Advanced Microeconomics for Policy Analysis I

Fall 2021

Problem Set 2

completeness: either u(x) >= u(y) or the opposite; represent degenerate lotteries that give either x or y with certainty, which mean in preferences that x weakly preferred to y or vice versa, etc.
use summation definition

transitivity: show if x >= y, y >=z, then x >= z. First, degen lot x >= degen lot y which means u(x) >= u(y) by definition of utility since utility of lotteries can be rep by expected utility form; then y>=z means degen lot y >= degen lot z which means u(y) >= u(z); then u(x) >= u(z) which means working backwards that x >= z

Expected Utility

Problem 1. Consider a set of alternatives A, and a preference relation over A, $\succsim$. Suppose $\succsim$ can be represented with the expected utility form. Show that it therefore satisfies rationality, continuity, and independence.

Problem 2. We will show that if the set of consequences X is finite and the rational preference relation $\succsim$ on the set of lotteries $\mathcal{L}(X)$ satisfies continuity and the independence axiom, then there are best and worst lotteries in $\mathcal{L}(X)$. That is, we can find lotteries $\overline{L}$ and $\underline{L}$ such that $\overline{L} \succsim L \succsim \underline{L}$ for all $L \in \mathcal{L}(X)$. (Notice that X is finite but $\mathcal{L}(X)$ is definitely not.)

a. Argue that there exists an utility function U that represents $\succsim$.

b. Argue that U takes the expected utility form.

c. Show that $\overline{L}$ and $\underline{L}$ are the lotteries degenerate at the best and worst consequences in X, respectively.

Problem 3. Consider a money manager with Bernoulli utility function $u(x) = \ln(x)$. He is deciding how to split an amount W between a risky stock and a government bond. The stock pays a random gross interest rate θ. The bond is a risk–free asset that promises a gross interest rate r with certainty. Show that the optimal fraction $\alpha \in [0, 1]$ of W he will invest in the risky asset does not depend on W.

Problem 4. Consider a taxpayer with income y who must report her income to the IRS. If there was symmetric information between the taxpayer and the IRS she would have to pay a fraction $t \in (0, 1)$ of her income, so the total amount would be ty. However, the IRS does not know the taxpayer's income. It must rather rely on the taxpayer's report. The taxpayer can report $x \in [0, y]$. When she reports x, she immediately pays tx to the IRS. After this, with a known, given probability π, the IRS audits a taxpayer. If this occurs, income is revealed perfectly. If $x < y$, the taxpayer has to pay not only the remaining amount she owes, but also $s > 0$ times the difference between what she should have paid and what she actually paid, with $1 - (1 + s)t > 0$.

a. For a given report x, specify the lottery that the taxpayer is facing. (Just to be perfectly clear: The outcomes/consequences are her final wealth, W, in each scenario.)

b. Suppose the taxpayer is an expected utility maximizer with strictly increasing Bernoulli utility function $u(W)$, where W is final wealth. Write down the corresponding maximization problem.

c. Characterize the optimal solution, x^*, to the maximization problem in b. Be careful with both bounds, 0 and y.

d. Provide a condition under which $x = 0$ is a solution. What can you say about the role played by the probability π in this condition?

e. Provide a condition under which $x = y$ is a solution. What can you say about the role played by the probability π in this condition?

f. Consider the case $u(W) = \ln(W)$.

   i. Assume the solution is interior and solve for the optimal report, x^*, explicitly.

   ii. Find an upper and a lower bound on π such that the assumption of an interior solution, "$x^* \in (0, y)$," is true. Suppose $s = 50\%$ and $t = 20\%$, what are the values of these bounds? Would you say it would take a lot of effort for the IRS to "induce" the taxpayer to truthfully reveal her income?

   iii. Assume the conditions you found in (ii) are satisfied. Compute partial derivatives in order to explain how the solution in (i) changes with each parameter of the problem. (Just focus on their signs.) Briefly explain each of them.