

HOMEWORK 03

3.14

KVL

$$V_a - R_a i_a - L_a s i_a - V_m$$

MOTOR CONSTANTS

$$T_m = K_t i_a, \quad v_m = K_e \dot{\theta}_m$$

$$V_a - R_a \left(\frac{T_m}{K_t} \right) - s L_a \left(\frac{T_m}{K_t} \right) - K_e s \theta_m$$

ROTATIONAL SYSTEM

$$\sum T = J_m \ddot{\theta}_m \rightarrow J_m \ddot{\theta}_m = T_m \xrightarrow{\text{L.T}} T_m = s^2 J_m \theta_m$$

$$\rightarrow V_a - R_a \frac{(s^2 J_m \theta_m)}{K_t} - s L_a \frac{(s^2 J_m \theta_m)}{K_t} - K_e s \theta_m$$

$$\Rightarrow V_a = (R_a + s L_a) \frac{s^2 J_m \theta_m}{K_t} - s K_e \theta_m$$

TRANSFER FUNCTION

$$G(s) = \frac{\dot{\theta}_m(s)}{V(s)} \rightarrow \frac{\dot{\theta}_m}{V_a} = \frac{K_t}{(R_a + s L_a) s^2 J_m - s K_e K_t}$$

$$\rightarrow G(s) = \frac{K_s}{(R_a + s L_a) s J_m - K_e K_t}$$

3.16

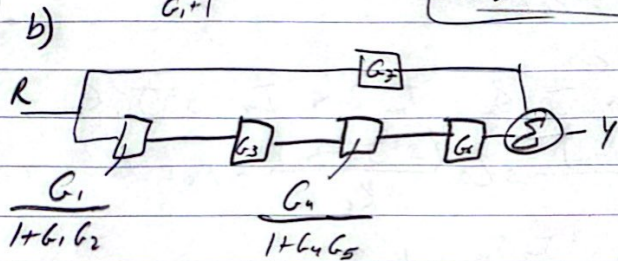
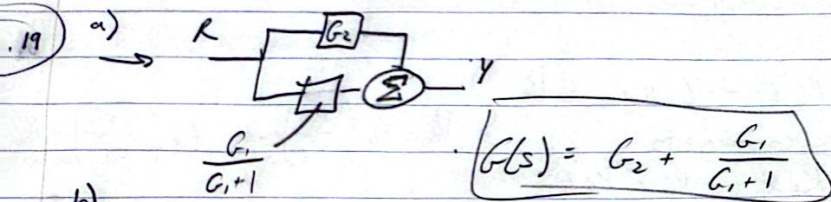
$$a) G(s)|_{s=0} = \frac{2}{0+0-2} \rightarrow \boxed{k = -1}$$

$$b) \lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} s G(s) \left(\frac{1}{s} \right) \rightarrow \lim_{s \rightarrow 0} s \left(\frac{2}{s^2 + s - 2} \right) \frac{1}{s}$$

$$= \lim_{s \rightarrow 0} \frac{2}{(s+2)(s-1)}$$

$$\rightarrow \boxed{y_{ss} = \infty}$$

FUT CANNOT BE USED IN AN UNSTABLE SYSTEM



c)

3.25 $\frac{Y(s)}{R(s)} = \frac{k}{s(s+2)} \Rightarrow G(s) = \frac{k}{s^2 + 2s + k}, M_p \leq 0.1$

$\omega_n^2 = k \Rightarrow \omega_n = \sqrt{k} \quad 2\zeta\omega_n = 2 \Rightarrow \zeta = \frac{1}{\sqrt{k}}$

$0.1 = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \Rightarrow k(0.1) = \frac{\pi\zeta}{\sqrt{1-\zeta^2}}$

... WOLFRAM... $\zeta = 0.591$ $k = 2.863$

3.26