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Conditional-logit Bayes estimators for consumer valuation of electric vehicle driving range



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ABSTRACT

Range anxiety – consumers' concerns about limited driving range – is generally considered an important barrier to the adoption of electric vehicles. If consumers cannot overcome these fears it is unlikely that they will consider purchasing an electric car. Hence, a successful introduction of low emission vehicles in the market requires a full understanding of consumer valuation of driving range. By analyzing experimental data on vehicle purchase decisions in California, I derive and study the statistical behavior of Bayes estimates that summarize consumer concerns toward limited driving range. These estimates are superior to marginal utilities as parameters of interest in a discrete demand model of vehicle choice. One of the empirical results is the posterior distribution of the willingness to pay for electric vehicles with improved batteries offering better driving range. Credible intervals for this willingness to pay, as well as both parametric and nonparametric heterogeneity distributions, are also analyzed.

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1. Introduction

Improvements in energy efficiency of car engines represent a pertinent step toward sustainability in personal transportation. However, ultra-low-emission vehicles still have numerous drawbacks when compared to their standard gasoline counterparts. For instance, high purchase price, limited driving range, lack of recharging stations and lengthy recharging may prevent consumers from adopting battery electric vehicles. Thus, private investments, subsidy programs, or both may be needed to support

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initial consumer acceptance of electric vehicles in the automotive market. In planning these investments and subsidies, it becomes critical to economically value the impact of the current limitations of ultra-low-emission vehicles on consumer conversion to alternative energy technologies.

To derive measures of consumer valuation of driving range of electric vehicles, I use stated-preference microdata on vehicle choice in California (Train and Hudson, 2000) and I revisit estimation of the conditional¹ logit model (McFadden, 1974), but from a Bayesian perspective. The logit model has been labeled as the workhorse of discrete choice modeling, as it represents an ideal, fully-specified model and actually was the most widely used model since the earliest developments in the field from the 1970s until the mid 1990s. Today, properties and limitations of the logit model are well understood, and more flexible models that account for more general covariance structures have been derived. However, there are at least two reasons to revisit estimation of the logit model. First, a logit model can form the kernel of a more complex model, such as a hybrid choice model with a logit kernel that accounts for unobserved heterogeneity. In fact, the logit model is the kernel of the mixed logit model (McFadden and Train, 2000), which is the current workhorse of discrete choice modeling. Second, within the extremely limited number of applications of the Bayesian approach to discrete choice modeling of travel behavior, there is practically no discussion of Bayesian estimation of the traditional logit with fixed parameters (without addressing random consumer heterogeneity). In addition, the estimation of heterogeneity distributions is being dominated by parametric assumptions, and almost exclusively uses frequentist estimators (one exception being Train and Sonnier, 2005, who use a Bayesian parametric estimator).

This paper adds to the empirical literature on Bayesian inference in discrete choice modeling by analyzing different samplers for Markov chain Monte Carlo (MCMC) estimation of conditional logit models, assuming not only a homogenous sample via fixed parameters but also accounting for both parametric and nonparametric heterogeneity distributions. Based on a convergence analysis, it was determined that an independence Metropolis Hastings sampler was faster and had better performance in terms of lower autocorrelation than a random walk or slice sampling. The analysis of this paper differs from previous research as I propose to study the posterior behavior of estimators of nonlinear transformations of the model parameters. This analysis is motivated by the possibility of deriving transformed parameters – such as willingness to pay – that are superior to the marginal utilities as parameters of interest in a discrete choice model, as these transformations have a direct economic interpretation. In addition, since willingness to pay is a parameter that is locally almost unidentified, it is not clear whether convergence of the marginal utilities will ensure convergence of their ratios. This first contribution is relatively general in scope in the sense that the methodology for obtaining and analyzing the results is applicable to any discrete choice problem. In fact, based on these results an independence Metropolis Hastings estimator was used as the kernel sampler for the population parameters of the hierarchical model that allows for random consumer heterogeneity.

Bayes estimators are applied in this paper to the problem of consumer valuation of electric vehicle attributes, with a special focus on driving range. I propose to derive from the estimates of the discrete choice demand model four specific driving-range-valuation measures, namely the willingness to pay for marginal driving range improvements (see Hidrue et al., 2011; Dimitropoulos et al., 2013), the compensating variation after improvements in range (cf. Dimitropoulos et al., 2013), the elasticity of the choice probability of buying an electric vehicle with respect to marginal range improvements, and a measure of range equivalency (cf. Hess et al., 2006). These four proposed measures are economically meaningful functions of the marginal utilities and – even though these functions can also be derived in a frequentist context – statistical inference (point and interval estimation) on these nonlinear functions is facilitated by the use of Markov chain Monte Carlo methods. This work combines technical and empirical contributions by using post-processing Sonnier et al., 2007 of the posterior distributions of the marginal utilities to identify robust confidence intervals for inference on the driving-range-valuation measures. In the context of parameter transformations, frequentist econometric methods

¹ Depending on the nature of the regressors, the econometric literature distinguishes the multinomial logit model from the conditional logit model. The multinomial logit model considers individual-specific regressors, whereas the conditional logit model considers alternative-specific attributes. Nevertheless, this distinction is somewhat artificial. In fact, in travel behavior modeling a multinomial logit model usually refers to what econometricians call conditional logit.

encounter problems due to parameters that may be weakly identified (Bolduc et al., 2010). Because the Bayesian approach is proving to be a compelling solution to this problem, I adopt these promising techniques to analyze willingness to pay and other economic measures derived from the estimates of a discrete demand model. Whereas working with a model in willingness-to-pay space (Train and Weeks, 2005) produces direct confidence intervals of willingness to pay, recasting the parameter space does not solve the problem of estimating intervals of other functions of the parameters, such as elasticities, choice probabilities, or welfare measures.

The empirical analysis of this work shows that the willingness to pay for driving range decreases with range and is around 100 \$/mile (cf. the median of 55 \$/mile found in Dimitropoulos et al. (2013)) for current market conditions – i.e. electric vehicles that can be driven for 100 miles on a single charge, whereas the marginal cost of producing a lithium-ion battery with improved range is about 160 \$/mile.²

The rest of the paper is organized as follows. Section 2 first overviews both the problem of driving range anxiety as well as econometric models of electric vehicle purchase decision. Section 3 introduces the data and derives measures for valuation of consumer concerns toward limited driving range. The data comes from stated vehicle-type choices, including low-emission alternatives, by consumers in California. Consumer valuation of driving range is then defined via nonlinear functions of the marginal utilities representing how consumers react to driving range improvements in terms of willingness to pay, compensating variation, and elasticity of the choice probability of buying an electric car. Section 4 presents the empirical analysis of the Bayes estimates of the marginal utilities and of the range valuation measures. Section 5 concludes and offers some insights for future work. In Appendix A, details regarding the derivation of the Bayes estimators are discussed.

2. Consumer adoption of electric vehicles

2.1. Electric vehicles and driving range anxiety

Electric vehicles are propelled by one or more electric motors that are powered by rechargeable electric vehicle batteries. Since standard vehicles that are powered by internal combustion rely on fossil fuels – with the concomitant negative externalities – electric vehicles have been identified as an alternative to promote sustainable personal transportation. Because electric vehicles have the potential for being charged using clean energy sources, a conversion to electric vehicles is usually regarded as a path toward an independent, cleaner, and more secure energy future. Even if the electricity comes from sources that are not environmentally friendly, such as coal, vehicles with zero tailpipe emissions still have associated benefits such as reducing direct pollution sources in an urban context. This includes reduced air pollutants and traffic noise in cities. Electric motors are in general more efficient than those propelled by internal combustion. Additionally, the conversion to clean energy could be boosted by an increasing penetration of electric vehicles. However, current technologies for electric vehicles have a series of limitations that may prevent broad consumer adoption.

In particular, current electric vehicles suffer from a limited driving range, which can be defined as the maximum distance allowed by a full battery charge. Consumer concerns toward the limited driving range of electric vehicles are increased because of long recharging times³ and lack of recharging stations.⁴ These concerns are summarized into what has been called *range anxiety* or the consumer fear that the electric vehicle battery will exhaust while driving. These fears represent a major problem for the introduction of electric vehicle into the market: if consumers continue to suffer driving range anxiety it is unlikely that they will consider the purchase of an electric car.

Both high purchase prices and high maintenance costs, including battery replacements, also appear as barriers for the adoption of electric vehicles. Table 1 displays three electric vehicles that are currently being offered in the US market.

² Dollars of 2005 for direct comparisons.

³ Recharging the electric vehicle battery with standard outlets can take 8 to 16 h.

⁴ Standard vehicles also have a limited driving range. However, the high availability of gas service stations as well as extremely short refueling times allow consumers to ignore driving range limits of fossil fuel-powered vehicles.

Table 1

Electric vehicles widely available in the US market in 2012. (Figures for a hybrid powertrain are added for reference.)

Make & model	MSRP ^a [US\$]	Estimated driving range	Energy efficiency ^b	In the US since	Units sold in the US ^c
Nissan LEAF	32,780	100 miles	99 [mpge]	2011	23,051
Chevrolet Volt ^d	40,280	35 miles	94 [mpge]	2011	37,702
Tesla Roadster	109,000	245 miles	120 [mpge]	2010	2350
Toyota Prius	24,000	536 miles	50 [mpg]	2000	1,309,543

Source: Specifications provided by the respective manufacturer.

The figures represent all-electric operation.

^a Starting manufacturer's suggested retail price does not include federal tax savings.^b EPA combined fuel economy.^c Until March 2013. In the case of the Tesla Roadster, the figure represents worldwide sales of the total fixed number that was produced.^d The Chevy Volt is technically an extended range electric vehicle that works on gas when the electric battery is exhausted.

