Logarithmic Functions

(*) $x + 3 = 6$ $\qquad$ $x + 123 = 764$

$\Rightarrow x = 6 - 3 = 3$

In order to solve an equation having "addition" we need "subtraction".

(*) $x \cdot 5 = 2$

$\Rightarrow x = \frac{2}{5}$

To solve an equation having multiplication, we need "division"

(*) ① $3^x = 9$

$\Rightarrow x = 2$

$\sqrt{2}$

② $3^x = 3$

$\Rightarrow x = 1$

③ $3^x = 4$

exact solution

$x = \log_3 4 \approx 1.261859\ldots\ldots$

approximate solution

(4) Solve: $2^x = 7$

$\Rightarrow \quad x = \log_2 7$

(5) Solve $6^x = 3$

$\Rightarrow \quad x = \log_6 3$

(6) Solve: $3 * (2^x) = 1$

$\Rightarrow \quad 2^x = \frac{1}{3}$

$x = \log_2 (1/3)$

<u>Definition</u>: we write

$y = \log_b x$

reads as: log base b of x

when: $b^y = x$

(b is also the base of the exponential)

Example: Solve for x:

(1) $2025^x = 2024$

$\Rightarrow \quad x = \log_{2025} 2024$

(2) $7^x = 10$

$\Rightarrow \quad x = \log_7 10$

(3) $7^{3x+1} = 10$

$\Rightarrow \quad 3x+1 = \log_7 10$

$\Rightarrow \quad 3x = \log_7 10 - 1$

$\Rightarrow \quad x = \dfrac{\log_7 10 - 1}{3}$

(4) $2 * (3^{1-x}) = 5$

Divide both sides by 2

$\Rightarrow \quad 3^{1-x} = 5/2$

$\Rightarrow \quad 1-x = \log_3 (5/2)$

$$\Rightarrow \quad 1 = \log_3 (5/2) + x$$

$$\Rightarrow \quad 1 - \log_3 (5/2) = x$$

$$\Rightarrow \quad x = 1 - \log_3 (5/2)$$

(*) Some property of log functions:

(1) $\log_b 1 = 0$

(2) $\log_b b = 1$

(3) $\log_b (a^x) = x \cdot \log_b a$

(4) $\log_b (x \cdot y) = \log_b x + \log_b y$

(5) $\log_b a = \dfrac{\log_c a}{\log_c b}$

Notice :

when the base is e $(2.71828 \ldots\ldots)$

we write $\ln x$ instead of $\log_e x$

$$\ln = \log_e$$

$$\log x = \log_{10} x$$

This means when the base is 10 we don't write the base.

(6) $$\log_b a = \frac{\ln a}{\ln b}$$

$\boxed{\text{Assignment: } 8}$

Solve:

(1) $3^x = 10$

(2) $4^x = 11$

(3) $6^x = 12$

(4) $6^{2x+1} = 12$

(5) $3 * 6^x = 12$