Linear Functions

Functions

function

Input → → output

x → → y

usually defined by an equation

Linear Function

linear

input → $y = mx + b$ → output

$x \rightarrow y = mx + b \rightarrow y$

Example :

input → output

$x \rightarrow y = 2x + 1 \rightarrow y$

$3 \rightarrow 2 * 3 + 1 \rightarrow 7$

$8 \rightarrow 2 * 8 + 1 \rightarrow 17$

In linear function,

slope

$$y = mx + b$$

x : input

y : output

$\underline{m}$: a constant (some known number)

called the slope of the linear function

$\underline{b}$: a constant called the intercept of the linear function

Graphs of Functions

The graph of a function is a collection of $\underline{\text{ALL}}$ the input and output pairs (x, y) presented on the x-y coordinates.

$\underline{\text{Example}}$: $y = 2x + 1$

Some input and output pairs :

1. $x = 0, \quad y = 2 * 0 + 1 = 1$

   $\Rightarrow \quad (x, y) = (0, 1)$ A

2. $x = 1, \quad y = 2 * 1 + 1 = 3$

   $(x, y) = (1, 3)$ B

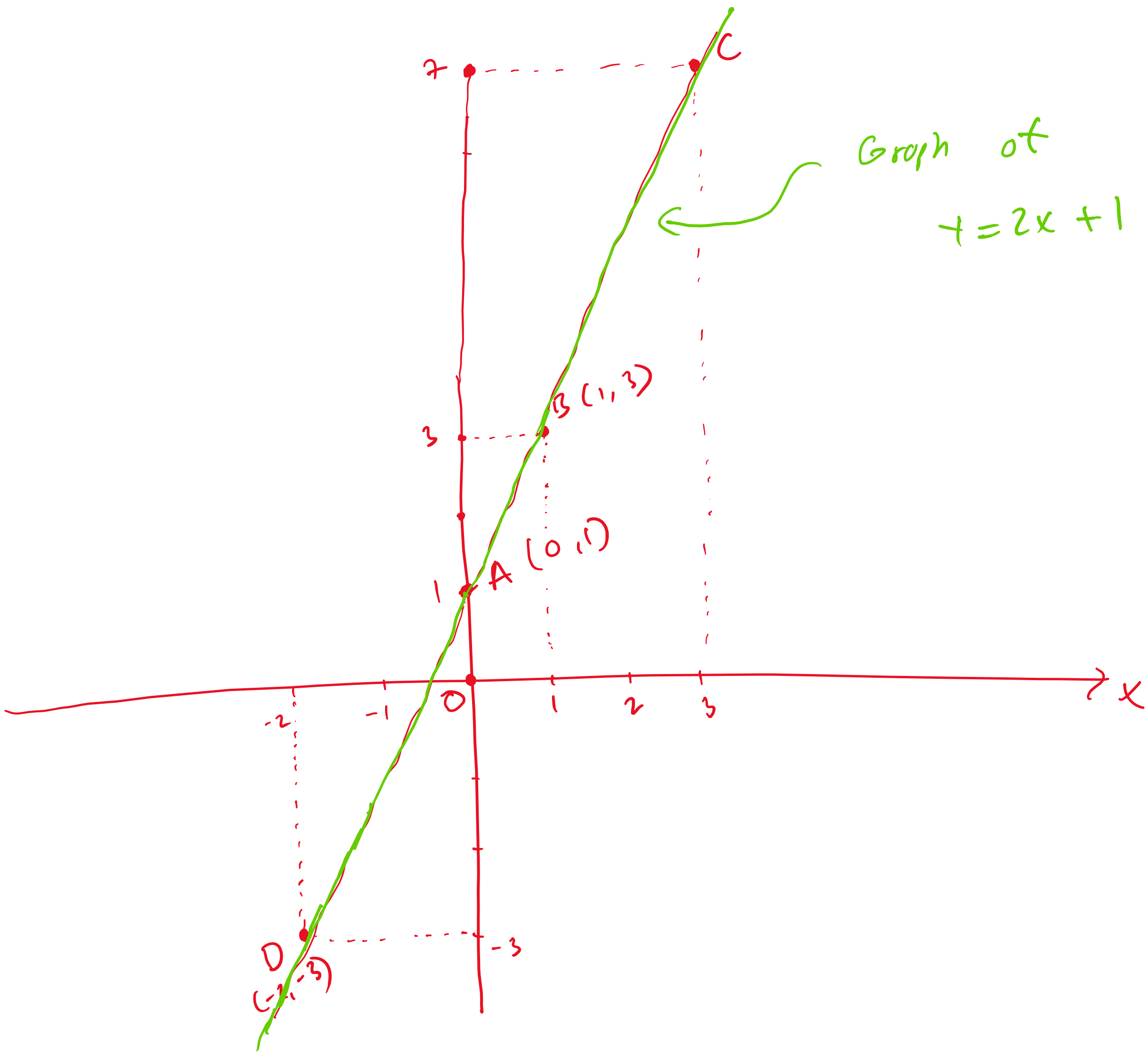
3. $x = 3, \quad y = 2 * 3 + 1 = 7$

   $(x, y) = (3, 7)$ C

④ $x = -2$, $y = 2 * (-2) + 1 = -3$

$(x, y) = (-2, -3)$

Let's present all of these pairs on x-y plane.



The graph of a linear function is a line. To graph a linear function, we just need to graph two points (pairs) then connect the two points to make the graph (line).

Practice

Graph the below linear functions. (just need two pairs and connect them to get the graph)

1. y = 3x + 4

2. y = -2x + 4

Slope

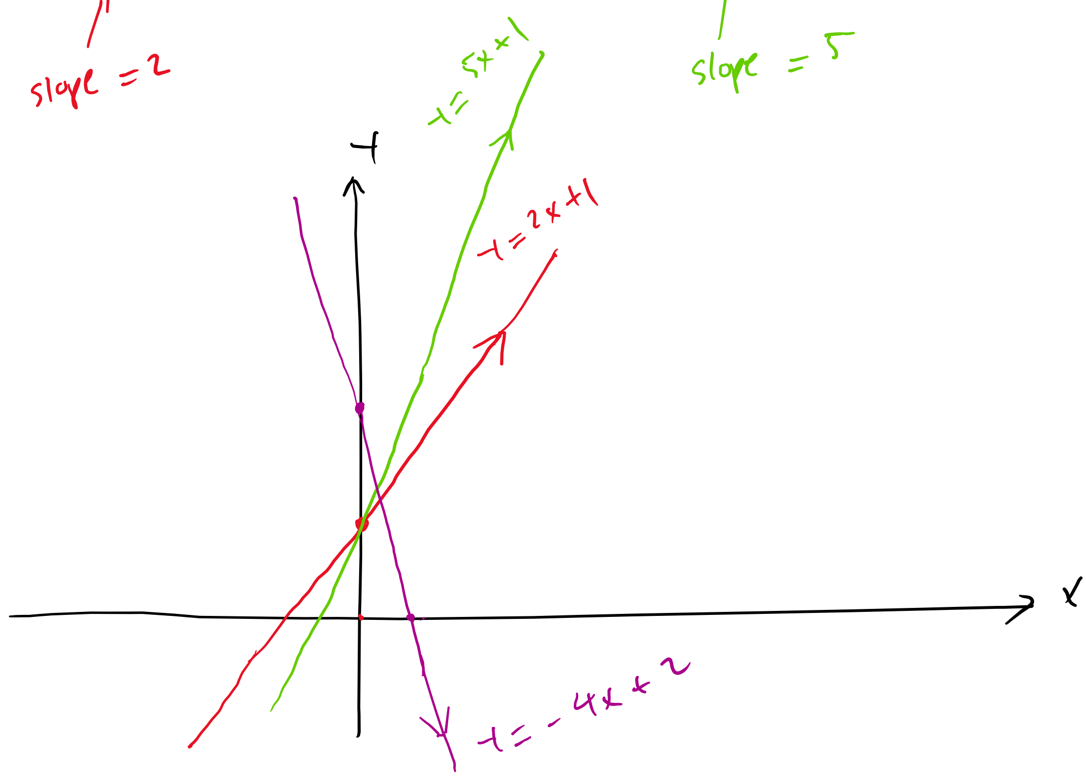
$$y = \underset{\text{slope}}{m}x + b$$

$y = 2x + 1$; $y = 5x + 1$; $y = -4x + 2$

slope = 2

slope = 5

slope



The slope tells the direction of the line. Lines with positive slopes go up (from left to right).