Note that all three electric vehicles have suggested retail prices that exceed the average price of new cars, which was 28,400 US\$ in 2010 (National Automobile Dealers Association, 2010). Although the actual expenditure is lower, because of a 7500 US\$ electric vehicle federal tax credit as well as additional state discounts, the cost of electric cars is still high for the offered body type. Even though the market for electric cars is at a very early stage, figures of deliveries for the Nissan LEAF and the Chevrolet Volt may disappoint enthusiasts who are expecting an electric car revolution.

2.2. Discrete choice models of electric vehicle purchase decisions

Microeconomic models of private vehicle purchase decisions involving alternative energy sources in the choice set have a long tradition (see Daziano and Chiew, 2012, for a review of these models). Just a couple of years after the first applications of discrete choice models to travel demand, the seminal work of Beggs and Cardell (1980) and Beggs et al. (1981) set the basis for later empirical research. When analyzing demand for electric vehicles using random utility maximization, non-standard attributes such as driving range (Bunch et al., 1993; Brownstone et al., 2000; Ewing and Chiew, 1998; Hidrue et al., 2011), availability of charging stations (Bunch et al., 1993; Brownstone et al., 2000; Achtnicht, 2012), and time for recharging (Golob et al., 1997; Ewing and Chiew, 1998) become relevant. This new set of attributes allows researchers to study consumer valuation of, for instance, improved electric batteries offering extended driving range.

Willingness to pay for marginal improvements in driving range is an economic measure of the tradeoff between purchase price and driving range of electric vehicles (see Dimitropoulos et al., 2013), which is a key input for welfare and cost-benefit analysis of investments in improving electric batteries. Inference on willingness to pay can be derived from the estimates of discrete choice models. Table 2

Table 2

Willingness to pay estimates for marginal improvements in driving range. Results from different studies in the US.

Main references	Market	WTP [US\$05/mile]		
		Mean est.	Min est.	Max est.
Beggs and Cardell (1980), Beggs et al. (1981)	US (1978)	85	61	132
Calfee (1985)	California (1980)	195	195	195
Bunch et al. (1993)	California (1991)	101	95	106
Brownstone et al. (2000)	California (1993)	99	58	202
Golob et al. (1997)	California (1994)	117	76	202
Tompkins et al. (1998)	US (1995)	64	44	102
Train and Hudson (2000) and Train and Sonnier (2005)	California (2000)	100	87	131
Hess et al. (2012)	California (2008)	43	36	49
Hidrue et al. (2011)	US (2009)	58	29	82
Nixon and Saphores (2011)	US (2010)	182	46	317

Source: Dimitropoulos et al. (2013).

provides a summary of different willingness-to-pay estimates for a one-mile improvement in driving that have been derived for the American market (estimates for other markets are also summarized in Dimitropoulos et al., 2013).

The figures for willingness to pay for driving range exhibit a high degree of variation, from 44 to 317 dollars per an additional mile in the distance allowed by a single charge. Both the average and median valuation of these studies are around 100 \$/mile.

3. A conditional logit model of vehicle choice in California

3.1. Data

To understand the tradeoffs that control the adoption of low-emission vehicles, including concerns toward limited driving ranges of electric cars, I use stated-preference data on vehicle choice in California as reported by Train and Hudson (2000).⁵

Participants of the survey were sampled from residents of California who were prospective buyers of a new vehicle. Each of the 500 respondents in the survey answered up to 15 experimental choice situations between three vehicle alternatives.⁶ The sample contains a total of 7437 observations. Respondents in the sample were provided with information on electric vehicles and on air quality in California. Additionally, respondents were provided with detailed information regarding performance of the vehicles, including top speed and total time required in second to reach a speed of 60 mph. For each vehicle, the experimental attributes were defined as:

- Energy source [gas internal combustion, electric, gas-electric hybrid]
- Purchase price [thousands of US dollars]
- Operating cost [US dollars per month]
- Driving range for electric vehicles [hundreds of miles]
- Performance [high, medium, low]
- Body type [mini car, small car, mid car, large car, small SUV, mid SUV, large SUV, compact PU, full PU, minivan]

The actual attribute values in the stated preference experiment were set following a randomized design: after defining the possible attribute levels, each choice situation was specified by randomly selecting from the levels of each attribute. Because the experiment was unlabeled and energy source (engine type) was drawn for each vehicle in each choice situation, the respondent faced a random combination of engine types at every choice experiment (e.g. two gasoline and one hybrid vehicle, or among one gasoline and two electric vehicles, or any other combination). Purchase prices and operating costs were drawn from a range of plausible prices and fuel costs for each combination of body type and energy source. For high performance, a top speed of 120 mph and 8 s to reach 60 mph were considered. Medium performance was set at a top speed of 100 mph and 12 s to reach 60 mph. Low performance was defined as having a top speed of 80 mph and 16 s to reach 60 mph. For electric vehicles, performance was randomly selected from the medium and low levels. A constant level of driving range was considered for both internal combustion engine and hybrid vehicles. In the case of electric cars, driving range was chosen from a set of 10 mile increments within the bounds of 60 and 200 miles. Finally, whereas in the first 7 experiments for each respondent body type was chosen randomly for each vehicle from the ten possible builds, in the last 8 experiments one body type was drawn and kept the same for all three vehicles in the choice situation. A summary of the randomized design of the choice experiment is displayed in Table 3, and Appendix B presents a sample of a choice situation as seen by respondents of the survey.

The experimental levels reflect relatively realistic choice situations when compared to current vehicles offered in the market. Note that the average experimental electric driving range is 130 miles,

⁵ Details regarding the design of the survey, and data collection and analysis, can be found in Train and Hudson (2000). This dataset has also been used by Sándor and Train (2004), Train and Sonnier (2005), and Hess et al. (2006)

⁶ The experimental design considered unlabeled alternatives.

Table 3

Data descriptive statistics resulting from the randomized experimental design.

	Gas		Hybrid		Electric		
	Purch. price [1000\$]	Op. cost [\$ /month]	Purch. price [1000\$]	Op. cost [\$ /month]	Purch. price [1000\$]	Op. cost [\$ /month]	Driving range [100 miles]
Mean	22.40	48.86	42.88	30.10	42.93	20.61	1.30
Stdv	7.37	11.99	17.50	10.20	17.66	9.58	0.40
Min	7.02	17.58	10.02	7.52	10.02	2.51	0.60
Max	38.93	72.11	97.30	52.31	97.26	42.30	2.00

with bounds that reflect current supply, and the average price of \$42,930 for the purchase of an electric vehicle in the experiment is of the same magnitude as the current price of the Volt. This shows that even though the data is somewhat dated, the experimental situation still accurately reflects current real-world situations.

3.2. Valuation of consumer concerns toward limited driving range

In this paper, a conditional logit model of vehicle purchase decisions is proposed to understand the factors that explain demand for low-emission vehicles. Thus, the random indirect utility is defined as $U_{int} = \mathbf{x}'_{int} \boldsymbol{\theta} + \varepsilon_{int}$, where $i \in \{1, 2, 3\}$, n represents the individual, and t the choice situation. The iid structure of the kernel covariance matrix is justified by the fact that the experiment was designed with unlabeled alternatives. However, to account for the repeated observation problem that is intrinsic to stated-preference experiments, I analyze two estimators that allow for individual-specific realizations of the random taste parameters for the choice situations answered by the same individual.

Regarding the attribute levels contained in \mathbf{x}_{int} , note that among the attributes only purchase price (pp), operating cost (oc), and driving range are continuous variables. The rest are defined as index functions. For energy source, gas internal combustion is set as base. The same applies for medium performance and mid-car body type. Since the experiment was unlabeled, the model does not contain alternative specific constants.

Previous studies – including [Sándor and Train \(2004\)](#), [Train and Sonnier \(2005\)](#), and [Hess et al. \(2006\)](#) who use the same vehicle choice data of this work – have specified marginal utilities of driving range that are constant. The underlying assumption is that both willingness to pay for and consumer surplus from improvements in driving range are constant. It is easy to argue that driving range should exhibit diminishing returns, meaning that an additional mile in driving range for a vehicle that currently offers 80 miles should provide more utility than an additional mile for a 200-mile range vehicle. Following the work of [Calfee \(1985\)](#), a logarithmic transformation of driving range was applied (see also [Kavalec, 1999](#); [Hess et al., 2012](#)). By using the definition of attributes discussed above, the dimension of both \mathbf{x}_{int} and $\boldsymbol{\theta}$ is 16.

The proposed demand model of vehicle choice can be used to derive insights into consumer valuation of driving range. Let θ_{pp} be the marginal utility of purchase price, θ_{oc} the marginal utility of operating costs, $\theta_{\ln range}$ the marginal utility of the logarithmic transformation of driving range of electric vehicles, and θ_{BEV} the constant specific to being electricity the energy source of the vehicle.⁷ It is then possible to define the following functions:

1. *Willingness to pay for driving range improvements* ($WTP_{\Delta range}$): marginal rate of substitution of driving range and purchase price

$$WTP_{\Delta range} = - \frac{\partial U_{BEV} / \partial range}{\partial U / \partial pp} = - \frac{\theta_{\ln range}}{\theta_{pp}} \frac{1}{range}. \quad (1)$$

⁷ $U = \theta_{BEV} + \theta_{pp} pp_{BEV} + \theta_{oc} oc_{BEV} + \theta_{\ln range} \ln range_{BEV} + \dots$

Note that because of the logarithmic transformation of range, $WTP_{\Delta range}$ is not just a parameter ratio but also a nonlinear function of the range level.

2. *Compensating variation after driving range improvements* ($CV_{\Delta range}$): welfare-analysis measure that represents the change in income needed to offset changes in utility after a change in driving range of $\Delta range$,

$$CV_{\Delta range} = -\frac{1}{\theta_{pp}} \left[\ln \sum_{i=1}^J \exp \left(\mathbf{x}_{int}^{(1)'} \boldsymbol{\theta} \right) - \ln \sum_{i=1}^J \exp \left(\mathbf{x}_{int}^{(0)'} \boldsymbol{\theta} \right) \right], \quad (2)$$

where $\mathbf{x}_{int}^{(1)} - \mathbf{x}_{int}^{(0)} = \ln(range + \Delta range) - \ln(range)$. (Eq. (2) is the well-known expression of the expected consumer-surplus variation for logit models, which measures the compensating variation when the marginal utility of income is assumed constant; see [Small and Rosen \(1981\)](#) and [de Jong et al. \(2007\)](#).) This measure is an important input for policymakers, as it can be used for evaluating welfare improving effects of subsidies for automakers to improve battery capacities, reduce cost of battery production, or both.

