Product Rule

$$[f(x) + g(x)]' = f'(x) + g'(x)$$

$$[f(x) - g(x)]' = f'(x) - g'(x)$$

$$[f(x) \cdot g(x)]' \neq f'(x) \cdot g'(x)$$

is diffrenent from

Here is the product rule:

$$[f(x) \cdot g(x)]' = f'(x) \cdot g(x) + g'(x) \cdot f(x)$$

Example: Find $f'(x)$

$$f(x) = (x^2 + x) \cdot (x^3 + 2)$$

$$f'(x) = (x^2 + x)' \cdot (x^3 + 2) + (x^3 + 2)' \cdot (x^2 + x)$$

$$= (2x + 1)(x^3 + 2) + (3x^2) \cdot (x^2 + x)$$

simplifying

$$= 2x^4 + 4x + x^3 + 2 + 3x^4 + 3x^3$$

$$= \boxed{5x^4 + 4x^3 + 4x + 2}$$

Example: Find $f'(x)$

$$f(x) = (2x+\sqrt{x}) \cdot (3\sqrt[4]{x} + \frac{1}{x})$$

product rule

$$f'(x) = (2x+\sqrt{x})' \cdot (3\sqrt[4]{x} + \frac{1}{x}) + (3\sqrt[4]{x} + \frac{1}{x})' \cdot (2x+\sqrt{x})$$

rewrite

$$= (2x + x^{1/2})'(3\sqrt[4]{x} + \frac{1}{x}) + (3x^{1/4} + x^{-1})'(2x+\sqrt{x})$$

$$= (2 + \frac{1}{2}x^{1/2-1})(3\sqrt[4]{x} + \frac{1}{x}) + (3\cdot\frac{1}{4}x^{1/4-1} - 1\cdot x^{-1-1})(2x+\sqrt{x})$$

$$= (2 + \frac{1}{2}x^{-1/2})(3\sqrt[4]{x} + \frac{1}{x}) + (\frac{3}{4}\cdot x^{-3/4} - x^{-2})(2x+\sqrt{x})$$

Assignment 14 — Part 1

Find $f'(x)$.

① $f(x) = (x^2 + 3x + 1) \cdot (x^3 + 4x^2 + 6)$

② $f(x) = (3\sqrt{x} + \frac{1}{x^2})(\sqrt[3]{x} - \frac{6}{x^3} + 7)$

Solution :

① $$f(x) = (x^2+3x+1)\cdot(x^3+4x^2+6)$$

$$f'(x) = (x^2+3x+1)'\cdot(x^3+4x^2+6) + \underbrace{(x^3+4x^2+6)'}\cdot(x^2+3x+1)$$

Power rule : $(x^n)' = n\cdot x^{n-1}$

$$f'(x) = (2\cdot x^{2-1}+3)(x^3+4x^2+6) + (3x^{3-1}+4\cdot 2x^{2-1})(x^2+3x+1)$$

$$= (2x+3)(x^3+4x^2+6) + (3x^2+8x)(x^2+3x+1)$$

② $$f(x) = \left(3\sqrt{x}+\frac{1}{x^2}\right)\left(\sqrt[3]{x}-\frac{6}{x^3}+7\right)$$

$$f'(x) = \left(3\sqrt{x}+\frac{1}{x^2}\right)'\cdot\left(\sqrt[3]{x}-\frac{6}{x^3}+7\right) + \left(\sqrt[3]{x}-\frac{6}{x^3}+7\right)'\cdot\left(3\sqrt{x}+\frac{1}{x^2}\right)$$

$$= \left(3x^{1/2}+x^{-2}\right)'\left(\sqrt[3]{x}-\frac{6}{x^3}+7\right) + \left(x^{1/3}-6x^{-3}+7\right)'\left(3\sqrt{x}+\frac{1}{x^2}\right)$$

$$\left[\sqrt[k]{x} = x^{1/k} \quad ; \quad \frac{1}{x^k} = x^{-k}\right]$$

$$= \left(3\cdot\frac{1}{2}x^{1/2-1}-2\cdot x^{-2-1}\right)\left(\sqrt[3]{x}-\frac{6}{x^3}+7\right) + \left(\frac{1}{3}x^{1/3-1}-6\cdot(-3)x^{-3-1}\right)\left(3\sqrt{x}+\frac{1}{x^2}\right)$$

$$= \left(\frac{3}{2}\cdot x^{-1/2}-2\cdot x^{-3}\right)\left(\sqrt[3]{x}-\frac{6}{x^3}+7\right)\left(\frac{1}{3}x^{-2/3}+18x^{-4}\right)\left(3\sqrt{x}+\frac{1}{x^2}\right)$$

Part 2 : Find $f'(x)$

① $f(x) = \left(7\sqrt{x} + \frac{1}{x}\right)\left(x^2 + 2x\right)$

② $f(x) = \left(x^6 - 3x + 1\right)\left(\sqrt[3]{x} + \frac{3}{x^5}\right)$