

## Lesson 11 Companion

### Identifying confounding variables:

For this example I am going to use the IOCT data set available in the files tab on our course Teams.

Every semester when I bring up this data set, I get several cadets discuss how unfair the IOCT is for the height-impaired among us. I'm not a particularly tall person but I never had any issues with the IOCT so I feel it's best to let the data decide if these complaints have merits. Let's start with a research question: What is the association between height and IOCT Time?

I like to draw causal diagrams when I look start looking at research questions and I find these are particularly important when I think that linear regression is going to be my approach.



Figure 1: Simple Linear Regression

One of the purposes of this diagram is to identify any potential confounding variables that we need to include in our analysis. You will recall that confounding variables are those that are “related both to the explanatory and to the response variable in such a way that its effects on the response variable cannot be separated from the effects of the explanatory variable.” This can be quite the definition so let's consider a concrete example here:

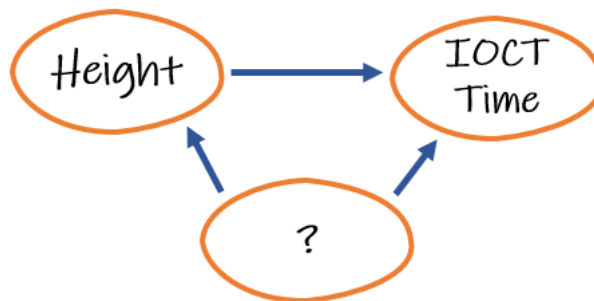


Figure 2: Considering a Confounding Variable

Is there a variable (“x”) that we could put in the third bubble that you would say “x” has an effect on *height* and an effect on *IOCT Time*? Ordinarily, you would be compose this diagram at the same time as designing your experiment or observational study. This would allow you to identify all the variables that it is important for you to collect data on. Obviously, as we are using a pre-assembled data set, we won't be able to add any

variables not available in the data set so let's consider some of those. After looking at the data set, I want to discuss two possibilities: *weight* and *sex*.

First let's consider *weight*. I've added *weight* to the causal diagram below along with the arrows that I think best dictate the relationship between the variables.

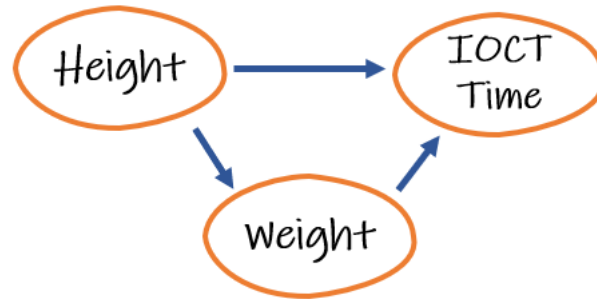


Figure 3: Mediator Variable

You will notice the the arrow between *height* and *weight* has been reversed from my the previous diagram. It seems to me that the “common sense” direction of this association is that *height* has an effect on *weight* and not the other way around. Could there be a relationship in the other direction? I suppose, but it seems much more likely that how tall you are provides the baseline for how much you are going to weigh. This means that *weight* is not likely a confounding variable and can be called a “mediator” (or one of the numerous other terms that are used depending on your statistical specialty).

Now let's consider *sex* as a confounding variable:

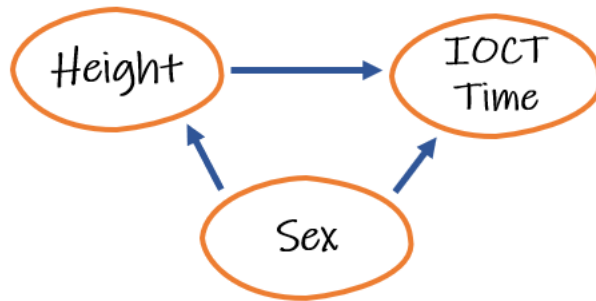


Figure 4: Sex as a Confounding Variable

Is it logical to say that *sex* has an effect on *height*? Well as most of those complaining about being “too short for the IOCT” seem to be of the female persuasion, I would say that is likely. Can we also say that *sex* has an effect on *IOCT Time*? Well, it's nothing I've explored yet but that seems to make a lot more sense than *IOCT Time* having an effect on *sex*.

