Exam 1 - Practice Problems

Table of contents

The exam will consist of two sections: Part 1: Manual calculation and Part 2: Excel calculation.

Part 1: Manual calculation

Problem 1.

Given a simulated dataset below. The tuition is in thousands.

Year (x)	Tuition (y)
2020	20
2021	21.5
2022	23
2023	24.5
2024	26

1. Calculate the differences of tuition in consecutive years and ratio of tuition for consecutive years to determine if the data is exponential or linear.

2. Write the equation of the model.

3. Use the model to predict the tuition in 2030.

4. What year the tuition will be more than 100k?

Solution

1. We have the following table. For a tuition difference, subtract the tuition of a year to the tuition of the previous year (for example: 21.5-20 = 1.5). For a tuition ratio, divide the tuition of a year to the tuition of the previous year (for example: 21.5/20 = 1.075)

Year (since 2020)	Tuition	Tuition Difference	Tuition Ratio
0	20		
1	21.5	1.5	1.075
2	23	1.5	1.069767
3	24.5	1.5	1.065217
4	26	1.5	1.061224

We observe that the tuition difference is a constant, hence the data is linear.

2. Let the equation be

$$y = mx + b.$$

Where y and x are the tuition and year, respectively.

We need to find the constant m and b. The slope m is always the different constant, so $m = 1.5$.

To find b, plug in a data point, say, $x = 0$ and $y = 20$, we have:

$$20 = b$$

Thus,

$$b = 20$$

.

Therefore, the equation is:

$$y = 1.5 * x + 20$$

3. To predict the tuition in 2030, plug $x = 2030 - 2020 = 10$ (since 2030 is 10 years since 2020) into the equation in 2. to find y.

$$y = 1.5 * 10 + 20 = 35$$

The model predicts the tuition in 2030 to be 35k.

4. To find the year the tuition passes 100k, plug $y = 100$ into the equation to find x

We have
$$100 = 1.5 * x + 20$$

Thus,
$$x = \frac{100 - 20}{1.5} = 53.33$$

.

Therefore the tuition will pass 100k in the year of $2020 + 53.33 = 2073.33$ or 2074. (round it up to the next year).

Problem 2.

Given a simulated dataset below. The tuition is in thousands.

Year (since 2010)	Tuition
0	20
1	22
2	24.2
3	26.62
4	29.282

1. Calculate the differences of tuition in consecutive years and ratio of tuition for consecutive years to determine if the data is exponential or linear.

2. Write the equation of the model.

3. Use the model to predict the tuition in 2030.

4. What year the tuition will be more than 100k?

Solution

1. We have the following table. For a tuition difference, subtract the tuition of a year to the tuition of the previous year (for example: 22-20 = 2). For a tuition ratio, divide the tuition of a year to the tuition of the previous year (for example: $22/20 = 1.1$)

Year	Tuition	Tuition Difference	Tuition Ratio
0	20		
1	22	2	1.1
2	24.2	2.2	1.1

3

Year	Tuition	Tuition Difference	Tuition Ratio
3	26.62	2.42	1.1
4	29.282	2.662	1.1

We observe that the tuition ratio is a constant, hence the data is exponential.

2. Let the equation be

$$y = a.b^x$$

Where y and x are the tuition and year, respectively.

We need to find the constant a and b. The base m is always the different ratio, so $b = 1.1$.

To find a, plug in a data point, say, $x = 0$ and $y = 20$, we have:

$$20 = a * 1.1^0$$

Thus,
$$a = 20$$

.

Therefore, the equation is:

$$y = 20 * 1.1^x$$

3. To predict the tuition in 2030, plug $x = 2030 - 2010 = 20$ into the equation in 2. to find y.

$$y = 20 * 1.1^{20} = 134.55$$

The model predicts the tuition in 2030 to be 134.55

4. To find the year the tuition passes 100k, plug $y = 100$ into the equation to find x

We have
$$100 = 20 * 1.1^x$$

Thus, $1.1^x = 100/20 = 5$.

$$x = \log_{1.1} 5 = 16.88632$$

Therefore the tuition will pass 100k in 17 years since 2010, which is 2027.

Part 2: Excel Calculation

Year	Population (000s)
2012	2.3
2013	2.51
2014	2.73
2015	2.98
2016	3.25

1. Model the dataset using exponential model and calculate the MAPE of the model.
2. Model the dataset using linear model and calculate the MAPE of the model.
3. Compare the models in term of MAPE to decide the better model. Use the better model to predict the population in 2017.

Solution