Linear Algebra

Course Overview

Linear algebra is the study of linear functions. At the most abstract, a linear function is a function that obeys certain rules. We will work on a more concrete level, mostly with matrices and vectors of real (or sometimes complex) numbers. We first look at solving linear equations, then analyze the structure of solutions to equations. From there we start to analyze the actions that linear functions take—what happens analytically and geometrically. We generalize to look at the structure of the set of linear functions. And of course we have plenty of applications.

Linear algebra is an *advanced* course. That means you are expected to be mature in your mathematics. You will answer questions in complete sentences, supporting your statements with valid mathematical arguments. You will read the book. You will do extra problems from the book to ensure you understand the concepts both computationally and theoretically. Linear algebra is one of the most beautiful structures in mathematics, and linearity is probably the most pervasive idea in all of mathematics. The ideas of linear algebra crop up in every branch of mathematics, so having a good understanding of the background here will go a long way to prying open other areas of mathematics.

So you will have to work hard. There will be frequent homework assignments, which will count toward your grade. There will be periodic tests. And of course there will be a final. You can expect the relative weight of homework to be about 1/3 of each quarter grade and tests to be about 2/3, depending on the number of tests we have. The final exam will count 20% of the course grade, while the homework and tests count 80%.

Resources

The text is excellent. The author has taught this course at MIT since forever, and has refined the text over the years to reflect both advances in the subject area and its applications as well as new insights as to how students learn. The text is kind of “chatty” but IMSA students should find it easy to read and learn from. It includes many demonstrations, a selection of fully-worked examples, and lots of extra problems for practice and to go beyond what we do in class.

This course is fully Web supported. Go to http://web.mit.edu/18.06/web and you will find a ton of extra material:

- Homework sets with solutions
- Sample exams
- Videos of course lectures
- Java applet demonstrations

This course was one of the first web-enabled courses ever, and has a lot of supporting material online. Take advantage of it!

Major Topics

Vectors
- Linear Combinations
- Length and Dot Product

Solving Linear Equations
- Systems of Equations
- Identifying Solutions
- Elimination

Matrix Notation
- Matrix Arithmetic
- Matrix Operations and Elimination
- Inverses
- Factorization **A** = **LU**

Vector Spaces

- Linear Combinations and Subspaces
- Nullspace, Column Space, and Row Space
- Independence, Basis, and Dimension
- Fundamental Subspaces of a Matrix

Orthogonaltiy

- Orthogonality and the Fundamental Subspaces
- Projections
- Least Squares
- Orthogonal Bases and Gram-Schmidt Algorithm

Determinants

- Defining determinants by desired properties
- Formulas: Pivots, Cofactors, etc.
- Applications: Inverses, Cramer's Rule, Volumes of Solids

Eigenvalues and Eigenvector Basics

- Diagonalizing a Matrix
- Difference and Differential Equations
- Special Matrices: Symmetric, Positive Definite
- The complex versions of things
- Complex special matrices (Hermitian, unitary)
- Similarity
- Schur's lemma; triangular form
- Singular Values

The above topics represent the core material of the course. Depending on the time remaining, and the interests of the group, we may tackle some of the following topics:

Linear Functions

- Linear Functions and Transformations Between General Vector Spaces
- Matrices for Transformations
- Change of Basis
- Jordan canonical form

Applications

- Markov Processes
- Linear Programming
- Fourier Series
- Fast Fourier Transform