3. *Elasticity of the choice probability of buying an electric vehicle with respect to marginal driving range improvements* ($E_{P_{BEV}, range}$): individual-specific measure of the percentage increase in the choice probability of choosing a battery electric vehicle ($P_{BEV,n}$) after a marginal improvement of a one-percent change in driving range

$$E_{P_{BEV}, range} = \theta_{\ln range} (1 - P_{BEV,n}). \quad (3)$$

Elasticities inform automakers about what to expect in terms of marginal improvements in market shares after improved batteries are introduced. Note that because of the logarithmic transformation of range, the resulting elasticity depends only on the parameter of the attribute of interest (in this case the logarithm of range) and the electric-vehicle choice probability.

4. *Range equivalency* (RE): although $WTP_{\Delta range}$ is perhaps the most relevant output to understand consumer valuation of driving range, this measure does not allow firms to answer directly vehicle-design questions such as the desired range that would make electric vehicles competitive based on consumer preferences. Assuming no differences in performance and body type, a measure of range equivalency is given by

$$RE = \exp \left(\frac{\theta_{pp}}{\theta_{\ln range}} (pp_j - pp_{BEV}) + \frac{\theta_{oc}}{\theta_{\ln range}} (oc_j - oc_{BEV}) + \frac{1}{\theta_{\ln range}} (\theta_j - \theta_{BEV}) + \ln range_j \right), \quad (4)$$

which is the value that solves equality between the utility of the electric vehicle and a benchmark vehicle j . Note that RE is a nonlinear function of the compensatory effects of the difference in vehicle attributes (purchase price, operating cost, and the energy source constants).

When using frequentist estimators, such as the maximum likelihood estimator, each of these functions result in a single value. These point estimates are just an asymptotic approximation of the Bayes decision. In fact, the Bayes estimator in general becomes more attractive than frequentist estimators not only because its properties are valid for finite samples, but also because it describes the posterior distribution of the measures of interest. In the Bayesian approach, the parameter vector $\boldsymbol{\theta}$ is random, and so are functions of $\boldsymbol{\theta}$. Thus, after the observation of the sample, it is possible to describe the posterior distributions of the proposed driving range measures $p(WTP_{\Delta range} | \mathbf{y}, \mathbf{X})$, $p(CV_{\Delta range} | \mathbf{y}, \mathbf{X})$, $p(E_{P_{BEV}, range} | \mathbf{y}, \mathbf{X})$, and $p(RE | \mathbf{y}, \mathbf{X})$. Microdata used in choice modeling usually is characterized by large samples. As a result, exact properties of the Bayes estimator become less attractive, as frequentist estimators are a very good approximation of Bayes estimators for larger samples. However, derivation of the posterior distributions of the functions defined above facilitates identification of robust confidence intervals for inference on the driving-range-valuation measures (cf. [Daly et al., 2011](#)). This is a strong enough motivation for the use of Bayesian methods, because standard frequentist methods in

Table 4

Point estimates of the population average marginal utilities.

Attribute	Fixed param. logit		Parametric heterogeneity		Nonparametric heterogeneity	
	$\hat{\theta}$	s.d.	$\hat{\theta}$	s.d.	$\hat{\theta}$	s.d.
Purchase price	−0.0533	0.002	−0.1879	0.008	−0.1228	0.004
Operating cost	−0.0266	0.002	−0.0693	0.004	−0.0478	0.003
ln(Driving range)	0.6082	0.095	1.7154	0.241	1.1706	0.138
Electric	−0.2648	0.114	−0.1890	0.256	−0.2088	0.178
Hybrid	0.4128	0.057	1.5134	0.127	1.0482	0.095
High performance	0.1220	0.037	0.3071	0.069	0.2076	0.054
Low performance	−0.3230	0.038	−0.7802	0.081	−0.5953	0.055
Mini	−1.7628	0.107	−5.3138	0.341	−3.6526	0.217
Small	−0.7877	0.099	−2.4697	0.247	−1.6888	0.161
Large	−0.3061	0.101	−0.7890	0.242	−0.4804	0.168
Small SUV	−0.4584	0.098	−1.4825	0.245	−0.9652	0.162
Medium SUV	0.2009	0.101	0.6701	0.214	0.4578	0.159
Large SUV	0.0122	0.112	−0.3488	0.283	−0.1608	0.158
Compact PU	−0.8039	0.099	−2.1453	0.228	−1.5284	0.141
Full PU	−0.4230	0.102	−1.2947	0.263	−0.8351	0.196
Minivan	−0.1884	0.101	−0.8557	0.247	−0.5617	0.167

the context of nonlinear functions of the model parameters face the problem of weak identification (Bolduc et al., 2010).

4. Empirical analysis of the Bayes estimators

4.1. Bayes estimates of vehicle purchase preferences

Three logit-based econometric models of vehicle choice behavior were considered, namely a conditional logit model with fixed parameters (no random consumer heterogeneity) and two models that account for a heterogeneity distribution of parameters. For the latter two models both a parametric and a nonparametric assumption were made for the heterogeneity distribution, leading to a parametric random parameter logit and a semiparametric random parameter logit, respectively. Table 4 presents the population point estimates of the choice model parameters for the three Bayes estimators considered. The results are based on 50,000 iterations after a burn-in of 5000 initial draws. Total length of the runs were determined using Raftery–Lewis estimates (Raftery and Lewis, 1992).⁸

The point estimates of the model with no consumer heterogeneity are based on a Metropolis–Hastings (MH) estimator with an independence proposal. That independence MH quickly navigates the parameter space was confirmed by analyzing convergence (Geweke, 1992; Raftery and Lewis, 1992) of the simulated posterior distribution of the willingness-to-pay for driving range improvements of three different Bayes estimators – Metropolis–Hastings with an independence proposal, Metropolis–Hastings with a random-walk proposal, and a slice sampler (see Section 3). Note that even though the performance of different Bayes estimators of logit models has been studied in previous research (Chib et al., 1998; Rossi et al., 2005; Frühwirth-Schnatter and Frühwirth, 2010; Scott, 2011), these studies consider only the posterior distribution of the alternative-specific marginal utilities θ_i of multinomial models, with no consideration of meaningful functions of the original parameter space.⁹

In the case of random consumer heterogeneity, two models were considered. The parametric model assumed a multivariate normal distribution of heterogeneity (correlation among the random taste

⁸ A run length control test based on the criterion of achieving a given level of accuracy for estimation of a pre-specified quantile of the marginal posterior density.

⁹ Given the large sample size, the point estimates were practically identical for the three estimators. However, on the one hand the slice sampler took as much as 100 times more computation time than the other two estimators to produce the posterior sample. On the other hand, the random-walk proposal exhibited poor mixing due to high correlations. Results of the convergence analysis are available upon request.

Table 5

Summary of the heterogeneity distributions.

Attribute	Parametric heterogeneity		Nonparametric heterogeneity selected quantiles of $p(\theta y, X)$				
	Std. dev. $\hat{\theta}$	s.d.	2.5%	25%	50%	75%	97.5%
Purchase price	0.26	0.002	−0.3393	−0.1823	−0.1097	−0.0522	0.0343
Operating cost	0.23	0.002	−0.1951	−0.0922	−0.0451	−0.0014	0.0856
ln(Driving range)	2.15	0.095	−0.9691	0.4202	1.1222	1.8508	3.3992
Electric	2.94	0.114	−3.8456	−1.4410	−0.2534	0.9293	3.3351
Hybrid	2.40	0.057	−2.0607	−0.0080	1.0317	2.0782	4.1351
High performance	1.21	0.037	−1.1080	−0.2350	0.2157	0.6715	1.5814
Low performance	1.09	0.038	−1.8442	−0.9916	−0.5677	−0.1505	0.6640
Mini	5.35	0.107	−10.6354	−5.9908	−3.6248	−1.3142	3.0383
Small	3.00	0.099	−5.2308	−2.8990	−1.7441	−0.6117	1.5789
Large	2.99	0.101	−4.2452	−1.6896	−0.4960	0.6634	3.0269
Small SUV	2.79	0.098	−4.5853	−2.1686	−1.0364	0.0783	2.3729
Medium SUV	2.44	0.101	−2.6160	−0.6101	0.3728	1.3510	3.3211
Large SUV	3.80	0.112	−5.1334	−1.8348	−0.1921	1.4288	4.5861
Compact PU	3.06	0.099	−5.4991	−2.8464	−1.5188	−0.2271	2.2547
Full PU	3.67	0.102	−5.3607	−2.2909	−0.8702	0.5035	3.3596
Minivan	3.72	0.101	−5.3266	−2.1404	−0.5411	1.0301	4.0656

variation of the parameters was allowed for). Aiming at letting the data tell the shape of the heterogeneity distribution, the nonparametric model of consumer heterogeneity considered a multivariate Dirichlet Process prior. (Brief explanation and estimators in [Appendix A](#).) For both the parametric random parameter logit and the semiparametric random parameter logit, the point estimates of [Table 4](#) represent population average marginal utilities.

As expected, on average consumers negatively perceive both purchase price and operating cost. Driving range is a desirable attribute of electric cars, and the posterior of its associated marginal utility discards the possibility of a zero value, indicating that driving range is a relevant attribute for vehicle choice. Regarding the effect of the energy source, note that on the one hand the constant for electric vehicles shows that consumers would be reluctant to adopt electric cars, everything else being equal among alternative vehicles. On the other hand, consumers show a very good perception of hybrid cars. The estimates for the rest of the attributes show expected marginal utilities. High performance is desired and consumers dislike low performance. The most preferred body type are medium SUVs and, in general, compact and small vehicles are negatively perceived.

