

Week 1 - AYU - Individual

Watch the videos to see how the similar problems are solve.

Parameter estimation

Problem 1.

x	2	3	5	6	1	9	10	15
y	1	4	6	4	4	3	20	25

Write the equation of the best fitted line.

- A. $y = -1.722 + 1.584x$
- B. $y = -1.722 - 1.584x$
- C. $y = 1.722 + 1.584x$
- D. $y = 1.584 + 1.722x$
- E. $y = -1.584 + 1.722x$

Problem 2.

The regression model is $y = \beta_0 + \beta_1x + \epsilon$. There are six observations. The summary statistics are:

$$\begin{aligned}\sum y_i &= 58, \\ \sum x_i &= 21, \\ \sum x_i^2 &= 91, \\ \sum x_i y_i &= 259, \\ \sum y_i^2 &= 754\end{aligned}$$

Calculate the least squares estimate of β_1 .

- (A) 3.0
- (B) 3.2
- (C) 3.4
- (D) 3.6
- (E) 3.8

Problem 3 (Similar Problem: Example 1.1.3)

The regression model is $y = \beta_0 + \beta_1 x + \epsilon$. You are given the follows.

$$\begin{aligned}n &= 10, \\ \bar{y} &= 21.1, \\ \bar{x} &= 7.5, \\ \sum x_i^2 &= 759, \\ \sum x_i y_i &= 2253, \\ \sum y_i^2 &= 7657\end{aligned}$$

Predict y_{11} given that $x_{11} = 20$

- A. 63.748
- B. 62.758
- C. 64.758
- D. 65.758
- E. The correct answer is not given by (A), (B), (C), or (D).

Problem 4 (Similar Problem: Example 1.1.4)

You are given the following summary statistics:

$$\begin{aligned}\bar{x} &= 6 \\ \bar{y} &= 13.75 \\ \sum (x_i - \bar{x})^2 &= 102 \\ \sum (x_i - \bar{x})(y_i - \bar{y}) &= 192 \\ \sum (y_i - \bar{y})^2 &= 503.5\end{aligned}$$

Determine the equation of the regression line, using the least squares method.

- (A) $y = 2.456 + 1.882x$
- (B) $y = 0.78 + 1.882x$
- (C) $y = 2.456 + 0.65x$
- (D) $y = 0.39 + 0.70x$
- (E) The correct answer is not given by (A), (B), (C), or (D).

Goodness of Fit

Problem 5 (Similar Problem: Example 1.2.1)

For a simple linear regression model the sum of squares of the residuals is

$$\sum_{i=1}^{25} e_i^2 = 300$$

and the R^2 statistic is 0.8. Calculate the total sum of squares (TSS) for this model.

- (A) 1000
- (B) 1200
- (C) 1500
- (D) 2000
- (E) 2500

Problem 6 (Similar Problem: Example 1.2.1)

For a simple linear regression model the total sum of squares (TSS) is 1000 and the R^2 statistic is 0.7. Calculate the sum of squares of the residuals for this model.

- (A) 300
- (B) 400
- (C) 500
- (D) 600
- (E) None of the above

Problem 7 (Similar Problem: Example 4.1.3 or SRM - Sample Question 11)

You are given the following results from a regression model.

Observation number (i)	y_i	$\hat{f}(x_i)$
1	1	4
2	2	3
3	6	7
4	8	9
5	4	6

Calculate the sum of squared errors (SSE).

- (A) 10
- (B) 12
- (C) 14
- (D) 16
- (E) 46

Problem 8 (Similar Problem Example 1.2.5 or SRM - Sample Question 44)

Two actuaries are analyzing dental claims for a group of $n = 100$ participants. The predictor variable is sex, with 0 and 1 as possible values.

Actuary 1 uses the following regression model:

$$Y = \beta + \epsilon$$

Actuary 2 uses the following regression model:

$$Y = \beta_0 + \beta_1 \times Sex + \epsilon$$

The residual sum of squares for the regression of Actuary 2 is 100,000 and the total sum of squares is 120,000.

Calculate the F-statistic to test whether the model of Actuary 2 is a significant improvement over the model of Actuary 1.

- (A) 20.5
- (B) 22.6
- (C) 19.6
- (D) 30.1
- (E) 34.5

Problem 9 (Similar Problem Example 1.2.5 or SRM - Sample Question 44)

Two actuaries are analyzing dental claims for a group of $n = 100$ participants. The predictor variable is sex, with 0 and 1 as possible values.

Actuary 1 uses the following regression model:

$$Y = \beta + \epsilon$$

Actuary 2 uses the following regression model:

$$Y = \beta_0 + \beta_1 \times Sex + \epsilon$$

Given $R^2 = .7$. Calculate the F-statistic to test whether the model of Actuary 2 is a significant improvement over the model of Actuary 1.

- (A) 120
- (B) 135
- (C) 147
- (D) 157
- (E) 240

Problem 10 (t-test)

The results of fitting ten observation by the regression model, $y = \beta_0 + \beta_1 x + \epsilon$, are given below.

Determine the test results of the hypothesis $H_0 : \beta_1 = 0$ against $H_a : \beta_1 \neq 0$.

	Estimate	Std. Error	t value	Pr(> t)
Intercept	-4.4916	6.6540	-0.675	0.51869
x	3.4122	0.7638	4.468	0.00209

- A. Reject at $\alpha = .2$
- B. Reject at $\alpha = .1$
- C. Reject at $\alpha = .05$
- D. Reject at $\alpha = .01$
- E. All (A), (B), (C), or (D) are correct.

Application of Linear Model.

Problem 11 (SRM - Sample Question 23)

Toby observes the following coffee prices in his company cafeteria:

- 12 ounces for 1.00
- 16 ounces for 1.20

- 20 ounces for 1.40

The cafeteria announces that they will begin to sell any amount of coffee for a price that is the value predicted by a simple linear regression using least squares of the current prices on size.

Toby and his co-worker Karen want to determine how much they would save each day, using the new pricing, if, instead of each buying a 24-ounce coffee, they bought a 48- ounce coffee and shared it.

Calculate the amount they would save.

- (A) It would cost them 0.40 more.
- (B) It would cost the same.
- (C) They would save 0.40.
- (D) They would save 0.80.
- (E) They would save 1.20.

Problem 12

Peter observes the following coffee prices in his company cafeteria:

- 1 bagel for 1.00 (USD)
- 2 bagel for 1.50 (USD)

The cafeteria announces that they will begin to sell any amount of bagels for a price that is the value predicted by a simple linear regression using least squares of the current prices.

With the new pricing model, how much Peter would save if he bought 10 bagels instead of 5 bagels twice?

- (A) It would cost him more so he would not save any money.
- (B) It would cost the same.
- (C) He would save 0.5 (USD)
- (D) He would save 1 (USD)
- (E) He would save 1.5 (USD)