boxed{5b_n + 8b_{n-1} + b_{n-2} = x_n\left(5G - \frac{12C}{h}\right) + x_{n-1}\left(8G - \frac{12C}{h}\right) + Gx_{n-2}}$$

3 - AC @ 1000 pts, sensitivity pts at r o/p nodes wrt 3 param.

	Sparse	L/U fact	f/b sub
a) Pertubation	1	4000	4000
b) Differentiation	1	1000	4000
c) Adjoint	1	1.000	6000

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All methods will have 1 sparse ordering.

a) Perturbation: L/U and f/b is 4k to account for Vout and the 3 points @ 1k f points. Since, ~~they~~ this method calculates for all ~~methods~~ output nodes there aren't any additional.

b) Differentiation: $\frac{dA}{dx}$ term can be reused so we only need 1k L/U factorizations. Therefore, we only add do multiple calculations for f/b sub. and it calculates it for all output nodes

c) Adjoint: This method one calculates for 1 o/p node so we must do an additional f/b for the other nodes.