

Operations research

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Project 5: Linear Programming Problems
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Index

1 Problem 1

2 Problem 2

3 Problem 3

4 Problem 4

5 Problem 5

6 Problem 6

7 Problem 7

8 Problem 8

9 Problem 9

10 Problem 10

Problem 1

Shale Oil, located on the island of Aruba, has a capacity of **600,000** barrels of crude oil per day. The final products from the refinery include two types of unleaded gasoline; regular and premium. The refining process encompasses four stages :

- (1) The pure crude flows through a distillation tower that produces a feedstock.
- (2) The feedstock output breaks up into two paths; the first path involves feedstock flowing into a cracker unit that refines the mixture into a gasoline stock. The second path has a portion of the feedstock flowing into the blender unit.
- (3) The gasoline stock (from the cracker unit) feeds into the blender.
- (4) The blender unit produces the final product, **regular or premium gasoline**.



Problem 1

cont

Both the regular and premium gasoline can be produced from either the feedstock or the gasoline stock during the blending process, although at different production costs. The company estimates that the net profit per barrel of regular gasoline is **6.20 \$** from feedstock and **8.80 \$** from gasoline stock. The corresponding profit values for the premium are **11.40 \$** from the feedstock and **10.30 \$** from the gasoline stock. According to design specifications, it takes five barrels of crude oil to produce one barrel of feedstock. The cracker units cannot use more than **30,000** barrels of feedstock per day. All remaining feedstock is used directly in the blender unit to produce the end product gasoline. The demand limits for regular and premium gasoline are **90,000** and **60,000 barrels per day**.

Problem 1 : decision variables

- X1 : Regular gasoline from crude oil
- X2 : Premium gasoline from crude oil
- X3 : Regular gasoline coming from the disintegration
- X4 : Premium gasoline from the disintegration

Problem 1 : Objective Function

Maximize

$$Z = 7.7X_1 + 10.40X_2 + 5.20X_3 + 12.3X_4$$

Problem 1 : Constraints

$$X_3 + X_4 \leq 40000$$

$$X_1 + X_3 \leq 80000$$

$$X_2 + X_4 \leq 50000$$

$$5X_1 + 5X_2 + 5X_3 + 5X_4 \leq 600000$$

$$X_1, X_2, X_3, X_4 \geq 0$$

Problem 1 : Full Model

Maximize

$$Z = 7.7X_1 + 10.40X_2 + 5.20X_3 + 12.3X_4$$

subject to :

$$X_3 + X_4 \leq 40000$$

$$X_1 + X_3 \leq 80000$$

$$X_2 + X_4 \leq 50000$$

$$5X_1 + 5X_2 + 5X_3 + 5X_4 \leq 600000$$

$$X_1, X_2, X_3, X_4 \geq 0$$

Problem 1 : Final Solution

$$X_1 = 70000$$

$$X_2 = 10000$$

$$X_3 = 0$$

$$X_4 = 40000$$

$$Z = 1135000$$

Index

- 1 Problem 1
- 2 Problem 2**
- 3 Problem 3
- 4 Problem 4
- 5 Problem 5

- 6 Problem 6
- 7 Problem 7
- 8 Problem 8
- 9 Problem 9
- 10 Problem 10

Problem 2

Hawaii Sugar Company produces brown sugar, processed sugar (white), powdered sugar and molasses with sugar cane syrup. The company buys 4000 tons of syrup a week and has a contract to deliver a minimum of 25 tons per week of each type of sugar. The production process starts with making brown sugar and molasses with the syrup. One ton of syrup produces 0.3 tons of brown sugar and 0.1 tons of molasses. Afterwards, the white sugar is made by processing brown sugar.



Problem 2

cont

It requires 0.1 tons of brown sugar to produce 0.8 tons of white sugar. Finally, powdered sugar is made from white sugar by means of a special milling process, which has 95 % conversion efficiency (1 ton of white sugar produces .95 tons of powdered sugar). The profits per ton of brown sugar, white sugar, powdered sugar and molasses are 150, 200, 230 and 35 \$, respectively.

Problem 2 : decision variables

X_1 = Tons brown sugar

X_2 = Tons white sugar

X_3 = Tons of powdered sugar

X_4 = Tons molasses

Problem 2 : Objective Function

Maximize

$$Z = 150X_1 + 200X_2 + 230X_3 + 35X_4$$

Problem 2 : Constrains

$$X1 \geq 25$$

$$X2 \geq 25$$

$$X3 \geq 25$$

$$X4 \geq 25$$

$$X4 \leq 400$$

$$0.76X1 + 0.95X2 + X3 \leq 912$$

Problem 2 : Full Model

Maximize

Maximize

$$Z = 150X_1 + 200X_2 + 230X_3 + 35X_4$$

subject to :

$$X_1 \geq 25$$

$$X_2 \geq 25$$

$$X_3 \geq 25$$

$$X_4 \geq 25$$

$$X_4 \leq 400$$

$$0.76X_1 + 0.95X_2 + X_3 \leq 912$$

Problem 2 : Solution with Lingo

The screenshot displays the Lingo software interface with two windows open: 'Lingo Model - #2.lng*' and 'Solution Report - #2.lng'.

Lingo Model - #2.lng*:

```

1 MAX = 150*X1 + 200*X2 + 230*X3 + 35*X4;
2
3 X1 >= 25;
4 X2 >= 25;
5 X3 >= 25;
6 X4 >= 25;
7 X4 <= 400;
8
9 0.76*X1 + 0.95*X2 + X3 <= 912;
10
11

```

Solution Report - #2.lng:

Global optimal solution found.

