

Exponential Functions

$f(x)$ is an exponential function if

$$f(x) = a \cdot b^x$$

b is the base of the exponential function.

$$b > 0 \quad \text{and} \quad b \neq 1$$

$$a \neq 0$$

[note: if $b = 1$, $f(x) = a \cdot 1^x = a$ (constant)]

Example:

$$\textcircled{1} \quad f(x) = 3^x$$

$$\textcircled{2} \quad f(x) = \left(\frac{1}{2}\right)^x$$

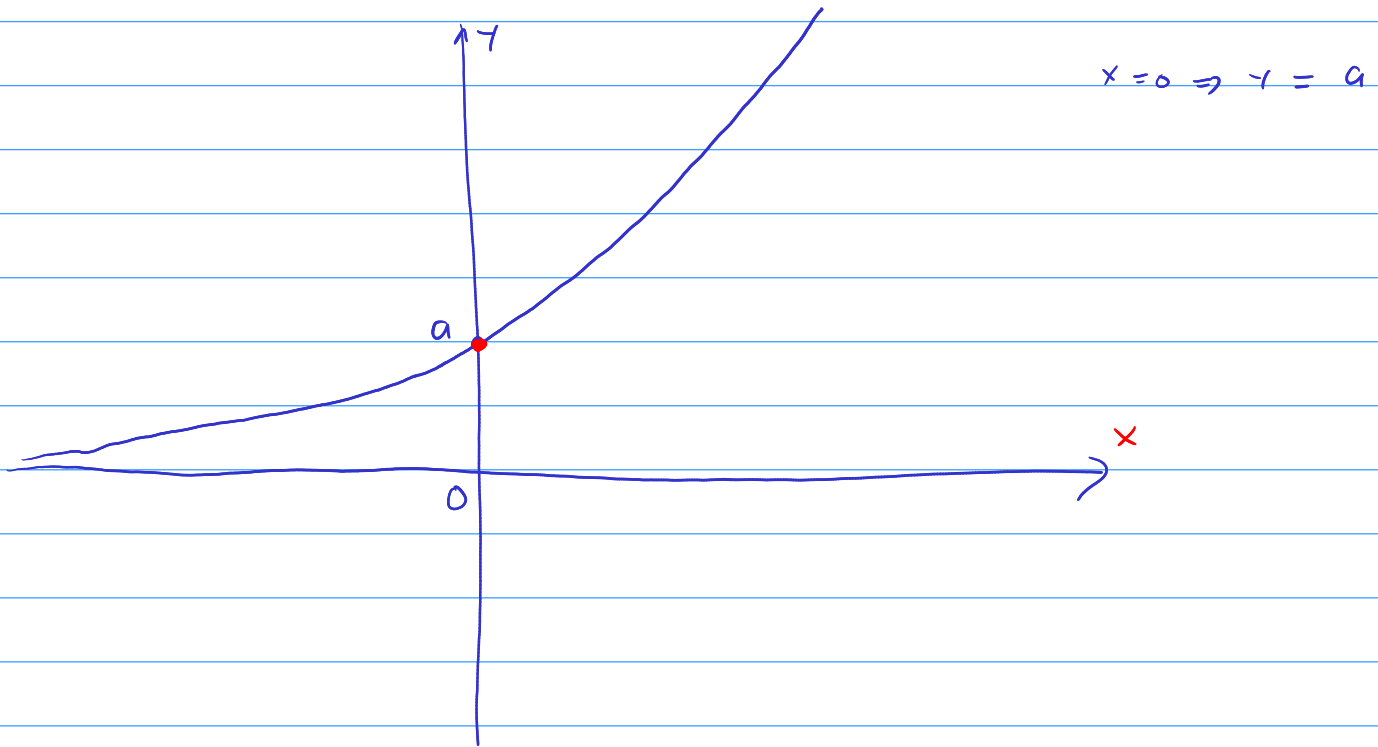
$$\textcircled{3} \quad f(x) = \underline{2024} \cdot (8)^x$$

