

# 1) Least Squares Estimator

$$\hat{\beta} = (X'X)^{-1} X'y \quad n=6$$

$$X = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & 2 \\ 1 & 3 & 4 \end{bmatrix} \quad X' = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 & 3 \\ 1 & 1 & 2 & 2 & 4 \end{bmatrix} \quad y = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 6 \\ 10 \end{bmatrix}$$

$$X'y = \begin{bmatrix} 2+3+4+6+10 \\ 2+3+8+18+30 \\ 2+3+8+12+40 \end{bmatrix} = \begin{bmatrix} 25 \\ 61 \\ 65 \end{bmatrix} \cdot \begin{bmatrix} 1.2 & -0.5 & 0 \\ -0.5 & .75 & -0.5 \\ 0 & -0.5 & 0.5 \end{bmatrix}$$

$$\begin{bmatrix} 30 - 30.5 + 0 \\ -12.5 + 45.75 - 32.5 \\ 0 - 30.5 + 32.5 \end{bmatrix} = \begin{bmatrix} -0.5 \\ .75 \\ 2 \end{bmatrix} = \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix} \quad \boxed{\hat{\beta}' = E}$$

## 2) $(X'X)^{-1} X'y = \hat{\beta}$

$$X = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \\ 1 & 0 & 0 \end{bmatrix} \quad X' = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 0 & 0 \\ 1 & 0 & 0 & 2 & 0 \end{bmatrix} \quad y = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 6 \\ 10 \end{bmatrix} \quad X'y = \begin{bmatrix} 2+3+4+6+10 \\ 2+3+8+0+0 \\ 2+0+0+12+0 \end{bmatrix}$$

$$\begin{bmatrix} 25 \\ 13 \\ 14 \end{bmatrix} \cdot \begin{bmatrix} 0.8285714 & -0.4857143 & -0.4 \\ -0.4857143 & 0.4571429 & 0.2 \\ -0.4 & 0.2 & 0.4 \end{bmatrix}$$

$$\begin{bmatrix} 20.714285 & -6.3142857 & -5.6 \end{bmatrix} = \begin{bmatrix} 8.74 \end{bmatrix} \quad \hat{\beta}_0 = 8.8$$

C

3)  $F = 20 \quad R^2 = ?$

$n = 105$

$p = 4$

$$R^2 = \frac{Fp}{Fp + n - p - 1} = R^2 = \frac{20(4)}{20(4) + 105 - 4 - 1} = 44$$

C

4.  $F = \frac{R^2/p}{(1-R^2)/(n-p-1)}$

$R^2 = .8$

$n = 20$

$p = 4$

$$F = \frac{.8/4}{(1-.8)/(20-4-1)} = 15$$

D

5.  $F = \frac{(RSS_0 - RSS_1)/q}{RSS_1/(n-p-1)}$

$q = 3$

$p = 5$

$n = 18$

$RSS_0 = 102$

$RSS_1 = 78$

$$\frac{(102-78)/3}{78/(18-5-1)} = 1.23 =$$

A

$$6. \frac{(R_F^2 - R_R^2)/q}{(1 - R_F^2)/(n - p - 1)}$$

$$R_F^2 = .94 \quad n = 42 \quad q = 2$$

$$R_R^2 = .915 \quad p = 4$$

$$\frac{(.94 - .915)/2}{(1 - .94)/(42 - 4 - 1)} = 7.7 = \boxed{C}$$

$$7. F\text{-ratio} =$$

$$RSS_0 = 27.7281 \quad q =$$

$$RSS_1 = 12.8156$$

$$DF_{full} = 7 \quad DF_{diff} = 2$$

$$DF_{red.} = 9$$

$$\frac{(27.7281 - 12.8156)/2}{12.8156/7} = 4.07$$

$\boxed{D}$

$$8. \quad \boxed{C} \quad III$$

$$p\text{-value} > .05$$

.5644 = not significant