[Table 5](#) summarizes the heterogeneity distributions. In the case of parametric heterogeneity, point estimates of the standard deviation of the individual realizations of the marginal utilities are derived (with a corresponding standard deviation of those point estimates). These standard deviations favor the hypothesis of unobserved consumer heterogeneity. In the case of nonparametric heterogeneity, [Table 5](#) summarizes selected quantiles of the nonparametrically-estimated heterogeneity posteriors. Since the semiparametric logit is not imposing any restriction on the heterogeneity distributions, this model should be preferred for analyzing consumer preferences. In the rest of the paper we analyze the results of all the three models for comparison purposes.

Note that in the most flexible model (i.e. the semiparametric logit with a nonparametric heterogeneity distribution) the quantiles show that changes in sign in the individual partworths are possible. In this sense, parametric assumptions that constraint the sign of the marginal utilities, such as lognormally distributed parameters as in [Train and Sonnier \(2005\)](#), would be misrepresenting the behavior revealed by the data. In addition, [Table 5](#) is summarizing the heterogeneity distribution for the whole sample, but the Bayes estimator actually produces a posterior distribution of the partworths for each individual in the sample. Both are shown in [Fig. 1](#) for selected parameters. The subsection below discusses the results of post-processing the MCMC samples for deriving the measures of driving-range valuation.

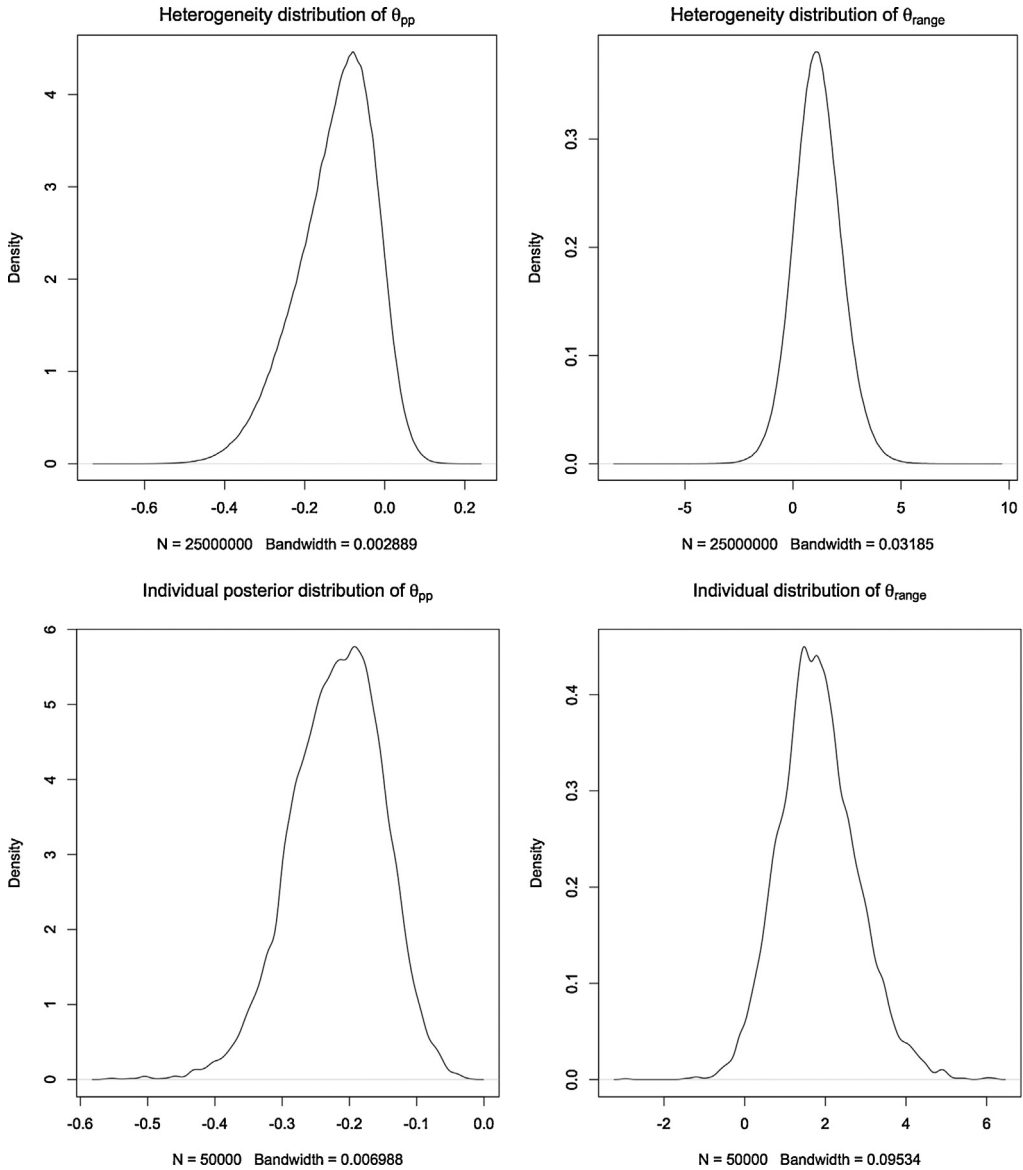


Fig. 1. Nonparametric estimate of the sample heterogeneity density (left) and a randomly selected individual partworth (right) for the parameters of purchase price and logarithm of driving range.

4.2. Estimates of consumer valuation of driving range

As mentioned at the end of Section 3.2, the posterior distributions of the marginal utilities (and individual partworths) can be post-processed to derive posterior distributions of the measures that summarize consumer valuation of driving range.

Because of the logarithmic transformation of range, the willingness to pay for driving range improvements ($WTP_{\Delta range}$) is a function of range. Thus, how much individuals are willing to pay for a marginal improvement in range depends on the value set as base. This means that one cannot

Table 6

Mean and selected quantiles of the posterior distribution of willingness to pay for different levels of range.

Quant.	Fixed param. logit	Parametric rand. param. logit	Semiparam. rand. param. logit	Parametric rand. param. logit*	Semiparam. rand. param. logit*
WTP _{Δrange} (50 miles)					
Mean	262.1	206.4	221.7	364.9	193.5
2.5%	185.5	150.5	166.0	−2.7	12.83
25%	234.6	186.5	191.0	213.3	125.2
50%	261.9	207.4	209.3	315.3	181.4
75%	289.5	226.2	229.9	450.9	247.2
97.5%	343.0	260.1	268.0	1036.5	456.1
WTP _{Δrange} (75 miles)					
Mean	174.7	137.6	141.1	243.3	129.0
2.5%	122.3	100.3	110.7	−1.8	8.6
25%	156.4	124.3	127.3	142.2	83.4
50%	174.6	138.2	139.5	210.2	120.9
75%	193.0	150.8	152.2	300.6	164.8
97.5%	228.7	173.4	178.6	691.0	304.1
WTP _{Δrange} (100 miles)					
Mean	129.0	103.2	107.8	182.5	96.8
2.5%	91.7	75.2	83.0	−1.3	6.4
25%	117.3	93.2	95.5	106.7	62.6
50%	130.9	103.7	104.7	157.7	90.7
75%	144.7	113.1	114.9	225.5	123.6
97.5%	171.5	130.0	134.0	518.3	228.1
WTP _{Δrange} (150 miles)					
Mean	86.0	68.8	71.9	121.6	64.5
2.5%	61.2	50.2	55.3	−0.9	4.3
25%	78.2	62.2	63.7	71.1	41.7
50%	87.3	69.1	69.8	105.1	60.5
75%	96.5	75.4	76.6	150.3	82.4
97.5%	114.3	86.7	89.3	345.5	152.0
WTP _{Δrange} (250 miles)					
Mean	52.4	41.3	42.3	73.0	38.7
2.5%	36.7	30.1	33.2	−0.5	2.6
25%	46.9	37.3	38.2	42.7	25.0
50%	52.4	41.5	41.9	63.1	36.3
75%	57.9	45.2	46.0	90.2	49.4
97.5%	68.6	52.0	53.6	207.3	91.2

* Randomly selected individual.

calculate a single value for WTP_{Δrange}, in contrast to the constant willingness to pay that is derived in a linear specification. The dependence on range is not a restriction, but the result of the sensible assumption of decreasing returns of range improvements.

Table 6 presents the nonparametric estimates of the WTP_{Δrange} posterior distribution, for different values of range, namely 50, 75, 100, 150, and 250 miles. 50 miles represent an expected value for all-electric range of plug-in hybrids and extended range electric vehicles, as well as the range of some electric micro-car prototypes. 75 miles is about the actual range obtained in current 100% electric vehicles under unfavorable conditions, such as cold weather. 100 miles is the expected range under ideal conditions for the Nissan LEAF (24 kWh lithium-ion battery). 150 miles is the expected driving range of the Tesla S with a 40 kWh electric battery (this model entered production in December 2012). 250 miles is about the distance achieved by the Tesla Roadster (53 kWh battery) after a single charge, as well as the expected range of the Tesla S with a 60 kWh battery. (The Tesla S with an 85 kWh battery is expected to run for 300 miles per charge.) For each of these values of range, the mean and selected quantiles of the WTP_{Δrange} posterior are calculated. Note that the first three columns of Table 6 are based on the population parameters, whereas the last two columns are derived using individual partworths of a randomly selected individual. To ease the comparison with the figures of Table 2 (as well as with the complete meta-analysis of Dimitropoulos et al. (2013)), all values are reported in [US\$05/mile].

