Second Derivative Test and Optimization

Second Derivative

Example 1: Find the second derivative for the following functions:

a. $f(x) = x^3 - 8x^2 + 9x - 11$	b. $f(x) = x^5 - 3x^4 + 2x^2 - 5$	c. $f(x) = x^2 - 3x + 1$

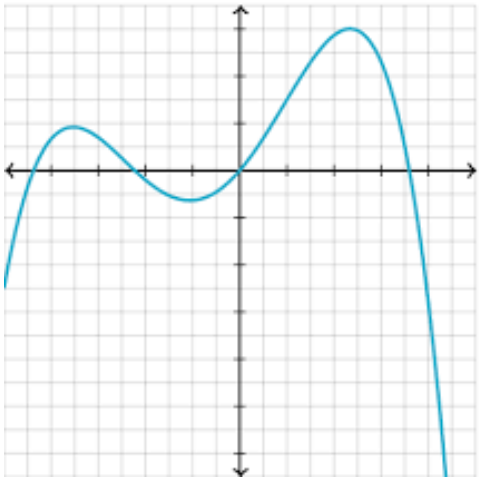
Critical Values:

Example 2: Find the critical values for the following functions:

a. $f(x) = x^3 - 3x^2 - 9x + 5$	b. $f(x) = x^2 - 6x + 1$
c. $f(x) = 4x^3 - 9x^2 - 12x + 3$	d. $f(x) = \frac{1}{3}x^3 - 9x + 24$

Second Derivative and Graphs

a. Concave Up –



b. Concave Down –

Example 3: Determine if there is a minimum or a maximum based on the given information.

a. $f''(2) = -3$	b. $f''(-1) = 5$
c. $f''(3) = 8$	d. $f''(-5) = -10$

Four Steps for Optimization

1. Find $f'(x)$
2. Find critical values (CV) by setting $f'(x) = 0$.
3. Find $f''(x)$
4. Evaluate $f''(x)$ for all critical values.

    a. $f''(\text{CV}) > 0$, meaning the graph is concave up, so there is a minimum at (x, y). Find the y value by plugging x back into ORIGINAL $f(x)$.

    b. $f''(\text{CV}) < 0$, meaning the graph is concave down, so there is a maximum at (x, y). Find the y value by plugging x back into ORIGINAL $f(x)$

For each function, use the 4-step optimization process to find the maximum/minimum values.

1. $f(x) = -x^3 + 3x^2 - 2$	2. $f(x) = x^2 - 2x - 1$
Step 1: Find the first derivative	Step 1: Find the first derivative
Step 2: Find the critical points	Step 2: Find the critical points
Step 3: Find the second derivative	Step 3: Find the second derivative
Step 4: Evaluate $f''(x)$ for all critical values. Find the coordinates of the maximums/minimums	Step 4: Evaluate $f''(x)$ for all critical values. Find the coordinates of the maximums/minimums

3. $f(x) = x^3 - 6x^2 + 9x - 7$	4. $f(x) = 2x^3 + 3x^2 + 4$
Step 1: Find the first derivative	Step 1: Find the first derivative
Step 2: Find the critical points	Step 2: Find the critical points
Step 3: Find the second derivative	Step 3: Find the second derivative
Step 4: Evaluate $f''(x)$ for all critical values. Find the coordinates of the maximums/minimums	Step 4: Evaluate $f''(x)$ for all critical values. Find the coordinates of the maximums/minimums

5. $f(x) = x^3 - 12x$	6. $f(x) = x^3 - 12x^2 + 45x + 7$
Step 1: Find the first derivative	Step 1: Find the first derivative
Step 2: Find the critical points	Step 2: Find the critical points
Step 3: Find the second derivative	Step 3: Find the second derivative
Step 4: Evaluate $f''(x)$ for all critical values. Find the coordinates of the maximums/minimums	Step 4: Evaluate $f''(x)$ for all critical values. Find the coordinates of the maximums/minimums