Now that we've identified a confounding variable we can now consider how to deal with it. Remember, because it is a confounding variable, we can say: the effects of *sex* on *IOCT Time* can not be separated from the effects of *height*. This feel rather final... like there isn't anything we can do about it. Except there is! We can control for this confounding variable by including as a covariate in our regression model. I know that sounds complicated but it's really not.

Before we build the model, however, let's do some exploratory data analysis.

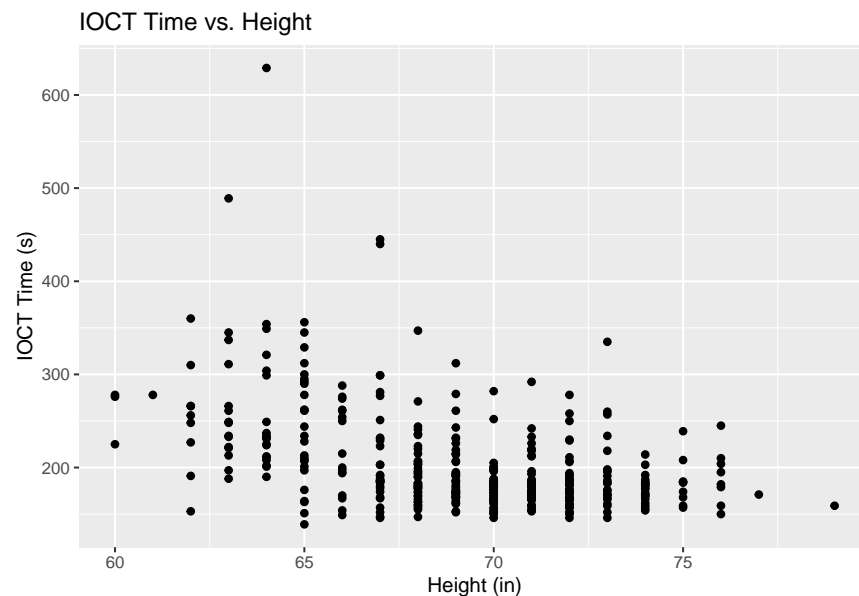
```
library(tidyverse)
```

```
IOCT = read.csv("IOCT_tab_data.csv")
```

```
head(IOCT)
```

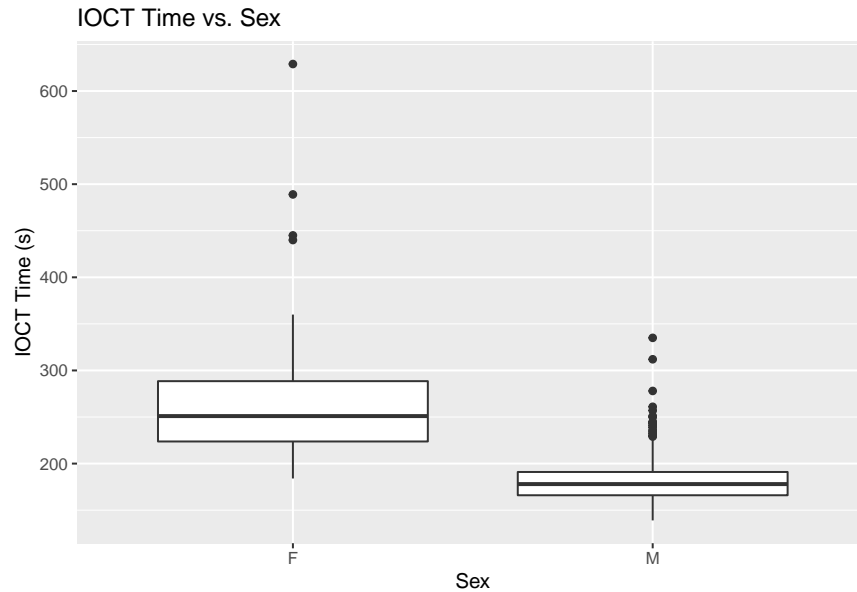
```
##   sex height weight IOCT_Time pu_score su_score run_score CS APFT_Score tab
## 1  F    67   134    203     108     112     112    0      332    1
## 2  F    63   121    197     123     118     125    1      366    1
## 3  M    72   176    150     125     109     125    0      359    1
## 4  F    68   168    235      65      86      72    0      223    0
## 5  F    65   149    210     108     114     115    1      337    1
## 6  M    74   176    203      96     100     100    0      296    0
```

```
IOCT %>%
  ggplot(aes(x = height, y = IOCT_Time)) +
  geom_point() +
  labs(x = "Height (in)", y = "IOCT Time (s)",
       title = "IOCT Time vs. Height")
```