$$\textcircled{4} \quad f(x) = -7 \cdot 6^x$$

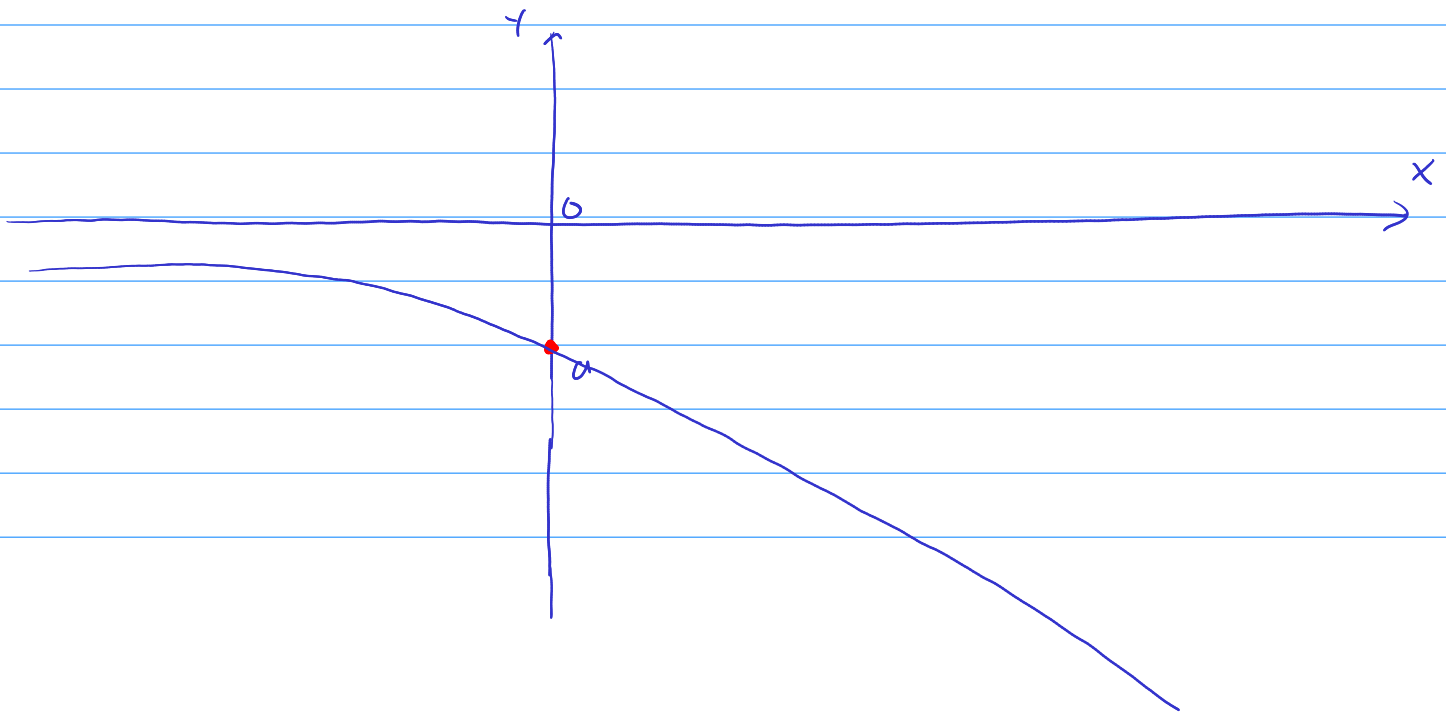
(*) Graphs of exponential functions.

$$f(x) = a \cdot b^x$$

Case 1 : $b > 1$ and $a > 0$

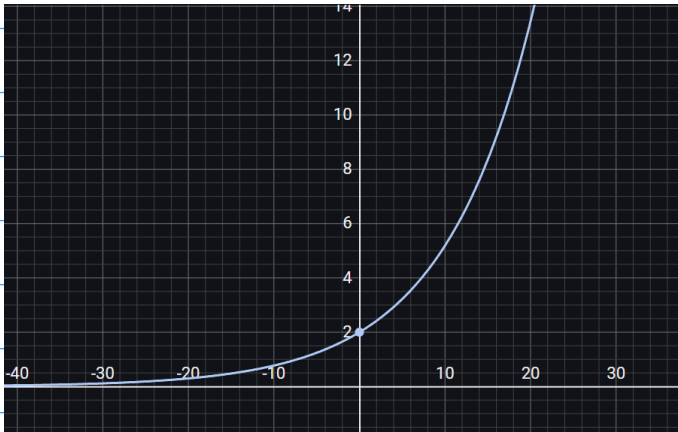


Case 2 : $b > 1$ and $a < 0$

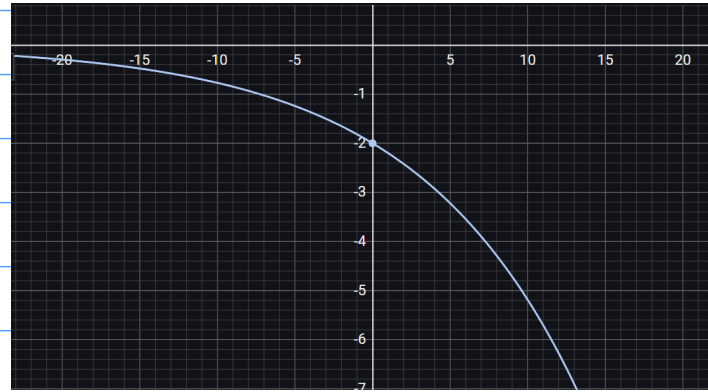


Example :

