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# Evaluating the Benefits of Non-marginal Reductions in Pollution Using Information on Defensive Expenditures[1]

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This article examines how the benefits of non-marginal pollution reductions can be evaluated using information on households' defensive expenditures to alleviate pollution. The most important conclusion is that upper and lower bounds to benefits can be derived with information only on the defensive expenditure technology. The article discusses the accuracy of these bounds and the appropriateness of the defensive expenditure model to several real-world pollution problems. © 1988 Academic Press, Inc.

Recent studies analyze the benefits of environmental improvements when households make defensive expenditures to alleviate pollution's effects (see [2, 7, 9, 15]). This literature focuses on the benefits of pollution reductions that are marginal, i.e., small. This article analyzes the relationship between defensive expenditures and the benefits of pollution reductions that are non-marginal, i.e., large.

If the household can take defensive measures against pollution, it can choose what I label the "quality of its personal environment" by choosing a level of defensive expenditures. I show that compensating and equivalent variation measures of the benefits of pollution can be derived by estimating demand for quality of the personal environment.

Available data may not permit estimation of household demand for quality of the personal environment. The most important result of this paper is that upper and lower bounds to benefits can be derived without this information. I show that a lower bound to benefits of a pollution reduction is the reduction in defensive expenditures needed to reach the originally chosen level of quality of the personal environment. An upper bound to benefits is the reduction in defensive expenditures needed to reach the personal environmental quality chosen after the pollution reduction. These upper and lower bounds allow benefit estimates to be made at a low cost.

The final sections of the article examine the accuracy of these bounds as approximations to true benefits and discuss the practical problems in applying the model.

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## 1. THE BENEFITS OF MARGINAL POLLUTION REDUCTIONS ARE GIVEN BY THE SAVINGS IN DEFENSIVE EXPENDITURES NEEDED TO KEEP PERSONAL ENVIRONMENTAL QUALITY CONSTANT

This section introduces the model. I show the result reached by others that benefits of marginal pollution reductions are equal to defensive expenditure savings.

Assume the household faces the problem,

$$\begin{array}{ll} \max\limits_{X,Q} U(X,Q) & U_X > 0 \\ & U_Q > 0 \\ \text{s.t. } X + D(Q,P) = Y & D_P > 0 \\ & D_Q > 0, \end{array} \tag{1}$$

where $Q$ is the quality of the individual's personal environment, that is, the environmental quality that directly affects utility; $P$ is the pollution level; $D(\ )$ is the defensive expenditure function showing the defensive expenditure needed to reach a particular personal environmental quality given pollution; $Y$ is income; and $X$ is the numeraire commodity. Subscripts indicate partial derivatives, with the signs assumed to be as shown.

The first-order conditions of this problem reduce to

$$U_Q/U_X = D_Q. \tag{2}$$

That is, the household chooses $Q$ and $X$ to equate the marginal value of personal environmental quality to its marginal cost.

To express the benefits of a small pollution reduction, note that the household's utility is a function of the model's two exogenous variables, pollution and income. The household's maximum attainable utility $v$, given by this indirect utility function, is equal to the Lagrangian of the household's maximization problem when $X$ and $Q$ are optimally chosen, or

$$v = V(P,Y) = U(X^*,Q^*) + \lambda(Y - X^* - D(Q^*,P)), \tag{3}$$

where $V(\ )$ is the indirect utility function, $X^*$ and $Q^*$ are the optimal choices, and $\lambda$ is the Lagrange multiplier. To find the income needed to keep utility constant as pollution changes, totally differentiate the Lagrangian with respect to $P$ and $Y$ and set the utility change to zero. This total differential is simplified using the envelope theorem. For marginal changes and when the maximization problem is smooth with respect to all variables, this theorem ensures that one can ignore the effects on a maximized function of changes in optimally chosen variables. The resulting needed compensation for a small pollution change is

$$\left.\frac{\partial Y}{\partial P}\right|_{v \text{ fixed}} = -V_P/V_Y = D_P. \tag{4}$$

So the benefit of a small reduction in pollution is $D_P$, the savings in defensive expenditures needed to reach the original level of personal environmental quality.

This result has been derived by others.[2] Because it measures the marginal benefits of reducing pollution, $D_P$ is also the optimal Pigovian tax.[3]

As pointed out by Courant and Porter [2], and Harrington and Portney [10], $D_P$ does not equal the actual change in defensive expenditures because $Q^*$ will change. The actual change in defensive expenditures is

$$\frac{dD}{dP} = D_P + D_Q\left(\frac{dQ^*}{dP}\right). \tag{5}$$

$D_P$ is measurable if one knows the defensive expenditure function, and $P$ and $Q^*$. Even if the $D$ function does not vary across households, the $D_P$ benefit measure varies because household choose $Q^*$.

## 2. THE BENEFITS OF NON-MARGINAL POLLUTION REDUCTION CAN BE MEASURED WITH ESTIMATES OF HOW HOUSEHOLDS ADJUST PERSONAL ENVIRONMENTAL QUALITY

Define a traditional expenditure function $e(P, v)$ giving the total household expenditure needed at pollution $P$ to reach utility $v$. The compensating and equivalent variation measures of the benefits of a non-marginal pollution reduction are

$$\begin{aligned} \mathrm{CV} &= e(P_0, v_0) - e(P_1, v_0) = Y - e(P_1, v_0) \\ \mathrm{EV} &= e(P_0, v_1) - e(P_1, v_1) = e(P_0, v_1) - Y. \end{aligned} \tag{6}$$

$v_0$ and $v_1$ are the equilibrium utility levels at $P_0$ and $P_1$. Substituting the expenditure function into the indirect utility function, and differentiating, one can derive

$$e_P = -V_P/V_y. \tag{7}$$

But $(-V_P/V_y)$ equals $D_P$. Hence CV can be expressed as

$$\mathrm{CV} = \int_{P_1}^{P_0} e_p(P, v_0)\,dP = \int_{P_1}^{P_0} D_P(Q[P, e(P, v_0)], P)\,dP. \tag{8}$$

