Exam 1 – Practice 1

Exam Guidelines This is an in-class, written exam with a 75-minute time limit.

- **Permitted Materials:** You may use a basic calculator and a formula sheet.
- **Formula Sheet Restrictions:** Your sheet must contain **formulas only**; no examples or worked problems are permitted. All sheets will be inspected at the start of the exam.
- **Prohibited Items:** Phones and all other smart devices are strictly forbidden.
- **Academic Integrity:** The use of AI is prohibited. Any AI usage will result in an automatic **F** for the exam and may lead to failing the entire course.

1. Graph the below line.

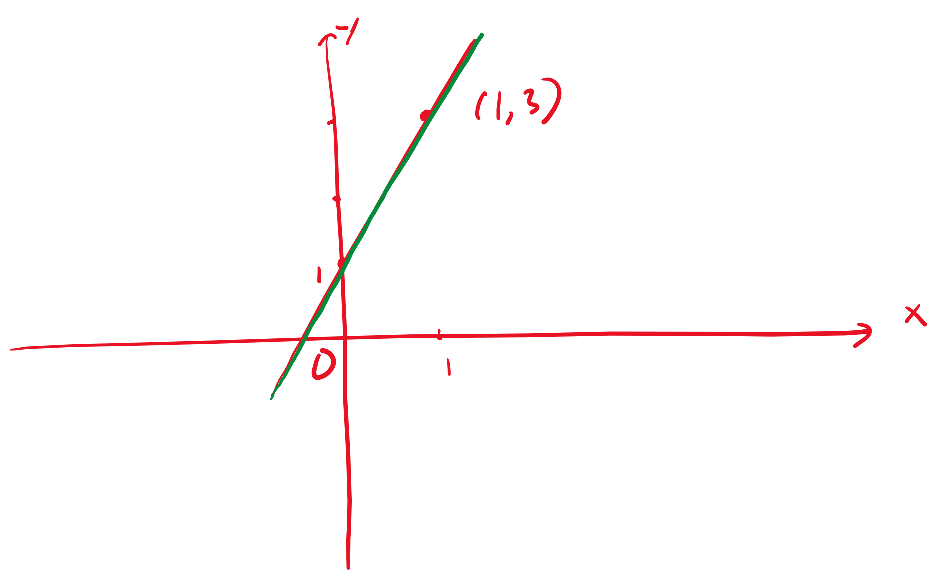
   a. $y = 2x + 1$

To graph a linear function, we just need to plot two points and connect them to make a line.

