Quiz 3 Practice

1. Based on data from Major League Baseball, the average price of a ticket to a major league game can be approximated by $p(x) = 0.03x^2 + 0.56x + 36.82$, where x is the number of years after 2008 and $p(x)$ is in dollars.
   a. Find the average price of a ticket to a major league game in 2015

$2015 - 2008 = 7 \qquad x = 7$

$0.03(7)^2 + 0.56(7) + 36.82 = \$42.21$

   b. Find average price of a ticket to a major league game in 2025

$2025 - 2008 = 17 \qquad x = 17$

$0.03(17)^2 + 0.56(17) + 36.82 = \$55.01$

   c. What is the average rate of change in the price of tickets to a major league game from 2015 to 2025? Include the correct units.

$$\frac{55.01 - 42.21}{17 - 7} = \frac{12.8}{10} = 1.28$$

$\$1.28$ / year

2. For a recently released novel, the function $y = 25000(0.85)^t$ models the number of books sold t months after the book was released.
   a. Find the number of books sold 2 months after it was released.

$y = 25000(0.85)^2 = 18062.5$ 18063 books

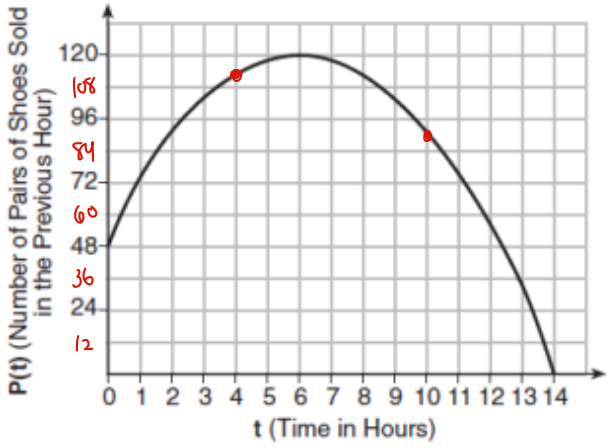
   b. Find the number of books sold 6 months after it was released.

$y = 25000(0.85)^6 = 9428.7$ 9429 books

   c. Find the average rate of change in the number of books sold between month 2 and month 6. Make sure to include correct units.

$$\frac{18063 - 9429}{2 - 6} = -2158.5 \text{ books / month}$$

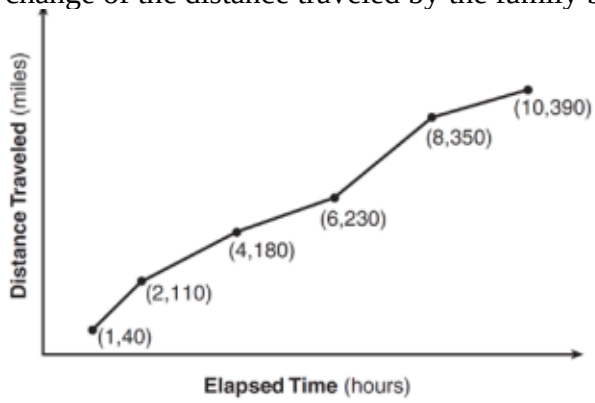
3. A manager wanted to analyze the online shoe sales for his business. He collected data for the number of pairs of shoes sold each hour over a 14-hour time period. Find the average rate of change of the number of pairs of shoes sold between hour 4 and hour 10.



(4, 112)
(10, 90)

$\frac{112-90}{4-10} = \frac{22}{-6} =$ -3.67 pairs of shoes / hr

4. The graph below shows the miles traveled by a family on a recent road trip. Find the average rate of change of the distance traveled by the family between hour 1 to hour 10.



$\frac{390-40}{10-1} = \frac{350}{9} =$ 38.89 miles / hr

Find the derivative.

5. $f(x) = 4x^3 - 6x^2 - 5x - 8$ $f'(x) = 12x^2 - 12x - 5$	6. $f(x) = 5x^8 - x^7 + 9x^5$ $f'(x) = 40x^7 - 7x^6 + 45x^4$
7. $y = \frac{x^8+6x^7+10x^5}{x^4}$ $y = \frac{x^8}{x^4} + \frac{6x^7}{x^4} + \frac{10x^5}{x^4}$ $y = x^4 + 6x^3 + 10x$ $y' = 4x^3 + 18x^2 + 10$	8. $y = (6x+1)^2$ $y = (6x+1)(6x+1)$ $y = 36x^2 + 6x + 6x + 1$ $y = 36x^2 + 12x + 1$ $y' = 72x + 12$