Objective value:	222677.5
Infeasibilities:	0.000000
Total solver iterations:	0
Elapsed runtime seconds:	0.01

Model Class: LP

Total variables:	4
Nonlinear variables:	0
Integer variables:	0
Total constraints:	7
Nonlinear constraints:	0
Total nonzeros:	12
Nonlinear nonzeros:	0

Variable	Value	Reduced Cost
X1	25.00000	0.000000
X2	25.00000	0.000000
X3	869.2500	0.000000
X4	400.0000	0.000000

Row	Slack or Surplus	Dual Price
1	222677.5	1.000000
2	0.000000	-24.80000
3	0.000000	-18.50000
4	844.2500	0.000000
5	375.0000	0.000000
6	0.000000	35.00000
7	0.000000	230.0000

Problem 2 : Final Solution

$$X_1 = 25$$

$$X_2 = 25$$

$$X_3 = 869$$

$$X_4 = 400$$

$$Z = 222677.5$$

Index

- 1 Problem 1
- 2 Problem 2
- 3 Problem 3**
- 4 Problem 4
- 5 Problem 5

- 6 Problem 6
- 7 Problem 7
- 8 Problem 8
- 9 Problem 9
- 10 Problem 10

Problem 3

Fox Companies plans **six possible construction projects** during the following **4 years**. The following table shows the expected revenues (at present value) and the cash disbursements for those projects. Fox is authorized to undertake any of the projects, partially or totally. A partial termination of a project will have income and proportional disbursements.



Problem 3

cont

<i>Máquina</i>	<i>Costo por hr (\$)</i>	<i>Tiempo de manufactura (hr) por unidad</i>				<i>Capacidad (hr)</i>
		<i>Producto 1</i>	<i>Producto 2</i>	<i>Producto 3</i>	<i>Producto 4</i>	
1	10	2	3	4	2	500
2	5	3	2	1	2	380
<i>Precio unitario de venta (\$)</i>		75	70	55	45	

Problem 3 : decision variables

- x_1 : Amount of project 1 to undertake
- x_2 : Amount of project 2 to undertake
- x_3 : Amount of project 3 to undertake
- x_4 : Amount of project 4 to undertake
- x_5 : Amount of project 5 to undertake
- x_6 : Amount of project 6 to undertake

Problem 3 : Objective Function

Maximize

$$Z = 32.40x_1 + 35.80x_2 + 17.75x_3 + 14.80x_4 + 18.20x_5 + 12.35x_6$$

Problem 3 : Constrains

Funds available in year 1.

$$10.5X_1 + 8.3X_2 + 10.2X_3 + 7.2X_4 + 12.3X_5 + 9.2X_6 \leq 60$$

Funds available in year 2.

$$14.4X_1 + 12.6X_2 + 14.2X_3 + 10.5X_4 + 10.1X_5 + 7.8X_6 \leq 70$$

Funds available in year 3.

$$2.2X_1 + 9.5X_2 + 5.6X_3 + 7.5X_4 + 8.3X_5 + 6.9X_6 \leq 35$$

Funds available in year 4.

$$2.4X_1 + 3.1X_2 + 4.2X_3 + 5X_4 + 6.3X_5 + 5.1X_6 \leq 20$$

positive values

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

Problem 3 : Full Model

Maximize

$$Z = 32.40X_1 + 35.80X_2 + 17.75X_3 + 14.80X_4 + 18.20X_5 + 12.35X_6$$

subject to :

$$10.5X_1 + 8.3X_2 + 10.2X_3 + 7.2X_4 + 12.3X_5 + 9.2X_6 \leq 60$$

$$14.4X_1 + 12.6X_2 + 14.2X_3 + 10.5X_4 + 10.1X_5 + 7.8X_6 \leq 70$$

$$2.2X_1 + 9.5X_2 + 5.6X_3 + 7.5X_4 + 8.3X_5 + 6.9X_6 \leq 35$$

$$2.4X_1 + 3.1X_2 + 4.2X_3 + 5X_4 + 6.3X_5 + 5.1X_6 \leq 20$$

Problem 3 Solution with Lingo

The screenshot shows the Lingo software interface with two windows open: 'Lingo Model - Lingo3.lng*' and 'Solution Report - Lingo3.lng'.

Lingo Model - Lingo3.lng*

```

1 MAX = 32.40*X1 + 35.80*X2 + 17.75*X3 + 14.80*X4 + 18.20*X5 + 12.35*X6;
2
3 10.5*X1 + 8.3*X2 + 10.2*X3 + 7.2*X4 + 12.3*X5 + 9.2*X6 <= 60;
4 14.4*X1 + 12.6*X2 + 14.2*X3 + 10.5*X4 + 10.1*X5 + 7.8*X6 <= 70;
5 2.2*X1 + 9.5*X2 + 5.6*X3 + 7.5*X4 + 8.3*X5 + 6.9*X6 <= 35;
6 2.4*X1 + 3.1*X2 + 4.2*X3 + 5*X4 + 6.3*X5 + 5.1*X6 <= 20;
7
8

```

Solution Report - Lingo3.lng

Global optimal solution found.