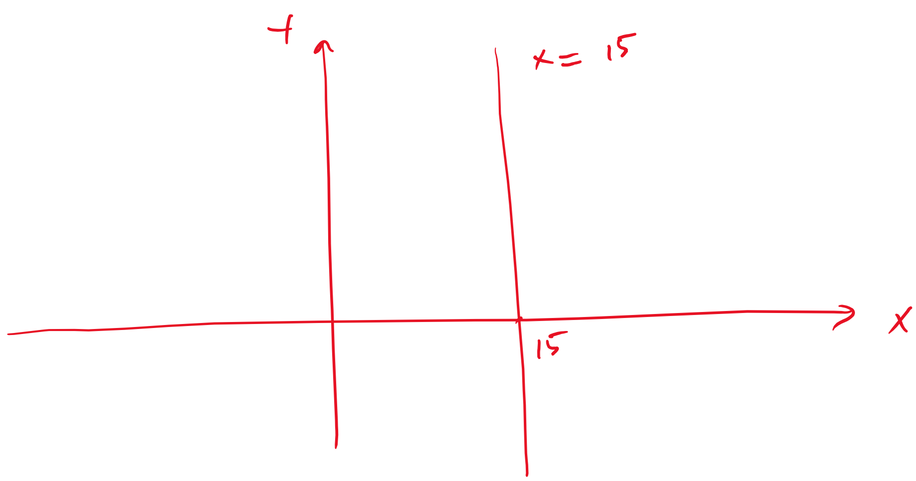
Lines with negative slopes go down.

Lines with greater positive slopes go up faster.

Lines with greater negative slopes go down faster.

Vertical Line

A vertical line is not a function. This means that a vertical line is not a linear function.



① x = 15 is not a function b/c there is no way to calculate output y.

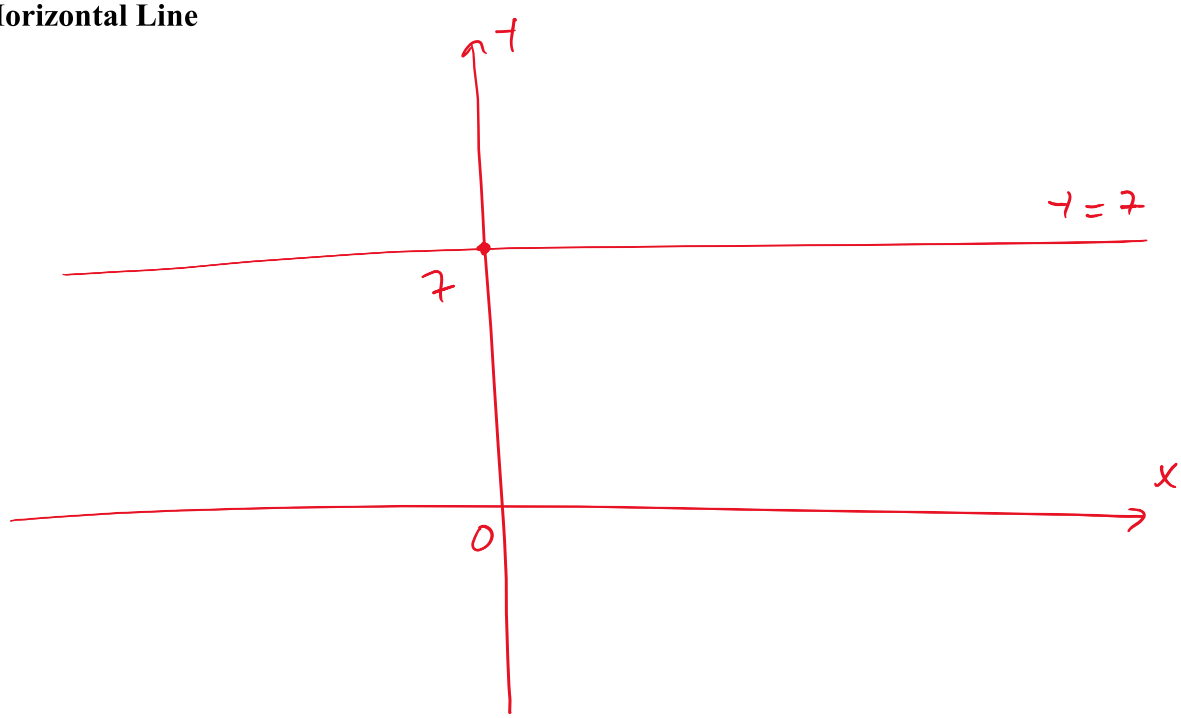
② In general a vertical has the equation:

x = some constant.

and they are not functions.