$Q(P, e)$ shows how $Q$ varies with the exogenous variables, $P$ and income $e$.[4] In this thought experiment, we change income as $P$ changes to keep utility constant. If $Q$ is adjusted in response to both the direct and the indirect (through $e$) effects of $P$, at each step $D_P$ will exactly equal $e_P$. The integral of this "compensated" $D_P$ function

[2] See Eqs. (7) and (12) in Courant and Porter [2], Eq. (17) in Harrington and Portney [10], and Eq. (9) in Gerking and Stanley [7]. Courant and Porter point out that their equation for marginal benefits is equal to defensive expenditure savings holding $Q$ constant. The other two articles do not provide this interpretation of their equations for marginal benefits, but a little algebra easily shows this result.

[3] See Shibata and Winrich [15]. As they point out, if $Q$ and $P$ are measured in the same units, one might mistakenly believe that $D_Q$ is the optimal Pigovian tax.

[4] Some readers have wondered whether defensive expenditures should be an argument in the $Q$ function. $D$ should not be in the $Q$ function, because this $Q$ function is the behavioral relationship between $Q$ and exogenous variables, not a production function that shows how $Q$ varies as $P$ and $D$ vary.

equals the compensating variation for the pollution reduction. A similar expression can be derived for equivalent variation, with $v_1$ substituted for $v_0$.

Expression (8) could be evaluated through iterative methods if the researcher knows $D(Q, P)$ and has estimates of the household demand function for personal environmental quality, $Q(P, Y)$.[5] One would start at $P_0$, where the household's income is $Y\,(= e(P_0, v_0))$ and the household chooses $Q_0$. We know $D_P(Q_0, P_0)$. For some small reduction $\Delta P$ in $P$, the CV benefit will be approximately $(\Delta P)[D_P(Q_0, P_0)]$. To go to the next step of the iteration, we keep the household's utility constant and allow for changes in the choice of $Q$. The household's utility is kept constant at $v_0$ by taking away $(\Delta P)D_P(Q_0, P_0)$ in income from the household. The household's adjustment in $Q$ is found by evaluating $Q\{P_0 - \Delta P, Y - (\Delta P)[D_P(Q_0, P_0)]\}$, i.e., the demand function for $Q$ at the new $P$ and $Y$. We can then evaluate $D_P[Q(P_0 - \Delta P, v_0), P_0 - \Delta P]$. This new $D_P$ is used to calculate the CV benefits of the next reduction in $P$. This iterative process continues until we reach $P_1$. The approximations involved can be made arbitrarily exact by taking small enough changes in $P$. This iterative process resembles Vartia's proposal [16] for evaluating Hicksian CV and EV measures of price changes.

The validity of this procedure for evaluating non-marginal benefits depends on the validity of the $D_P$ expression. If households face a small number of discrete choices of $Q$, they cannot equate the marginal benefits and costs of more $Q$. The envelope theorem will not apply, because small changes in a household's optimal $Q^*$ and $X^*$ cannot be ignored even for small changes in $P$. The $D_P$ marginal benefit measure will be incorrect, and this approach to measuring non-marginal benefits will not work. Defensive options often may be limited; for example, a household seeking to reduce the effects of toxic waste on its water supply might be able to defend itself only by a water filter, bottled water, or moving away. But with a sufficient number of defensive options, one can treat these options as approximating a smooth defensive expenditure function. In the toxic waste example, if several qualities of water filters are available, the defensive expenditure function may be close enough to smooth that $D_P$ will be a valid marginal benefit measure.

When the defensive expenditure technology is not sufficiently smooth, benefits of non-marginal pollution reductions still can be estimated using a conditional logit model. If household $i$ faces several discrete choices of $Q$, the conditional logit model specifies the probability of household $i$ choosing defensive option $k$ among all defensive options $j$ as

$$\Pr(ik) = \exp W\left[Y - D(Q_k, P), Q_k\right] \Big/ \sum_j \exp W\left[Y - D(Q_j, P), Q_j\right], \tag{9}$$

where $W[\ ]$ is the observable portion of the household's direct utility function. Equation (9) can be estimated via maximum likelihood.

The conditional logit model depends on assumptions about the distribution of unobserved disturbances that affect household utility. These assumptions are outlined in McFadden [13, 14]. In particular, the logit model assumes that the utility

[5] Other demand shifters in addition to $P$ and $Y$ could be included in the estimation. Ideally data would be available over the range $(P_0, P_1)$, so that benefit estimation does not require extrapolation to values of the independent variables that one never observes. Just *et al.* [12, pp. 173–175] make the general point that one should prefer benefit measurement methods that do not require extrapolation.

disturbances are independent across the options. But in the defensive expenditure case, one would expect households with higher utility for one defensive option to have higher utility for other defensive options. For example, a household that has greater utility than expected from installing a water filter to protect itself from polluted drinking water probably also will have greater utility from using bottled water than one would predict. This statistical problem can be dealt with by using a modified approach called nested logit [14] that allows for a fixed correlation within particular subsets of the household's options, such as, for example, all defensive options. But, it is difficult within a logit model to allow for more general patterns of correlation among the disturbance terms.

The logit model yields estimates of utility function parameters. With these estimates, one can simulate how household utility changes as pollution is reduced. Hanemann [8] discusses the complexities that arise when using discrete choice models for welfare analysis.