Objective value:	181.4045
Infeasibilities:	0.000000
Total solver iterations:	3
Elapsed runtime seconds:	0.01
Model Class:	LP
Total variables:	6
Nonlinear variables:	0
Integer variables:	0
Total constraints:	5
Nonlinear constraints:	0
Total nonzeros:	30
Nonlinear nonzeros:	0

Variable	Value	Reduced Cost
X1	2.053539	0.000000
X2	3.208654	0.000000
X3	0.000000	17.57395
X4	0.000000	14.62354
X5	0.000000	11.17044
X6	0.000000	10.81414

Row	Slack or Surplus	Dual Price
1	181.4045	1.000000
2	11.80601	0.000000
3	0.000000	2.059743
4	0.000000	0.9834983
5	5.124679	0.000000

Problem 3 : Final Solution

$$X_1 = 2.05$$

$$X_2 = 3.20$$

$$X_3 = 0$$

$$X_4 = 0$$

$$X_5 = 0$$

$$X_6 = 0$$

$$Z = 181.40$$

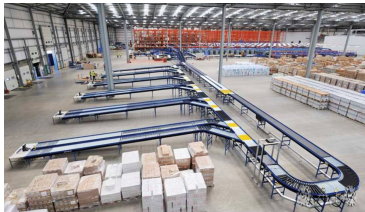
Index

- 1 Problem 1
- 2 Problem 2
- 3 Problem 3
- 4 Problem 4**
- 5 Problem 5

- 6 Problem 6
- 7 Problem 7
- 8 Problem 8
- 9 Problem 9
- 10 Problem 10

Problem 4

Manufacturing Acme received a contract to deliver housing windows for the following 6 months. The successive demands for the six periods are 100, 250, 190, 140, 220 and 110, respectively. The cost of production per window varies from month to month, depending on the costs of labor, materials and services. Acme estimates that the production cost per window, during the following 6 months, will be 50, 45, 55, 48, 52 and 50 dollars, respectively. To take advantage of fluctuations in the cost of manufacturing, Acme could choose to produce more than necessary in a given month, and save surplus units to deliver in later months. However, this will result in a storage cost of 8 \$ per window and per month, evaluated with the inventory raised at the end of the month.



Problem 4 : decision variables

i = month number in this case 1,2,3,4,5,6 for 6 months

X_i = Number of units produced in month i

I_i = Units left in the end of month inventory i

Problem 4 : Objective Function

Minimize

$$Z = 50X_1 + 45X_2 + 55X_3 + 48X_4 + 52X_5 + 50X_6 + 8(W_1 + W_2 + W_3 + W_4 + W_5 + W_6)$$

Problem 4 : Constrains

$$x_1 - W_1 = 100$$

$$W_1 + X_2 - W_2 = 250$$

$$W_2 + X_3 - W_3 = 190$$

$$W_3 + X_4 - W_4 = 140$$

$$W_4 + X_5 - W_5 = 220$$

$$I_5 + X_6 = 110$$

Problem 4 : Full Model

Minimize

$$Z = 50X_1 + 45X_2 + 55X_3 + 48X_4 + 52X_5 + 50X_6 + 8(W_1 + W_2 + W_3 + W_4 + W_5 + W_6)$$

subject to :

$$x_1 - W_1 = 100$$

$$W_1 + X_2 - W_2 = 250$$

$$W_2 + X_3 - W_3 = 190$$

$$W_3 + X_4 - W_4 = 140$$

$$W_4 + X_5 - W_5 = 220$$

$$I_5 + X_6 = 110$$

Problem 4 : Solution with Lingo

The screenshot shows the Lingo software interface with two windows open: 'Lingo Model - #4.lng*' and 'Solution Report - #4.lng'.

Lingo Model - #4.lng*

```

1 MIN = 50*X1 + 45*X2 + 55*X3 + 48*X4 + 52*X5 + 50*X6 + 8*(W1 + W2 + W3 + W4 + W5 + W6);
2
3 X1 - W1 = 100;
4 W1 + X2 - W2 = 250;
5 W2 + X3 - W3 = 190;
6 W3 + X4 - W4 = 140;
7 W4 + X5 - W5 = 220;
8 W5 + X6 = 110;
9
10
11
12

```

Solution Report - #4.lng

Global optimal solution found.
 Objective value: 49980.00
 Infeasibilities: 0.000000
 Total solver iterations: 0
 Elapsed runtime seconds: 0.01

Model Class: LP

Total variables: 12
 Nonlinear variables: 0
 Integer variables: 0

Total constraints: 7
 Nonlinear constraints: 0

Total nonzeros: 28
 Nonlinear nonzeros: 0

Variable	Value	Reduced Cost
X1	100.0000	0.000000
X2	440.0000	0.000000
X3	0.000000	2.000000
X4	140.0000	0.000000
X5	220.0000	0.000000
X6	110.0000	0.000000
W1	0.000000	13.00000
W2	190.0000	0.000000
W3	0.000000	13.00000
W4	0.000000	4.000000
W5	0.000000	10.00000
W6	0.000000	8.000000

Row	Slack or Surplus	Dual Price
1	49980.00	-1.000000
2	0.000000	-50.00000
3	0.000000	-45.00000
4	0.000000	-53.00000
5	0.000000	-48.00000
6	0.000000	-52.00000
7	0.000000	-50.00000

Problem 4 : Final Solution

$$X_1 = 100$$

$$X_2 = 440$$

$$X_3 = 0$$

$$X_4 = 140$$

$$X_5 = 220$$

$$X_6 = 110$$

$$W_1 = 100$$

$$W_2 = 440$$

$$W_3 = 0$$

$$W_4 = 140$$

$$W_5 = 220$$

$$W_6 = 110$$

$$Z = 49980$$

Index

- 1 Problem 1
- 2 Problem 2
- 3 Problem 3
- 4 Problem 4
- 5 Problem 5**

- 6 Problem 6
- 7 Problem 7
- 8 Problem 8
- 9 Problem 9
- 10 Problem 10

Problem 5

Investor Doe has four potential opportunities to invest a total of 100,000 \$. The following table provides the cash flow for four investments.

