

## Average Rate of Change and Instantaneous Rate of Change

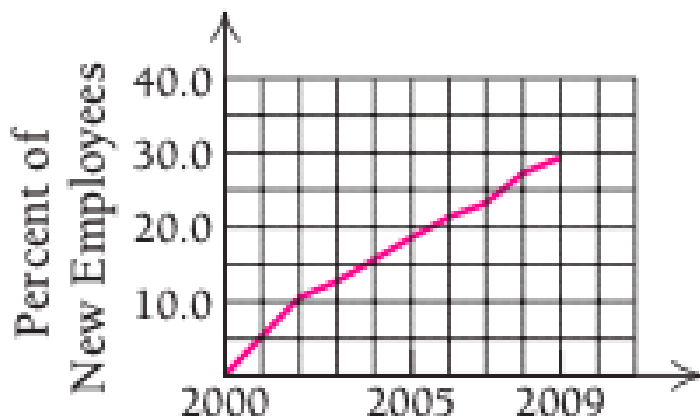
Let's say that a car travels 120 mi in 2 hr. Its *average rate of change (speed)* is 120 mi/2 hr, or 60 mi/hr (60 mph). Suppose that you are driving on the highway, and you look down at the speedometer to see that you are traveling at 60 mph. That is your *instantaneous rate of change*.

These are two quite different concepts. The first you are probably familiar with. The second involves Calculus. To understand instantaneous rate of change, we first need to develop a solid understanding of average rate of change.

### Average Rate of Change:

**Example 1:** Use the graph to estimate the average rate of change of the percentage of new employees on the following intervals:

#### Education

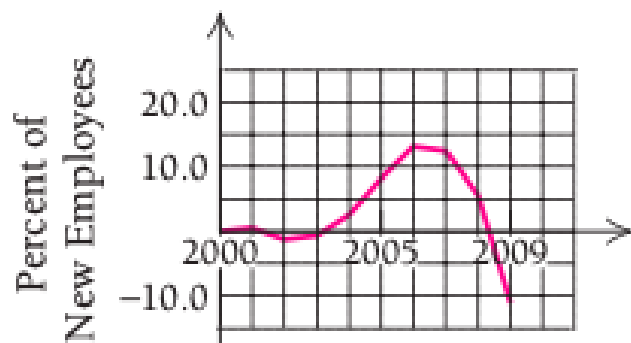


a. 2000 to 2005

b. 2005 to 2009

c. 2000 to 2009

#### Construction



a. 2000 to 2005

b. 2005 to 2009

c. 2000 to 2009

**Example 2:** When a balance of \$5000 is owed on a credit card and interest is begin charged at a rate of 22% per year, the total amount owed,  $A(t)$ , after  $t$  years is given by  $A(t) = 5000(1.21)^t$ . Assume that the credit card holder cannot make any payments on this card.

- a. Find the total amount owed after 2 years
- b. Find the total amount owed after 5 years
- c. What is the average rate of change in the total amount owed from 2 years to 5 years? Include the correct units.

**Example 3:** A company that manufactures sunglasses has a revenue function of  $R(x) = -0.01x^2 + 100x$ , where  $x$  is the number of pairs of sunglasses sold and  $R(x)$  is in dollars.

- a. What is the revenue when 100 pairs of sunglasses are sold?
- b. What is the revenue when 300 pairs of sunglasses are sold.
- c. Find the average rate of change in the company's revenue. Include the correct units.

**Example 4:** The same company in Example 3, has a cost function of  $C(x) = -0.05x^2 + 50x$ , where  $x$  is the number of pairs of sunglasses produced and  $C(x)$  is in dollars.

- a. What is the cost when 100 pairs of sunglasses are produced?
- b. What is the cost when 300 pairs of sunglasses are produced
- c. Find the average rate of change in the company's cost. Include the correct units.

**Example 5:** On October 14, 2012, Austrian skydiver Felix Baumgartner broke a world record for a high-altitude dive when he ascended 127,850 feet in a helium balloon and then went into a free fall lasting more than 4 minutes.

His elevation (in feet) above sea-level,  $t$  seconds after stepping off the balloon can be approximated by  $f(t) = 127,850 - 16t^2$

- a. Let's see if we can estimate his velocity exactly 30 seconds after leaving the balloon.

What is his average velocity between  $t = 20$  and  $t = 30$ ?

What is his average velocity between  $t = 30$  and  $t = 40$ ?

- b. Let's take an interval even closer to 30.

Find the average velocity between  $t = 29$  and  $t = 30$ .

Find the average velocity between  $t = 30$  and  $t = 31$ .

- c. Felix's actual velocity at 30 seconds was -960 ft/sec. Which of the estimates was closest? How could we get an even better estimate?

## Derivative:

### Derivative Rules

#### Constant

#### Power Rule

**Example 6:** Find the derivative

a. $y = x^7$	b. $f(x) = -3x$
c. $y = 12$	d. $f(x) = 2x^{15}$
e. $y = -5x^3 + 2x^2 - 3x + 1$	f. $f(x) = 3x^2 - 2x + 3$
g. $y = 2x + 3x^2$	h. $f(x) = 6 - 3x^3 + 6x^4$

**Example 7:** Simplify and then find  $f'(x)$  for each of the following functions.

a.  $f(x) = (5x + 1)^2$

b.  $f(x) = \frac{3x^4 - 3x^2 - 2x}{x}$

c.  $f(x) = (x + 3)(x + 2)$

d.  $f(x) = \frac{x^5 - 5x^3}{x^2}$

**Example 8:** Find the derivative of each function and then evaluate at the given  $x$ -value.

a. If $f(x) = x^2 + 4x - 5$ , find $f'(10)$	b. If $f(x) = x^3 + 2x - 11$ , find $f'(-2)$
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### Product Rule

**Example 9:** Find  $y'$  given that  $y = (2x^2 - 3x)(3x^4 - 3)$

**Example 10:** Find  $y'$  given that  $y = (2x^5 + x - 1)(3x - 2)$

## Quotient Rule

**Example 11:** Find  $f'(x)$  given that  $f(x) = \frac{2x^3+1}{3x^4-x+3}$

**Example 12:** Given that  $y = \frac{4x^2}{x^2-5x}$

**Derivative of  $f(x) = e^x$**

**Example 13: Find the following derivatives**

a. $y = 3e^x$	b. $y = e^{8x}$
c. $f(x) = e^{-x^2+4x-7}$	d. $f(x) = 4e^{5x} + x^2 - 3x + 7$
e. $y = e^{3x-2}$	f. $f(x) = e^{-x}$
g. $y = (x + 3)e^{x^2-x}$	h. $f(x) = (x^2 + 8x)e^x$

i.  $y = \frac{e^{-3x}}{x^3}$

j.  $f(x) = \frac{e^{2x}}{1+e^{2x}}$

### Derivative of logarithmic functions

**Example 14: Find the following derivatives**

a.  $y = 3 \ln x$

b.  $y = \ln (3x)$

c.  $f(x) = \ln (x^2 - 5)$

d.  $f(x) = \ln (4x^3 - 5x^2 + 2x - 6)$

e.  $y = 6 \ln(2x^2 + 1)$

f.  $y = \ln(x^3 + 2)$

g.  $y = x^5 \ln(2x)$

h.  $y = (10x^4 + 3x^3) \ln(7x + 2)$

i.  $f(x) = \frac{\ln x}{x^3}$

j.  $f(x) = \frac{x+5}{\ln(8x)}$