Advanced Microeconomics for Policy Analysis I

Fall 2021

Problem Set 1

Problem 1. Consider a set of k elements, $A = \{a_1, a_2, \ldots, a_k\}$, with $a_i \in \mathbb{R}^L_+$ for each $i = 1, 2, \ldots, k$. Show that a consumer with complete and transitive preferences over A is able to rank all the elements in A according to her preferences, from the most to the least preferred one. (Do not worry about indifference.)

Problem 2. Assume that A, the set of alternatives, is finite, that $\succsim$ is rational, and that there is no pair of alternatives $x \neq y$ with $x \sim y$.

a. Show that $C^*(\cdot, \succsim)$ is resolute.

b. Show that $C^*(\cdot, \succsim)$ is contraction consistent.

Problem 3. Show that, if $\succsim$ is transitive, then both $\succ$ and $\sim$ are transitive as well.

Problem 4. Suppose $\succsim$ is transitive. Show that, if $x \succ y$ and $y \succsim z$, then $x \succ z$.

Problem 5. Consider the set of alternatives $A = \{\textit{an orange}, \textit{an apple}, \textit{a peach}\}$, and the following preference relation over the elements of A:

$$\begin{aligned}\succsim = \{&(orange, orange), (orange, apple), (apple, apple),\\ &(apple, peach), (peach, orange), (peach, peach)\}.\end{aligned}$$

a. Is this preference relation complete?

b. Is it transitive?

Problem 6. Let X be a finite set. For a set S, we write $|S|$ to denote the number of elements in S.

Let $V : X \to \mathbb{R}$ be some real-valued function on X. Check whether the following choice correspondences satisfy contraction consistency.

[Keep in mind: In order to prove contraction consistency, consider two sets $A, B \in X$, with $A \subset B$. If $x \in A$ and $x \in C(B)$, you want to check that $x \in C(A)$.]

a. $C(A) = \{x \in A : \text{the number of } y \in X \text{ with } V(x) \geq V(y) \text{ is at least } |X|/2\}$. If this set is empty, then $C(A) = A$.

b. $D(A) = \{x \in A : \text{the number of } y \in A \text{ with } V(x) \geq V(y) \text{ is at least } |A|/2\}$

c. $E(A) = \{x \in A : y \succsim x \text{ for all } y \in A\}$, where $\succsim$ is complete.

Problem 7. Consider the set of integers $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$, and the following preference relation over $\mathbb{Z}$: for any two elements $a, b \in \mathbb{Z}$, $a \succsim b$ if, and only if, $a \geq b$.

a. Is $\succsim$ rational?

b. Does $u(x) = x^2$ represent $\succsim$?

c. Does $v(x) = x$ represent $\succsim$?

d. Briefly discuss (b) and (c), in light of the fact that "any strictly increasing transformation of a function that represents a preference relation still represents the same preference relation."