Exam 2 Review

1. The feasible region determined by a system of constraints is given. Find the maximum and minimum of the objective function.

(2,4) $C = -2(2)+5(4) = 16$

(4,0) $C = -2(4)+5(0) = -8$

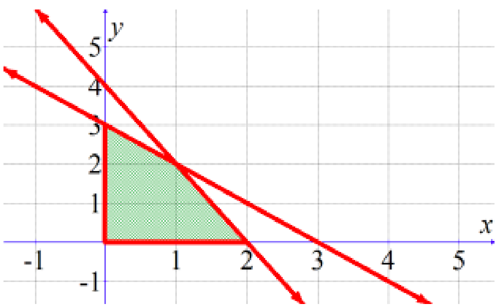
(6,0) $C = -2(6)+5(0) = -12$

$C = -2x + 5y$

Maximum: 16 @ (2,4)

Minimum: −12 @ (6,0)

2. The feasible region determined by a system of constraints is given. Find the maximum and minimum of the objective function.



(0,0) $C = 5(0)-3(0) = 0$

(0,3) $C = 5(0)-3(3) = -9$

(2,0) $C = 5(2)-3(0) = 10$

(1,2) $C = 5(1)-3(2) = -1$

$C = 5x - 3y$

Maximum: 10 @ (2,0)

Minimum: −9 @ (0,3)

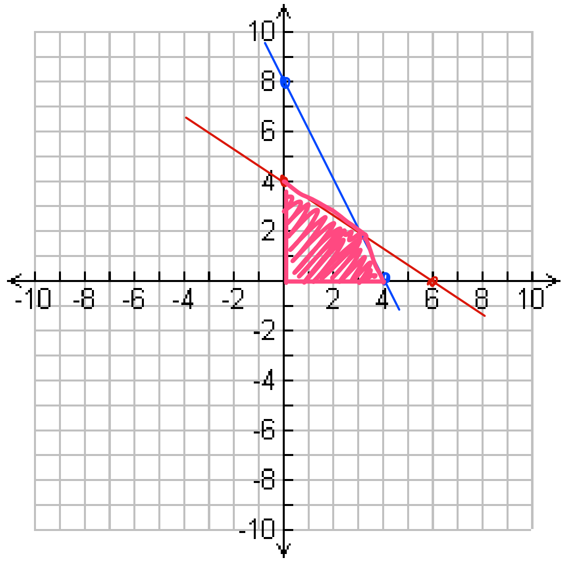
3. Given the following inequalities:

$2x + 3y \leq 12$ (6,0) (0,4)

$2x + y \leq 8$ (4,0) (0,8)

$x \geq 0$

$y \geq 0$



a. Graph the constraints

b. Find the vertices of the feasibility region

(0,4) (3,2)

(0,0) (4,0)

c. Test each vertex in the objective function $C = 4x + 3y$ to find the minimum and maximum values

(0,4) C= 12

(0,0) C=0

(3,2) C= 18

(4,0) C= 16

maximum: 18 @ (3,2) minimum: 0 @ (0,0)

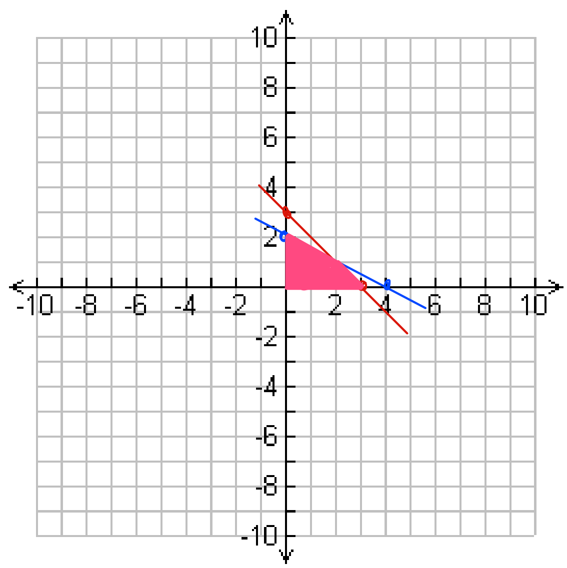
4. Given the following inequalities:

