1.2 Counting

events

To place probabilities on countably many possibilities, we must know how to count all such subsets. ← need Tools

THM FUNDAMENTAL THEOREM of COUNTING

> if a job consists of **k separate tasks**, the ith of which can be done by **n_i ways**, for $i=1,\dots,k$
> Then, the entire job can be done in
>
> **$n_1 \times n_2 \times \dots \times n_k$ ways**

EXAMPLES

1. Lottery: FROM $\{1,\dots,44\}$ pick six different numbers for a ticket.

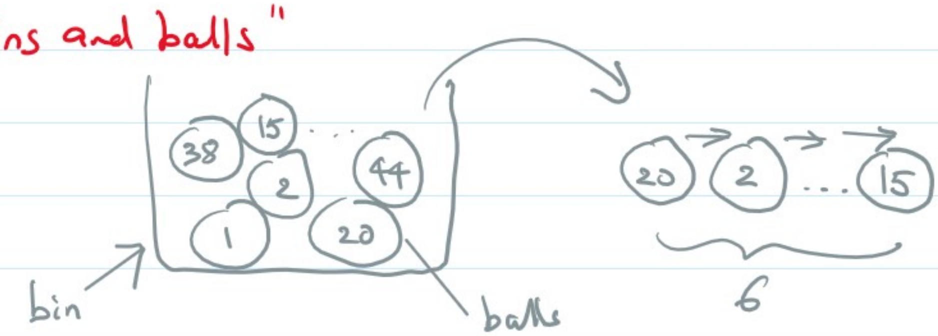
   How many possible tickets?

$$44 \times 43 \times 42 \times 41 \times 40 \times 39$$

$$= \frac{44 \times \dots \times 2 \times 1}{38 \times \dots \times 2 \times 1} = \frac{44!}{38!}.$$

ordered, without replacement

SAMPLING TERMINOLOGY "bins and balls"



2. $\boxed{\text{Ordered, with Replacement}}$: The six numbers need not differ

How many tickets?

$$44 \times 44 \cdots \times 44 = 44^6.$$

3. $\boxed{\text{Unordered, without Replacement}}$: the six numbers be distinct, but their order irrelevant

$$\frac{44 \times 43 \times \cdots \times 39}{6 \times 5 \times \cdots \times 1} = \frac{44!}{38!\,6!}$$

$$= \binom{44}{6} = \binom{44}{38}$$

"n choose r" : $$\binom{n}{r} := \frac{n(n-1)\cdots(n-r+1)}{r!\,(n-r)!}$$

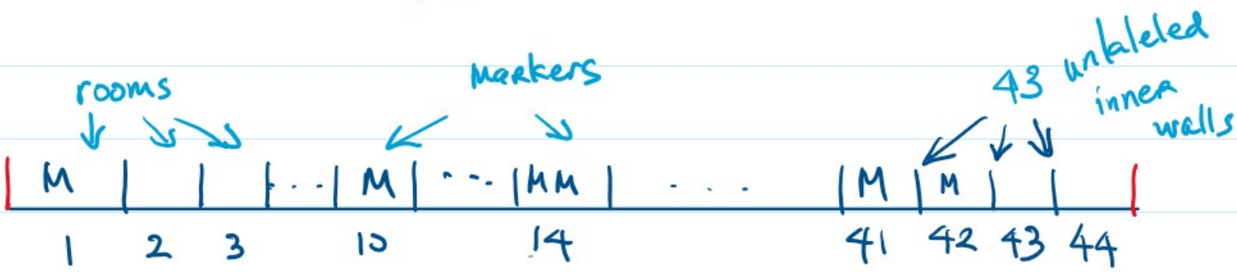
$$= \binom{n}{n-r}.$$

4. $\boxed{\text{Unordered, with replacement}}$

the six numbers need not be distinct, order irrelevant

This is more tricky to count, answer $\neq \frac{44^6}{6!}$.

- Lay out the 44 numbers in a fixed order, and create 44 "rooms", one for each number
- Rooms are separated by walls (45 in totals)
- Place the 6 markers "M" in rooms where the numbers have been chosen



Above: $\{1, 10, 14, 14, 41, 42\}$ were chosen.

Key observations:

\# tickets $\equiv$ \# unique ways to place 6 markers to 44 ROOMS

= ways to arrange 6 markers with 43 inner walls

$$= \binom{6+43}{6}$$

SUMMARY

Arrangements of Size r from n

	w/o Replacement	w/ placement
ordered	$n!/(n-r)!$	n^r
unordered	$\binom{n}{r}$	$\binom{n+r-1}{r}$