## 3. A LOWER (UPPER) BOUND FOR THE BENEFITS OF NON-MARGINAL POLLUTION REDUCTIONS IS GIVEN BY THE SAVINGS IN DEFENSIVE EXPENDITURES NEEDED TO REACH THE INITIAL (FINAL) LEVEL OF PERSONAL ENVIRONMENTAL QUALITY

### *3.1. The Basic Result and the Intuition Behind It*

The exact benefit measures described in Section 2 require the estimation of household demand for personal environmental quality, which is costly in both time and data requirements. This section develops two benefit measures that are lower and upper bounds to true benefits. These only require information on the defensive expenditure function and household choices before and after the pollution reduction.

The pollution reduction decreases defensive expenditures needed to reach any particular $Q$. The reduction in defensive expenditure needed to reach $Q_0$, the originally chosen level of personal environmental quality, will be $D(Q_0, P_0) - D(Q_0, P_1)$. Define the restricted expenditure function $e(P, v; \bar{Q})$ as the function giving the minimum expenditure needed to reach utility $v$ when pollution is $P$ and personal environmental quality is restricted to $\bar{Q}$. The savings in defensive expenditures needed to reach $Q_0$, or $\mathrm{DS}(Q_0)$ for short, can be expressed as

$$\begin{aligned} e(P_0, v_0) - e(P_1, v_0; Q_0) &= X(P_0, v_0) + D(Q_0, P_0) \\ &\quad - [X(P_1, v_0; Q_0) + D(Q_0, P_1)] \\ &= X_0 + D(Q_0, P_0) - [X_0 + D(Q_0, P_1)] \\ &= D(Q_0, P_0) - D(Q_0, P_1) \equiv \mathrm{DS}(Q_0), \end{aligned} \tag{10}$$

where $X(P, v; \bar{Q})$ is the $X$ needed to reach utility $v$ when pollution is $P$ and $Q$ is restricted to $\bar{Q}$. The key step is that $X(P_0, v_0)$ must equal $X(P_1, v_0; Q_0)$ because $Q_0$ and $v_0$ are the same in both cases, and only $Q$ and $X$ directly affect utility.

Comparing expression (10) with expression (6) for compensating variation, we note that

$$\begin{aligned} \mathrm{CV} &= e(P_0, v_0) - e(P_1, v_0) \\ &= D(Q_0, P_0) - D(Q_0, P_1) + e(P_1, v_0; Q_0) - e(P_1, v_0) \\ &\geqslant D(Q_0, P_0) - D(Q_0, P_1) \end{aligned} \tag{11}$$

because

$$e(P_1, v_0; Q_0) \geqslant e(P_1, v_0).$$

$e(P_1, v_0; Q_0)$ must be greater than $e(P_1, v_0)$, because the expenditure needed to reach some utility level with restrictions must be greater than the expenditure needed without restrictions. Thus, the savings in defensive expenditure needed to reach the initial $Q$ are a lower bound to CV.

The intuition for this result is that adjustment increases utility. Suppose pollution is reduced, but we force the household to retain the same level of personal environmental quality. Despite this restriction, the household gains a benefit equal to the savings in defensive expenditures. If adjustment in $Q$ is allowed, the household must be even better off.

A similar procedure can be used to derive an upper bound for the equivalent variation measure of the benefits of a pollution reduction:

$$\begin{aligned} \mathrm{EV} &= e(P_0, v_1) - e(P_1, v_1) \\ &= D(Q_1, P_0) - D(Q_1, P_1) + e(P_0, v_1) - e(P_0, v_1; Q_1) \\ &\leqslant D(Q_1, P_0) - D(Q_1, P_1) \equiv \mathrm{DS}(Q_1) \end{aligned} \tag{12}$$

because

$$e(P_0, v_1; Q_1) \geqslant e(P_0, v_1).$$

The savings in defensive expenditure at the final chosen level of $Q$, $\mathrm{DS}(Q_1)$, are thus an upper bound to the EV measure of benefits. For intuition, imagine that we begin at the final level of $P_1$ with optimal adjustment to $Q_1$. We then imagine reversing the pollution reduction and increasing $P$ to $P_0$. If the household must keep the same $Q_1$, the loss is equal to the increased defensive expenditures. But if adjustment is allowed, the utility cost is reduced. The defensive expenditure changes holding $Q_1$ constant are therefore greater in absolute value than the CV measure of the loss from pollution increases. This CV measure has the same absolute value as the EV measure of the gain from pollution reduction.[6]

These lower and upper bounds are analogous to Laspeyres and Paasche measures of the benefits of a price reduction.[7] The difference is that $D(Q, P)$ is usually a non-linear function of $Q$. If $D(Q, P)$ is linear in $Q[D(Q, P) = Qf(P)]$, then these lower and upper bounds are Laspeyres and Paasche measures of the benefits of a

[6]As this implies, the defensive expenditure increase at the original level of $Q$ is greater in absolute value than the CV measure of the loss from a pollution increase, while the defensive expenditure increase at the final $Q$ is less in absolute value than the EV measure of the loss.

price reduction. The defensive expenditure savings at $Q_0$ are then

$$D(Q_0, P_0) - D(Q_1, P_0) = Q_0[f(P_0) - f(P_1)] \\ = Q_0[\Delta \text{ in price of } Q], \tag{13}$$

which is a Laspeyres measure of the real income increase caused by a decline in the price of $Q$. Similarly, the savings in defensive expenditure at the final $Q$ are exactly a Paasche measure of the real income increase if the $D$ function is linear.

These defensive expenditure savings need not be close to the actually observed change in defensive expenditure, $D(Q_0, P_0) - D(Q_1, P_1)$. If a pollution reduction increases the chosen personal environment quality, actual changes in defensive expenditure will always be less than the savings in defensive expenditure at the original level of $Q_0$, and hence less than CV, or

$$D(Q_0, P_0) - D(Q_1, P_1) \leqslant D(Q_0, P_0) - D(Q_0, P_1) \leqslant \mathrm{CV}. \tag{14}$$

The observed change in defensive expenditure is thus a less accurate lower bound to the CV benefit measure than the $\mathrm{DS}(Q_0)$ measure.