③ Some time we call vertical lines are "a linear functions" with undefined / infinity slopes.

Horizontal Line



① A horizontal line has the equation :

$$y = \text{some constant}$$

② A horizontal line has the slope $= 0$

$$y = \underbrace{m}_{=0} \cdot x + b$$

Equations of the linear passing through two given points.

The equation of the line that passes two points

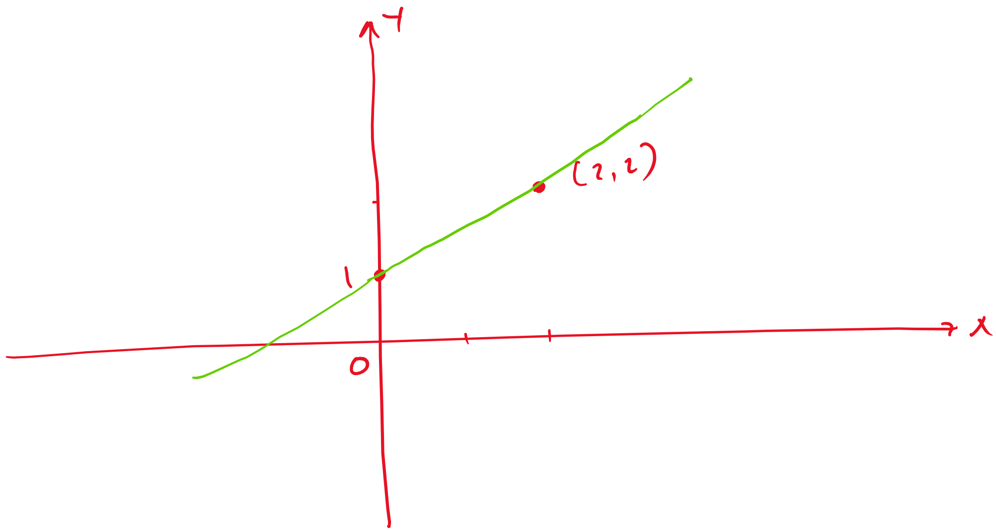
(x_1, y_1) and (x_2, y_2) is

$$\boxed{y = \frac{y_2 - y_1}{x_2 - y_1} \cdot (x - x_1) + y_1}$$

Example

x_1 y_1 x_2 y_2

Equations of the linear passing through two points (0, 1) and (2, 2)



$$y = \frac{y_2 - y_1}{x_2 - y_1} \cdot (x - x_1) + y_1$$

$$y = \frac{2 - 1}{2 - 0} \cdot (x - 0) + 1$$

$$\boxed{y = \frac{1}{2} x + 1}$$

The slope of the line passes through two given points: (x_1, y_1) and (x_2, y_2)

is

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad \frac{\text{difference in } y}{\text{difference in } x}$$

slope

Example 1: Find the slope of the line that passes through the following points:

$(-12,-5)$ and $(0,-8)$	$(-3,5)$ and $(-3,-4)$	$(4,5)$ and $(-1,5)$
$m = \frac{-8-(-5)}{0-(-12)} = \frac{-3}{12}$ $= -1/4$	$m = \frac{-4-5}{-3-(-3)} = \frac{-9}{0}$ or undefined (vertical line)	$m = \frac{5-5}{-1-4} = 0$ (horizontal line)

The equation of the line with a given slope passes through a given point.

Given slope : m

Given point : (x_1, y_1)

$$y = m(x - x_1) + y_1$$

m x_1 y_1

Example

Write the equation of the line with slope of 2000 and passes through the point (1, 2)

Given slope : m

Given point : (x_1, y_1)

$$y = m(x - x_1) + y_1$$

$$\Rightarrow \quad y = 2000(x - 1) + 2$$

$$\Rightarrow \quad y = 2000x - 2000 + 2$$

$$\Rightarrow \quad y = 2000x - 1998$$

Practice

1. Write the equation of the line passing through (1, 2) and (5, 0)

2. Write the equation of the line with the slope of -2 and passing through (2, 3)

Formulas

1. $$\boxed{y = \frac{y_2 - y_1}{x_2 - y_1} \cdot (x - x_1) + y_1}$$

2. $$\boxed{y = m(x - x_1) + y_1}$$