Exponential / Logarithmic Functions

U.S. Investment Abroad In 1980, direct U.S. business investment abroad was about 13.5 billion dollars. From 1980 through 2010, that investment[12] grew at an average annual rate of 11.24%.

a. Make an exponential model that shows the U.S. direct investment abroad A, in billions of dollars, t years after 1980.

b. From 1980, how long did it take for U.S. investments abroad to double?

(a) $A(t) = P * (1 + r)^t$

growth rate

$A(t)$: the amount of money at the time t (since 1980)

P: initial amount of money (Principal)

$$A(t) = 13.5 * (1 + 11.24\%)^t$$

(b) We need to find t so that

$$A(t) = 2 * 13.5$$

$$13.5 * (1.1124)^t = 2 * 13.5$$

$$1.1124^t = 2$$

$$t = \log_{1.1124} 2 = \frac{\ln 2}{\ln 1.124}$$

$t \simeq 6.507$ (years)

(c) In what year, the investment will be 100 (billions).

we need to find t so that

$$A(t) = 100$$

$$13.5 * (1.1124)^t = 100$$

$$1.1124^t = \frac{100}{13.5} \simeq 7.407$$

$$\Rightarrow \quad t = \log_{1.1124} 7.407 = \frac{\ln 7.407}{\ln 1.1124}$$

$t \simeq 18.8$ (years)

Example

If tuition at a college is increasing by 6.6% each year, how many years will it take for tuition to double?

r

$$A(t) = P(1+r)^t$$

$A(t)$: tuition at the time t

P: Initial tuition ($t=0$)

r: growth rate

we need to find t so that

$$A(t) = 2P$$

$$P(1+.066)^t = 2P$$

$$\Rightarrow \quad 1.066^t = 2$$

$$\Rightarrow \quad t = \log_{1.066} 2 = \frac{\ln 2}{\ln 1.066} \approx 10.845$$

Assignment 9

Suppose you invest $\$1000$ to an SP500 index ETF.

(a) How long does it take for your investment to be $\$100,000$.

(b) How long does it tall to double the investment.

(c) How long does it take to triple the investment.

Use Google to find the average annual growth of SP 500.