### 3.2. *Are CV and EV Completely Bounded by the DS Measures?*[8]

If CV $\leqslant$ EV, as one would ordinarily expect for a utility gain, then these two benefit measures provide upper and lower bounds to both the CV and EV measures of the benefits of a pollution reduction. Under this assumption, the overall relationship between these four benefit measures for a pollution reduction is

$$\mathrm{DS}(Q_0) \leqslant \mathrm{CV} \leqslant \mathrm{EV} \leqslant \mathrm{DS}(Q_1), \tag{15}$$

where $\mathrm{DS}(Q)$ indicates the savings in defensive expenditure needed to reach $Q$ when $P$ changes from $P_0$ to $P_1$.

A perusal of Eq. (8) indicates that CV $\lesseqgtr$ EV as $e_{PV} \gtreqless 0$. But $e_{PV} = D_{PQ}Q_Y e_V$, where subscripts indicate partial derivatives, and double subscripts indicate second partials. $e_V$ must be positive. Hence, if $Q$ is a normal good ($Q_Y > 0$), the sign of $D_{PQ}$ determines the sign of $e_{PV}$ and the relationship between CV and EV. $D_{PQ} > 0$ implies CV < EV, and the DS measures bound both CV and EV. But if $D_{PQ} < 0$, CV > EV; under these conditions, while $\mathrm{DS}(Q_0)$ must be less than CV, it is possible that EV is less than $\mathrm{DS}(Q_0)$. Similarly, while $\mathrm{DS}(Q_1)$ must be an upper bound to EV, CV might be greater than $\mathrm{DS}(Q_1)$.

[7]Some readers of this article have inquired whether the $\mathrm{DS}(Q_0)$ and $\mathrm{DS}(Q_1)$ measures are the same as the Hicksian compensating surplus and equivalent surplus measures. If this were true, Eq. (15) would reexpress Hicks' result [11] that compensating and equivalent surpluses bound compensating and equivalent variation. But the $\mathrm{DS}(Q_0)$ and $\mathrm{DS}(Q_1)$ measures are not the same as the Hicksian surplus measures. The Hicksian surplus measures give the willingness to pay for some exogenous change in price (or other exogenous variable) when the household is not permitted to change its consumption of the commodity in response to the compensation. The household is, however, permitted to adjust to the change in price. The $\mathrm{DS}(Q_0)$ and $\mathrm{DS}(Q_1)$ measures permit no household adjustment whatsoever. Just *et al.* [12, pp. 136–142] discuss the four Hicksian surplus measures.

[8]This discussion is in response to a very insightful comment on the original draft of this article by Winston Harrington, who pointed out that Eq. (15) cannot hold for some defensive expenditure functions and some behavioral reactions to changes in $P$.

An unpublished paper by this author discusses the case of $D_{PQ} < 0$ in more detail. But the following points indicate that this case is not a significant problem for the model:

(1) Even if the DS measures do not completely bound both measures, having a lower bound to CV and an upper bound to EV is still useful. Which is more useful depends on whether CV or EV is a better measure of benefits. This in turn depends on the appropriate assignment of property rights in the case of pollution.

(2) As long as CV and EV are close, even if CV > EV, the DS measures must approximately bound both CV and EV. If CV and EV are not close, a benefit analyst faces the difficult task of deciding which measure is appropriate.

(3) It can be shown [1] that even if $D_{PQ} < 0$, CV and EV will be close unless $D_{PQ}$ is large in absolute value. But if $D_{PQ}$ is negative and large, the marginal price of $Q$ increases a great deal as pollution is reduced. This will cause households to respond oddly to pollution reductions: the household will choose lower values of $Q$. Hence, large negative values of $D_{PQ}$ are only plausible for pollutants for which one believes this behavior is plausible.

(4) A large negative value of $D_{PQ}$ also implies that the benefits of marginal pollution reductions ($D_P$) decline rapidly as household income (and hence $Q$) increases. Thus this type of technology would lead to political concern over this pollutant being more pronounced in lower income households than in upper income households. This does not seem to fit observed political behavior.

(5) Finally, as detailed in [1], if one expects decreasing returns to scale in the final amount of pollution reduction by defensive measures, $D_{PQ} > 0$ seems a more plausible technology. As pollution is increased, the increased scale of the required defensive activity would be expected to increase the marginal cost of additional $Q$.

Ultimately, the sign of $D_{PQ}$ is an empirical matter, which must be determined for the particular pollutant and technology being analyzed. The point is that a large negative $D_{PQ}$ is probably unusual and in any event is not a disaster for the model.

### 3.3. *The Key Advantage of These Bounds for Benefit Estimates*

The key advantage to these lower and upper bounds is that they require less information and fewer assumptions than more exact measures. For example, the DS measures do not require an assumption that the defensive expenditure function is smooth,[9] or the logit model's assumptions about the error term. Furthermore, the DS($Q_0$) measure only requires knowledge about how the defensive expenditure function is affected by changes in $P$ at $Q_0$, while the CV or EV measures require information on the behavioral responses of the household's chosen $Q$. It might seem

[9]One might think that the DS measures require the differentiability of the defensive expenditure function, as problem (1) assumes that $D_P$, $D_Q$, $U_X$, and $U_Q$ exist. But these assumptions are not needed for the DS measures to be valid bounds. If one considers the equations used to derive the DS bounds (Eqs. (6), (10), (11), and (12)), they require the existence of household expenditure functions and restricted expenditure functions, but not their differentiability. Epstein [4] has shown that the existence of household expenditure functions when the household faces a non-linear budget constraint does not require the differentiability of the budget constraint. Hence, the budget constraint, $X + D(P, Q)$, does not need to be differentiable for household expenditure functions to exist and for the DS bounds to be valid.

that the DS($Q_1$) measure also requires information on the household's behavioral responses to changes in $P$. But one may be able to guess $Q_1$ without knowing exactly how the household's chosen $Q$ will change along the path from $P_0$ to $P_1$.

