Public Policy 42000
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Problem Set 1

due October 10

1. Limits of averages and laws of large numbers

For each of the following 4 stochastic processes, calculate the limiting expected value and variance of $\bar{x}_T = \sum_{t=0}^{T-1} x_t$ as $T \Rightarrow \infty$. Relate to laws of large numbers.

i.

$x_t = x_{t-1},\ t > 0$
$x_t = -1$ with probability $1-p$, 1 with probability p

ii.

$x_t = -x_{t-1},\ t > 0$
$x_t = -1$ with probability $1-p$, 1 with probability p

iii.

$x_t = \varepsilon_t,\ t \geq 0$
$\varepsilon_t = -1$ with probability $1-p$, 1 with probability p
$\text{cov}\left(\varepsilon_t \varepsilon_{t+k}\right) = 0$ if $k \neq 0$

iv.

$x_t = \xi + \varepsilon_t,\ t \geq 0$

$\xi \sim N\left(0, \sigma^2\right)$

$\varepsilon_t = -1$ with probability $1-p$, 1 with probability p

$\text{cov}\left(\varepsilon_t \varepsilon_{t+k}\right) = 0$ if $k \neq 0$

2. Covariance information

For case 4, what is the relationship between the limiting sample covariance of x_t and x_{t+1} and the true covariance between them for

i. Case 1.i

ii. Case 1.iv

Explain.