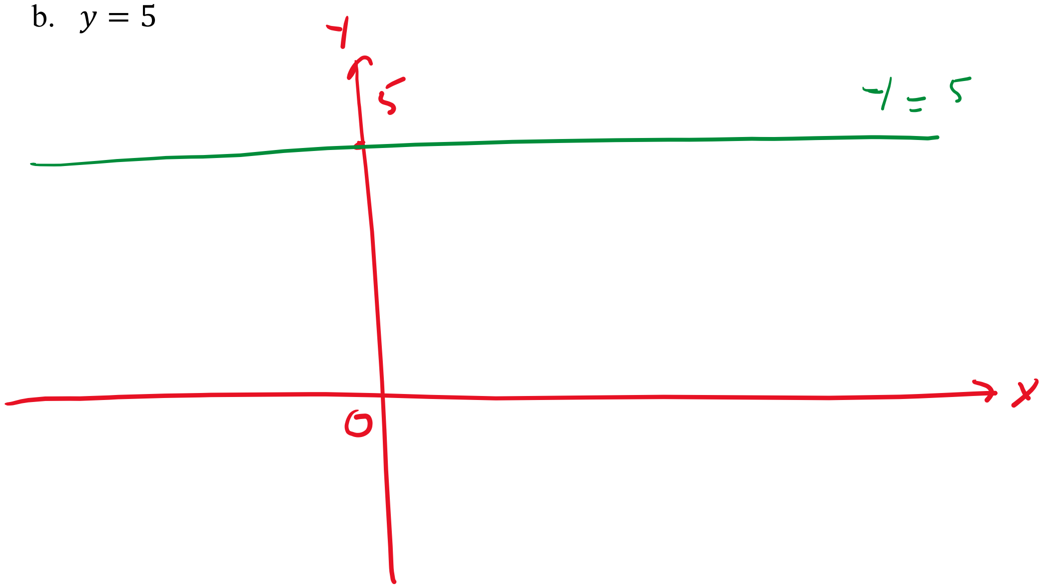
Choose any value for x, says,

Choose x = 0, y = 2*0+1 =1. We have a point (0,1)

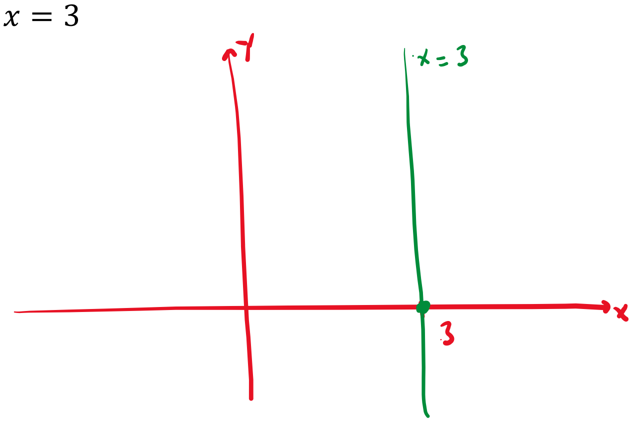
Choose x = 1, y = 2*1+1 = 3. We have a point (1, 3).



b. $y = 5$



c. $x = 3$



2. Write the equation of the line

a. passing through two points (1, 2) and (4, 3)

Formula: The line passing (x_1, y_1) and (x_2, y_2) is

$$y = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1) + y_1$$

$$\underset{x_1}{(1,} \underset{y_1}{2)} \quad \underset{x_2}{(4,} \underset{y_2}{3)}$$

$$\Rightarrow \quad y = \frac{3-2}{4-1} \cdot (x-1) + 2$$

$$\Rightarrow \quad y = \frac{1}{3}(x-1) + 2$$

$$\Rightarrow \quad y = \frac{1}{3}x - \frac{1}{3} + 2$$

$$\Rightarrow \quad \boxed{y = \frac{1}{3}x + \frac{5}{3}}$$

b. with the slope of $\overset{m}{3}$ and passing through $(\overset{x_1}{1}, \overset{y_1}{2})$