$x + y \leq 3$ (3,0) (0,3)

$x + 2y \leq 4$ (4,0) (0,2)

$x \geq 0$

$y \geq 0$



a. Graph the constraints

b. Find the vertices of the feasibility region

(0,2) (3,0)

(0,0) (2,1)

c. Test each vertex in the objective function $C = 2x - y$ to find the minimum and maximum values

(0,2) C= -2

(0,0) C=0

(3,0) C= 6

(2,1) C= 3

maximum: 6 @ (3,0)

minimum: -2 @ (0,2)

5. A shoe manufacturer makes outdoor and indoor soccer shoes. There is a two-step manufacturing process for both kinds of shoes, cutting and sewing. Each pair of outdoor shoes requires 2 hours of cutting and 1 hour or sewing. Indoor shoes require 1 hour of cutting and 3 hours of sewing. The company has only 40 hours of labor available for cutting and 60 hours available for sewing. Outdoor shoes make a profit of $20 per pair and indoor shoes make a profit of $15 per pair. How many pairs of each shoe should be made to maximize profit? What is the maximum profit?
   a. Define the variables

      x = outdoor soccer shoes y = indoor soccer shoes

   b. Write the constraints

      cutting: $2x + y \leq 40$

      sewing: $x + 3y \leq 60$

      $x \geq 0$

      $y \geq 0$

   c. Write the objective function

      $C = 20x + 15y$

6. The Acme Class Ring Company designs and sells two types of rings: the VIP and the SST. They can produce up to 24 rings each day using up to 60 total man-hours of labor. It takes 3 man-hours to make one VIP ring, versus 2 man-hours to make one SST ring. How many of each type of ring should be made daily to maximize the company's profit, if the profit on a VIP ring is $30 and on an SST ring is $40?
   a. Define the variables

      x = VIP ring y = SST ring

   b. Write the constraints

      $x + y \leq 24$

      $3x + 2y \leq 60$

      $x \geq 0$

      $y \geq 0$

   c. Write the objective function

      $C = 30x + 40y$

7. You invest $5,000 in an account that pays 2.8% interest.
   a. How much will you have in the account in 5 years if interest is compounded monthly?

   $$A = 5000\left(1 + \frac{0.028}{12}\right)^{12(5)} = \boxed{\$5750.43}$$

   b. How much interest will you earn?

   $$5750.43 - 5000 = \boxed{\$750.43}$$

   c. How much will you have in the account in 5 years if interest is compounded continuously?

   $$A = 5000e^{0.028(5)} = \boxed{\$5751.37}$$

   d. How much interest will you earn?

   $$5751.37 - 5000 = \boxed{\$751.37}$$

   e. Which account earned more interest? How much more?

   $\boxed{\text{Continuous}}$ $\boxed{\text{\$0.94 more interest}}$

8. You invest $4,000 in an account that pays 3.25% interest.
   a. How much will you have in the account in 10 years if interest is compounded monthly?

   $$A = 4000\left(1 + \frac{0.0325}{12}\right)^{12(10)} = \boxed{\$5533.69}$$

   b. How much interest will you earn?

   $$5533.69 - 4000 = \boxed{\$1533.69}$$

   c. How much will you have in the account in 10 years if interest is compounded continuously?

   $$A = 4000e^{0.0325(10)} = \boxed{\$5536.12}$$

   d. How much interest will you earn?

   $$5536.12 - 4000 = \boxed{\$1536.12}$$

   e. Which account earned more interest? How much more?

   $\boxed{\text{Continuous}}$ $\boxed{\text{\$2.43 more}}$

9. You invest \$6,000 in an account that pays 3.9% annual interest compounded monthly. When will you have double the amount of money in the account?

$A = 12000$

$P = 6000$

$r = 0.039$

$n = 12$

$t = ?$

$$\frac{12000}{6000} = \frac{6000\left(1+\frac{0.039}{12}\right)^{12t}}{6000}$$

$$2 = (1.00325)^{12t}$$

$$\log(2) = \log(1.00325)^{12t}$$

$$\frac{\log(2)}{\log(1.00325)} = \frac{12t\log(1.00325)}{\log(1.00325)}$$

$$\frac{213.62}{12} = \frac{12t}{12}$$

$$\boxed{t = 17.80 \text{ yrs}}$$

10. You invest \$2,000 in an account that pays 2.8% annual interest compounded quarterly. When will you have triple the amount of money in the account?