<i>Proyecto</i>	<i>Flujo de efectivo (\$ miles) al iniciar el</i>				
	<i>Año 1</i>	<i>Año 2</i>	<i>Año 3</i>	<i>Año 4</i>	<i>Año 5</i>
1	-1.00	0.50	0.30	1.80	1.20
2	-1.00	0.60	0.20	1.50	1.30
3	0.00	-1.00	0.80	1.90	0.80
4	-1.00	0.40	0.60	1.80	0.95

Problem 5

cont

The information in this table can be interpreted as follows :
for project 1, 1.00 \$ invested at the beginning of year 1, it will yield 0.50 \$ at the beginning of year 2, 0.30 \$ at the beginning of year 3, 1.80 \$ at the beginning of year 4 and 1.20 \$ at the beginning of year 5. The remaining elements can be interpreted in analogous A case without transactions is indicated by a 0.00 element. Juan also has the option of investing in a bank account that produces 6.5% per year. The funds accumulated in a year can be reinvested in the following years.



Problem 5 : decision variables

X_i = Dollars invested in the project i , $i = 1, 2, 3, 4$

Y_j = Dollars invested in the bank per year j , $j = 1, 2, 3, 4, 5$

Problem 5 : Objective Function

Maximize

$$Z = Y_5$$

Problem 5 : Constrains

$$X_1 + X_2 + X_4 + Y_1 \leq 100\,000$$

$$0.5X_1 + 0.6X_2 - X_3 + 0.4X_4 + 1.065Y_1 - Y_2 = 0$$

$$0.3X_1 + 0.2X_2 - 0.8X_3 + 0.6X_4 + 1.065Y_2 - Y_3 = 0$$

$$1.8X_1 + 1.5X_2 - 1.9X_3 + 1.8X_4 + 1.065Y_3 - Y_4 = 0$$

$$1.2X_1 + 1.3X_2 - 0.8X_3 + 0.95X_4 + 1.065Y_4 - Y_5 = 0$$

$$X_i \geq 0$$

Problem 5 : Full Model

Maximize

$$Z = Y_5$$

subject to :

$$X_1 + X_2 + X_4 + Y_1 \leq 100\,000$$

$$0.5X_1 + 0.6X_2 - X_3 + 0.4X_4 + 1.065Y_1 - Y_2 = 0$$

$$0.3X_1 + 0.2X_2 - 0.8X_3 + 0.6X_4 + 1.065Y_2 - Y_3 = 0$$

$$1.8X_1 + 1.5X_2 - 1.9X_3 + 1.8X_4 + 1.065Y_3 - Y_4 = 0$$

$$1.2X_1 + 1.3X_2 - 0.8X_3 + 0.95X_4 + 1.065Y_4 - Y_5 = 0$$

$$X_i \geq 0$$

Problem 5 Solution with Lingo

The screenshot displays the Lingo software interface with two windows: 'Lingo Model - #5.lng*' and 'Solution Report - #5.lng'.

Lingo Model - #5.lng*

```

1 MAX = Y5;
2
3
4 X1 + X2 + X4 + Y1 <= 100000;
5 0.5*X1 + 0.6*X2 - X3 + 0.4*X4 + 1.065*Y1 - Y2 = 0;
6 0.3*X1 + 0.2*X2 - 0.8*X3 + 0.4*X4 + 1.065*Y2 - Y3 = 0;
7 1.8*X1 + 1.5*X2 - 1.9*X3 + 1.8*X4 + 1.065*Y3 - Y4 = 0;
8 1.2*X1 + 1.3*X2 - 0.8*X3 + 0.95*X4 + 1.065*Y4 - Y5 = 0;
9
10
11
12
13

```

Solution Report - #5.lng

Global optimal solution found.

Objective value:	406124.2
Infeasibilities:	0.000000
Total solver iterations:	6
Elapsed runtime seconds:	0.01
Model Class:	LP
Total variables:	9
Nonlinear variables:	0
Integer variables:	0
Total constraints:	6
Nonlinear constraints:	0
Total nonzeros:	29
Nonlinear nonzeros:	0

Variable	Value	Reduced Cost
Y5	406124.2	0.000000
X1	100000.0	0.000000
X2	0.000000	0.2121275
X4	0.000000	0.3052746E-01
Y1	0.000000	2.774776
X3	0.000000	4.938830
Y2	50000.00	0.000000
Y3	83250.00	0.000000
Y4	268661.2	0.000000

Row	Slack or Surplus	Dual Price
1	406124.2	1.000000
2	0.000000	4.061242
3	0.000000	-1.207950
4	0.000000	-1.134225
5	0.000000	-1.065000
6	0.000000	-1.000000

Problem 5 : Final Solution

$$X1 = 100000$$

$$X2 = 0$$

$$X3 = 0$$

$$X4 = 0$$

$$Y1 = 0$$

$$Y2 = 50000$$

$$Y3 = 83250$$

$$Y4 = 268661.2$$

$$Z = 406124$$

Index

- 1 Problem 1
- 2 Problem 2
- 3 Problem 3
- 4 Problem 4
- 5 Problem 5

- 6 Problem 6
- 7 Problem 7
- 8 Problem 8
- 9 Problem 9
- 10 Problem 10

Problem 6

Toolco has signed a contract with AutoMate to provide their discount automotive stores with chisels and wrenches. The weekly demand of AutoMate is 1500 wrenches and 1200 chisels. Toolco's current capacity is not large enough to produce units the required units and must work overtime and possibly outsource to other tool shops.

The result is an increase in the cost of production per unit, as shown in the following table. The market restricts the wrenches to a ratio of at least 2 : 1 with the chisels.