For example, consider the cleanup of a toxic waste dump that reduces risk to zero. Then the household obviously will choose zero defensive expenditures and a risk of zero. Suppose the household before the cleanup is reducing some risk from toxic waste by undertaking defensive expenditures of \$200. Furthermore, suppose some expert determines that the household could have obtained complete protection from the risk at a cost of \$1000. Then \$200 is an underestimate of the CV benefit measure for cleaning up the toxic waste dump, while \$1000 is an overestimate of the EV benefit measure.

## 4. THE ACCURACY OF THE DS MEASURES AS APPROXIMATIONS TO TRUE BENEFITS

This section assesses the accuracy of the DS measures as approximations to true benefits. I consider linear defensive expenditure functions before examining more general defensive expenditure functions. The basic result is that the DS measures are good approximations to benefits if the household's change in $Q$ in response to changes in $Q$'s price is small, either because the change in $Q$'s price is small, or because the household is not very responsive to price changes.

### *4.1. Linear Defensive Expenditure Functions*

The accuracy of these approximations is easily determined if budget constraints are linear in $Q$, that is, if $D(Q, P) = f(P)Q$. The benefit measures can then be depicted in an ordinary demand diagram such as Fig. 1.

In the figure, $f(P_0)$ and $f(P_1)$ are the prices of $Q$ when pollution is $P_0$ and $P_1$. $Q^{\mathrm{m}}$ and $Q^{\mathrm{h}}$ are the Marshallian and Hicksian curves for $Q$. The various benefit

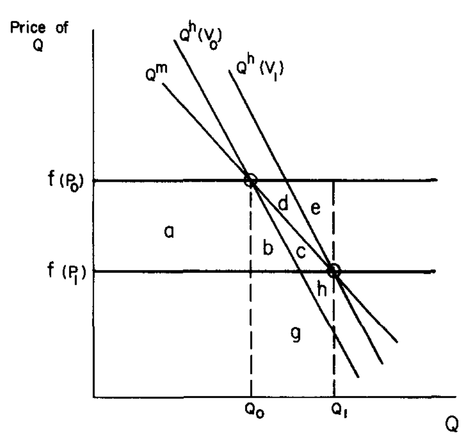


FIG. 1. Comparison of four benefit measures when the defensive expenditure function is linear.

measures correspond to the following areas in the diagram:

$$\begin{aligned} \mathrm{DS}(Q_0) &= a \\ \mathrm{CV} &= a+b \\ \mathrm{CS} &= a+b+c \\ \mathrm{EV} &= a+b+c+d \\ \mathrm{DS}(Q_1) &= a+b+c+d+e \end{aligned} \tag{16}$$

Actual change in defensive expenditure $= -a + g + h$.

The percentage error in $\mathrm{DS}(Q_0)$ as a measure of CV thus is equal to area $b$ as a proportion of area $a$. The accuracy of $\mathrm{DS}(Q_1)$ as an approximation to EV depends on the size of area $e$ relative to area $(a + b + c + d + e)$.

This question resembles Willig's [18] question about the accuracy of CS (consumer surplus) as a measure of CV and EV. In Fig. 1, Willig was comparing the size of areas $c$ or $d$ to area $(a + b + c)$.

Methods similar to Willig's show that the percentage errors in the DS measure of benefits will be approximated by[10]

$$[\mathrm{CV} - \mathrm{DS}(Q_0)]/\mathrm{DS}(Q_0) = \text{area } b/\text{area } a \approx \frac{1}{2}\left(\frac{\Delta \text{ price}}{\text{price}}\right)(-\varepsilon), \tag{17}$$

where $\varepsilon$ is the Hicksian price elasticity of demand. A similar formula applies to the error in $\mathrm{DS}(Q_1)$ as a measure of EV. These formulas differ from the analogous equation in Willig in that here the error depends on the percentage price change and the price elasticity of demand, while Willig found that the error in CS depended on the percentage change in real income as prices change, and the income elasticity of demand. While price and income elasticities could be of similar size, a given percentage change in prices usually results in a much smaller percentage change in real income. Area $b$ will be larger than area $c$ or $d$. $\mathrm{DS}(Q_0)$ will therefore not be as good an approximation to CV as CS.

Linear defensive expenditure functions are not as unusual as one might suppose. The units in which $Q$ is measured are arbitrary. Hence, if $D(Q, P) = f(P)g(Q)$ (i.e., $P$ has the same percentage effect on $D$ at all $Q$) then $g(Q)$ could be defined as the "true" measure of $Q$, making the budget constraint linear.

### *4.2. The General Case: A Hedonic Approach Is Useful for Examining Possible Approximation Errors*

The case of a non-linear defensive expenditure function can be shown in a hedonic diagram. To use hedonic methods, define a bid function $W(Q, v, Y)$ as the

[10] This is essentially an approximation that treats the Hicksian demand curves as linear for small changes in the price. While this approximation is inexact, it is still much more accurate than using marginal valuations to measure non-marginal benefits. In the present context, the marginal benefit measure at the original equilibrium is $-f_P(P_0)Q_0$; extrapolating to the total change from $P_0$ to $P_1$ yields a measure of $-(P_0 - P_1)f_P(P_0)Q_0$. This is incorrect because (1) $Q$ will change; (2) $f_P$ will change unless the $f$ function is linear. The $\mathrm{DS}(Q_0)$ measure allows for $f$ to be non-linear; the Hicksian measure allows a first-order approximation to how $Q$ will adjust.

