Exam 2 – Practice 1 Math 110.

Exam Guidelines This is an in-class, written exam with a 75-minute time limit.

- **Permitted Materials:** You may use a basic calculator and formula sheets.
- **Formula Sheet Restrictions:** Your sheets must contain formulas only; no examples or worked problems are permitted. All sheets will be inspected at the start of the exam.
- **Prohibited Items:** Phones and all other smart devices are strictly forbidden.
- **Academic Integrity:** The use of AI is prohibited. Any AI usage will result in an automatic F for the exam and may lead to failing the entire course.
- Show **ALL** your work for credits.

1. Solve each quadratic by factoring or the quadratic formula.
   a. $2x^2 = 4x$

$$2x^2 = 4x$$

$$2x^2 - 4x = 0$$

factoring

$$2x \cdot (x - 2) = 0$$

$$\Leftrightarrow \left[\begin{array}{l} 2x = 0 \\ x - 2 = 0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = 0 \\ x = 2 \end{array}\right.$$

b. $x^2 - 6x + 10 = 0$

$1 \cdot x^2 - 6x + 10 = 0$

$a = 1, \quad b = -6, \quad c = 10$

$b^2 - 4ac = (-6)^2 - 4 \cdot 1 \cdot 10$

$= -4 < 0$

$\Rightarrow$ No solution.

c. $x^2 - 6x + 9 = 0$

$a = 1, \quad b = -6, \quad c = 9$

$b^2 - 4ac = (-6)^2 - 4 \cdot 1 \cdot 9 = 0$

$$\left[\begin{array}{l} x = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a} = \dfrac{6 + 0}{2} = 3 \\ x = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a} = \dfrac{6 - 0}{2} = 3 \end{array}\right. \Leftrightarrow \boxed{x = 3}$$

d. $x^2 + 7x = -10$

$$x^2 + 7x = -10$$

$$\underbrace{x^2 + 7x + 10}_{} = 0$$

we look for 2 numbers : sum = 7

product = 10

$\Rightarrow$ 2 ; 5

$$(x+2)\cdot(x+5) = 0$$

$$\Leftrightarrow \left[\begin{array}{l} x+2=0 \\ x+5=0 \end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l} x=-2 \\ x=-5 \end{array}\right.$$

2. Graph of the quadratic functions. Label the vertex and another point.

a. $y = x^2 + 2x + 3$

$a = 1, \quad b = 2, \quad c = 3$

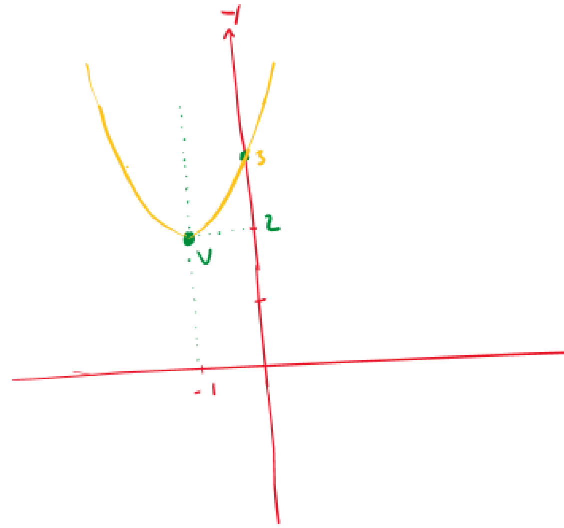
(*) $V = \left(-\frac{b}{2a}, \frac{4ac - b^2}{4a} \right)$

$= \left(\frac{-2}{2 \cdot 1}, \frac{4 \cdot 1 \cdot 3 - 2^2}{4 \cdot 1} \right)$

$= (-1, 2)$

(*) Another point:

pick $x = 0 \Rightarrow y = 3$



b. $y = -2x^2 + 4x + 4$

$a = -2$, $b = 4$, $c = 4$

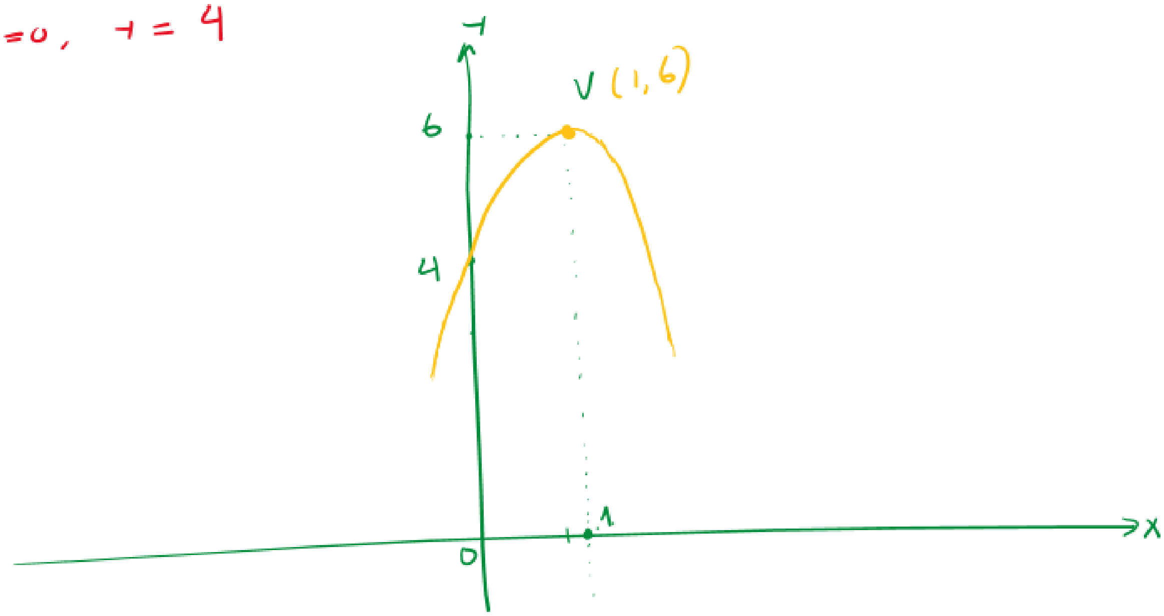
Vertex: $V = \left(-\frac{b}{2a} , \frac{4ac - b^2}{4a}\right)$

$= \left(\frac{-4}{2\cdot(-2)} , \frac{4(-2)4 - 4^2}{4\cdot(-2)}\right)$

$= (1, 6)$

Another point:

pick $x = 0$, $y = 4$



3. Suppose that in a monopoly market the total cost per week of producing a high-tech product is given by $C = 3600 + 100q + 2q^2$. Suppose further that the sales price function for this product is

$p = 500 - 2q$.

a. Find the revenue function in term of q.

Revenue = price × quantity

$\Leftrightarrow \quad R = p \cdot q$

$= (500 - 2q) \cdot q$

$\Leftrightarrow \quad R = 500q - 2q^2$

$\Leftrightarrow \quad \boxed{R = -2q^2 + 500q}$

b. Find the number of units that will **maximize the revenue.**

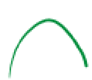
Formula: Maximizing | Minimizing a quadratic.

$y = ax^2 + bx + c$

⊛ If $a > 0$: ⊕ y is minimized when $x = -\frac{b}{2a}$

⊕ y has no maximum value.

⊛ If $a < 0$: ⊕ y is maximized when $x = -\frac{b}{2a}$



⊕ y has no minimum value

we have. $R = -2q^2 + 500q$

$a = -2$, $b = 500$; $c = 0$

R is maximized when $q = -\frac{b}{2a} = -\frac{500}{2*(-2)}$

$\Rightarrow \boxed{q = 125}$

c. Find the profit function

$$\text{Profit} = \text{Revenue} - \text{Cost}$$
$$= -2q^2 + 500q - (3600 + 100q + 2q^2)$$
$$= -2q^2 + 500q - 3600 - 100q - 2q^2$$
$$= \boxed{-4q^2 + 400q - 3600}$$

d. Find the number of units that will give **break-even** for the product

Profit = 0

$$\text{profit} = 0$$
$$\Leftrightarrow -4q^2 + 400q - 3600 = 0$$
$$\Leftrightarrow -4\cdot(q^2 - 100q + 900) = 0$$
$$\Leftrightarrow q^2 - 100q + 900 = 0$$

$a = 1, \quad b = -100, \quad c = 900$

$$\Rightarrow q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{(quadratic formula)}$$

$$q = \frac{100 \pm \sqrt{(-100)^2 - 4\cdot 1\cdot 900}}{2}$$

$$q = \frac{100 \pm 80}{2}$$

$$q = \frac{100 + 80}{2} = 90$$

$$q = \frac{100 - 80}{2} = 10$$

There are 2 break - even points: $q = 10$, $q = 90$

e. Find **the maximum profit** and the number of products need to maximize the profit.

