

Derivative of Exponential and Logarithmic Functions

① $[b^x]' = b^x \cdot \ln b$

Special case : $b = e = 2.71828 \dots$

$[e^x]' = e^x$

② $(\log_b x)' = \frac{1}{x \cdot \ln b}$

Special case: $b = e = 2.71828 \dots$

$(\ln x)' = \frac{1}{x}$

Example: Find $f'(x)$

① $f(x) = 2025^x \Rightarrow f'(x) = 2025^x \cdot \ln 2025$

② $f(x) = 3^x \Rightarrow f'(x) = 3^x \cdot \ln 3$

③ $f(x) = (1/2)^x \Rightarrow f'(x) = (1/2)^x \cdot \ln(1/2)$

④ $f(x) = x^3 \cdot 7^x$

$(x^3)' = 3x^2$; $(7^x)' = 7^x \cdot \ln 7$

product rule

$$\Rightarrow f'(x) = (x^3)' \cdot 7^x + (7^x)' \cdot x^3$$

$$f'(x) = 3x^2 7^x + 7^x (\ln 7) \cdot x^3$$

$$\textcircled{3} \quad f(x) = \frac{e^x}{x^2 + 2}$$

quotient rule

$$\Rightarrow f'(x) = \frac{(e^x)' \cdot (x^2 + 2) - (x^2 + 2)' \cdot e^x}{(x^2 + 2)^2}$$

$$= \frac{e^x \cdot (x^2 + 2) - (2x) \cdot e^x}{(x^2 + 2)^2}$$

$$\textcircled{4} \quad f(x) = \log_7 x$$

$$\Rightarrow f'(x) = \frac{1}{x \cdot \ln 7}$$

$$\textcircled{5} \quad f(x) = \log_{19} x \Rightarrow f'(x) = \frac{1}{x \cdot \ln 19}$$

$$\textcircled{6} \quad f(x) = 6 \log_8 x \Rightarrow f'(x) = 6 \cdot \frac{1}{x \cdot \ln 8} = \frac{6}{x \cdot \ln 8}$$

$$\textcircled{7} \quad f(x) = 4 \ln x - 9 \log_{20} x + 7 \log_{2025} x$$

$$\Rightarrow f'(x) = \frac{4}{x} - \frac{9}{x \cdot \ln 20} + \frac{7}{x \cdot \ln 2025}$$

⑧ $f(x) = x \cdot \ln x$

$$f'(x) = \underset{\substack{\uparrow \\ \text{product} \\ \text{rule.}}}{(x)' \cdot \ln x + (\ln x)' \cdot x}$$

$$f'(x) = \ln x + \left(\frac{1}{x} \cdot x \right) = \ln x + 1$$

⑨ $f(x) = \frac{3^x + \ln x}{e^x + x^7}$

$$\begin{aligned} f'(x) &= \frac{(3^x + \ln x)' \cdot (e^x + x^7) - (e^x + x^7)' \cdot (3^x + \ln x)}{(e^x + x^7)^2} \\ &= \frac{(3^x \cdot \ln 3 + \frac{1}{x})(e^x + x^7) - (e^x + 7x^6)(3^x + \ln x)}{(e^x + x^7)^2} \end{aligned}$$

Assignment: Find $f'(x)$

① $f(x) = 11^x$

③ $f(x) = \left(\frac{1}{20}\right)^x$

⑤ $f(x) = 6^x - 7^x + 8^x$

② $f(x) = 200^x$

④ $f(x) = e^x$

⑥ $f(x) = 9^x + x^9$

$$(7) f(x) = x e^x$$

$$(9) f(x) = (x^2 + 3x) \cdot (3^x + 4^x)$$

$$(11) f(x) = \log_{17} x$$

$$(13) f(x) = 6 \ln x - 3 \log_2 x$$

$$(15) f(x) = \frac{3x}{e^x + 1}$$

$$(17) f(x) = \frac{2^x + e^x}{\log_2 x}$$

$$(8) f(x) = x^9 \cdot 9^x$$

$$(10) f(x) = x^3 \cdot (e^x + x)$$

$$(12) f(x) = \log_{19} x$$

$$(14) f(x) = \frac{\ln x}{x}$$

$$(16) f(x) = \frac{e^x}{4 \ln x}$$

$$(18) f(x) = \frac{3^x}{x^3 + \log_6 x}$$