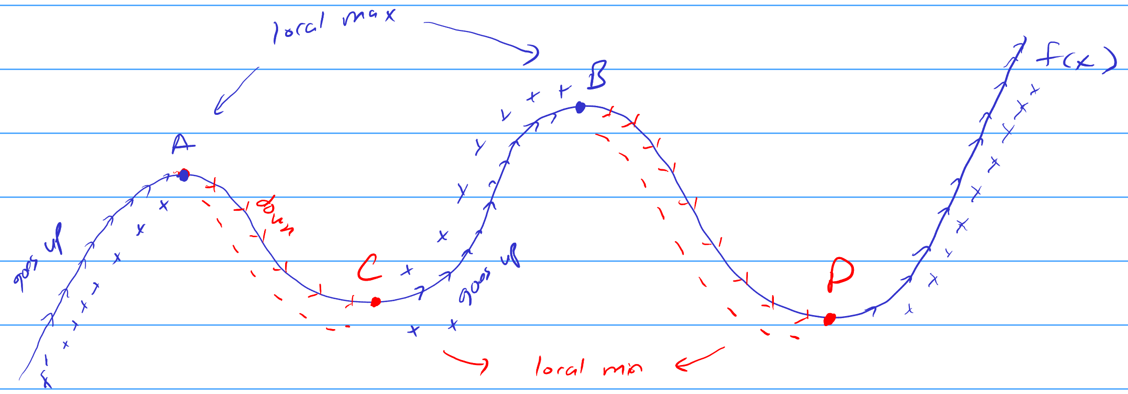
(*) Meaning and Applications of Derivatives.



If $f'(a) < 0 \Rightarrow f(x)$ is decreasing at $x = a$

If $f'(a) > 0 \Rightarrow f(x)$ is increasing at $x = a$

If $f'(a) = 0 \Rightarrow f(x)$ could potentially a local max or local min.

① If $f'(x)$ changes the sign from $(-)$ to $(+)$ at $x = a$ then $f(x)$ is [locally] minimized at $x = a$.

② If $f'(x)$ changes the sign from $(+)$ to $(-)$ at $x = a$ then $f(x)$ is locally maximized at $x = a$.

Notice : $f'(x)$ "usually" changes the sign when $f'(x) = 0$

Example :

The per-day cost function for the manufacture of portable MP3 players is given by

$$C(x) = 128,000 + 30x + x^3,$$

where x is the number of MP3 players manufactured per day. Assume that the company cannot manufacture more than 100 MP3 players per day. How many MP3 players should be manufactured in order to minimize average cost?

x : Number of items manufactured, $x \leq 100$

$C(x)$: cost to make x items.

Average cost to make 1 item is : $\dfrac{C(x)}{x}$

We need to find x to minimize $\dfrac{C(x)}{x} = f(x)$

The plan:

① Find and simplify $f'(x)$ ✓

② Solve for $f'(x) = 0$ ✓

(optional) ③ Verify that $f'(x)$ change the sign at the solution of $f'(x) = 0$.

④ Conclude that $f(x)$ is minimized at the solution of $f'(x) = 0$.

Step 1 : Find and simplify $f'(x)$.

$$f(x) = \frac{128{,}000 + 30x + x^3}{x}$$

quotient rule

$$f'(x) = \frac{(128{,}000 + 30x + x^3)' \cdot x - (x)' \cdot (128{,}000 + 30x + x^3)}{x^2}$$

$$f'(x) = \frac{(30 + 3x^2) \cdot x - 1 \cdot (128{,}000 + 30x + x^3)}{x^2}$$

$$\Rightarrow \quad f'(x) = \frac{\cancel{30x} + 3x^3 - 128{,}000 - \cancel{30x} - x^3}{x^2}$$

$$\Rightarrow \quad \boxed{f'(x) = \frac{2x^3 - 128{,}000}{x^2}}$$

Step 2: Solve for $f'(x) = 0$

$$\frac{2x^3 - 128{,}000}{x^2} = 0$$

$$\Leftrightarrow \quad 2x^3 - 128{,}000 = 0$$

$$\Leftrightarrow \quad 2x^3 = 128{,}000$$

$$\Leftrightarrow \quad x^3 = \frac{128{,}000}{2} = 64{,}000$$

$$\Leftrightarrow \quad x = \sqrt[3]{64000} = 40.$$

Step 3: Verify that $f'(x)$ changes the sign at the solution of $f'(x) = 0$.

(optional)

Step 4: Conclude that the average cost is minimized when the company make 40 items.

Assignment :

The per-day cost function for the manufacture of portable MP3 players is given by

$$C(x) = 686,000 + 30x + x^3,$$

where x is the number of MP3 players manufactured per day. Assume that the company cannot manufacture more than 100 MP3 players per day. How many MP3 players should be manufactured in order to minimize average cost?