Quotient Rule

$$\left[\frac{\text{Top}}{\text{Bot}}\right]' = \frac{\text{Top}' \cdot \text{Bot} - \text{Bot}' \cdot \text{Top}}{(\text{Bot})^2}$$

OR

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)\,g(x) - g'(x)\cdot f(x)}{[g(x)]^2}$$

Example : Find $h'(x)$

① $h(x) = \dfrac{x^3+1}{x^2-2}$

$$h'(x) = \frac{(x^3+1)'\cdot(x^2-2) - (x^2-2)'\cdot(x^3+1)}{(x^2-2)^2}$$

$$= \frac{3x^2\cdot(x^2-2) - (2x)(x^3+1)}{(x^2-2)^2}$$

② Find $h'(x)$ and simplify the answer.

$$h(x) = \frac{x^5+2x+1}{x^5-2x+1}$$

$$h'(x) = \frac{(x^5+2x+1)'\cdot(x^5-2x+1) - (x^5-2x+1)'\cdot(x^5+2x+1)}{(x^5-2x+1)^2}$$

$$h'(x) = \frac{(5x^4 + 2)(x^5 - 2x + 1) - (5x^4 - 2)(x^5 + 2x + 1)}{(x^5 - 2x + 1)^2}$$

$$h'(x) = \frac{(5x^9 - 10x^5 + 5x^4 + 2x^5 - 4x + 2) - (5x^9 + 10x^5 + 5x^4 - 2x^5 - 4x - 2)}{(x^5 - 2x + 1)^2}$$

$$h'(x) = \frac{5x^9 - 10x^5 + 5x^4 + 2x^5 - 4x + 2 - 5x^9 - 10x^5 - 5x^4 + 2x^5 + 4x + 2}{(x^5 - 2x + 1)^2}$$

$$h'(x) = \frac{-16x^5 + 4}{(x^5 - 2x + 1)^2}$$

③ $$h(x) = \frac{\sqrt{x} + 1}{\sqrt{x} + x}$$

$$h'(x) = \frac{(\sqrt{x} + 1)' \cdot (\sqrt{x} + x) - (\sqrt{x} + x)' \cdot (\sqrt{x} + 1)}{(\sqrt{x} + x)^2}$$

Note: $(\sqrt{x})' = (x^{1/2})' = \frac{1}{2} x^{1/2 - 1} = \frac{1}{2} x^{-1/2}$

$$h'(x) = \frac{\frac{1}{2} x^{-1/2} \cdot (\sqrt{x} + x) - (\frac{1}{2} x^{-1/2} + 1)(\sqrt{x} + 1)}{(\sqrt{x} + x)^2}$$

④ $h(x) = \dfrac{\left(\frac{2}{x} + 3x\right)}{\sqrt[3]{x}}$

$$h'(x) = \frac{\left(\frac{2}{x} + 3x\right)' \cdot \sqrt[3]{x} - \left(\sqrt[3]{x}\right)' \cdot \left(\frac{2}{x} + 3x\right)}{\left(\sqrt[3]{x}\right)^2}$$

Note:

$$\left(\frac{2}{x}\right)' = \left(2x^{-1}\right)' = 2 \cdot (-1)\, x^{-1-1} = -2x^{-2}$$

$$\left(\sqrt[3]{x}\right)' = \left(x^{1/3}\right)' = \frac{1}{3} x^{1/3 - 1} = \frac{1}{3} x^{-2/3}$$

$$h'(x) = \frac{\left(-2x^{-2} + 3\right)\left(\sqrt[3]{x}\right) - \frac{1}{3} x^{-2/3}\left(\frac{2}{x} + 3x\right)}{\left(\sqrt[3]{x}\right)^2}$$

⑤ $h(x) = \dfrac{1}{3x + 2}$

$$h'(x) = \frac{(1)' \cdot (3x + 2) - (3x + 2)' \cdot 1}{(3x + 2)^2}$$

$$= \frac{0 \cdot (3x + 2) - 3 \cdot 1}{(3x + 2)^2} = \frac{-3}{(3x + 2)^2}$$

Assignment . Find $h'(x)$

① $h(x) = \dfrac{x^3 + 3x}{x^2 + 4}$

② $h(x) = \dfrac{x^2 + 1}{x^2 - 1}$ (simplify)

③ $h(x) = \dfrac{2x^4 + x}{x + 4}$

④ $h(x) = \dfrac{x^2 + 3x + 1}{x^2 + 4x + 1}$

⑤ $h(x) = \dfrac{x^{2025} + 4x^3}{x^{2002} - 6}$

⑥ $h(x) = \dfrac{3x + 1}{4x + 1}$

⑦ $h(x) = \dfrac{\sqrt{x} + 2}{\sqrt{x} + 3}$

(8) $h(x) = \dfrac{\sqrt[4]{x} + 1}{x + 2}$

(9) $h(x) = \dfrac{\frac{1}{x} + 1}{x}$

(10) $h(x) = \dfrac{\frac{1}{x^2} + 1}{3x}$