$$y = 2 \times (1.1)^x$$



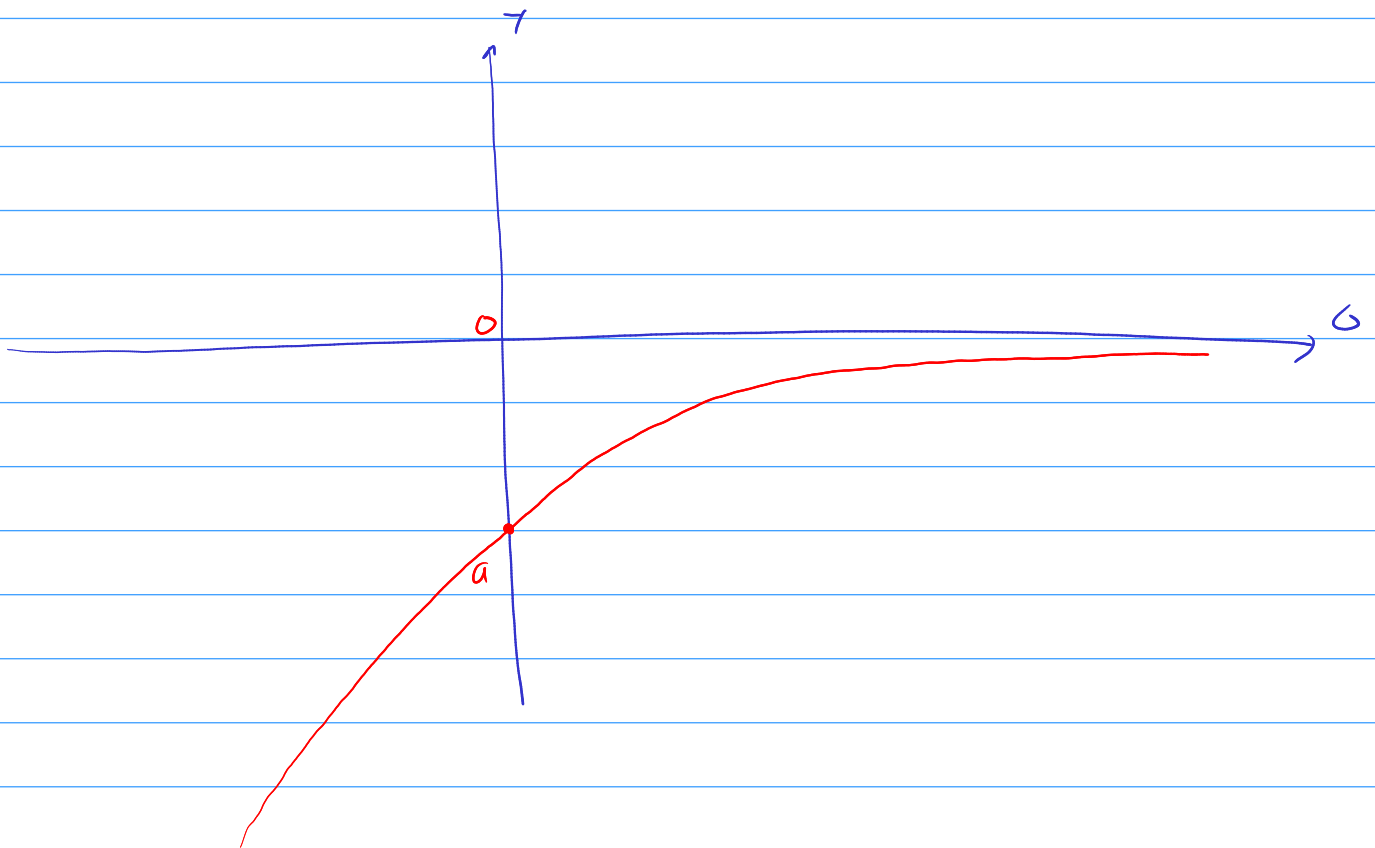
$$y = -2 \times (1.1)^x$$



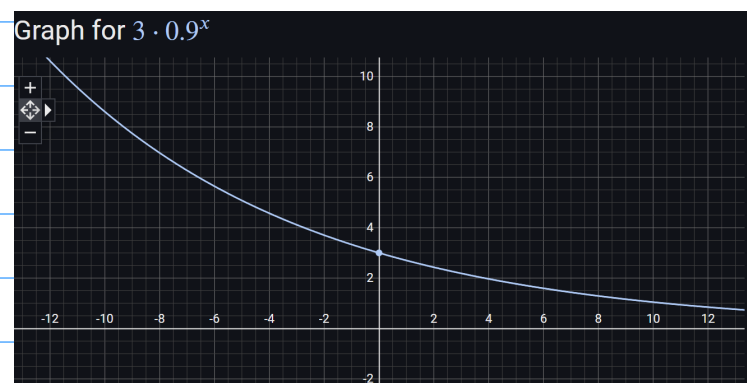
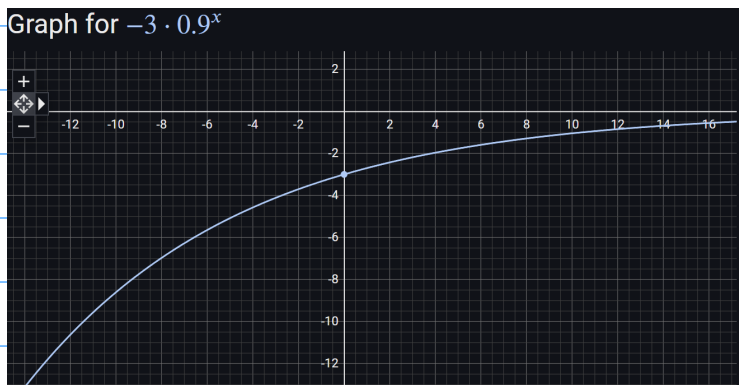
Case 3 : $0 < b < 1$ and $a > 0$



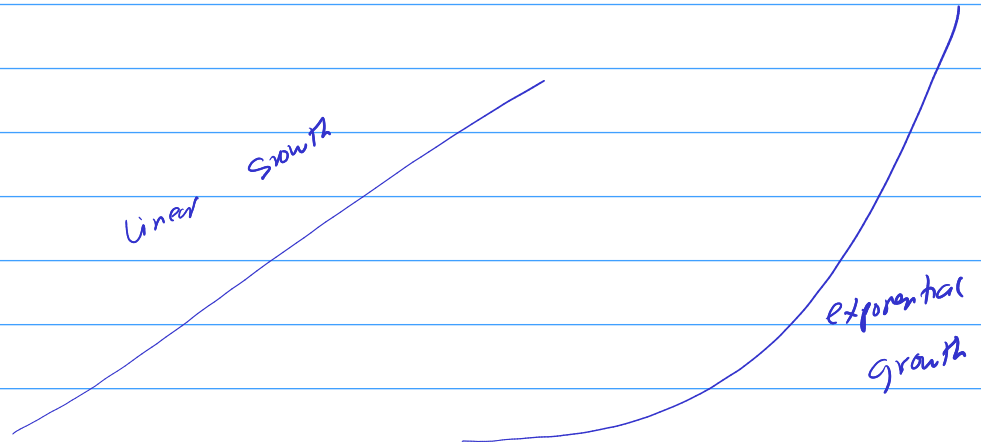
Case 4 : $0 < b < 1$ and $a < 0$



Example :



Linear vs. Exponential Growth.



Simple Interest vs. Compound Interest.

⊛ Simple Interest: The interest is only apply to the principal

Suppose we deposit \$ 100 to a bank with simple interest of 5% yearly.

The principal: $P = 100$

The interest rate: $r = 5\%$

Year 1: You earn: $P.r = 100 \times 5\% = \$5$

$$\Rightarrow \underbrace{\text{Your total money}}_A: 100 + 5 = \$105$$

$$A = 100 + \underbrace{100 \times 5\%}_{\text{interest}} = 100(1 + 1 \cdot 5\%) = \underline{105}$$

Year 2: You earn the same amount $\$5$

$$\rightarrow \text{total money: } 105 + 5 = \underline{110}$$