Problem 6

cont

Herramienta	Tipo de producción	Producción semanal (unidades)	Costo unitario (\$)
Llaves	Normal	0–550	2.00
	Tiempo extra	551–800	2.80
	Subcontratadas	801– ∞	3.00
Cinceles	Normal	0–620	2.10
	Tiempo extra	621–900	3.20
	Subcontratados	901– ∞	4.20

Problem 6 : decision variables

- K_n : Number of keys produced in normal time
- K_e : Quantity of keys produced in extra time
- K_s : Number of keys by subcontract
- C_n : Amount of Chisels produced in normal time
- C_e : Quantity of Chisels produced in extra time
- C_s : Quantity of Chisels by subcontract

Problem 6 : Objective Function

Minimize

$$Z = 2K_n + 2.8K_e + 3K_s + 2.1C_n + 3.2C_e + 4.2C_s$$

Problem 6 : Constrains

$$-2K_n - 2K_e - 2K_s + C_n + C_e + C_s \geq 0 \quad K_n + K_e + K_s \geq 1500$$

$$C_n + C_e + C_s \geq 1200$$

$$K_n \leq 550$$

$$C_n \leq 620$$

$$K_n + K_e \leq 800$$

$$C_n + C_e \leq 900$$

Problem 6 : Full Model

Minimize

$$Z = 2K_n + 2.8K_e + 3K_s + 2.1C_n + 3.2C_e + 4.2C_s$$

subject to :

$$-2K_n - 2K_e - 2K_s + C_n + C_e + C_s \geq 0 \quad K_n + K_e + K_s \geq 1500$$

$$C_n + C_e + C_s \geq 1200$$

$$K_n \leq 550$$

$$C_n \leq 620$$

$$K_n + K_e \leq 800$$

$$C_n + C_e \leq 900$$

Problem 6 : Solution with Lingo

The screenshot shows the Lingo software interface with two windows: 'Lingo Model - #6.lng*' and 'Solution Report - #6.lng'.

Lingo Model - #6.lng*

```

1 MIN = 2*Kn + 2.8*Ke + 3*Ks + 2.1*Cs + 3.2*Ce + 4.2*Ca;
2
3 -2*Kn-2*Ke-2*Ks+Cs+Ce+Ca >= 0;
4 Kn + Ke + Ks >= 1500;
5 Cs + Ce + Ca >= 1200;
6 Kn <= 550;
7 Cs <= 620;
8 Kn + Ke <= 800;
9 Cs + Ce <= 900;
10
11
12

```

Solution Report - #6.lng

Global optimal solution found.
 Objective value: 14918.00
 Infeasibilities: 0.000000
 Total solver iterations: 5
 Elapsed runtime seconds: 0.01

Model Class: LP

Total variables: 6
 Nonlinear variables: 0
 Integer variables: 0

Total constraints: 8
 Nonlinear constraints: 0

Total nonzeros: 24
 Nonlinear nonzeros: 0

Variable	Value	Reduced Cost
KN	550.0000	0.000000
KE	250.0000	0.000000
KS	700.0000	0.000000
CN	620.0000	0.000000
CE	280.0000	0.000000
CS	2100.000	0.000000

Row	Slack or Surplus	Dual Price
1	14918.00	-1.000000
2	0.000000	-4.200000
3	0.000000	-11.40000
4	1800.000	0.000000
5	0.000000	0.800000
6	0.000000	1.100000
7	0.000000	0.200000
8	0.000000	1.000000

Problem 6 : Final Solution

$$K_n = 550$$

$$K_e = 250$$

$$K_s = 700$$

$$C_n = 620$$

$$C_e = 280$$

$$C_s = 2100$$

$$Z = 14918$$

Index

- 1 Problem 1
- 2 Problem 2
- 3 Problem 3
- 4 Problem 4
- 5 Problem 5

- 6 Problem 6
- 7 Problem 7**
- 8 Problem 8
- 9 Problem 9
- 10 Problem 10

Problem 7

In **two machines**, four products are processed sequentially. The following table shows the relevant data of the problem.

<i>Máquina</i>	<i>Costo por hr (\$)</i>	<i>Tiempo de manufactura (hr) por unidad</i>				<i>Capacidad (hr)</i>
		<i>Producto 1</i>	<i>Producto 2</i>	<i>Producto 3</i>	<i>Producto 4</i>	
1	10	2	3	4	2	500
2	5	3	2	1	2	380
<i>Precio unitario de venta (\$)</i>		75	70	55	45	

P1 : Product 1

P2 : Product 2

P3 : Product 3

P4 : Product 4



Problem 7 : Objective Function

Maximize

$$Z = 30P_1 + 30P_2 + 20P_3 + 15P_4$$

Problem 7 : Constrains

$$2P_1 + 3P_2 + 4P_3 + 2P_4 \leq 500$$

$$3P_1 + 2P_2 + P_3 + 2P_4 \leq 380$$

Problem 7 : Full Model

Maximize

$$Z = 30P1 + 30P2 + 20P3 + 15P4$$

subject to :

$$2P1 + 3P2 + 4P3 + 2P4 \leq 500$$

$$3P1 + 2P2 + P3 + 2P4 \leq 380$$

Problem 7 : Solution with Lingo

The screenshot shows the Lingo software interface. The main window displays the Lingo Model - #7.ing* with the following code:

```

1 MAX = 30*P1+30*P2+20*P3+15*P4;
2
3
4 2*P1+3*P2+4*P3+2*P4 <= 500;
5 3*P1+2*P2+P3+2*P4 <= 380;
6
7

```

A Solution Report - #7.ing* window is open, displaying the following information:

Global optimal solution found.
 Objective value: 5280.000
 Infeasibilities: 0.000000
 Total solver iterations: 2
 Elapsed runtime seconds: 0.01

Model Class: LP

Total variables: 4
 Nonlinear variables: 0
 Integer variables: 0

Total constraints: 3
 Nonlinear constraints: 0

Total nonzeros: 12
 Nonlinear nonzeros: 0

Variable	Value	Reduced Cost
P1	28.00000	0.000000
P2	148.0000	0.000000
P3	0.000000	10.00000
P4	0.000000	9.000000

Row	Slack or Surplus	Dual Price
1	5280.000	1.000000
2	0.000000	6.000000
3	0.000000	6.000000

Problem 7 : Final Solution

$$P1 = 28$$

$$P2 = 148$$

$$P3 = 0$$

$$P4 = 0$$

$$Z = 5280$$

Index

- 1 Problem 1
- 2 Problem 2
- 3 Problem 3
- 4 Problem 4
- 5 Problem 5

- 6 Problem 6
- 7 Problem 7
- 8 Problem 8**
- 9 Problem 9
- 10 Problem 10

Problem 8

A manufacturer produces **three models, I, II and III**, of a certain product, using the raw materials A and B. The following table shows the data for the problem.

