

1.2 Counting (cont)

if we can count the number of outcomes of an experiment, say N , and if such outcomes are equally probable, then the probability assigned to each outcome can be set to be $\frac{1}{N}$

in math, if $|S| = n$
set $P(\{a\}) = \frac{1}{n} \quad \forall a \in S.$

EXAMPLES

- ① How many ways to choose a 5-card hand from a deck of 52 cards?

$$\binom{52}{5}$$

← unordered, w/o replacement

if cards are well-shuffled and randomly dealt

$$P(\text{a hand}) = \frac{1}{\binom{52}{5}} = 1/2598,960.$$

② What is the prob. of obtaining 4 aces from the 5-card hand?

$$P(\text{4 aces}) = \frac{48}{\binom{52}{5}} \approx \frac{1}{50,000}.$$

ways to select the fifth card

③ What about a "four of a kind"?

$$P(\text{'4 of a kind'}) = 13 \times 48 \cdot \frac{1}{\binom{52}{5}}.$$

e.g. {8D, 8H, 8C, 8S, *}

④ Prob of having exactly one pair

$$\# \text{ such hands} \text{ is } m = 13 \times \binom{4}{2} \times \binom{12}{3} \times 4^3$$

$$P(\text{'having exactly one pair in a 5-card hand'}) = \frac{m}{\binom{52}{5}}.$$

EXAMPLES (Sampling w/ Replacement)

"Uniform" Sampling $r=2$ items from $n=3$ items with replacement

ordered	(1,1)	(2,2)	(3,3)	(1,2)	(2,1)	(1,3)	(3,1)	(2,3)	(3,2)
unordered	{1,1}	{2,2}	{3,3}	{1,2}	{1,3}	{2,3}			
probability	1/9	1/9	1/9	2/9	2/9	2/9			

Question: Can you come up with a different sampling mechanism according to which the unordered outcomes are given equal probabilities ($1/6$)?