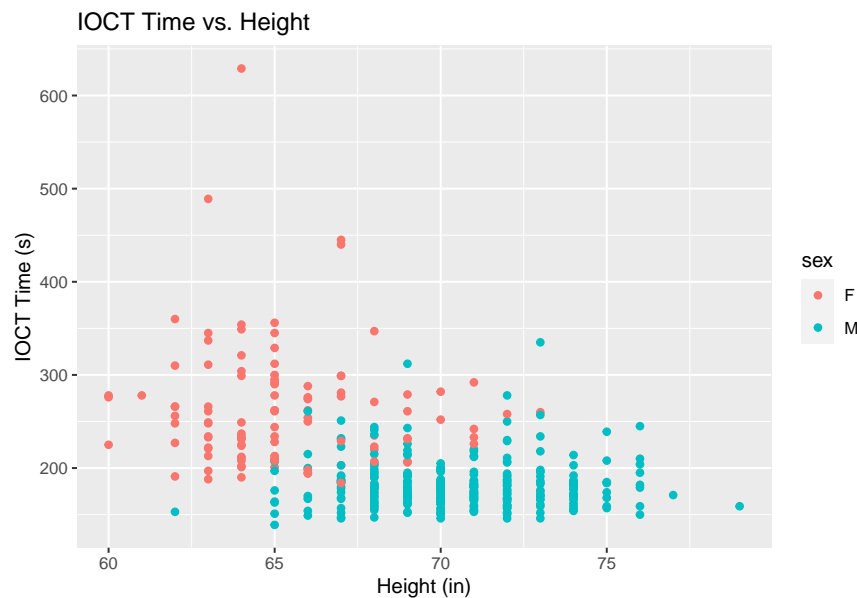
Considering this scatter plot, I would say there is a moderate negative linear relationship between *height* and *IOCT Time*. There are several unusual data points, however, the cadet with the *IOCT Time* of over 600 seconds is probably going to be influential in its effect on the slope of our regression line.

```
IOCT %>%
  ggplot(aes(x = sex, y = IOCT_Time)) +
  geom_boxplot() +
  labs(x = "Sex", y = "IOCT Time (s)",
       title = "IOCT Time vs. Sex")
```



This box plot demonstrates that the median male time is noticeably lower than the median female time. It also shows that the variability of the male time is also a lot less. We can also identify that our influential point is a female cadet.

```
IOCT %>%
  ggplot(aes(x = height, y = IOCT_Time, color = sex)) +
  geom_point() +
  labs(x = "Height (in)", y = "IOCT Time (s)",
       title = "IOCT Time vs. Height")
```



There isn't a lot of additional information gained from this plot but I wanted to demonstrate how you can include a color to differentiate categories by using a color.

### The “outlier problem”:

Another common thread of questions when we utilize this data set is what to do about the outlier. I’m sure the lady who recorded the over 10-minute time on the IOCT is a great person, but her data point is really cramping my modeling style. Can’t we just remove it then? Well, in true statistics fashion, I’m going to answer this question with a set of other questions: Do you think there was a data collection error for this data point? Assuming not, what are you sacrificing by removing this cadet’s time? If you start removing data points because you don’t like them, where will the madness end?