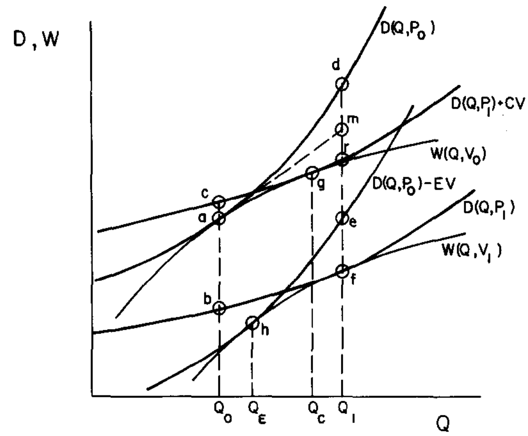


FIG. 2. Comparison of four benefit measures in a hedonic defensive expenditure diagram.

solution to

$$U(Y - W, Q) = v. \tag{18}$$

One can show that at the chosen $Q^*$, $W = D$ and $W_Q = D_Q$, that is, the bid function is tangent to the defensive expenditure function.

Figure 2 shows the household's choice at two different levels of $P$ and the various benefit measures of a pollution reduction. Facing $D(Q, P_0)$, the household maximizes utility by choosing the lowest attainable bid function, $W(Q, v_0)$, with the tangency occurring at point $a$ and $Q = Q_0$. When pollution drops to $P_1$, the defensive expenditure function shifts downward. I show the defensive expenditure function shifting downward and flattening ($D_{PQ} > 0$), but it could get steeper ($D_{PQ} < 0$). Facing this new defensive expenditure function, the household again chooses the lowest feasible bid function, here $W(Q, v_1)$, with the tangency at point $f$ and the chosen $Q$ at $Q_1$.

The DS($Q_0$) measure of the benefits of this pollution reduction is line segment $ab$. The DS($Q_1$) measure of the benefits is line segment $df$. To show the CV measure requires further manipulation. CV is the income that if taken away from the household when pollution is $P_1$ results in the same utility as at pollution $P_0$. A reduction in income is equivalent to increasing the $D$ function everywhere. Thus the CV measure of benefits can be found by shifting the $D(Q, P_1)$ function upward in a parallel fashion until it is tangent to $W(Q, v_0)$, with CV equal to the parallel shift. In the diagram, this "compensated" tangency occurs at point $g$, with $Q$ at $Q_C$. The CV parallel shift is shown at $Q = Q_0$ as line segment $cb$, and the DS($Q_0$) measure of $ab$ can be seen to be less than CV. A similar procedure shows that the EV measure is $de$, and the DS($Q_1$) measure of $df$ is greater than EV.

The diagram indicates that the error in the DS benefit measures results from "compensated" price responses to shifts in the $D$ function. If these compensated price responses are small—i.e., the movement from $a$ to $g$ or $f$ to $h$ is small—then the error will not be large. In algebra, the CV measure of benefits is

$$\mathrm{CV} = \mathrm{DS}(Q_0) + [W(Q_C, v_0) - W(Q_0, v_0)] - [D(Q_C, P_1) - D(Q_0, P_1)]. \tag{19}$$

That is, the CV measure of benefits is equal to the savings in defensive expenditure, plus the net gains to the household from adjusting from $Q_0$ to $Q_C$ along a given bid curve, equal to the household willingness to pay minus the increase in defensive expenditure from $Q_0$ to $Q_C$. DS($Q_0$) is less of an underestimate if $Q_C$ is close to $Q_0$. A similar equation can be developed to show that if $Q_E$ is close to $Q_1$, DS($Q_1$) will be close to EV.

The size of these compensated price responses depends on two factors: how the defensive expenditure function shifts and household preferences. The compensated price response will be small if the shift in $D$ does not greatly change the marginal price of $Q$, or if the household does not respond greatly to compensated changes in the marginal price (i.e., the bid curves $W$ are highly curved). The compensated price response will also be small if corner solutions cause $Q_0$ and $Q_1$ to be close.

An unpublished paper by this author [1] considers extreme examples and demonstrates the following propositions:

(1) If the defensive expenditure function shifts down everywhere by the same amount with a pollution reduction ($D_{PQ} = 0$), then all four benefit measures (DS($Q_0$), DS($Q_1$), CV, EV) coincide.

(2) If the bid function allows no substitution between $Q$ and money (Leontief preferences), then DS($Q_0$) = CV, and DS($Q_1$) = EV.

(3) If corner solutions, such as an infinite price for additional $Q$ at some level of $Q$ (e.g., when the level of $Q$ signifies a perfectly clean environment), result in the same choice of $Q$ before and after the pollution reduction, all four benefit measures coincide.

A final insight from Fig. 2 is that one can sometimes develop a closer upper bound to CV than DS($Q_1$). This is important because the DS($Q_1$) measure could be infinite. In other words, it may be impossible for the household on its own, when pollution is $P_0$, to purchase the level of personal environmental quality that it chooses when pollution is $P_1$. But if the defensive expenditure function is convex, and the bid function $W$ is concave, then an upper bound closer to CV than to DS($Q_1$) can be derived. This is shown in Fig. 2. Extend a tangent to $W(Q, v_0)$ at point $a$ to the vertical line through $Q_1$. The vertical distance $mf$ is greater than CV (equal to $cb$ or $rf$) but less than DS($Q_1$) (equal to $df$). The unpublished paper by this author [1] derives this upper bound algebraically.[11]

## 5. PROBLEMS IN EMPIRICAL IMPLEMENTATION OF THE MODEL

In this section I show how the model might be applied and problems that could arise. Table I lists different types of pollution and possible defensive measures that could fall within this model's scope.

[11] The algebraic proof depends on $Q_1$ being close to $Q_C$, i.e., on income effects being small. This type of approximation is similar to Freeman's [5, 6] suggestions for approximating non-marginal WTP in a hedonic model where one only knows marginal WTP. A similar procedure could be used to develop a closer lower bound to EV than DS($Q_0$). Although these bounds may sometimes be closer, they require a lot of assumptions about the convexity and concavity of the $D$ and $W$ functions and about the magnitude of income effects.