$A = 6000$

$P = 2000$

$r = 0.028$

$n = 4$

$t = ?$

$$\frac{6000}{2000} = \frac{2000\left(1+\frac{0.028}{4}\right)^{4t}}{2000}$$

$$3 = (1.007)^{4t}$$

$$\log(3) = \log(1.007)^{4t}$$

$$\frac{\log(3)}{\log(1.007)} = \frac{4t\log(1.007)}{\log(1.007)}$$

$$\frac{157.49}{4} = \frac{4t}{4}$$

$$\boxed{t = 39.37 \text{ years}}$$

11. How much money must you deposit today in an account that earns 1.89% interest compounded continuously if you want to have \$1000 in 2 years?

$A = 1000$

$P = ?$

$r = 0.0189$

$t = 2$

$$1000 = Pe^{0.0189(2)}$$

$$\frac{1000}{1.038523507} = \frac{P(1.038523507)}{1.038523507}$$

$$\boxed{P = \$962.91}$$

12. How much money must you deposit today in an account that earns 2.6% interest compounded continuously if you want to have $2000 in 4 years?

$A = 2000$

$P = ?$

$r = 0.026$

$t = 4$

$$2000 = Pe^{0.026(4)}$$

$$\frac{2000}{1.109600455} = \frac{P(1.109600455)}{1.109600455}$$

$$\boxed{P = \$1802.45}$$

13. What interest rate do you need if you want to turn $2000 into $5000 in 5 years with interest compounded continuously?

$A = 5000$

$P = 2000$

$r = ?$

$t = 5$

$$\frac{5000}{2000} = \frac{2000e^{r(5)}}{2000}$$

$$2.5 = e^{5r}$$

$$\ln(2.5) = \ln e^{5r}$$

$$\ln(2.5) = 5r\ln(e)$$

$$\frac{\ln(2.5)}{5} = \frac{5r}{5}$$

$$\boxed{r = 0.1833 = 18.33\%}$$

14. What interest rate do you need if you want to double your initial investment of $750 in 3 years if interest is compounded continuously?

$A = 1500$

$P = 750$

$r = ?$

$t = 3$

$$\frac{1500}{750} = \frac{750e^{r(3)}}{750}$$

$$2 = e^{3r}$$

$$\ln(2) = \ln(e)^{3r}$$

$$\ln(2) = 3r\ln(e)$$

$$\frac{\ln(2)}{3} = \frac{3r}{3}$$

$$\boxed{r = 0.2310 = 23.10\%}$$

15. Assembly-line operations tend to have a high turnover of employees, forcing the companies involved to spend much time and effort in training new workers. It has been found that a worker who is new to the operation of a certain task on the assembly line will produce $P(t)$ items on day t, where $P(t) = -25e^{-0.3t} + 25$.

a. Graph this equation by identifying and labeling the horizontal asymptote, t-intercept, and y-intercept.

$y = -25e^{-0.3(0)} + 25 = 0$

$(0,0)$

$y = 25$

$-25e^{-0.3t} + 25 = 0$

$\frac{-25e^{-0.3t}}{-25} = \frac{-25}{-25}$

$e^{-0.3t} = 1$

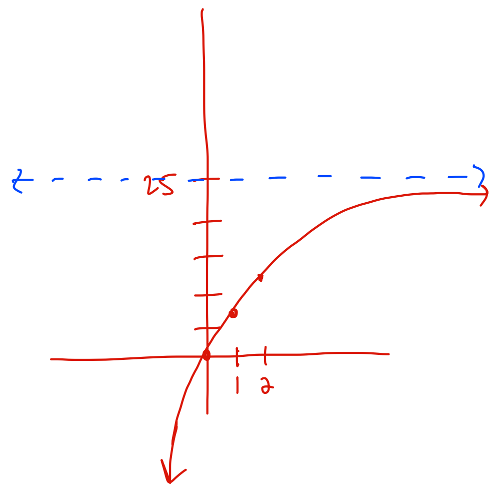
$\ln e^{-0.3t} = \ln 1$

$-0.3t = 0$

$t = 0$

$(0,0)$

1	6.48
2	11.28



b. How many items will be produced on the eighth day?