In Bayesian econometrics, the mean of the posterior is the point estimate (when using a quadratic loss). Thus, the point estimate of the population average $WTP_{\Delta range}$ evaluated at 100 miles is 129 [\$/mile] for a fixed parameter logit, 103.2 for the parametric random parameter logit, and 107.8 for the semiparametric random parameter logit. Note that 100 [\$/mile] is the mean estimate among the previous studies that have used the same data (Train and Hudson, 2000; Train and Sonnier, 2005; Hess et al., 2006). Recall that the mean range in the sample is 130 miles. The fixed parameter logit produces a point estimate of exactly 100 [\$/mile] in that case, whereas the parametric model of heterogeneity produces 79.3, and the result of the semiparametric model of heterogeneity is 81.4. In general, the values for the population willingness to pay is lower for the two models that consider random heterogeneity. In fact, the two models of consumer heterogeneity produce comparable results. In addition, uncertainty in the determination of the population averages is low. (Bounds of the area containing 95% of the posterior mass are relatively tight; these results can be seen in Fig. 2.) However, variability in the individual partworths is high (for instance, see Fig. 3). In addition, note that for a randomly chosen individual – in the case of normally distributed heterogeneity – the individual posterior variance is so high that zero is always contained in the 95% posterior central mass. Note also that a direct implication of the logarithmic transformation of range is that the willingness to pay decreases in the same percentage as the ratio between the increase in range and the target range. For example, the figures of $WTP_{\Delta range}$ are 80% lower for 250 miles than for 50 miles.

Interestingly, given the current lower bound of the cost of lithium-ion batteries (475 [\$/kWh]), at 100 miles the marginal cost of producing batteries with an additional mile of range is 160 [\$/mile]. All of the population $WTP_{\Delta range}$ point estimates at 100 miles are below that marginal cost. In addition, only for the fixed parameter logit is the marginal cost within the 95% credible interval. The upper bound of the 95% credible interval for the models that consider consumer heterogeneity is in the order of 130–135 [\$/mile], which is still lower than the marginal cost. Thus, expected revenues based on range improvements will not match incurred production costs unless the unit cost of the batteries goes down. The average $WTP_{\Delta range}$ derived from the meta-analysis of Dimitropoulos et al. (2013) is 67 [\$/mile] for all markets considered in the study (cf. the willingness to pay derived in this work for 150 miles), a figure that is even lower than those derived here. At the same time, given the high uncertainty in the determination of individual-level $WTP_{\Delta range}$ (see Fig. 4), the marginal cost is in general within the 95% credible interval of individual partworths.

The rest of the functions ($CV_{\Delta range}$, $E_{P_{BEV, range}}$, and RE) depend on individual-specific values of the vehicle attributes. In the case of revealed preference data, the correct methodology would be to calculate $CV_{\Delta range, n}$, $E_{P_{BEV, range, n}}$, and RE_n for all individuals in the sample ($n \in \{1, \dots, N\}$),¹⁰ and then aggregate these values accounting for appropriate weights. However, aggregation in the case of stated preference data is not representative of an actual market. In this latter case, it is more sensible to consider the results of a representative individual, either considering the average choice situation of the experimental design or a particular choice scenario of interest.

In this work, the representative individual is assumed to be choosing among an internal combustion engine vehicle (ICV), a battery electric vehicle (BEV), and a hybrid electric vehicle (HEV). Average fuel economies for new passenger vehicles and average gasoline costs were taken to build the attribute values for each vehicle alternative. According to the US Department of Transportation (2010), the average new vehicle fuel efficiency was 33.7 mpg in 2010. An estimate of 45 mpg is considered for hybrid vehicles.¹¹ To derive the electric car savings in operating costs, I use a fuel equivalency of 100 mpg as suggested by the US Environmental Protection Agency (2011). This calculation is based on the US national average electricity rate of 11 US¢/kWh and a performance of 3 miles per kWh. BEV driving range was set at 100 miles, which is about the average of the range under ideal conditions of 100% electric vehicles currently offered in the market. Purchase prices were set at \$20,000 for the ICV, \$27,500 for the HEV, and \$37,500 for the BEV. These prices are a rough average of current market conditions.

¹⁰ Considering individual partworths in the case of random consumer heterogeneity.

¹¹ This figure reflects, for instance, the average fuel economy of the 2012 models of the Toyota Prius, Honda Civic Hybrid, and Ford Fusion Hybrid.

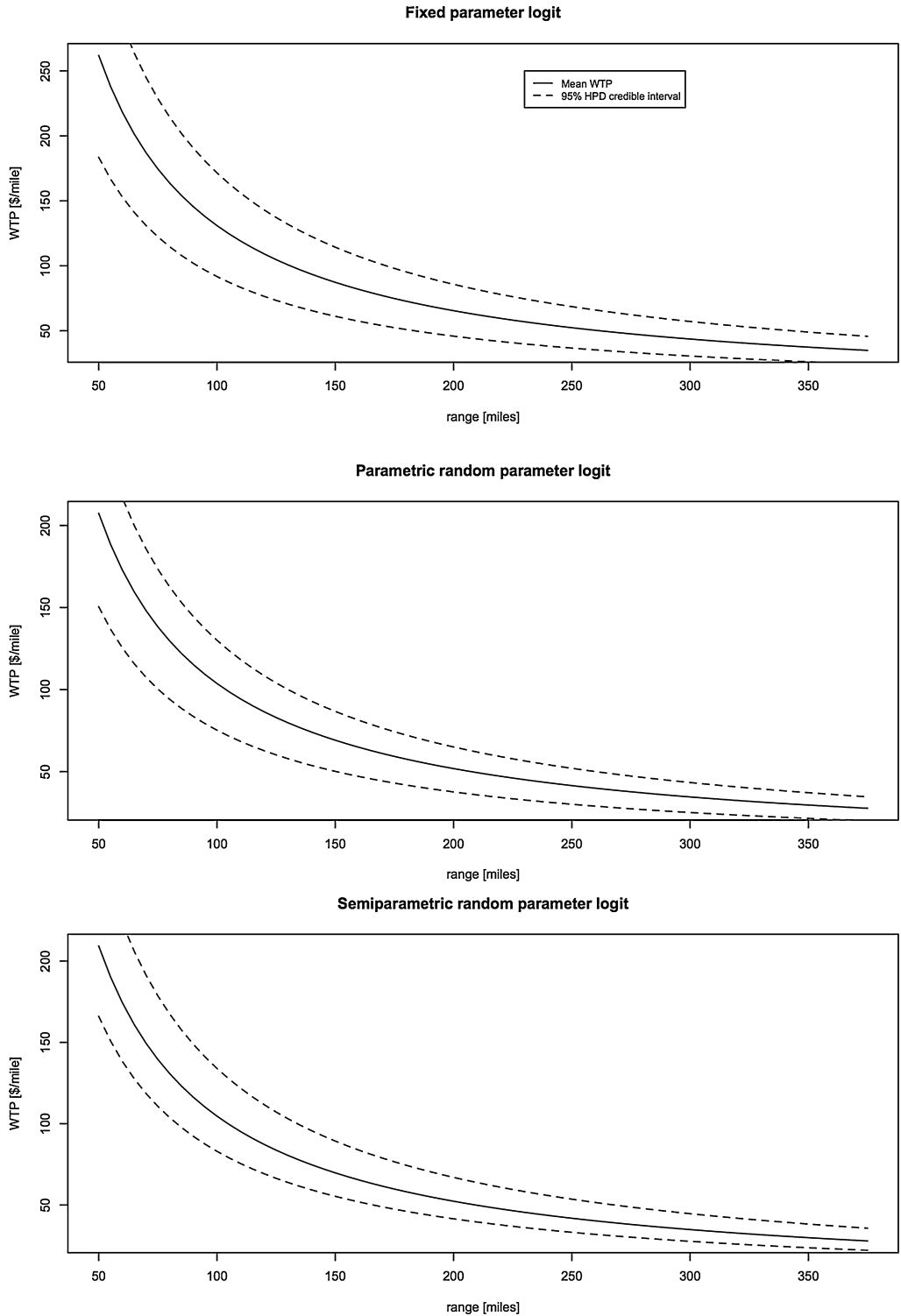


Fig. 2. Mean and 95% HPD credible interval bounds of the WTP for driving range improvements based on the population average estimates.

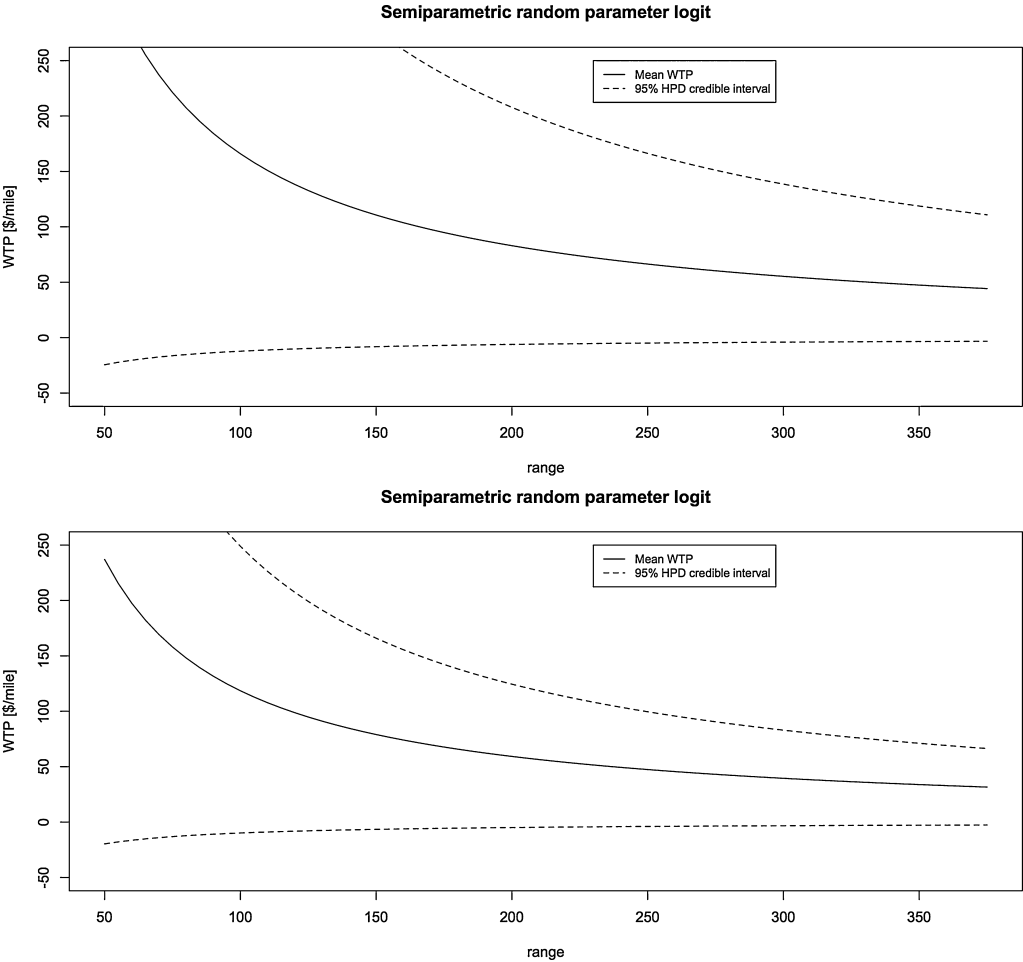


Fig. 3. Mean and 95% HPD credible interval bounds of the WTP for driving range improvements based on two randomly selected individuals.