I think these are all important questions to consider before you think about removing data points. As for me, unless there was a clear measurement error (like a weight of five pounds for a cadet), I like to leave them in until I’ve assessed their impact on my model.

### Hypothesis:

As we two explanatory variables in this question, we have two sets of hypotheses that we are testing. Notice that we use language to describe “controlling” for the other variable when we define these hypotheses.

#### Height

$H_0$  : Controlling for sex, there is no association between height and IOCT time ( $\beta_1 = 0$ )

$H_a$  : Controlling for sex, there is an association between height and IOCT time ( $\beta_1 \neq 0$ )

#### Sex

$H_0$  : Controlling for height, there is no association between sex and IOCT time ( $\beta_2 = 0$ )

$H_a$  : Controlling for height, there is an association between sex and IOCT time ( $\beta_2 \neq 0$ )

### Multiple linear regression:

Now on to building our multiple linear regression model.

```
model = lm(IOCT$IOCT_Time ~ IOCT$height + IOCT$sex)
```

```
summary(model)
```

```
##
## Call:
## lm(formula = IOCT$IOCT_Time ~ IOCT$height + IOCT$sex)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -79.45 -20.52  -4.57   11.37  364.36
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  290.0195    51.1840   5.666 2.88e-08 ***
## IOCT$height  -0.3966     0.7818  -0.507   0.612
## IOCT$sexM    -80.1894     6.2191 -12.894 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 40.87 on 381 degrees of freedom
## Multiple R-squared:  0.4466, Adjusted R-squared:  0.4437
## F-statistic: 153.8 on 2 and 381 DF, p-value: < 2.2e-16
```

Composing our linear model’s equation from this output would give us:

$\hat{y} = -0.3966x_1 - 80.1894x_2 + 290.0195$   
where  $x_1$ : height in inches and  $x_2$ : sex (0 is female, 1 is male)

To interpret our coefficients, we would say:

If sex is fixed, for every one inch increase in height, we expect a 0.3966 second decrease in IOCT time. Also, if height is fixed, males have a 80.1894 second faster IOCT time on average.

### Indicator encoding

How did I know that “0” was *female* and “1” was *male* for our model? Well, as you know from your reading, there are two different types of encoding for categorical variables in the regression: indicator and effects. *R* uses indicator encoding meaning that it chooses an indicator variable(s) (**IOCT\$sexM** in this case) to represent your categories. You’ll probably hear me call this “one-hot encoding” because I’m most familiar with it in machine learning contexts. It’s probably not strictly “one-hot encoding” but it’s close. Here is an example:

Let’s say your variable is *color* and it has four values: red, white, blue, green. To represent these four categories using indicator variables in our model we would use three indicator variables: **colorR**, **colorW**, **colorB**. How can three variables represent four categories? Check out the table below:

	colorR	colorW	colorB
Red	1	0	0
White	0	1	0
Blue	0	0	1
Green	0	0	0

As you can see, at most one indicator variable is “hot” (value of 1) at a time. We can represent our fourth category by turning them all “cold.” You can also see that “red” is represented by **colorR** being hot... this is how I knew that “male” was 1 for the indicator variable in our regression model.

