

1.2 Counting (cont)

if we can count the number of outcomes of an experiment, say N , and if such outcomes are equally probable, then the probability assigned to each outcome can be set to be $\frac{1}{N}$

in math, if $|S| = N$
set $P(\{a\}) = \frac{1}{N} \quad \forall a \in S.$

EXAMPLES

① How many ways to choose a 5-card hand from a deck of 52 cards?

$$\binom{52}{5}$$

← unordered, w/o replacement

if cards are well-shuffled and Randomly dealt

$$P(\text{a hand}) = \frac{1}{\binom{52}{5}} = 1/2598,960.$$

② what is the prob. of obtaining 4 aces from the 5-card?

ways to select the fifth card

$$P(4 \text{ aces}) = \frac{48}{\binom{52}{5}} \approx 1/50,000.$$

③ what about a "four of a kind"?

$$P(\text{'4 of a kind'}) = 13 \times 48 \cdot \frac{1}{\binom{52}{5}}.$$

e.g. $\{8D, 8H, 8C, 8S, *\}$

④ Prob of having exactly one pair

$$\# \text{ such hands is } m = 13 \times \binom{4}{2} \times \binom{12}{3} \times 4^3$$

$$P(\text{'having exactly one pair in a 5-card hand'}) = \frac{m}{\binom{52}{5}}.$$

EXAMPLES (Sampling w/ Replacement)

"Uniform" Sampling $r=2$ items from $n=3$ items with replacement

ordered	$(1,1)$	$(2,2)$	$(3,3)$	$(1,2)$	$(2,1)$	$(1,3)$	$(3,1)$	$(2,3)$	$(3,2)$
unordered	$\{1,1\}$	$\{2,2\}$	$\{3,3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$			
probability	$1/9$	$1/9$	$1/9$	$2/9$	$2/9$	$2/9$			

Question: Can you come up with a different sampling mechanism according to which the unordered outcomes are given equal probabilities ($1/6$)?