A first result that can be derived using the attribute values of the representative individual is the choice probabilities of each alternative. Point estimates and standard deviations derived from the posterior distribution of the choice probabilities are shown in Table 7. (The assumption is that the representative individual has marginal utilities that correspond to the population average.)

Note that the resulting choice probabilities are very different depending on the model. In particular, the probability of the representative individual choosing the electric car is much lower in the models that account for random taste variations. Given the attribute levels, the choice probabilities of the

Table 7
Point estimates of the population average choice probabilities.

Attribute	Fixed param. logit		Parametric heterogeneity		Semiparametric heterogeneity	
	$\hat{\theta}$	s.d.	$\hat{\theta}$	s.d.	$\hat{\theta}$	s.d.
P_{ICV}	0.2639	0.001	0.1315	0.012	0.1907	0.011
P_{BEV}	0.2169	0.012	0.0478	0.001	0.1000	0.012
P_{HEV}	0.5192	0.011	0.8215	0.017	0.7094	0.015

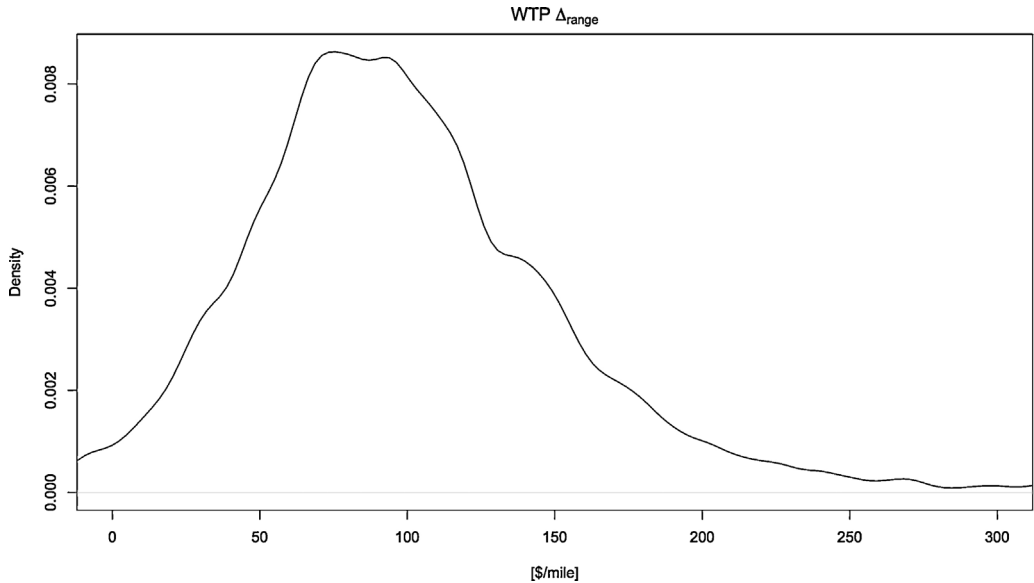


Fig. 4. Nonparametric estimate of the posterior density of WTP for driving range improvements of a randomly selected individual (evaluated at 100 miles).

Table 8

Point estimates of the population average valuation of driving range.

Attribute	Indep MH		Parametric heterogeneity		Semiparametric heterogeneity	
	$\hat{\theta}$	95% C.I.	$\hat{\theta}$	95% C.I.	$\hat{\theta}$	95% C.I.
WTP _{Δrange} (100 miles)	129.00	[89.5,169.2]	103.19	[75.2,130.0]	107.77	[86.1,137.8]
WTP _{Δrange} (150 miles)	86.00	[59.7,112.8]	68.79	[50.2,86.7]	71.85	[57.4,91.9]
CV _{Δrange} (100 → 150 miles)	1105.18	[759.1,1474.6]	248.96	[158.2,361.7]	468.79	[339.6,622.6]
$E_{PEV,range}$	1.34	[0.97,1.69]	1.63	[1.17,2.06]	1.54	[1.11,1.93]
RE _{ICV}	139.76	[107.9,180.8]	184.21	[147.8,236.0]	178.50	[140.9,229.7]
RE _{HEV}	447.58	[299.6,747.1]	559.35	[379.8,926.3]	580.67	[396.3,878.8]

model with fixed parameters seem less plausible. The hybrid is the most likely alternative to be chosen in all models, a result that is explained not only by the energy efficiency gains at a relatively lower price (compared to the BEV), but also by the value of the constant for hybrid technologies.¹² Not only does the HEV appear as more attractive in the models accounting for random consumer heterogeneity, but in these models the BEV is much less attractive.

The differences in the choice probabilities among the different models also have an important impact on the resulting compensating variation. Table 8 contains the expected compensating variation for improvements in driving range from 100 to 150 miles. The population point estimates are \$1105 for the model with fixed parameters, \$249 for the parametric random parameter logit, and \$469 for the semiparametric random parameter logit. If the choice probabilities are not taken into account, all three models produce an estimate around \$5000 (cf. the average compensating variation of \$3800 found in Dimitropoulos et al. (2013), who calculate the definite integral of the willingness to pay). Note that given current production costs, the additional 50 miles would imply an increase of roughly \$13,500 in purchase price.

¹² Everything else being equal, the odds of choosing HEV with respect to ICV are 1.51 for the fixed parameter logit, 4.54 for the parametric logit with random heterogeneity (cf. 4.47 according to the results of Train and Sonnier (2005)), and 2.85 for the semiparametric logit with random heterogeneity.

Regarding the elasticities of the choice probability of buying an electric vehicle with respect to a one percent change in driving range, for the representative consumer this one percent change represents an improvement of an additional mile (electric driving range going from 100 to 101 miles). The result is that the choice probability is elastic: the point estimates of the percentage change in the choice probability is greater than 1%. Note, however, that in the case of fixed parameters, one cannot reject the hypothesis of the choice probability being inelastic. At the same time, the 95% credible interval of both the parametric and semiparametric random parameter logit models indicate that the choice probability is elastic.

The range equivalency measure proposed in this paper (Eq. (4)) can be compared to the estimator used in Hess et al. (2006) for a linear specification of range.¹³ If the latter estimator is used, an electric vehicle is perceived as equivalent to a gasoline vehicle if the driving range of the electric vehicle reaches 330 miles (i.e. range-equivalency of a linear-in-range model reflects *driving range parity*.) RE is extended in this paper to better represent the tradeoffs among higher purchase prices, limited range, and efficiency gains. In addition, nonlinearities in the valuation of driving range are taken into account. With the range equivalency measure proposed here and when the corresponding tradeoffs with respect to internal combustion are considered, RE is much lower than driving range parity, meaning that consumers are willing to accept a lower driving range that is compensated by lower operating costs. When electric vehicles are compared to internal combustion, RE point estimates of the models that account for consumer heterogeneity are about 180 miles, almost double the currently offered electric driving range. Automakers often defend the 100 mile range by arguing that 100 miles is enough to cover daily needs. (45 miles is the daily average driven distance according to the NHTS data analyzed in Pearre et al. (2011).) However consumer preferences not only reflect needs but also desires. That 100 miles is not even within the 95% credible interval of the range equivalency with respect to internal combustion in any of the models shows that consumers are indeed expecting more than what the market now offers. In fact, when a hybrid is considered as benchmark, RE goes up to a value that resembles range parity. The population point estimate of RE is in the order of 560–580 miles for the models that consider taste heterogeneity. As a reference, the highway driving range of the Prius is 570 miles. Thus, even when accounting for operating cost savings, given the current difference in purchase price and intrinsic consumer preferences for hybrids, consumers would expect a range that matches that of a hybrid to make electric vehicles fully competitive.

I have now derived and discussed post-processed estimates of the three different models. Analyzing posterior distributions provides interesting insights, such as robust point and interval estimates of functions of parameters such as the willingness to pay for range improvements. However, the analysis of posterior distributions is correct only as long as the parameters are unbiased. Sources of bias include measurement errors and unobserved consumer heterogeneity. Note that the results of the fixed parameter logit model differ in general from the estimates of both the parametric and semi parametric random parameter logit models. Because the latter two models address the problems of repeated observations of a single individual as well as unobserved taste variations, both are superior to the model with fixed parameters. Thus, the estimates of the models that allow for consumer heterogeneity should be considered as more reliable. In fact, both produce relatively similar results. For example, the 95% credible interval of the willingness to pay of driving range improvements at 150 miles is [50.2 \$/mile, 86.7 \$/mile] for the parametric random parameter logit and [53.3 \$/mile, 89.3 \$/mile] for the semiparametric random parameter logit, but [61.2 \$/mile, 114.3 \$/mile] for the fixed parameter logit. However, some of the estimates of the two random parameter logit models are somewhat different. For instance, the choice probability of choosing an electric vehicle for a representative individual is 0.05 for the case of parametric heterogeneity, but 0.10 for the case of nonparametric heterogeneity. Since the heterogeneity distribution of the semiparametric logit is assumption-free and revealed by the data, leading to estimators that are robust against miss-specification, results of this model should be preferred.

