Quotient Rule

$$\left[\frac{TOP}{bot} \right]' = \frac{(top)' \cdot Bot - (Bot)' \cdot Top}{(Bot)^2}$$

OR

$$\left[\frac{f(x)}{g(x)} \right]' = \frac{f'(x) \cdot g(x) - g'(x) \cdot f(x)}{[g(x)]^2}$$

Example : Find $h'(x)$

① $h(x) = \frac{x^2 + 3x}{x^2 - 4}$

$$h'(x) = \frac{(x^2 + 3x)' \cdot (x^2 - 4) - (x^2 - 4)' \cdot (x^2 + 3x)}{(x^2 - 4)^2}$$

$$= \frac{(2x + 3)(x^2 - 4) - (2x)(x^2 + 3x)}{(x^2 - 4)^2}$$

② $h(x) = \frac{x + 2}{3x + 7}$

$$\Rightarrow h'(x) = \frac{(x+2)' \cdot (3x + 7) - (3x + 7)' \cdot (x+2)}{(3x + 7)^2}$$

$$h'(x) = \frac{1 \cdot (3x+7) - 3(x+2)}{(3x+7)^2}$$

③ (Simplify the answer.)

$$h(x) = \frac{4x+1}{4x-1}$$

$\rightarrow$
$$h'(x) = \frac{(4x+1)'(4x-1) - (4x-1)' \cdot (4x+1)}{(4x-1)^2}$$

$$= \frac{4(4x-1) - 4 \cdot (4x+1)}{(4x-1)^2}$$

$$= \frac{16x - 4 - 16x - 4}{(4x-1)^2}$$

$$= \boxed{\frac{-8}{(4x-1)^2}}$$

④ $\quad h(x) = \dfrac{\sqrt{x} + x^9}{\sqrt{x} + 2}$

$\Rightarrow h'(x) = \dfrac{\left(\sqrt{x} + x^9\right)' \cdot \left(\sqrt{x} + 2\right) - \left(\sqrt{x} + 2\right)' \cdot \left(\sqrt{x} + x^9\right)}{\left(\sqrt{x} + 2\right)^2}$

$\left[\text{Note:} \quad \left(\sqrt{x}\right)' = \left(x^{1/2}\right)' = \dfrac{1}{2} x^{1/2 - 1} = \dfrac{1}{2} x^{-1/2}\right]$

$\Rightarrow h'(x) = \dfrac{\left(\frac{1}{2} x^{-1/2} + 9 x^8\right)\left(\sqrt{x} + 2\right) - \frac{1}{2} x^{-1/2} \cdot \left(\sqrt{x} + x^9\right)}{\left(\sqrt{x} + 2\right)^2}$

⑤ $\quad h(x) = \dfrac{\left(\frac{3}{x^2} + 4x\right)}{\frac{4}{x} + 6}$

$\Rightarrow h'(x) = \dfrac{\left(\frac{3}{x^2} + 4x\right)' \cdot \left(\frac{4}{x} + 6\right) - \left(\frac{4}{x} + 6\right)' \cdot \left(\frac{3}{x^2} + 4x\right)}{\left(\frac{4}{x} + 6\right)^2}$

$\left[\text{Note:} \quad \left(\dfrac{3}{x^2}\right)' = \left(3x^{-2}\right)' = 3 \cdot (-2) \cdot x^{-2-1} = -6x^{-3}\right.$

$\left(\dfrac{4}{x}\right)' = \left(4x^{-1}\right)' = 4 \cdot (-1) \, x^{-1-1} = -4x^{-2} \Big]$

$$\Rightarrow h'(x) = \frac{\left(-6x^{-3} + 4\right)\left(\frac{4}{x} + 6\right) - \left(-4x^{-2}\right)\left(\frac{3}{x^2} + 4x\right)}{\left(\frac{4}{x} + 6\right)^2}$$

(5)

$$h(x) = \frac{3}{x+1}$$

$$\Rightarrow h'(x) = \frac{(3)'\cdot(x+1) - (x+1)'\cdot 3}{(x+1)^2}$$

$$= \frac{0\cdot(x+1) - 1\cdot 3}{(x+1)^2} = \frac{-3}{(x+1)^2}$$

Assignment : Find $h'(x)$

① $h(x) = \dfrac{x^3 + 3x}{x^2 + 4}$

② $h(x) = \dfrac{x^2 + 1}{x^2 - 1}$ (<mark>simplify</mark>)

③ $h(x) = \dfrac{2x^4 + x}{x + 4}$

④ $h(x) = \dfrac{x^2 + 3x + 1}{x^2 + 4x + 1}$

⑤ $h(x) = \dfrac{x^{2025} + 4x^3}{x^{2002} - 6}$

⑥ $h(x) = \dfrac{3x + 1}{4x + 1}$

⑦ $h(x) = \dfrac{\sqrt{x} + 2}{\sqrt{x} + 3}$

$$8) \quad h(x) = \frac{\sqrt[4]{x} + 1}{x + 2}$$

$$9) \quad h(x) = \frac{\frac{1}{x} + 1}{x}$$

$$10) \quad h(x) = \frac{\frac{1}{x^2} + 1}{3x}$$

$$11) \quad h(x) = \frac{2025}{x^3 + 1}$$

$$12) \quad h(x) = \frac{x^2 + x + 1}{x^2 - x - 1} \qquad (\text{simplify})$$