Applications of Liner Functions

Example

Total Cost The *total cost* C for a manufacturer during a given time period is a function of the number N of items produced during that period. To determine a formula for the total cost, we need to know two things. The first is the manufacturer's *fixed costs*. This amount covers expenses such as plant maintenance and insurance, and it is the same no matter how many items are produced. The second thing we need to know is the cost for each unit produced, which is called the *variable cost*.

Suppose that a manufacturer of widgets has fixed costs of \$1500 per month and that the variable cost is \$20 per widget (so it costs \$20 to produce 1 widget).

x : number of items produced.

F : Fixed cost

v : variable cost (cost to produce 1 item)

The total cost : $C(x)$

$$C(x) = F + v \cdot x$$

$$C(x) = 1500 + 20 \cdot x$$

1500: Fixed cost (F)

20: variable cost (v)

$C(x)$ is a liner function of x

(*) Find the total cost to produce 10 (items)

$$C(10) = 1500 + 20 \cdot 10 = 1700$$

(*) with a buget of 3000, how many items can be produced ?

we need to find x so that

$$C(x) = 3000$$

$$\Rightarrow \quad 1500 + 20x = 3000$$

$$\Rightarrow \quad 20x = 3000 - 1500$$

$\Rightarrow \quad 20x = 1500$

$x = \frac{1500}{20} = 75$

(*) Revenue: Suppose we sell all x items

the price per each item is s (dollars)

Revenue function: $R(x) = x \cdot s$ (dollars)

In this example the price for 1 item is 40 ($)

$R(x) = 40x$

(Notice that the $R(x)$ is also linear)

(*) The break-event point.

This is when the revenue is equal the cost.

$R(x) = C(x)$

Let's find the break-event point in this example.

$40x = 1500 + 20x$

$\Rightarrow \quad 40x - 20x = 1500$

$\Rightarrow \quad 20x = 1500$

$x = \frac{1500}{20} = 75$

(*) Profit: revenue ↓ cost ↙

$$P(x) = R(x) - C(x)$$

In this example, how many items needed to be sold to make a profit of 10,000.

$$P(x) = 10,000$$

$$\Rightarrow R(x) - C(x) = 10,000$$

$$\Rightarrow 40x - (1500 + 20x) = 10,000$$

$$\Rightarrow \underline{40x} - 1500 - \underline{20x} = 10,000$$

$$\Rightarrow 20x - 1500 = 10,000$$

$$\Rightarrow 20x = 11,500$$

$$\Rightarrow x = \frac{11,500}{20} = \boxed{575}$$

Example :

Consider a manufacter with the fixed cost of 1000 and the variable cost of 10 ($).

① Write the equation of the total cost

② with a budget of 10000, how many items can be produced ?

③ If the company sells an item for $30, what is the break – event point ?

④ Find the number of items needed to be sold to have a profit of 20,000.

① $C(x) = 1000 + 10x$

② $10,000 = 1000 + 10x$

$\Rightarrow 9000 = 10x$

$\Rightarrow \boxed{x = 900}$

③ $R(x) = 30 \cdot x$

$R(x) = C(x)$

$30x = 1000 + 10x$

$\Rightarrow 30x - 10x = 1000$

$\Rightarrow 20x = 1000$

$x = \frac{1000}{20} = \boxed{50}$

④ $\text{Profit} = 20,000$

$\Rightarrow P(x) = 20,000$

$\Rightarrow R(x) - C(x) = 20,000$

$\Rightarrow 30x - (1000 + 10x) = 20,000$

$\Rightarrow 30x - 1000 - 10x = 20,000$

$\Rightarrow 20x = 20000 + 1000$

$\Rightarrow 20x = 21000$

$\Rightarrow \boxed{x = 10,500}$

Assignment 2 :

Consider a manufacter with the fixed cost of 3000 and the variable cost of 5 ($).

(1) Write the equation of the total cost

(2) with a budget of 10,000$, how many items can be produced ?

(3) If the company sells an item for $25 what is the break – event point ?

(4) Find the number of items needed to be sold to have a profit of 100, 000.