¹³ In a linear-in-range model, an electric vehicle is valued equally to an internal combustion vehicle if range equals $-\theta_{BEV}/\theta_{range}$, *ceteris paribus*.

5. Conclusions

Electric vehicles currently offered in the market suffer from a rather limited driving range. Current technology translates into relatively affordable electric vehicles that can be driven for about 100 miles on a single charge. 100% electric vehicles that are being sold (Nissan LEAF) or will be introduced in the market soon (Ford Focus) offer a driving range of 75–100 miles. This may be a major concern for consumers, who may be reluctant to convert to electric vehicles despite potential economies, as well as environmental benefits of such a decision. In this paper I have derived and studied the posterior distribution of four functions of the parameters of a vehicle choice model that are useful in understanding consumer concerns toward limited driving range of electric vehicles. The first function is the willingness to pay for marginal driving range improvements. The second function is the compensating variation after driving range improvements, which is an important input for policymaking. The third function is the elasticity of the choice probability of buying an electric vehicle with respect to marginal driving range improvements. The last function is a measure of the desired driving range (range equivalency) that would make electric cars fully competitive. Estimates for all four functions were found using Bayesian econometrics. When using frequentist methods, one finds point estimates for the proposed measures that are an asymptotic approximation of the Bayes point estimates. Even though the Bayes estimators are exact, microdata is characterized by comparatively large sample sizes. Nevertheless, a clear benefit of the Bayesian approach in the case of microdata is the possibility of post-processing the estimates to obtain posterior distributions of functions of the original parameters. In this paper, post-processing has been used for deriving robust credible intervals of the four measures of consumer valuation of driving range. Three different models were considered, namely a conditional logit model with fixed parameters, a logit model with parametric heterogeneity distributions, and a logit model with nonparametric heterogeneity distributions. The latter model should be preferred as is the most flexible one, since it does not impose any assumption regarding the density of the unobserved taste variations.

The informative value of the proposed measures has been validated using stated preference data on vehicle choice in California. The data comprises behavioral purchase intentions of vehicles that are differentiated in several attributes, including energy source and driving range. A first result is that the data favored the hypothesis of a nonlinear valuation of driving range. After assuming a logarithmic transformation of range, a decreasing willingness to pay was estimated. The population point estimates of the willingness to pay for the most flexible model considered in this paper are 141.1 US\$05/mile (95% credible interval of [110.7,178.6]) at 75 miles, and 107.8 US\$05/mile [83.0,134.0] at 100 miles. These figures are higher than the willingness to pay found in recent studies, which report an average of around 65–70 \$/mile (Dimitropoulos et al., 2013; Hidrue et al., 2011). But these latter lower valuations are in line with the willingness to pay that was derived in this work at 150 miles. An interesting comparison is that the willingness to pay at 100 miles is lower than the marginal cost of producing a battery with improved range, which is 160 \$/mile at 100 miles.

The evaluation of the other three functions depends on attribute levels, so posteriors were obtained for a representative individual facing a decision among an internal combustion engine vehicle, a hybrid electric vehicle, and a battery electric vehicles. Attributes of these three vehicles reflect current average values. A result that is in line with the cost of production being lower than the willingness to pay is that the compensating variation of improvements in driving range is much lower than the cost of producing that improvement. The expected consumer benefits of the improvement are even lower due to probabilities of choosing an electric car being very low. For example, for an improvement from 100 to 150 miles – implying an increase in production costs of \$13,500, the expected compensating variation is in the range of \$250–\$1,100. An additional result is range equivalency, which measures the desired driving range level for which a consumer would perceive the electric vehicle and a benchmark car as being equally attractive. In contrast to range equivalency calculated in previous studies, in this paper differences in purchase price and operating cost are considered in the derivation of the measure. When an internal combustion vehicle is taken as benchmark, the population point estimate of range equivalency for the more flexible models is in the range of 180–185 miles. An interesting exercise that combines the different measures for valuation of driving range is the compensating variation from 100 miles to range equivalency. The result is around \$7900, a value that is surprisingly little above the current federal tax credit of \$7500 for the purchase of electric vehicles.

However, range equivalency when calculated with respect to hybrids reaches range parity, i.e. a figure around 570 miles. This result shows that hybrid vehicles may become the strongest competitor preventing the adoption of electric vehicles. Hybrid vehicles offer the best of both worlds: higher efficiency and environmental benefits without a limiting driving range. Thus, to assess the potential success of the electric car, a deeper comparison has to be made with what appears to be its strongest competitors: the plug-in hybrid and the extended range electric vehicle. Future empirical research should focus on this comparison via the combination of stated and revealed preference data, to avoid any potential bias due to the gap between purchase intentions and actual behavior.

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Appendix A. Bayes estimators of the conditional logit model

A.1. Fixed parameter logit

Consider a standard choice situation where for each individual $n \in \{1, \dots, N\}$ an exogenous random sample contains independent choice indicators y_{in} for each discrete alternative i in the set C_n .¹⁴ The choice indicator y_{in} identifies whether alternative i was chosen by individual n or not. This observed choice is the base of a revealed-preference mechanism that manifests an underlying truncated indirect utility function $U_{in} = \mathbf{x}'_{in}\boldsymbol{\theta} + \varepsilon_{in}$, where \mathbf{x}_{in} is a vector of attributes or constituent characteristics of the discrete alternatives, $\boldsymbol{\theta}$ is a vector of unknown parameters representing consumer tastes, and ε_{in} is an error term. The link between the binary observations y_{in} and the indirect utility function is given by the measurement equation $y_{in} = 1 \text{ if } U_{in} = \max_{j \in C_n} U_{jn}$. Under the assumption $\varepsilon_{in} \stackrel{iid}{\sim} \text{EV1}(0, \lambda)$, the parametric model that generates the data is such that

$$P_{in} = \frac{\exp(\lambda^{-1} \mathbf{x}'_{in} \boldsymbol{\theta})}{\sum_{j \in C_n} \exp(\lambda^{-1} \mathbf{x}'_{jn} \boldsymbol{\theta})}, \quad (\text{A.1})$$

where P_{in} are the choice probabilities derived from the model. For identification of $\boldsymbol{\theta}$ it is necessary to normalize the scale of utility $\lambda = 1$.

Since the elements $\mathbf{y}_n | \mathbf{X}_n$ in the sample space¹⁵ and the choice probabilities in Eq. (A.1) define a dominated parametric conditional model, frequentist point estimation is based on maximizing the likelihood

$$\ell(\boldsymbol{\theta}; \mathbf{y} | \mathbf{X}) = \prod_{n=1}^N \prod_{i=1}^{J_n} \left[\frac{\exp(\mathbf{x}'_{in} \boldsymbol{\theta})}{\sum_{j \in C_n} \exp(\mathbf{x}'_{jn} \boldsymbol{\theta})} \right]^{y_{in}}. \quad (\text{A.2})$$

In a Bayesian context, the parameters $\boldsymbol{\theta}$ of the model are assumed to have a *prior distribution* $p(\boldsymbol{\theta})$ that describes the probability distribution of $\boldsymbol{\theta}$ before the observation of the sample data $\mathbf{y} | \mathbf{X}$. Considering $\boldsymbol{\theta}$ to be a random variable is what distinguishes the Bayesian approach from frequentist statistics. This notion is fundamental for Bayesian inference and is derived from the concept of subjective probabilities.¹⁶ The combination of the prior distribution $p(\boldsymbol{\theta})$ with the information coming in via

¹⁴ C_n contains a total of J_n available alternatives.

¹⁵ $\mathbf{y}_n = (y_{1n}, \dots, y_{J_n n})'$, $\mathbf{X}_n = (\mathbf{x}'_{1n}, \dots, \mathbf{x}'_{J_n n})$.

¹⁶ Subjective probabilities measure the beliefs about the occurrence of a particular event.

the sample determines the *posterior distribution* of the parameters $p(\theta|\mathbf{y}, \mathbf{X})$. The posterior and prior distributions are related following Bayes' theorem according to

$$p(\theta|\mathbf{y}, \mathbf{X}) = \frac{\ell(\theta; \mathbf{y}|\mathbf{X})p(\theta)}{p(\mathbf{y}|\mathbf{X})},$$

where $p(\mathbf{y}|\mathbf{X})$ is the distribution of the data. For inference purposes Bayes' theorem is rewritten as $p(\theta|\mathbf{y}, \mathbf{X}) \propto \ell(\theta; \mathbf{y}|\mathbf{X})p(\theta)$ or

$$p(\theta|\mathbf{y}, \mathbf{X}) \propto p(\theta) \prod_{n=1}^N \prod_{i=1}^{J_n} \left[\frac{\exp(\mathbf{x}'_{in}\theta)}{\sum_{j \in C_n} \exp(\mathbf{x}'_{jn}\theta)} \right]^{y_{in}}, \quad (\text{A.3})$$

an expression that emphasizes the Bayesian notion of updating knowledge through evidence.

Even though Bayesian inference is focused on the posterior distribution $p(\theta|\mathbf{y}, \mathbf{X})$, the calculation of the first and second moments of the posterior are of fundamental interest. In fact, the posterior mean $\hat{\theta} = \mathbb{E}(\theta|\mathbf{y}, \mathbf{X})$ is the Bayes decision for the point estimation problem that minimizes the Bayes risk with a quadratic loss function. Additionally, with a quadratic loss function, the posterior variance $\mathbb{E}[\text{var}(\theta|\mathbf{y}, \mathbf{X})]$ is an unbiased estimator of the precision of the Bayes decision $\hat{\theta} = \mathbb{E}(\theta|\mathbf{y}, \mathbf{X})$.