<i>Materia prima</i>	<i>Requerida por unidad</i>			<i>Disponibilidad</i>
	<i>I</i>	<i>II</i>	<i>III</i>	
<i>A</i>	2	3	5	4000
<i>B</i>	4	2	7	6000
Demanda mínima	200	200	150	
Utilidad por unidad (\$)	30	20	50	

The labor time for model I is double that for II and triple of III. All factory personnel can produce the equivalent of **1500 model I units**. Market needs specify the 3 : 2 : 5 relationships of the productions of the three respective models.



Problem 8 : decision variables

X_1 = Units to be produced from the model product I

X_2 = Units to be produced of the product model II

X_3 = Units to be produced of the model product III

Problem 8 : Objective Function

Maximize

$$Z = 30X_1 + 20X_2 + 50X_3$$

Problem 8 : Constrains

Restriction of raw material A : $2X_1 + 3X_2 + 5X_3 \leq 4000$

Restriction of raw material B : $4X_1 + 2X_2 + 7X_3 \leq 6000$

Time restriction : $X_1 + 0.5X_2 + 0.33333 X_3 \leq 1500$

Production ratio in models I and II : $2X_1 - 3X_2 = 0$

Production ratio in models II and III : $5X_2 - 2X_3 = 0$

Restriction of demand of the model I, II, III :

$X_1, X_2, X_3 \geq 200$

$X_1, X_2, X_3 \geq 0$

Problem 8 : Full Model

Maximize

$$Z = 30X_1 + 20X_2 + 50X_3 \text{ [1em]}$$

subject to :

$$2X_1 + 3X_2 + 5X_3 \leq 4000$$

$$4X_1 + 2X_2 + 7X_3 \leq 6000$$

$$X_1 + 0.5X_2 + 0.33333 X_3 \leq 1500$$

$$2X_1 - 3X_2 = 0 \quad 5X_2 - 2X_3 = 0 \quad X_1, X_2, X_3 \geq 200$$

$$X_1, X_2, X_3 \geq 0$$

Problem 8 : Solution with Lingo

File Edit Solver Window Help

Lingo Model - #8.Ing* x Solution Report - #8.Ing x

Lingo Model - #8.Ing*

```

1 Max = 30*X1 + 20*X2 + 50*X3;
2
3 2*X1 + 3*X2 + 5*X3 <= 4000;
4 4*X1 + 2*X2 + 7*X3 <= 6000;
5 X1 + 0.5*X2 + 0.333333*X3 <= 1500;
6 2*X1 - 3*X2 = 0;
7 5*X2 - 2*X3 = 0;
8
9 X1 >= 200;
10 X2 >= 200;
11 X3 >= 150;
12
13

```

Solution Report - #8.Ing

Global optimal solution found.

Objective value:	41081.08
Infeasibilities:	0.000000
Total solver iterations:	0
Elapsed runtime seconds:	0.02

Model Class: LP

Total variables:	3
Nonlinear variables:	0
Integer variables:	0
Total constraints:	9
Nonlinear constraints:	0
Total nonzeros:	19
Nonlinear nonzeros:	0

Variable	Value	Reduced Cost
X1	324.3243	0.000000
X2	216.2162	0.000000
X3	540.5405	0.000000

Row	Slack or Surplus	Dual Price
1	41081.08	1.000000
2	0.000000	10.27027
3	486.4865	0.000000
4	887.3876	0.000000
5	0.000000	4.729730
6	0.000000	0.6756757
7	124.3243	0.000000
8	16.21622	0.000000
9	390.5405	0.000000

Problem 8 : Final Solution

$$X1 = 324$$

$$X2 = 216$$

$$X3 = 540$$

$$Z = 41081.08$$

Index

- 1 Problem 1
- 2 Problem 2
- 3 Problem 3
- 4 Problem 4
- 5 Problem 5

- 6 Problem 6
- 7 Problem 7
- 8 Problem 8
- 9 Problem 9**
- 10 Problem 10

Problem 9

HiRise Construction can bid on two one-year projects. The quarterly cash flow (in millions of dollars) is provided in the following table for the **two projects**.

<i>Proyecto</i>	Flujo de efectivo (en millones de \$) al				
	<i>1/1/08</i>	<i>4/1/08</i>	<i>7/1/08</i>	<i>10/1/08</i>	<i>12/31/08</i>
I	−1.0	−3.1	−1.5	1.8	5.0
II	−3.0	−2.5	1.5	1.8	2.8

HiRise has cash funds of **1 million** at the beginning of each quarter and can borrow an amount equal to **10%** of the annual nominal interest rate. This means that if the **amount borrowed in quarter i is B_i** then.

$$0 \leq B_i \leq 1, i = 1, 2, 3, 4.$$

Any borrowed money must be paid at the end of the quarter. The excess cash can earn a quarterly interest at a nominal annual rate of **8%**. All the money accumulated at the end of a quarter is invested in the following quarter.



