Public Policy 42000
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Problem Set 4

due October 31

1. Predictions with Random Walks

Consider the process

$$x_t = x_{t-1} + \varepsilon_t \quad -\infty < t < \infty$$
$$E\left(\varepsilon_t \varepsilon_{t-r}\right) = 0 \ \forall r \neq 0$$

and the associated first difference

$$\Delta x_t = x_t + x_{t-1}$$

i. Calculate the sequence of linear predictions $x_{t+k|t}$

Suppose that, for this process

$$\varepsilon_t = \phi \varepsilon_{t-1} + \eta_t$$
$$E\left(\eta_t \eta_{t-r}\right) = 0 \ \forall r \neq 0$$

ii. Calculate the sequence of linear predictions $x_{t+k|t}$

iii. In what senses are these prediction problems deviations from the Wiener-Kolmogorov formulas?

2. True, False, or Uncertain and Explain

Assume throughout that x_t is 0 mean, second order stationary, finite variance.

i. If x_t is projected against x_{t-1}, the associated projection coefficient α must obey $|\alpha| < 1$.

ii. The projection of x_t against $H_{t-1}(x)$, $x_{t|t-1}$ will obey the condition $\operatorname{var}(x_{t|t-1}) \leq \operatorname{var}(x_t)$

iii. The forecast error $x_t - x_{t|t-k}$ has the property that $\operatorname{var}(x_t - x_{t|t-k})$ is increasing in $k \geq 1$.

iv. The forecast error $x_t - x_{t|t-k}$ is serieally uncorrelated, i.e. $\operatorname{cov}(x_t - x_{t|t-k}, x_{t-r} - x_{t-r|t-r-k}) = 0 \; r > 0$