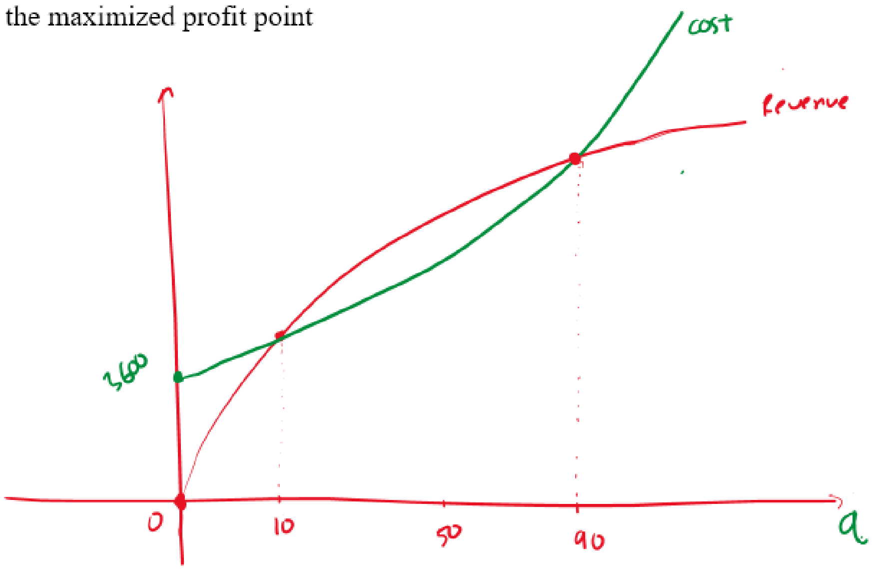
$$\text{Profit} = -4q^2 + 400q - 3600$$

$a = -4$; $b = 400$, $c = -3600$

Profit is maximized when $q = -\frac{b}{2a} = -\frac{400}{2\cdot(-4)} = 50$

Notice: The profit is ALWAYS maximized at the mid-point of the 2 break-even points. $\left(\frac{10+90}{2} = 50\right)$

f. Graph the revenue function and the cost function label the break-even points, fixed cost, and the maximized profit point



4. On a certain route, an airline carries 8000 passengers per month, each paying \$50. A market survey indicates that for each \$1 increase in the ticket price, the airline will lose 100 passengers.

a. What is the airline's current revenue?

Revenue = price x quantity

$= 50 * 8000 = 400,000$

b. Create an income (revenue) function if "x" is defined as the number of \$1 price increases

	current	increasing the price by x
price	50	$50 + x$
Quantity / passengers	8000	$8000 - 100x$
Revenue	$50 \cdot 8000$	$(50 + x) \cdot (8000 - 100x)$

New Revenue $= (50 + x) \cdot (8000 - 100x)$

c. Find the number of \$1 price increases that will maximize the revenue.

we need to find x that maximizes

$$(50 + x) \cdot (8000 - 100x)$$

$$R = 50 * 8000 - 50 * 100x + 8000x - 100x^2$$

$$R = -100x^2 + 3000x + 400,000$$

$a = -100, \quad b = 3000, \quad c = 400,000$

R is maximized when $x = -\dfrac{b}{2a} = -\dfrac{3000}{2 * (-100)}$

$\boxed{x = 15}$

d. Find the new ticket price (that will maximize the revenue)

New price = current price + x

$= 50 + 15 = \boxed{65}$

e. Find the number of passengers at that price in d.

New number of passengers $= 8000 - 100x$

$= 8000 - 100 * 15$

$= 6500$

f. Find the new maximum income (income at that price in d)

new Revenue $= 65 \times 6500 = 422500$

5. If the supply function for a commodity is given by $p = 10q^2 + 2q$ and the demand function is given by $p = 150 - 6q^2$, find the point of market equilibrium (Supply equals Demands).

$10q^2 + 2q = 150 - 6q^2$

$\Leftrightarrow \underline{10q^2} + 2q + \underline{6q^2} - 150 = 0$

$\Leftrightarrow 16q^2 + 2q - 150 = 0$

$a = 16$; $b = 2$; $c = -150$

$$q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 16 \cdot (-150)}}{2 \cdot 16}$$

$$= \frac{-2 \pm 98}{32}$$

$\nearrow \frac{-2 - 98}{32} = -3.125$

$\searrow \frac{-2 + 98}{32} = 3$

Since q is non-negative, $\boxed{q = 3}$

The price at $q = 3$ is $10q^2 + 2q$

$$p = 10 * 3^2 + 2 * 3$$

$$\boxed{p = 96}$$