Problem 9 : decision variables

P_i = Fraction undertaken from project i , $i = 1, 2$

B_j = Millions of dollars loaned in semester j , $j = 1, 2, 3, 4$

S_j = Surplus million dollars at the beginning of semester j , $j = 1, 2, 3, 4, 5$

Problem 9 : Objective Function

Maximize

$$Z = S5$$

Problem 9 : Constrains

$$P1 + 3P2 + S1 - B1 = 1$$

$$3.1P1 + 2.5P2 - 1.0S1 + S2 + 1.025B1 - B2 = 1$$

$$1.5P1 - 1.5P2 - 1.02S2 + S3 + 1.025B2 - B3 = 1$$

$$-1.8P1 - 1.8P2 - 1.02S3 + S4 + 1.025B3 - B4 = 1$$

$$-5P1 - 2.8P2 - 1.02S4 + S5 + 1.025B4 = 1$$

$$0 \leq P1 \leq 1$$

$$0 \leq P2 \leq 1$$

$$0 \leq B_j \leq 1, j = 1,2,3,4$$

Problem 9 : Full Model

Maximize

$$Z = S5$$

subject to :

$$P1 + 3P2 + S1 - B1 = 1$$

$$3.1P1 + 2.5P2 - 1.0S1 + S2 + 1.025B1 - B2 = 1$$

$$1.5P1 - 1.5P2 - 1.02S2 + S3 + 1.025B2 - B3 = 1$$

$$-1.8P1 - 1.8P2 - 1.02S3 + S4 + 1.025B3 - B4 = 1$$

$$-5P1 - 2.8P2 - 1.02S4 + S5 + 1.025B4 = 1$$

$$0 \leq P1 \leq 1$$

$$0 \leq P2 \leq 1$$

$$0 \leq B_j \leq 1, j = 1,2,3,4$$

Problem 9 : Solution with Lingo

The screenshot displays the Lingo software interface with two windows: 'Lingo Model - #9.lng*' and 'Solution Report - #9.lng'.

Lingo Model - #9.lng*

```

1 MAX = S5;
2
3 P1 + 3*P2 + S1 - B1 = 1;
4 3.1*P1 + 2.5*P2 - 1.0*S1 + S2 + 1.025*B1 - B2 = 1;
5 1.5*P1 - 1.5*P2 - 1.02*S2 + S3 + 1.025*B2 - B3 = 1;
6 -1.8*P1 - 1.8*P2 - 1.02*S3 + S4 + 1.025*B3 - B4 = 1;
7 -5*P1 - 2.8*P2 - 1.02*S4 + S5 + 1.025*B4 = 1;
8 P1 <= 1;
9 P2 <= 1;
10 B1 <= 1;
11 B2 <= 1;
12 B3 <= 1;
13 B4 <= 1;
14
15

```

Solution Report - #9.lng

Global optimal solution found.
 Objective value: 5.829528
 Infeasibilities: 0.000000
 Total solver iterations: 7
 Elapsed runtime seconds: 0.02

Model Class: LP

Total variables: 11
 Nonlinear variables: 0
 Integer variables: 0

Total constraints: 12
 Nonlinear constraints: 0

Total nonzeros: 34
 Nonlinear nonzeros: 0

Variable	Value	Reduced Cost
S5	5.829528	0.000000
P1	0.7102148	0.000000
P2	0.000000	0.3239211
S1	0.2897852	0.000000
B1	0.000000	0.3071854E
S2	0.000000	0.5993862E
B2	0.9118808	0.000000
S3	0.000000	0.1583725
B3	1.000000	0.000000
S4	1.253387	0.000000
B4	0.000000	0.5000000E

Row	Slack or Surplus	Dual Price
1	5.829528	1.000000
2	0.000000	1.228742
3	0.000000	1.228742
4	0.000000	1.198772
5	0.000000	1.020000
6	0.000000	1.000000
7	0.2897852	0.000000
8	1.000000	0.000000
9	1.000000	0.000000
10	0.8811925E-01	0.000000
11	0.000000	0.1532725
12	1.000000	0.000000

Problem 9 : Final Solution

$$S1 = 0.28$$

$$S2 = 0$$

$$S3 = 0$$

$$S4 = 1.25$$

$$S5 = 5.82$$

$$P1 = 0.81$$

$$P2 = 0$$

$$B1 = 0$$

$$B2 = 0.91$$

$$B3 = 1$$

$$B4 = 0$$

$$Z = 5.829528$$

Problem 9 : Final Analysis

The solution shows that $BiSi = 0$, meaning tha you cannot borrow and also end up with surplus un any quarter. The result makes sense because the cost of borrowing 2.5% is higher than the return on surplus funds 2%

Index

- 1 Problem 1
- 2 Problem 2
- 3 Problem 3
- 4 Problem 4
- 5 Problem 5

- 6 Problem 6
- 7 Problem 7
- 8 Problem 8
- 9 Problem 9
- 10 Problem 10**

Problem 10

In anticipating your child's considerable college spending, a couple has initiated an annual investment program on the day the child turned eight, which will continue until he turns 18. The couple estimates that they can invest the following amounts at the beginning of each year :

Año	1	2	3	4	5	6	7	8	9	10
Cantidad (\$)	2000	2000	2500	2500	3000	3500	3500	4000	4000	5000

Problem 10

cont

To avoid unpleasant surprise, the couple chooses to invest with security in the following options :

- 1) Savings insured with annual return of 7.5% (profit)
- 2) **Government bonds for 6 years**, which produce 7.9% and have a current market price equal to 98% of the nominal value. (1-5) years.
- 3) **Municipal bonds at 9 years**, which produce 8.5% with a current market price of 1.02 times the nominal value. (1-2) years.