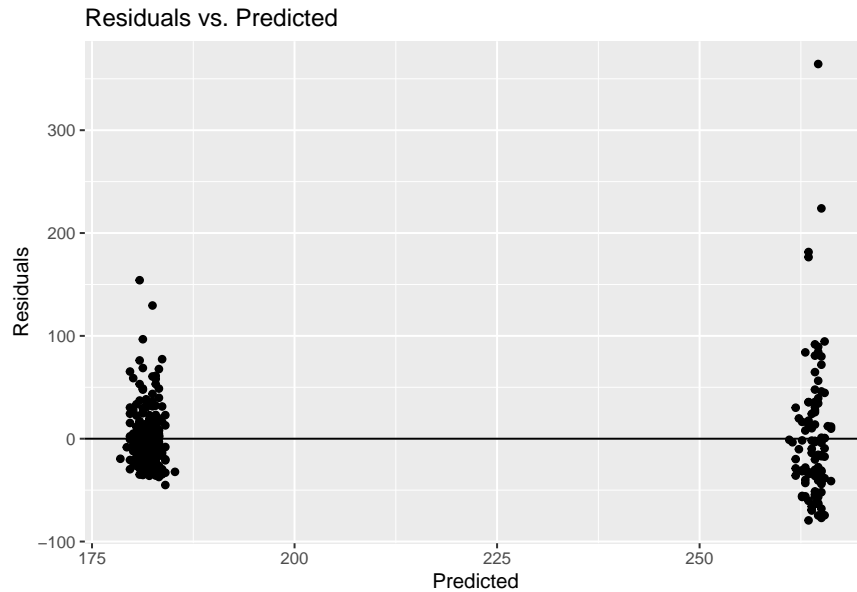
This indicator encoding is why I also caution you to try to stick with quantitative variables for your project. You can see how adding a few categorical variables with a few categories each can add a lot of terms to your model very quickly. Add the categorical variables that you need to for your research question... just be smart about it.

### Statistical inference

Now that we have built a multiple regression model we have slope estimates that depict the association between the variables in our sample. What can we say about the true relationship between these variables? I always prefer to use the theory-based approach (especially considering the model summary provides us the p-values) but we need to check the **validity conditions** first.

Linearity and Equal Variance:

```
model %>%  
  ggplot(aes(x = .fitted, y = .resid)) +  
  geom_point() +  
  labs(x = "Predicted", y = "Residuals",  
       title = "Residuals vs. Predicted") +  
  geom_hline(yintercept = 0)
```



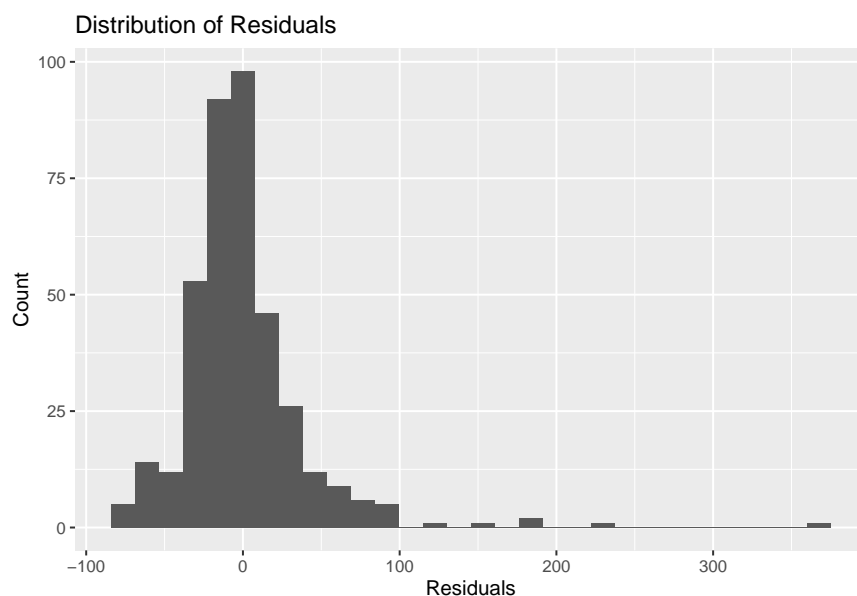
Full stop! If you see this sort of output on one of your diagnostic plots, you have an issue and should investigate it further. How? We'll talk more about this next class but, for now, let's finish with our validity conditions.

**Independence:**

I believe it is reasonable to assume that the height, sex, and IOCT time of one cadet is independent of another cadet.