$p(8) = -25e^{-0.3(8)} + 25 = 22.73$ $\boxed{\text{22 items}}$

c. Interpret the meaning of the y-intercept.

On day 0, the worker can make 0 items

d. Interpret the meaning of the horizontal asymptote.

The worker will never be able to make 25 items or more in a day

e. On what day will the worker be able to produce 15 items?

$15 = -25e^{-0.3t} + 25$
$-25 \qquad\qquad -25$

$\frac{-10}{-25} = \frac{-25e^{-0.3t}}{-25}$

$0.4 = e^{-0.3t}$

$\ln(0.4) = \ln e^{-0.3t}$

$\ln(0.4) = -0.3t \ln e$

$\frac{\ln(0.4)}{-0.3} = \frac{-0.3t}{-0.3}$

$3.05 = t$

$\boxed{\text{3rd day}}$

16. The number of words per minute that an average person can type is given by $W(t) = -30e^{-0.5t} + 60$, where t is time in months after the beginning of a typing class.

a. Graph this equation by identifying and labeling the horizontal asymptote, *t*-intercept, and y-intercept.

$y = -30e^{-0.5(0)} + 60$

$y = 30$

$(0, 30)$

$y = 60$

$-30e^{-0.5t} + 60 = 0$

$\frac{-30e^{-0.5t}}{-30} = \frac{-60}{-30}$

$e^{-0.5t} = 2$

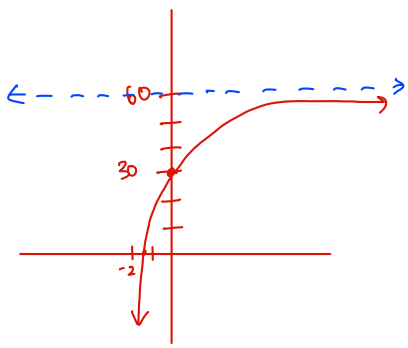
$\ln(e)^{-0.5t} = \ln(2)$

$-0.5t \ln e = \ln(2)$

$\frac{-0.5t}{-0.5} = \frac{\ln(2)}{-0.5}$

$t = -1.39$

$(-1.39, 0)$



b. How many words per minute can the average person type after 4 months?

$W(4) = -30e^{-0.5(4)} + 60 = \boxed{55.94 \text{ words per minute}}$

c. Interpret the meaning of the y-intercept.

Before starting the typing class, the average person can type 30 words per minute.

d. Interpret the meaning of the horizontal asymptote.

Even with the typing class, the average person will never type 60 or more words per minute.

e. When will the average person be able to type 50 words per minute?

$50 = -30e^{-0.5t} + 60$

$-60 \qquad\qquad -60$

$\frac{-10}{-30} = \frac{-30e^{-0.5t}}{-30}$

$\frac{1}{3} = e^{-0.5t}$

$\ln(1/3) = \ln(e)^{-0.5t}$

$\ln(1/3) = -0.5t \ln(e)$

$\frac{\ln(1/3)}{-0.5} = \frac{-0.5t}{-0.5}$

$t = 2.20$

$\boxed{\text{2.2 months after the start of the typing class}}$

17. In 2000, the world population was 6.09 billion. The world population grew at an annual rate of 1.18%.

   a. Write an exponential model for the world population, in billions, *t* years after 2000.

$y = 6.09(1 + 0.0118)^t$ $\boxed{y = 6.09(1.0118)^t}$

   b. What is the estimated world population for 2025?