Formula: The line with slope m and passing (x_1, y_1) is

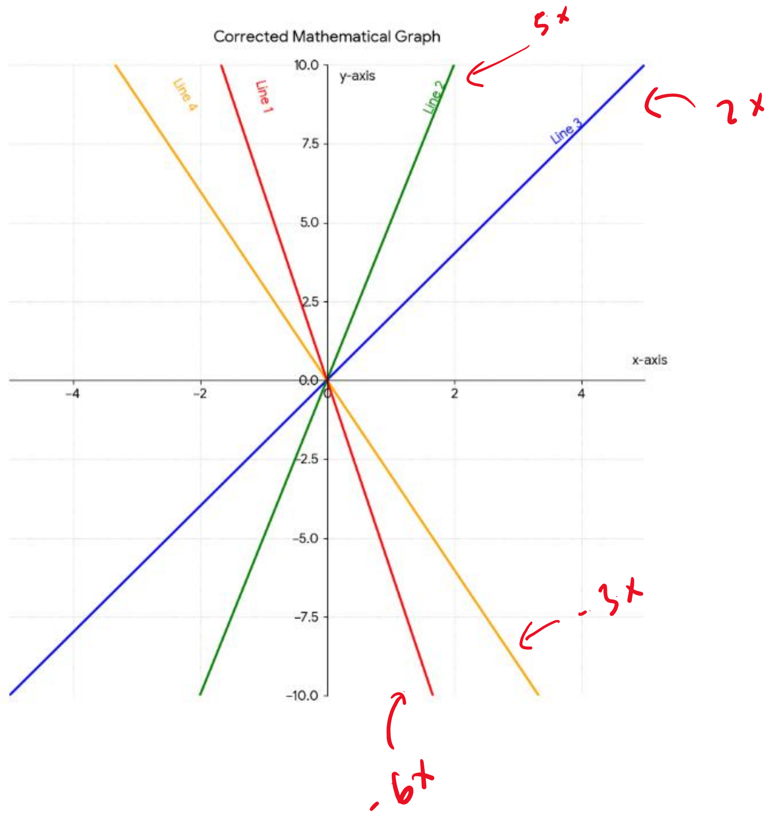
$$y = m(x - x_1) + y_1$$

$$\Rightarrow \quad y = 3(x-1) + 2$$

$$\Rightarrow \quad y = 3x - 3 + 2$$

$$\boxed{y = 3x - 1}$$

3. Match each equation to its corresponding graph.



a. $y = 2x$
b. $y = 5x$
c. $y = -3x$
d. $y = -6x$

Notice:

- A line with positive slope goes up. The greater the slope, the faster it goes up.
- A line with negative slope goes down. The smaller the slope, the faster it goes down.

4. You manage a local craft shop that sells handmade artisan coffee mugs.

Demand

Market research indicates that you can sell 150 mugs per month if they are priced at \$10 each, but you will only sell 50 mugs per month if the price is increased to \$20 each.

Supply

On the other side, your supplier is willing to provide 30 mugs per month if the retail price is \$8 each but will increase production to 130 mugs per month if the retail price reaches \$18 each.

a. Write the linear demand and supply functions.

Demand :

	P	Qd	
(P_1)	10	150	(Q_1)
(P_2)	20	50	(Q_2)

Equation:

$$Qd = \frac{Q_2 - Q_1}{P_2 - P_1} \cdot (P - P_1) + Q_1$$

$$\Rightarrow Qd = \frac{50 - 150}{20 - 10} \cdot (P - 10) + 150$$

$$\Rightarrow Qd = \frac{-100}{10} \cdot (P - 10) + 150$$

$$\Rightarrow Qd = -10 (P - 10) + 150$$

$$\Rightarrow Qd = -10P + 100 + 150$$

$$\Rightarrow \boxed{Qd = -10P + 250}$$

Supply:

	P	Qs	
(P_1)	8	30	(Q_1)
(P_2)	18	130	(Q_2)

Equation: $Qs = \frac{Q_2 - Q_1}{P_2 - P_1} \cdot (P - P_1) + Q_1$

$\Rightarrow Qs = \frac{130 - 30}{18 - 8} \cdot (P - 8) + 30$

$\Rightarrow Qs = \frac{100}{10} (P - 8) + 30$

$\Rightarrow Qs = 10(P - 8) + 30$

$\Rightarrow Qs = 10P - 80 + 30$

$\Rightarrow \boxed{Qs = 10P - 50}$

b. Find the equilibrium point. At what price must the mugs be sold for supply to exactly equal demand?