Problem 10 : decision variables

A_i = savings assured $i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$

BG_i = Government bonds $i = 1, 2, 3, 4, 5$

BM_i = Municipal Bonds $i = 1, 2$

Problem 10 : Objective Function

Maximize

$$Z = 1.075A_{10} + 1.079B_{g5} + 1.085B_{m2}$$

Problem 10 : Constrains

Year 1 : $2000 = A_1 + 0.98Bg_1 + 1.02Bm_1$

Year 2 : $2000 + 1.075A_1 = A_2 + 0.98Bg_2 + 1.02Bm_2$

Year 3 : $2500 + 1.075A_2 = A_3 + 0.98Bg_3 + 1.02Bm_3$

Year 4 : $2500 + 1.075A_3 = A_4 + 0.98Bg_4 + Bm_4$

Year 5 : $3000 + 1.075A_4 = A_5 + 0.98Bg_5 + Bm_5$

Year 6 : $3500 + 1.075A_5 = A_6$

Year 7 : $3500 + 1.075A_6 + 1.079 \cdot 0.98Bg_1 = A_7$

Year 8 : $4000 + 1.075A_7 + 1.079 \cdot 0.98Bg_2 = A_8$

Year 9 : $4000 + 1.075A_8 + 1.079 \cdot 0.98Bg_3 = A_9$

Year 10 : $5000 + 1.075A_9 + 1.079 \cdot 0.98Bg_4 + 1.085Bm_1 = A_{10}$

Problem 10 : Full Model

Maximize

$$Z = 1.075A_{10} + 1.079Bg_5 + 1.085Bm_2$$

subject to :

$$2000 = A_1 + 0.98Bg_1 + 1.02Bm_1$$

$$2000 + 1.075A_1 = A_2 + 0.98Bg_2 + 1.02Bm_2$$

$$2500 + 1.075A_2 = A_3 + 0.98Bg_3 + 1.02Bm_3$$

$$2500 + 1.075A_3 = A_4 + 0.98Bg_4 + Bm_4$$

$$3000 + 1.075A_4 = A_5 + 0.98Bg_5 + Bm_5$$

$$3500 + 1.075A_5 = A_6$$

$$3500 + 1.075A_6 + 1.079 \cdot 0.98Bg_1 = A_7$$

$$4000 + 1.075A_7 + 1.079 \cdot 0.98Bg_2 = A_8$$

$$4000 + 1.075A_8 + 1.079 \cdot 0.98Bg_3 = A_9$$

$$5000 + 1.075A_9 + 1.079 \cdot 0.98Bg_4 + 1.085Bm_1 = A_{10}$$

Problem 10 : Solution with Lingo

Lingo Model - Lingo10.lng*			Solution Report - Lingo10.lng		
Lingo Model - Lingo10.lng*			Global optimal solution found.		
1 MAX = A10*1.075+Bg5*1.079+Bm2*1.085;			Objective Value:	45858.29	0.000000
2			Infeasibilities:	0.000000	
3 2000=A1+0.98*Bg1+1.02*Bm1;			Total solver iterations:	0	
4			Elapsed runtime seconds:	0.02	
5 2000+A1*1.075+A2+0.98*Bg2+1.02*Bm2;			Model Class:	LP	
6			Total variables:	20	
7			Nonlinear variables:	0	
8 2500+A2*1.075+A3+0.98*Bg3+1.02*Bm3;			Integer variables:	0	
9			Total constraints:	11	
10 2500+A3*1.075+A4+0.98*Bg4+1.02*Bm4;			Nonlinear constraints:	0	
11			Total nonzeros:	37	
12			Nonlinear nonzeros:	0	
13 3000+A4*1.075+A5+0.98*Bg5+1.02*Bm5;					
14					
15 3500+A5*1.075=A6;					
16					
17					
18 3500 + A6*1.075 + 0.98*Bg1*1.079=A7;					
19					
20					
21 4000+A7*1.075+0.98*Bg2*1.079 = A8;			Variable	Value	Reduced Cost
22			A10	42658.88	0.000000
23			BG5	0.000000	0.434355
24 4000+A8*1.075+0.98*Bg3*1.079=A9;			BH2	0.000000	0.8705834
25			A1	2000.000	0.000000
26			BG1	0.000000	0.6076592
27 5000+A9*1.075+0.98*Bg4*1.079+1.02*Bm1*1.085=A10;			BH1	0.000000	0.9125457
28			A2	4150.000	0.000000
29			BG2	0.000000	0.5652643
			A3	6961.250	0.000000
			BG3	0.000000	0.5258273
			BH3	0.000000	1.819147
			A4	9983.344	0.000000
			BG4	0.000000	0.4891417
			BH4	0.000000	1.692230
			A5	13732.09	0.000000
			BH5	0.000000	1.574168
			A6	18262.00	0.000000
			A7	23131.65	0.000000
			A8	28866.53	0.000000
			A9	35031.52	0.000000
			Row	Slack or Surplus	Dual Price
			1	45858.29	1.000000
			2	0.000000	-2.061032
			3	0.000000	-1.917239
			4	0.000000	-1.783478
			5	0.000000	-1.659049
			6	0.000000	-1.543302
			7	0.000000	-1.435629
			8	0.000000	-1.335469
			9	0.000000	-1.242297
			10	0.000000	-1.155625
			11	0.000000	-1.075000

Problem 10 : Final Solution

$$A1 = 2000$$

$$A2 = 4150$$

$$A3 = 6961$$

$$A4 = 9983.34$$

$$A5 = 13732.09$$

$$A6 = 18262$$

$$A7 = 231313.65$$

$$A8 = 28866$$

$$A9 = 35031$$

$$A10 = 42658.88$$

Problem 10 : Final Solution

cont

$$BG1 = 0$$

$$BG2 = 0$$

$$BG3 = 0$$

$$BG4 = 0$$

$$BG5 = 0$$

$$BM1 = 0$$

$$BM2 = 0$$

$$BM3 = 0$$

$$BM4 = 0$$

$$Z = 45858.29$$

References

Hamdy A. Taha. (2004). Introducción a la programación Lineal. En Investigación de operaciones(11-67). México : PEARSON Education.