$2025 - 2000 = 25$ $t = 25$

$y = 6.09(1.0118)^{25} = \boxed{8.17 \text{ billion}}$

   c. In what year did the world population hit 7 billion?

$$\frac{7}{6.09} = \frac{6.09(1.0118)^t}{6.09}$$

$$\frac{7}{6.09} = (1.0118)^t$$

$$\log(7/6.09) = \log(1.0118)^t$$

$$\log(7/6.09) = t\log(1.0118)$$

$$\frac{\log(7/6.09)}{\log(1.0118)} = t$$

$$11.87 = t$$

2011

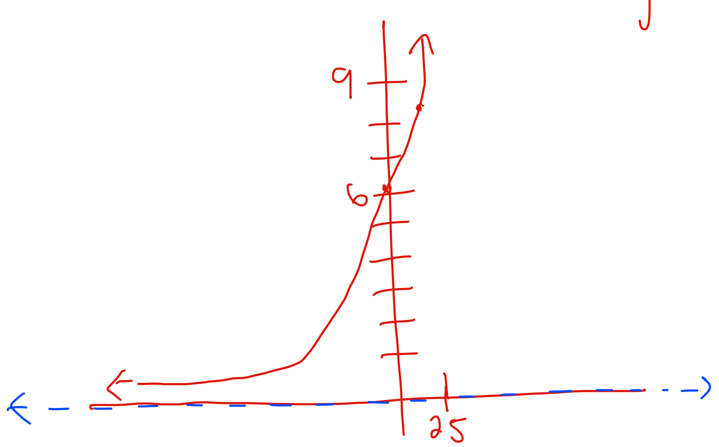
   d. Graph the equation. Find the horizontal asymptote, *x*-intercept, and *y*-intercept.

$y = 0$ none 6.09

growth

another point

(25, 8.17)



18. The first year of a charity walk event had an attendance of 500. The attendance, *y*, increases by 5% each year.

a. Write an exponential model for number of people attending the charity walk.

$y = 500(1+0.05)^t$ $\boxed{y = 500(1.05)^t}$

b. How many people will attend in the 10th year?

$y = 500(1.05)^{10} = \boxed{814 \text{ people}}$

c. When will there be 2000 people in attendance?

$$\frac{2000}{500} = \frac{500(1.05)^t}{500}$$

$$4 = (1.05)^t$$

$$\log(4) = \log(1.05)^t$$

$$\frac{\log(4)}{\log(1.05)} = \frac{t\log(1.05)}{\log(1.05)}$$

$\boxed{28.41 \text{ years}}$

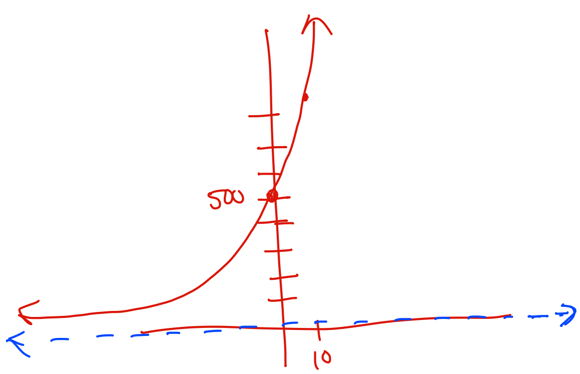
d. Graph the equation. Find the horizontal asymptote, *x*-intercept, and *y*-intercept.

$y=0$ none $(0, 500)$

growth

another point

$(10, 814)$



19. For a new novel, the function $y = 25000(0.72)^t$ models the number of books sold t months after the book was released.

   a. Is the number of books sold increasing or decreasing? By what percentage?

$\boxed{\text{decreasing}}$

$1 - r = 0.72$

$-r = -0.28$

$r = 28$

$\boxed{28\%}$

   b. How many more books were sold in month 3 than month 5?

month 3: $y = 25000(0.72)^3 = 9331$ books

month 5: $y = 25000(0.72)^5 = 4837$ books

$9331 - 4837 = \boxed{4494 \text{ books}}$

20. The amount g (in trillions of cubic feet) of natural gas consumed in the United States from 1940 to 1970 can be modeled by $y = 2.91(1.07)^t$ where $t = 0$ represents 1940.

   a. Is the consumption of natural gas increasing or decreasing? By what percentage?

$\boxed{\text{Increasing}}$

$1 + r = 1.07$

$r = 0.07$

$\boxed{7\%}$

   b. How much more natural gas, in trillions of cubic feet, was consumed in 1960 than 1950?

1960 $t = 20$ $y = 2.91(1.07)^{20} = 11.26$

1950 $t = 10$ $y = 2.91(1.07)^{10} = 5.72$

$11.26 - 5.72 = \boxed{5.54 \text{ trillion cubic feet}}$