Equilibrium point: $Qd = Qs$

$\Rightarrow -10P + 250 = 10P - 50$

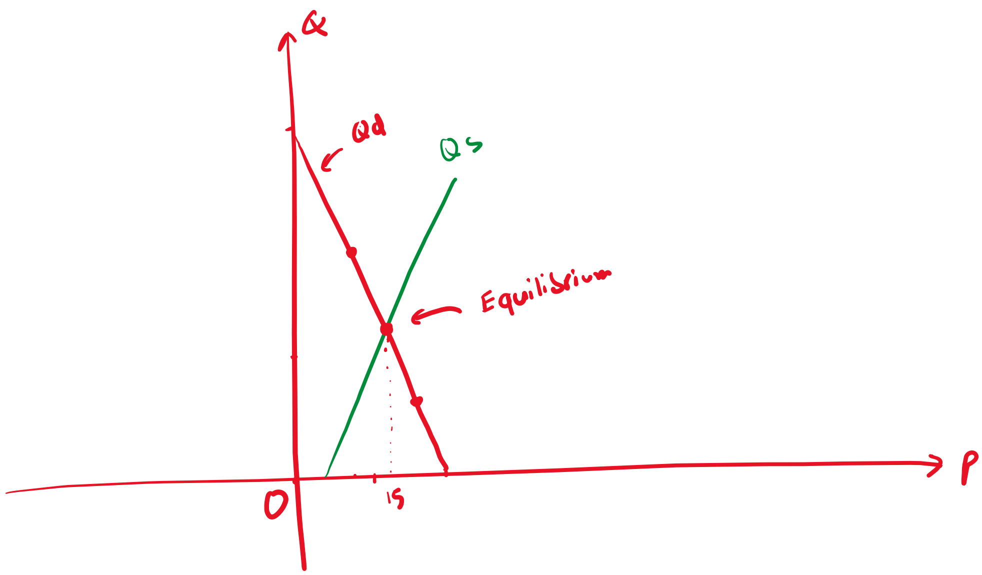
$\Rightarrow 250 + 50 = 10P + 10P$

$$\Rightarrow \quad 300 = 20p$$

$$\Rightarrow \quad p = \frac{300}{20} = 15$$

At $p = 15$, the demand equals supply.

c. Graph both the demand and supply functions on the same axis.



Q_d:
$(10, 150)$; $(20, 50)$

Q_s
$(8, 30)$; $(18, 130)$

5. A company that prints custom T-shirts has fixed monthly costs of \$60,000 and variable costs of \$30 per T-shirt produced. Each T-shirt sells for \$90.

a. Find the cost function.

$$C = mq + b$$

m : variable cost per item

b : fixed cost

$$\Rightarrow \quad C = 30q + 60\,000$$

b. Find the revenue function.

$$R = p \cdot q$$

$$\Rightarrow \quad R = 90 \cdot q$$

c. Graph and clearly label the cost and revenue functions on the same set of axes. Identify and label the break-even point.

Break even: $R = C$

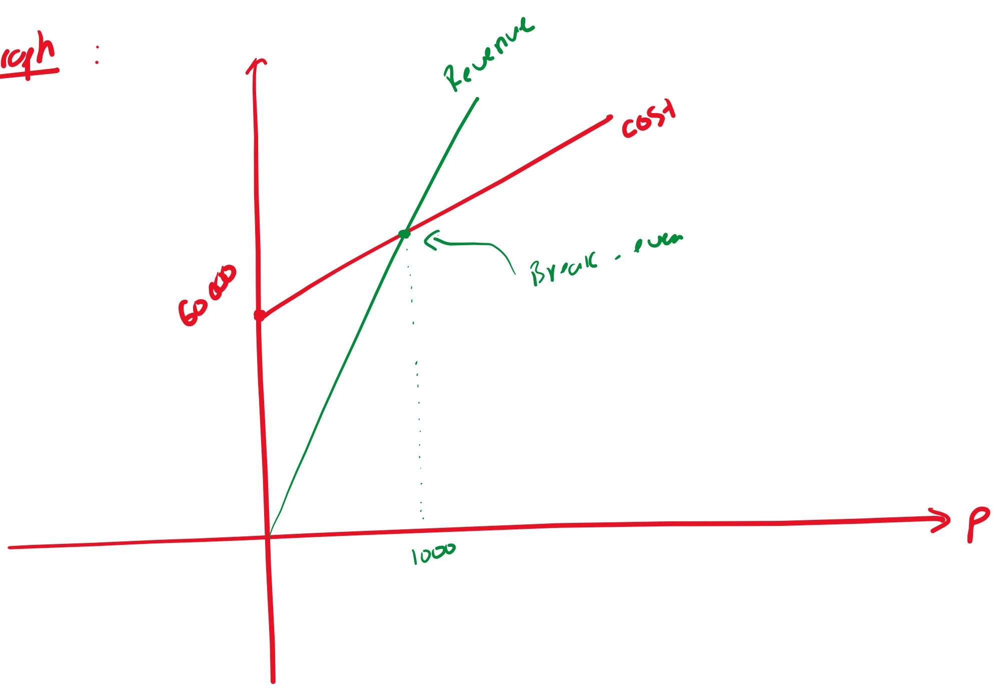
$$\Rightarrow \quad 90 \cdot q = 30q + 60\,000$$