TABLE I
Examples of Defensive Measures against Pollutants

| Pollutant | Defensive measures |
|---|---|
| Air pollution | Clean or repaint exterior of house; install air purifiers or air conditioners; visit the doctor more frequently; move away from pollution source |
| Water pollution | New well; bottled water; water purifiers; move away |
| Hazardous waste | Similar to both water and air pollution depending upon medium by which hazardous waste affects households |
| Noise pollution | Storm windows; thicker walls; move away |
| Radon in well water | Filter or aerate water; bottled water; increase house air ventilation; move away |
| Radon in soil underneath house | Ventilate crawlspace of house; seal foundation of house; use thicker concrete in basement; increase house air ventilation; move away |

The defensive expenditure model relies on four important assumptions that may prove questionable in some of these applications:

(1) Defensive expenditures are perfect substitutes for pollution reduction and have no value other than alleviating pollution.

(2) There are no significant adjustment costs associated with reducing the level of investment in defensive measures.

(3) The defensive expenditure function is a technical relationship that can be discovered using scientific information, without requiring behavioral estimates.

(4) Pollution can be influenced by the government.

Next, I will explain how these assumptions may be violated in various applications and why the violations create problems. These problems can be surmounted, but increase the model's complexity.

First, some effects of pollution cannot be controlled by defensive measures. For example, it may be that some health effects of air pollution cannot be alleviated by defensive measures, even if effects on a house's exterior are easy to remedy with soap or fresh paint. This can be modeled by assuming personal environmental quality is a vector $\mathbf{Q}$ of variables, only some of which ($\mathbf{Q}_d$) can be alleviated with defensive measures. Only the $\mathbf{Q}_d$ variables enter the defensive expenditure function; the other variables $\mathbf{Q}_n$ affect utility but are simply functions of $P[\mathbf{Q}_n = f(P)]$.

A similar problem arises if household defensive measures serve other purposes. For example, air conditioners may filter air pollutants out of the interior of the home, but are also valued for cooling. This case can be modeled by letting defensive expenditure $g = D(Q, P)$ enter the utility function directly, in addition to $Q$.

These two cases differ from the pure defensive expenditure model in that pollution enters the utility function directly, via the functions $\mathbf{Q}_n = f(P)$ and $g = D(Q, P)$. The pure defensive expenditure model assumes that pollution only indirectly affects utility by altering the budget constraint.

The model can account for these additional effects of pollution, but the model becomes more complex or less accurate. Consider the DS($Q_0$) measure if some effects $\mathbf{Q}_n$ of pollution cannot be defended against. In the modified model, in which $Q_0$ is the original chosen level for only those $\mathbf{Q}_d$ that enter the defensive expenditure function, DS($Q_0$) is more of an underestimate of CV than it was in the simple model. DS($Q_0$) now overlooks not only the benefits of adjusting to lower levels of pollution, but also the benefits of the reductions in $\mathbf{Q}_n$ as $P$ is reduced. Algebraically, it can be shown that DS($Q_0$) equals

$$\begin{aligned}\mathrm{DS}(Q_0) = \mathrm{CV} &+ \left[e(P_1, v_0) - e(P_1, v_0; Q_0)\right] \\ &+ \left[e(P_1, v_0; Q_0) - e(P_1, v_0; Q_0, f_0)\right].\end{aligned} \tag{20}$$

As shown in Section 3, the second term is less than zero because expenditures needed to reach a given utility without restrictions are less than those needed with restrictions. The third term contains a new function, $e(P_1, v_0; Q_0, f_0)$. This function gives the expenditures needed to reach utility $v_0$ when pollution is $P_1$, the $\mathbf{Q}_d$ vector is restricted to $Q_0$, and we imagine that some magical wand has kept $\mathbf{Q}_n$ at $f_0 = f(P_0)$ even though pollution has declined to $P_1$. This third term also must be less than zero. This is because expenditures must be higher to reach utility $v_0$ when $\mathbf{Q}_n$ is artificially kept low at $f(P_0)$ rather than being allowed to increase to $f(P_1)$. As a result, DS($Q_0$) is still less than CV, but by more than before, due to the introduction of the third term.

Similar reasoning shows that DS($Q_1$) need no longer be an upper bound to EV. Suppose we reverse the pollution decrease, increasing pollution from $P_1$ to $P_0$. The increase in defensive expenditures holding $Q$ constant at $Q_1$ overstates the cost to households because adjustment in $\mathbf{Q}_d$ will make households better off. But DS($Q_1$) understates the cost of the pollution increase inasmuch as this measure overlooks the decrease in $\mathbf{Q}_n$ as pollution increases.

The results limit the usefulness of the DS measures. If some consequences of pollution cannot be defended against, DS($Q_0$) will be a less accurate lower bound to CV, and DS($Q_1$) is no longer necessarily an upper bound to EV. An upper bound to EV, and a more accurate lower bound to CV, can be derived with estimates of the household's willingness to pay for changes in $\mathbf{Q}_n$. But these estimates of WTP may be difficult to obtain.

Similar arguments show that the model is more complicated when defensive measures provide other types of benefits. In this case, DS($Q_1$) will still be an upper bound to EV, but will be less accurate, while DS($Q_0$) is no longer necessarily a lower bound to CV. Consider the DS($Q_0$) measure. As pollution decreases, the household savings in defensive spending, holding $Q$ constant at $Q_0$, overlooks the benefits of adjusting $Q$. But this measure also overlooks the loss due to the independent value of defensive measures. For example, the DS($Q_0$) measure overlooks the utility loss due to less cooling of the house when the air conditioner is eliminated. Hence DS($Q_0$) may no longer be a lower bound to CV. Similar intuition shows that the DS($Q_1$) measure is a less accurate upper bound. (The algebra showing these results is available from the author.) Better bounds to benefits could be developed with information on a household's valuation of defensive measures for non-defensive reasons. But such adjustments complicate the simplicity of the DS measures.