**Normally-distributed residuals:**

```
model %>%
  ggplot(aes(x = .resid)) +
  geom_histogram() +
  labs(x = "Residuals", y = "Count",
       title = "Distribution of Residuals")
```



For this validity condition we are assessing the normality of the distribution of our residuals. Because I perceive that we may have some issues here, I'm going to borrow some code from my *Normality Testing* document:

```
RSE = summary(model)[[6]][[1]]

sd_count = as.data.frame(model$residuals) %>%
  mutate(sd_away = ceiling(abs(model$residuals / RSE))) %>%
  group_by(sd_away) %>%
  summarise(count = n())

paste("Within 1SD:", sd_count[1,2] / nrow(IOCT))

## [1] "Within 1SD: 0.838541666666667"

paste("Within 2SD:", (sd_count[1,2] + sd_count[2,2]) / nrow(IOCT))

## [1] "Within 2SD: 0.96875"

paste("Within 3SD:", (sd_count[1,2] + sd_count[2,2] + sd_count[3,2]) / nrow(IOCT))

## [1] "Within 3SD: 0.984375"
```

While we see a higher distribution of points within one standard deviation than we'd expect, these values match our "68-95-99 Rule" fairly well. Much better than I expected by looking at the histogram by itself. Here is a perfect example that, just because you have outliers, doesn't mean your residuals are normally distributed.

In total, except for the first diagnostic plot, our validity conditions look pretty good. I have a good idea what is going on with the first plot but I'm not going to tell you until next lesson on interaction terms. For the purposes of discussing the p-values, let's **assume** that our validity conditions are met.

### Theory-based approach:

Because you are all seasoned statisticians, I'm going to skip right to utilizing the wonderful output of `lm()` to obtain our standardized statistic value ( $t$ ) and our p-value.

```
summary(model)

##
## Call:
## lm(formula = IOCT$IOCT_Time ~ IOCT$height + IOCT$sex)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -79.45 -20.52  -4.57   11.37  364.36
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  290.0195     51.1840   5.666 2.88e-08 ***
## IOCT$height   -0.3966      0.7818  -0.507   0.612
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## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
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## Residual standard error: 40.87 on 381 degrees of freedom
## Multiple R-squared:  0.4466, Adjusted R-squared:  0.4437
## F-statistic: 153.8 on 2 and 381 DF, p-value: < 2.2e-16
```



Our model summary states that the p-value for our sex slope parameter estimate ( $\beta_2$ ) is **less than**  $2 \times 10^{-16}$  (also known as “basically 0”). This suggests this sample provides very strong evidence against that null hypothesis and, further, suggests there is an association between sex and IOCT time.

For height, the p-value is 0.612 which means there is little to no evidence against the null hypothesis. This means that, controlling for sex, this sample doesn’t provide evidence to discount that there is no association between height and IOCT time. Does this mean that all the cadets who complain about the IOCT being unfair for short people are wrong? You may have heard the phrase “absence of evidence is not evidence of absence.” What this means in this context is that we don’t have evidence against the null hypothesis, but this isn’t evidence of a lack of association. Further exploration could include composing confidence intervals for your slope parameters and/or collecting a different sample of data and repeating your analysis.

### Confidence intervals

For the height parameter:

```
beta_1 = -0.3966

t_crit = abs(qt(0.025, df = nrow(IOCT) - 2))

standard_error = 0.7818

lower = beta_1 - t_crit * standard_error

upper = beta_1 + t_crit * standard_error

paste("(", lower, ",", upper, ")")

## [1] "( -1.93377008363736 , 1.14057008363736 )"
```

For the sex parameter:

```
beta_2 = -80.1894

t_crit = abs(qt(0.025, df = nrow(IOCT) - 2))

standard_error = 6.2191

lower = beta_2 - t_crit * standard_error

upper = beta_2 + t_crit * standard_error

paste("(", lower, ",", upper, ")")

## [1] "( -92.4173540383079 , -67.9614459616921 )"
```

Why these confidence intervals shouldn’t surprise us in the fact that one of them contains zero and the other doesn’t, they do provide further insight into plausible values for our height slope parameter. While the sample does not provide evidence against the null hypothesis, the confidence interval suggests that it is plausible that there is an association between height and IOCT time. We see that it is plausible that this association is positive or negative and also that it’s fairly slight: (at a “maximum”) for every one inch increase in height, we expect a 1.4 (1.9) second increase (decrease) in IOCT time. I suppose for those short cadets, those extra seconds matter but it’s not a huge effect in the grand scheme of things.