$$\Rightarrow \quad 90q - 30q = 60\,000$$

$$\Rightarrow \quad 60 \cdot q = 60\,000$$

$$\Rightarrow \quad q = \frac{60\,000}{60} = 1000$$

Graph:



d. Find the profit function.

$$\text{Profit} = R - C$$

$$= 90q - (30q + 60\,000)$$

$$= 90q - 30q - 60\,000$$

$$= 60q - 60\,000$$

e. How much profit will the company make by producing and selling 1,500 T-shirts?

$$q = 1500$$

$$\Rightarrow \quad \text{Profit} = 60 * 1500 - 60000$$

$$= 30\,000$$

f. How many T-shirts must be produced and sold in order to obtain a profit of $60,000?

$$\text{Profit} = 60\,000$$

$$\Rightarrow \quad 60q - 60000 = 60000$$

$$\Rightarrow \quad 60q = 60000 + 60000$$

$$\Rightarrow \quad 60 \cdot q = 120\,000$$

$$\Rightarrow \quad q = \frac{120\,000}{60} = \boxed{2000}$$

6. Two investment options that earn simple interest are available.

Investment A starts with \$1,200 and earns simple interest at an annual rate of 4%.
Investment B starts with \$2,000 and earns simple interest at an annual rate of 2%.

a. Write a linear equation that represents the total amount of money in each investment after t years.

Formula: $A = P + Prt$

- A = total amount after t years
- P = principal (initial amount)
- r = annual interest rate (decimal)
- t = time in years

Investment A: $P = 1200$, $r = .04$

$\Rightarrow A = 1200 + 1200 * .04\, t$

$\Rightarrow \boxed{A = 1200 + 48t}$

Investment B: $P = 2000$; $r = .02$

$\Rightarrow B = 2000 + 2000 * .02\, t$

$\Rightarrow \boxed{B = 2000 + 40t}$

b. How much money will there be in Investment A in 3 years?

$t = 3$

$\Rightarrow A = 1200 + 48 * 3$

$\Rightarrow A = 1344$

c. When will investment A reaches $1488?

$$A = 1488$$

$$\Rightarrow \quad 48t + 1200 = 1488$$

$$\Rightarrow \quad 48t = 1488 - 1200$$

$$\Rightarrow \quad 48t = 288$$

$$\Rightarrow \quad t = \frac{288}{48} = 6$$

$$\Rightarrow \quad \boxed{t = 6}$$

d. Determine which investment grows faster and explain your answer by comparing the slopes of the two equations.

$$A = 1200 + 48t \qquad (\text{slope} = 48)$$

$$B = 2000 + 40t \qquad (\text{slope} = 40)$$

A grows faster due to greater slope.

e. Determine whether the two investments will ever have the same total value. If so, find when this occurs.

$$A = B$$

$$\Rightarrow \quad 1200 + 48t = 2000 + 40t$$

$$\Rightarrow \quad 48t - 40t = 2000 - 1200$$

$$\Rightarrow \quad 8t = 800$$

$$\Rightarrow \quad \boxed{t = 100}$$

f. Plot both investment functions on the same coordinate system.