A second problem with the model is that reducing the investment in defensive measures may require significant adjustment costs. Many defensive measures are costly to reverse once undertaken: moving away from the source of the pollution, putting in a new well to avoid water pollution, building thicker walls in a new house to mitigate noise pollution, and using thicker concrete in a house basement to minimize radon contamination. The defensive expenditure savings from reversing these investments will be less than the defensive expenditures avoided by not undertaking the investment.

These adjustment costs can be included in the model by treating previous defensive investment as a variable that shifts the defensive expenditure function. The DS measures will still be bounds to benefits, but benefits and the DS measure may vary depending on the previous defensive investments undertaken by the household. Consider as an example the installation of pollution controls that would minimize groundwater contamination at a hazardous waste site and reduce the risk from the hazardous waste site to zero. Suppose that the controls are being considered before the hazardous waste site is open, or, alternatively, before any nearby residents are aware of the true hazards. Assume that with full information and without controls, all nearby residents would move away from the site, reducing their exposure to zero.

In this example, the $\mathrm{DS}(Q_0)$ and $\mathrm{DS}(Q_1)$ measures of the benefits of pollution control *before* residents have moved away are equal to the moving costs avoided by the nearby residents, and hence exactly measure $\mathrm{CV} = \mathrm{EV}$. These moving costs include financial and search costs, the psychological costs of leaving a familiar neighborhood, and any other sources of economic "rent" households obtain from their current location.[12]

The benefits of pollution control *after* the site is open and publicized, and all nearby residents have moved away, will be quite different. After the control measures are adopted and the groundwater is clean, some households will move back near the site. The surplus gained from this move is a DS measure of the benefits from the cleanup. But there is no reason for the total household surplus from the move back to be close to the surplus lost from moving out. The moving costs given up in the move out are not recovered in the move back, and the move back will require additional moving costs. Furthermore, not all the households who moved out will move back, and some new households will move into the neighborhood.

The costs of adjusting downward defensive investment lead to some surprising patterns in the distribution of benefits. A pollution reduction may provide less benefits to households who have made extensive defensive investments, and hence presumably care or know more about pollution, than to households who have adopted more easily reversible defensive measures. If affected households pay for pollution reduction, households who have made irreversible defensive investments might question the fairness of their paying again for dealing with pollution, while neighbors who have not made such investments gain greater benefits. Furthermore, the distribution of benefits will vary considerably depending on the timing of the pollution reduction as this will affect who has made defensive investments. Both the

[12]Several studies [3, 17] indicate that moving costs of all types amount to about 15% of permanent income for the average household, in the sense that the average household seems willing to forego indefinitely an income gain of 15% to remain at its current location.

timing of pollution reduction and the fairness of schemes for paying for pollution reduction require special attention from policymakers when defensive investments are difficult to reverse.

A third problem is that the defensive expenditure function may not be a well-known technical relationship, but a function that must be estimated using information on household behavior. While the DS measures will still be valid, they no longer have as much of an advantage over other benefit measures in ease of calculation. A recent article by Gerking and Stanley [7] encountered this problem in estimating a health production function. In my notation, Gerking and Stanley were seeking to determine the shape of a defensive expenditure function $D(\mathbf{Q}, P, \mathbf{T})$, where $\mathbf{Q}$ is a vector of health characteristics of the individual, $P$ is air pollution, and $\mathbf{T}$ is a vector of other individual characteristics that may affect health. Because we do not know the exact effects of pollution, individual characteristics, or defensive expenditures (in this case, medical expenditures) on health, this function must be estimated. A further complexity arises because $\mathbf{Q}$ is endogenous; the individual's choice of health depends on unobserved individual characteristics that are in the residual in the defensive expenditure function. Income and exogenous price variation are correctly used by Gerking and Stanley as instrumental variables for the $\mathbf{Q}$ variables.

A final problem is that the government may not control the pollution level. In this case, the DS and other benefit measures can still be used to measure the benefits of government-induced change in some other exogenous variable. The best example of this situation is radon. The government cannot directly control the radon exposure of an individual house. Radon is a naturally occurring phenomenon; furthermore, radon is a private good, in that each house's radon problem only affects that house's residents. But government may play a role in supplying a related public good, information on defensive measures against radon. Defensive information is another variable that enters the defensive expenditure function. The DS measures can bound the benefits of providing this information. The DS($Q_0$) measure would now be

$$\mathrm{DS}(Q_0) = D(Q_0, P_0, I_0) - D(Q_0, P_0, I_1), \tag{21}$$

where $I_0$ and $I_1$ are the amounts of information on defensive measures before and after the government intervention. For example, suppose that a household faced with a severe radon problem would have moved away if it had little information on defensive measures. If the government information provision enables the household to protect itself without moving away, the DS($Q_0$) measure would be the difference between moving costs and the costs of the alternative defensive measure. If the household chooses the same $Q_0$ before and after the information is provided, the DS($Q_0$) measure is an exact measure of both the CV and EV from the information.

## 6. CONCLUSION

This article has proposed a relatively simple procedure for bounding the benefits of pollution reduction, using information on the household's possible defensive measures against pollution. An important area of future research is to implement the suggested procedure in a variety of empirical settings. These empirical studies should be designed so that the bounds can be compared with the more exact, but costly, benefit measures that are also developed in this article. While I have described various factors that theoretically would be expected to affect the accuracy of these bounds, specific empirical studies would provide better information. Such information is important to policy researchers who must make decisions about the most cost-effective method to estimate the benefits of environmental policies. In some cases, the quick and dirty methods proposed here may be close enough for policy purposes.

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