Advanced Microeconomics for Policy Analysis I

Fall 2021

Problem Set 3

Risk Aversion

Problem 1. Consider an expected utility maximizer with Bernoulli utility function $u(x) = -(x-y)^2$ defined on $X = \mathbb{R}$, where $y \in \mathbb{R}$ is just some number. The interpretation is that this DM has a preferred outcome, y. No lottery would make her happier than $x = y$ with certainty. (Think now of x as being a stochastic policy outcome -e.g., the inflation rate-, rather than money.) Consider a lottery over X characterized by a cdf F. All you know about F is that $\mathbb{E}_F[x] = \mu$ and $\text{Var}_F(x) = \sigma^2$.

a. Is our DM is risk–averse? Why/Why not? In case "yes", is she weakly or strictly risk–averse?

b. Compute expected utility. In particular, show that it is a function of y, μ, and σ^2. In light of your answer to part (a), discuss the role of σ^2 in the expression of expected utility.

c. Consider some fixed utility level $U < 0$. Characterize an indifference curve on the plane (σ^2, μ) for such utility level. The final outcome here should be an expression for μ in terms of σ^2 (and y, and U). Carefully draw two of these indifference curves, indicating in what directions the utility of our DM increases.

d. Compute the certainty equivalent of F, C_F. Is it unique? In case your answer is "no": in class we said it is unique (Theorem 3.3)! Explain yourself, then.

e. Suppose we have a second DM, with Bernoulli utility function $v(x) = -(x-z)^2$, with $z > y$. Can you tell whether this DM is more or less risk averse than the DM with u?

Problem 2. In class, we derived a measure of absolute risk aversion. As we argued, this is useful for comparing preferences over risk between two different decision makers. It is also in our interest to analyze how risk aversion varies with wealth for a given individual. To do this, it is useful to consider risk tolerance measured as a percentage of initial wealth. The **Arrow–Pratt coefficient of relative risk aversion** is defined as

$$\gamma(x) := -\frac{u''(x)}{u'(x)}x.$$

Now consider the following Bernoulli utility functions:

1. $u_1(x) = -\exp^{-\alpha x}$, with $\alpha > 0$;
2. $u_2(x) = \frac{x^{1-\sigma}}{1-\sigma}$, with $\sigma > 0$;
3. $u_3(x) = \frac{(x-x_0)^{1-\sigma}}{1-\sigma}$ for $x > x_0$;
4. $u_4(x) = x - \delta x^2$, with $\delta > 0$, for $x \in [0, 1/\delta]$.

In each case, compute $\lambda(x)$ and $\gamma(x)$, and analyze how these coefficients change when x increases.

Problem 3. Consider an expected utility maximizer with Bernoulli utility function $u(x) = \frac{x^\alpha}{\alpha}$, with $\alpha \in [0, 1]$. She faces the following lottery: with probability $p \in (0, 1)$ she gets a high prize, x_h; with probability $1 - p$, she gets a low prize, $x_l \in (0, x_h)$.

a. Compute the certainty equivalent for this DM as a function of p, x_l, x_h, and α.

b. How does the certainty equivalent change with α? To make things easier, assume $x_l = 0$. Also, the following fact will be useful: the derivative of $f(z) = a^{1/z}$, where $a > 0$ is just some number, is $f'(z) = -a^{1/z}\frac{\ln(a)}{z^2}$. Briefly explain your result. (It may be helpful to take a look at your results in Problem 5, part 2.)

Stochastic Dominance

Problem 4. First, consider a continuous uniform distribution $U\left[c - \frac{a}{2}, c + \frac{a}{2}\right]$, with $a > 0$. Its density, then, is equal to $\frac{1}{a}$ for $x \in \left[c - \frac{a}{2}, c + \frac{a}{2}\right]$ and 0 otherwise. Verify that the expected value is equal to c, and the variance is equal to $\frac{a^2}{12}$.

Now consider the following risk averse agent: $u(x) = -\frac{1}{x}$. In each one of the following two scenarios, she faces a pair of lotteries. Recall that the antiderivative/primitive integral of $\frac{1}{x}$ is $\ln(x)$.

a. L_1 is a uniform distribution $U[2, 8]$ and L_3 is a uniform distribution $U[4, 5]$.

   i. Which lottery has a larger expected value?
   ii. Does our DM prefer the lottery with higher expected value?
   iii. Briefly discuss your results in i and ii in relation with the concept of first–order stochastic dominance.

b. L_1 is a uniform distribution $U[2, 8]$ and L_2 is a uniform distribution $U[1, 13]$.

   i. Which lottery has a smaller variance?
   ii. Does our DM prefer the lottery with smaller variance?
   iii. Briefly discuss your results in i and ii in relation with the concept of second–order stochastic dominance.