Week 4 - AYU - Individual

Random Walks

Problem 1 (Sample - Question 4) You are given:

i) The random walk model

$$y_t = y_0 + c_1 + c_2 + c_3 + ... + c_t,$$

where $c_i, (i = 1, 2, ..., t)$ denote observations from a white noise process.

ii) The following ten observed values of c_t:

t	1	2	3	4	5	6	7	8	9	10
y_t	2	5	10	13	18	20	24	25	27	30

iii) $y_0 = 0$

Calculate the standard error of the 9 step-ahead forecast, $\hat{y}_{19}$.

(A) 4/3
(B) 4
(C) 9
(D) 12
(E) 16

Problem 2 (Sample - Question 55) You are given the following eight observations from a time series that follows a random walk model:

t	0	1	2	3	4	5	6	7
y_t	3	5	7	8	12	15	21	22

You plan to fit this model to the first five observations and then evaluate it against the last three observations using one-step forecast residuals. The estimated mean of the white noise process is 2.25.

Let F be the mean error (ME) of the three predicted observations.

Let G be the mean square error (MSE) of the three predicted observations.

Calculate the absolute difference between F and G, $F - G$

(A) 3.48
(B) 4.31
(C) 5.54
(D) 6.47
(E) 7.63

Problem 3 (Sample - Question 21) A random walk is expressed as

$$y_y = y_{t-1} + c_t$$

where

$E(c_t) = \mu_t$ and $Var(c_t) = \sigma_c^2$

Determine which statements is/are true with respect to a random walk model.

I. If $\mu_c \neq 0$, then the random walk is nonstationary in the mean.
II. If $\sigma_c^2 = 0$, then the random walk is nonstationary in the variance.
III. If $\sigma_c^2 > 0$, then the random walk is nonstationary in the variance.

(A) None
(B) I and II only
(C) I and III only
(D) II and III only
(E) The correct answer is not given by (A), (B), (C), or (D).

Problem 4 (Sample - Question 31) Determine which of the following indicates that a nonstationary time series can be represented as a random walk

I. A control chart of the series detects a linear trend in time and increasing variability.

II. The differenced series follows a white noise model.

III. The standard deviation of the original series is greater than the standard deviation of the differenced series.

(A) I only
(B) II only
(C) III only
(D) I, II and III
(E) The correct answer is not given by (A), (B), (C), or (D).

Autoregressive

Problem 5 (Sample - Question 22) A stationary autoregressive model of order one can be written as

$$y_t = \beta_0 + \beta_1 y_{t-1} + \epsilon, t = 1, 2, ...$$

Determine which of the following statements about this model is false

(A) The parameter β_0 must not equal 1.
(B) The absolute value of the parameter β_1 must be less than 1.
(C) If the parameter $\beta_1 = 0$, then the model reduces to a white noise process.
(D) If the parameter $\beta_1 = 1$, then the model is a random walk.
(E) Only the immediate past value, y_{t-1}, is used as a predictor for y_t .

Problem 6 (Sample - Question 58) You are given the following six observed values of the autoregressive model of order one time series

$$y_t = \beta_0 + \beta_1 y_{t-1} + \epsilon_t, \text{ with } Var(\epsilon_t) = \sigma^2.$$

t	1	2	3	4	5	6
y_t	31	35	37	41	45	51

The approximation to the conditional least squares method is used to estimate β_0 and β_1

Calculate the mean squared error s^2 that estimates σ^2

(A) 13
(B) 21
(C) 22
(D) 26
(E) 35

Problem 7 (Sample - Question 64) You are given a stationary AR(1) model,

$$y_t = \beta_0 + \beta_1 y_{t-1} + \epsilon_t, t = 1, 2, ..., T.$$

Determine which or the following is always true.

(A) $\beta_0 \neq 0$
(B) $\beta_0 = 1$
(C) $\beta_1 = 0$
(D) $\beta_1 = 1$
(E) $|\beta_1| < 1$

Problem 8 You are given

$$y_t = .6y_{t-1} - 5 + \epsilon$$
$$y_T = 7$$

Calculate the three step ahead forecast of y_{T+5}

Smoothing

Problem 9 (Sample - Question 46) A time series was observed at times 0, 1, ..., 100. The last four observations along with estimates based on exponential and double exponential smoothing with $w = 0.8$ are:

Time (t)	97	98	99	100
Observation (y_t)	96.9	98.1	99.0	100.2
Estimates $(\hat{s}(1)_t)$	93.1	94.1	95.1	
Estimates $(\hat{s}(2)_t)$	88.9	89.9		

All forecasts should be rounded to one decimal place and the trend should be rounded to three decimal places.

Let F be the predicted value of y_{102} using exponential smoothing with $w = 0.8$.

Let G be the predicted value of y_{102} using double exponential smoothing with $w = 0.8$.

Calculate the absolute difference between F and G, $F - G$

(A) 0.0
(B) 2.1
(C) 4.2
(D) 6.3
(E) 8.4