9. $f(x) = e^{6x^3-2x^2+3x-1}$

$f'(x) = e^{6x^3-2x^2+3x-1} \cdot (18x^2-4x+3)$

10. $f(x) = \ln(10x^5 - 4x^3 + 2)$

$f'(x) = \frac{1}{10x^5-4x^3+2} \cdot 50x^4 - 12x^2$

$f'(x) = \frac{50x^4-12x^2}{10x^5-4x^3+2}$

11. $y = (3x^4 - x^2)(x^3 - 4x)$

$f(x) = 3x^4-x^2 \quad g(x) = x^3-4x$

$f'(x) = 12x^3-2x \quad g'(x) = 3x^2-4$

$y' = (3x^4-x^2)(3x^2-4) + (x^3-4x)(12x^3-2x)$

$y' = 9x^6-12x^4-3x^4+4x^2+12x^6-2x^4-48x^4+8x^2$

$y' = 21x^6-65x^4+12x^2$

12. $y = \frac{x^3-4}{6x^2+3}$

$f(x) = x^3-4 \quad g(x) = 6x^2+3$

$f'(x) = 3x^2 \quad g'(x) = 12x$

$y' = \frac{(6x^2+3)(3x^2)-(x^3-4)(12x)}{(6x^2+3)^2}$

$y' = \frac{18x^4+9x^2-12x^4+48x}{(6x^2+3)^2}$

$y' = \frac{6x^4+9x^2+48x}{(6x^2+3)^2}$

13. $f(x) = \frac{e^{9x}}{7x^2}$

$f(x) = e^{9x} \quad g(x) = 7x^2$

$f'(x) = e^{9x} \cdot 9 \quad g'(x) = 14x$

$f'(x) = \frac{7x^2(e^{9x} \cdot 9) - e^{9x}(14x)}{(7x^2)^2}$

$f'(x) = \frac{63x^2e^{9x}-14xe^{9x}}{49x^4}$

14. $f(x) = 6x^4 \ln(2x)$

$f(x) = 6x^4 \quad g(x) = \ln(2x)$

$f'(x) = 24x^3 \quad g'(x) = \frac{1}{2x} \cdot 2 = \frac{2}{2x} = \frac{1}{x}$

$f'(x) = 6x^4\left(\frac{1}{x}\right) + \ln(2x)(24x^3)$

$f'(x) = 6x^3 + 24x^3\ln(2x)$

15. $y = 4x^3 + 8x^2 - 9x + 14$

$y' = 12x^2 + 16x - 9$

16. $y = 5x^5 + 8x^4 - 2x$

$y' = 25x^4 + 32x^3 - 2$

17. $f(x) = \frac{3x^6+7x^5-3x^3}{x^2}$

$f(x) = \frac{3x^6}{x^2} + \frac{7x^5}{x^2} - \frac{3x^3}{x^2}$

$f(x) = 3x^4 + 7x^3 - 3x$

$f'(x) = 12x^3 + 21x^2 - 3$

18. $f(x) = (8x - 3)(x + 5)$

$f(x) = 8x^2 + 40x - 3x - 15$

$f(x) = 8x^2 + 37x - 15$

$f'(x) = 16x + 37$

19. $f(x) = e^{10x^2-6x+5}$

$f'(x) = e^{10x^2-6x+5} \cdot (20x - 6)$

20. $f(x) = \ln(8x^3 - 9)$

$f'(x) = \frac{1}{8x^3-9} \cdot 24x^2$

$f'(x) = \frac{24x^2}{8x^3-9}$

21. $y = (3x^2 - 9x)(x^5 - 11)$

$f(x) = 3x^2 - 9x$ $\quad g(x) = x^5 - 11$

$f'(x) = 6x - 9$ $\quad g'(x) = 5x^4$

$y' = (3x^2 - 9x)(5x^4) + (x^5 - 11)(6x - 9)$

$y' = 15x^6 - 45x^5 + 6x^6 - 9x^5 - 66x + 99$

$y' = 21x^6 - 54x^5 - 66x + 99$

22. $y = \frac{2x^4+x}{3x^2-9}$

$f(x) = 2x^4 + x$ $\quad g(x) = 3x^2 - 9$

$f'(x) = 8x^3 + 1$ $\quad g'(x) = 6x$

$y' = \frac{(3x^2 - 9)(8x^3 + 1) - (2x^4 + x)(6x)}{(3x^2 - 9)^2}$

$y' = \frac{24x^5 + 3x^2 - 72x^3 - 9 - 12x^5 - 6x^2}{(3x^2 - 9)^2}$

$y' = \frac{12x^5 - 72x^3 - 3x^2 - 9}{(3x^2 - 9)^2}$

23. $f(x) = \frac{3x^3}{e^{5x}}$

$f(x) = 3x^3 \qquad g(x) = e^{5x}$

$f'(x) = 9x^2 \qquad g'(x) = e^{5x} \cdot 5$

$$f'(x) = \frac{e^{5x}(9x^2) - 3x^3(5e^{5x})}{(e^{5x})^2}$$

$$\boxed{f'(x) = \frac{9x^2e^{5x} - 15x^3e^{5x}}{(e^{5x})^2}}$$

24. $f(x) = 9x^2 \ln(4x)$

$f(x) = 9x^2 \qquad g(x) = \ln(4x)$

$f'(x) = 18x \qquad g'(x) = \frac{1}{4x} \cdot 4 = \frac{4}{4x} = \frac{1}{x}$

$f'(x) = 9x^2\left(\frac{1}{x}\right) + \ln(4x)(18x)$

$$\boxed{f'(x) = 9x + 18x\ln(4x)}$$

25. If $f(x) = 7x^4 - 5x^3 + 4x - 10$, find $f'(2)$

$f'(x) = 28x^3 - 15x^2 + 4$

$f'(2) = 28(2)^3 - 15(2)^2 + 4$

$$\boxed{f'(2) = 168}$$

26. If $f(x) = 3x^5 - 4x^3 + 5x^2 - 7x$, find $f'(1)$

$f'(x) = 15x^4 - 12x^2 + 10x - 7$

$f'(1) = 15(1)^4 - 12(1)^2 + 10(1) - 7$

$$\boxed{f'(1) = 6}$$