$$\begin{aligned} A &= \underbrace{100}_{\text{principal}} (1 + \underbrace{5\%}_{\text{rate}}) + \underbrace{100}_{\text{principal}} \cdot \underbrace{5\%}_{\text{rate}} \\ &= 100 (1 + 2 \cdot 5\%) \end{aligned}$$

Year 3:

$$A = 100 (1 + 3 \cdot 5\%)$$

Year t :

$$\boxed{A = 100 (1 + t \cdot 5\%)} \quad (\text{Linear function})$$

This is a Linear Growth

(*) Compound Interest : The interest is applied to the current amount of money.

$$\begin{aligned}\text{Year 1 : } 100 + 100 \cdot 5\% &= 100 (1 + .05) \\ &= 100 \times 1.05\end{aligned}$$

Year 2 :

$$100 \times 1.05 \times 1.05 = 100 \times (1.05)^2$$

Year 3 :

$$A = 100 \times (1.05)^3$$

Year t :

$$A = 100 (1.05)^t \quad (\text{Exponential Growth})$$

Assignment :

(1) Plot $y = 3 + (1.02)^x$

(2) Plot $y = 2 + (.8)^x$

(3) Find the total amount of money you have if you invest \$ 2000 after 5 years if .

(a) the interest rate is 6% and it is a simple interest .

(b) the interest rate is 6% and it is a compound interest .