An important class of Bayes estimators uses conjugate distributions, for which both the posterior and the prior belong to the same family of distributions.¹⁷ In the case of the conditional logit model, however, there is no general conjugate prior (cf. *Koop and Poirier, 1993*). When this occurs, a Bayes estimator can be derived using the iterative Metropolis-Hastings algorithm.¹⁸ Let Θ be the parameter space. Using Metropolis-Hastings, a candidate $\theta^{cand} \in \Theta$ is drawn from the *transition probability* $q(\theta^{cand}|\theta^{curr})$ of generating candidate θ^{cand} given $\theta^{curr} \in \Theta$, such that $\theta^{curr} \sim p(\theta, \mathbf{y}|\mathbf{X})$. The candidate realization θ^{cand} is then compared to the current $\theta^{curr} \in \Theta$ through the *acceptance ratio*:

$$\alpha = \min \left\{ 1, \frac{\ell(\theta^{cand}; \mathbf{y}|\mathbf{X})p(\theta^{cand})q(\theta^{cand}|\theta^{curr})}{\ell(\theta^{curr}; \mathbf{y}|\mathbf{X})p(\theta^{curr})q(\theta^{curr}|\theta^{cand})} \right\}. \quad (\text{A.4})$$

Starting with an arbitrary value $\theta^{(0)}$, at the g th iteration of the Metropolis-Hastings algorithm the candidate is accepted as the new $\theta^{(g)} = \theta^{cand}$ with probability α , whereas the old one is preserved $\theta^{(g)} = \theta^{curr}$ with probability $1 - \alpha$.

Consider the following asymptotic approximation to the posterior distribution of the conditional logit model (*Scott, 2003*):

$$p(\theta|\mathbf{y}, \mathbf{X}) \propto |\mathcal{I}(\theta)|^{\frac{1}{2}} \exp \left(1/2(\theta - \hat{\theta}_{MLE})' \mathcal{I}(\theta)(\theta - \hat{\theta}_{MLE}) \right), \quad (\text{A.5})$$

where $\hat{\theta}_{MLE}$ is the maximum likelihood estimator of θ , i.e. the value $\hat{\theta}_{ML}(\mathbf{y}|\mathbf{X})$ that maximizes the likelihood function $\ell(\theta; \mathbf{y}|\mathbf{X})$ once \mathbf{y} is observed, and where $\mathcal{I}(\theta)$ is the Fisher information matrix of the conditional logit model.¹⁹

Based on the asymptotic approximation to the posterior in Eq. (A.4), *Rossi et al. (2005)* propose two transition processes for updating θ in the Metropolis-Hastings algorithm. For a *random-walk* Metropolis chain, the candidate realization is defined as $\theta^{cand} = \theta^{curr} + \varepsilon$, where $\varepsilon \sim \mathcal{N}(0, s^2 \mathcal{I}^{-1})$ and s^2 is the precision. For an *independence* Metropolis, the candidate realization is found using $\theta^{cand} \sim \text{MSt}(\nu, \hat{\theta}_{MLE}, \mathcal{I}^{-1})$, i.e. θ^{cand} is drawn from a multivariate t distribution with mean $\hat{\theta}_{MLE}$, dispersion \mathcal{I}^{-1} , and ν degrees of freedom.²⁰

¹⁷ In the case of conjugate distributions, inference is based on direct sampling from the known parametric form of the posterior.

¹⁸ Metropolis-Hastings belongs to the class of Markov chain Monte Carlo (MCMC) methods, which are stochastic sampling algorithms based on constructing a Markov chain that has the posterior distribution as its equilibrium distribution. The estimators are based on numerical approximations from posterior simulators.

¹⁹ Instead of considering the maximum likelihood estimator, the approximation (A.4) can also be evaluated at the posterior mode (*Chib et al., 1998*).

²⁰ This is a generalization of the estimator proposed by *Chib et al. (1998)*.

Consider $p(\theta) \sim \mathcal{N}(\check{\mathbf{b}}, \check{\mathbf{B}}^{-1})$, a multivariate normal prior on θ with mean $\check{\mathbf{b}}$ and precision $\check{\mathbf{B}}$. To include the effect of the prior precision it is possible to extend both transition processes. Thus, it is possible to consider the following general transition process $\theta^{cand} \sim \mathcal{N}(\theta^{curr}, \mathbf{S}[\check{\mathbf{B}} + \mathcal{I}(\hat{\theta}_{MLE})]^{-1}|\mathbf{S})$ for the random walk, and $\theta^{cand} \sim \text{MSt}(\nu, \hat{\theta}_{MLE}, \mathbf{S}[\check{\mathbf{B}} + \mathcal{I}(\hat{\theta}_{MLE})]^{-1}|\mathbf{S})$ for independence Metropolis-Hastings, where \mathbf{S} is a diagonal matrix with elements that adjust the covariance matrix of the candidate to get satisfactory acceptance ratios. The implication here is that both \mathbf{S} and ν work as *tuning parameters*.

I will discuss now the implementation of the *slice sampler* (Wakefield et al., 1991; Damien et al., 1999; Neal, 2003), which is a special case of the Metropolis-Hasting algorithm.²¹ The slice sampler exploits data augmentation methods, where auxiliary variables are summed up to the parameter space in order to get full conditional distributions of standard form while keeping the marginal posterior of interest. Consider the posterior (A.3) of the conditional logit model, which depends on the choice probabilities through the likelihood function as well as on the prior. When introducing the auxiliary variables of the slice sampler, the likelihood function can be rewritten as an associated completion where the choice probabilities truncate the parameter space to reflect information summarized by the choice indicators. Then, with a normally distributed prior, the posterior can be simulated from a truncated multivariate normal distribution. An attractive feature of the slice sampler is that it does not require tuning.

A.2. Random parameter logit

Consider now an extension to the model discussed above where the parameters are individual-specific, leading to the following choice probability

$$P_{int|\theta_n} = \frac{\exp(\lambda^{-1} \mathbf{x}'_{int} \theta_n)}{\sum_{j \in C_n} \exp(\lambda^{-1} \mathbf{x}'_{jnt} \theta_n)}, \quad (\text{A.6})$$

where $t \in \{1, \dots, T\}$ indexes the choice situation within the sequence of choices by the same individual, accounting for the panel structure of the data.

This model represents a solution to the problem of repeated observations as well as to the problem of unobserved consumer heterogeneity. In this model, each consumer has his or her own set of parameters. To avoid the curse of dimensionality, θ_n can be assumed to have a certain distribution that represents heterogeneity in tastes that is found in the sample. A Bayesian estimator can be derived by adopting the Markov chain Monte Carlo sampler of a hierarchical conditional logit model with random parameters (Train and Sonnier, 2005; Burda et al., 2008).

On the one hand, if the heterogeneity distribution is assumed continuous and parametric, a Gibbs sampler with a Metropolis-Hastings step can be derived as in Train and Sonnier (2005). On the other hand, to represent a general assumption-free continuous heterogeneity distribution, the hierarchical semiparametric logit estimator that is adopted in this paper is based on a Dirichlet Process (DP) prior for the random mixing distribution of the parameters. A Dirichlet Process $\mathcal{DP}(G_0, \alpha)$, where G_0 is the baseline distribution and α the concentration or scale parameter, is a stochastic process that can be interpreted as the generalization of the Dirichlet distribution to infinite dimensions. Dirichlet processes produce random distributions that have the baseline G_0 as support of the output distribution. Because of the flexibility of the output distribution, Dirichlet process have been widely used as priors for nonparametric density representations.

For non-panel versions of discrete choice models, θ_n is drawn directly from the Dirichlet process (Burda et al., 2008). For panel microdata, the heterogeneity distribution can be represented as a mixture of multivariate normal distributions with an unknown mixing distribution following a Dirichlet process (Lancelot and Lau, 2004). This is the approach adopted in the estimator used in this paper. Consider $\theta_n \sim \mathcal{N}(\mu_n, \mathbf{T}_n)$, where a Dirichlet Process $\mathcal{DP}(G_0, \alpha)$ acts as prior for the nonparametric mixing joint distribution of the mean μ_n and covariance matrix \mathbf{T}_n . A normal-inverse-Wishart distribution is

²¹ The slice sampler is a subclass of Gibbs sampling. A Gibbs sampler approximates the joint posterior by sampling from a conditional decomposition of the posterior.

set as prior for the baseline G_0 (see Lancelot and Lau, 2004). Unlike the estimator derived in Lancelot and Lau (2004), where θ_n is updated using a Metropolis-Hastings procedure that is analog to the one used in Train and Sonnier (2005), in this paper the Metropolis-Hastings step of the Gibbs sampler is based on the independence chain that was described for the fixed parameter logit model. The estimator for the parametric random parameter logit model was taken as a particular case of the semiparametric estimator, where the heterogeneity distribution is defined as a single multivariate normal distribution (cf. Train and Sonnier, 2005).

Appendix B. Choice situation sample

See Fig. B.5.

	Vehicle A	Vehicle B	Vehicle C
Vehicle Type	Large SUV	Mini Car	Compact PU
Engine Type	Gasoline	Electric	Hybrid
Performance	Top Speed 80MPH	Top Speed 80MPH	Top Speed 80MPH
	0-60: 16 Seconds	0-60: 16 Seconds	0-60: 16 Seconds
Total Purchase Price*	\$36,298	\$16,594	\$33,025
Operating Cost (less routine maintenance)	\$56.70/mo	\$7.85/mo	\$29.10/mo
Range (in miles)	300 - 500	140 - 150	400 - 700

*Total Purchase Price is the amount customers can expect to pay for the vehicle new at the dealership

☐
☐
☐

Fig. B.5. Sample of a choice situation presented to respondents of the stated preference experiment.

Source: Train and Hudson (2000).

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