Advanced Microeconomics for Policy Analysis I

Fall 2021

Problem Set 4

Preferences over commodity bundles

Problem 1. Suppose u is a utility representation of preferences $\succsim$ and f is a strictly increasing function. Show that if $f \circ u$ is concave, then preferences are convex.

Problem 2. Suppose $u(x_1, x_2)$ and $v(x_1, x_2)$ are two utility functions. Prove that if u and v are quasiconcave, then $m(x_1, x_2) = \min\{u(x_1, x_2), v(x_1, x_2)\}$ is quasiconcave as well.

Solving the consumer's problem

Problem 3. Consider a consumer with preferences over $\mathbb{R}^2_+$ represented by the utility function $u\left(x\right) = \alpha_1 \sqrt{x_1} + \alpha_2 \sqrt{x_2}$, with $\alpha_1, \alpha_2 > 0$. All prices are strictly positive.

a. Are assumptions A1-A5 satisfied?

b. Is the condition $\lim_{x_i \to 0} \frac{\partial u(x)}{\partial x_i} = \infty$ satisfied in this case?

c. Apply Proposition 4.1 to characterize the solution to the consumer's problem.

d. Find the solution to the consumer's optimization problem.

Problem 4. Consider a consumer with preferences over $\mathbb{R}^n_+$ represented by the utility function $u(x) = x_1$. All prices are strictly positive. Find the solution to the consumer's optimization problem.

Problem 5. Consider a consumer with preferences over $\mathbb{R}^2_+$ represented by the utility function $u(x) = \max\{ax_1, ax_2\} + \min\{x_1, x_2\}$, with $a \in (0, 1)$. All prices are strictly positive. Find the solution to the consumer's optimization problem.

Problem 6. Consider a consumer with preferences over $\mathbb{R}^4_+$ represented by the utility function $u(x_1, x_2, x_3, x_4) = \min\{x_1 \cdot x_2, x_3 \cdot x_4\}$. All prices are strictly positive.

a. Argue that, at the optimum, $x_1 x_2 = x_3 x_4$.

b. Argue that, at the optimum, the budget constraint is satisfied with equality.

c. Argue that there cannot be a corner solution.

d. Incorporate the results from (a) and (b) into the consumer's maximization problem. You will be left with a different but equivalent maximization problem. Hint: this new maximization problem has *two* constraints: the budget constraint and another one. Moreover, the objective function is different than the original one (by part (a)).

e. Set up the Lagrangian and derive the Kuhn–Tucker conditions for a solution. Recall the result in (c).

f. Find the solution to the consumer's optimization problem.