PPHA 42200: Problem Set 4*

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Part I: Theory (GMM and Optimization)

1. Suppose you estimate a parameter using maximum likelihood, method of moment, GMM with uniform weights, and GMM with optimal weight matrix. Assume that all of the required assumptions are satisfied for each estimation. Denote the variance of the estimator from each estimation by V_{ML}, V_{MM}, V_{GMM}, V_{OGMM}. Is it possible to use the inequality sign ($\leq$) to rank these variances from the lowest to the highest in general? Explain why.

2. Consider a random variable x and a GMM objective function $f(\theta) = \bar{x}\,(\theta - \ln(\theta))$, where θ is a parameter, and a scalar $\bar{x} > 0$ is the sample mean of x. Recall the formula for the Newton-Rapson method is $\theta_{t+1} = \theta_t + (-H_t^{-1})g_t$, where g_t and H_t are the first and second derivatives of $f(\theta)$, evaluated at θ_t.

   (a) Using calculus, find θ^* that minimizes the GMM objective function.

   (b) You use the Newton-Rapson method with the initial value of $\theta_0 = \frac{1}{2}$. Calculate θ_1 and θ_2. Does this suggest that this iteration process makes θ_t approach θ^*?

   (c) You use the Newton-Rapson method with the initial value of $\theta_0 = 2$. Calculate θ_1, θ_2, and θ_3. Does this suggest that this iteration process makes θ_t approach θ^*?

3. Consider that we have population moment conditions:

$$E[\psi(x, y, \theta)] = E\begin{pmatrix} 3x - \theta \\ 12y - \theta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

   (a) Show the sample moment conditions implied by the population moment conditions.

   (b) Show the GMM objective function with the identity matrix $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ as the weight matrix.

   (c) Solve the optimization problem to find a GMM estimator $\hat{\theta}_{gmm}$.

   (d) Suppose that x and y are independent and $Var(x) = Var(y) = 1$. Calculate $Var(\hat{\theta}_{gmm})$.

   (e) Suppose that x and y are independent and $Var(x) = Var(y) = 1$. Instead of the identity matrix, you want to use the following matrix as a weight matrix.

*Note: The questions in part II are based on Anderson (2008). Also, for part II, Jens Hainmueller's website will be helpful to run synthetic control methods (`<http://web.stanford.edu/~jhain/synthpage.html>`).

$$M = \begin{pmatrix} c & 0 \\ 0 & 1 \end{pmatrix}.$$

What is c that minimizes the variance of $\hat{\theta}_{gmm}$? Calculate the new variance of $\hat{\theta}_{gmm}$. Is this smaller or larger than the variance in question (d)?

Part II: Empirical Analysis (Synthetic Control)

This empirical portion of this problem set is based off of a data set used in Anderson (2008). That paper examined the effects of light trucks on traffic fatalities, but in doing so it also collected data on primary and secondary seat belt laws as additional covariates. This problem set will focus on the question of whether primary seat belt laws save lives. (A primary belt law stipulates that law enforcement can ticket a driver for not wearing a seat belt, regardless of whether he has broken any other laws. A secondary belt law stipulates that law enforcement can only ticket a driver for not wearing a seat belt if the driver has already been pulled over for simultaneously breaking a different traffic law.)

1. We now apply the synthetic control methods from Abadie et al (2010). Some preliminaries: Abadie et al have created a downloadable "canned" command to run the synthetic control method. To download the command for Stata you will need to have an updated version of Stata and be running Stata on a Mac, PC or Unix/Linux. R code is also available. In Stata type 'update all' and then 'update swap'. Next, go to the website below and follow the instructions. There is also downloadable code for Matlab at `<http://web.stanford.edu/~jhain/synthpage.html>` (note: copying and pasting this URL may not reproduce the tilde)

   (a) We created an aggregate "treatment" state (state number 99 or "TU") which combines the (population weighted) data from the first 4 states to have a primary seatbelt law (CT, IA, NM, TX). Please use this state as the "treatment" state in the synthetic control analysis.

      i. Compare the average pre-period log traffic fatalities per capita of the TU site to that of the average of all the “control” states. Next, graph the pre-period log traffic fatalities by year for the pre-period for both the TU and the average of the control group. Interpret.

      ii. Compare the dependent variable between the TU site and each control state for the years before the treatment (Note: you can calculate MSE for the years). Which control state best matches the TU? Now compare this state's covariates with the TU covariates. Do they appear similar? What might this imply for in terms of using this state as the counterfactual state?

   (b) Apply the synthetic control method using the available covariates and pre-treatment outcomes to construct a synthetic control group.

      i. Discuss the synthetic control method including its benefits and potential drawbacks.

      ii. Use the pre-programmed command provided by Abadie et al to apply the synthetic control method. (You are free to use either Stata, Matlab, or R but answers will be provided in Stata only!) Please be sure to state precisely what the command is doing and how you determined your preferred specification.

(c) Graphical interpretation and treatment significance.

   i. Generate graphs plotting the gap between the TU and the synthetic control group under both your preferred specification and a few other specifications you tried.

   ii. Compare the graph plotting the gap between the TU and the synthetic control group under your preferred specification with the graphs plotting the gap between each control state and its "placebo" treatment. Do you conclude that the treatment was significant? Why or why not?