

Quadratic Functions

$$y = ax^2 + bx + c \quad (a \neq 0)$$

is not

Example:

$$y = \underbrace{2x^2}_{\text{quadratic term}} + \underbrace{4x}_{\text{linear term}} + \underbrace{6}_{\text{constant term}}$$

$$y = 4x^2 + 2026$$

$$y = 6x^2 + x$$

1. Solving Quadratics Equations

$$ax^2 + bx + c = 0 \quad (a \neq 0)$$

a. Factoring

Example: $2x^2 = 4x$

$$2x^2 = 4x$$

$$\Leftrightarrow 2x^2 - 4x = 0$$

$$\Rightarrow 2x \cdot (x - 2) = 0$$

factoring

$$\Leftrightarrow \begin{cases} 2x = 0 \\ x - 2 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} x = 0 \\ x = 2 \end{cases} \quad (\text{Two solutions})$$

Example: $x^2 + 7x = -10$

$$x^2 + 7x = -10$$

$$\Leftrightarrow x^2 + 7x + 10 = 0$$

$$\Leftrightarrow (x + 2)(x + 5) = 0$$

$$\left[\begin{array}{l} \text{note: we looked for 2 numbers:} \\ \text{sum} = 7 \\ \text{product} = 10 \\ \rightarrow 2 \text{ and } 5 \end{array} \right]$$

$$\Leftrightarrow \begin{cases} x + 2 = 0 \\ x + 5 = 0 \end{cases} \quad \Leftrightarrow \begin{cases} x = -2 \\ x = -5 \end{cases}$$

Example: $x^2 - 4x + 3 = 0$

$$x^2 - 4x + 3 = 0$$

$$\Leftrightarrow (x - 1)(x - 3) = 0$$

$$\left[\begin{array}{l} \text{sum} = -4 \\ \text{product} = 3 \end{array} \rightarrow -1; -3 \right]$$

$$\Leftrightarrow \begin{cases} x - 1 = 0 \\ x - 3 = 0 \end{cases} \quad \Leftrightarrow \begin{cases} x = 1 \\ x = 3 \end{cases}$$

Example: $x^2 - 25 = 0$

$$x^2 - 25 = 0$$

$$\Rightarrow x^2 = 25$$

$$\Rightarrow x = \pm \sqrt{25}$$

$$\Rightarrow x = \pm 5 \quad \text{or} \quad \begin{cases} x = 5 \\ x = -5 \end{cases}$$

Example: $x^2 - x = 12$

b. Quadratic Formula

$$ax^2 + bx + c = 0$$

case 1: $b^2 - 4ac < 0$

No solution!

case 2: $b^2 - 4ac \geq 0$

$$\text{solution(s)} : \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example:

$$1 \cdot x^2 + 4x + 1 = 0$$

$\downarrow \quad \quad \downarrow \quad \quad \downarrow$
 $a=1 \quad b=4 \quad c=1$

$$b^2 - 4ac = 4^2 - 4 \cdot 1 \cdot 1 = 12$$

$$\text{solution: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{12}}{2}$$

$$x = \frac{-4 + \sqrt{12}}{2} ; x = \frac{-4 - \sqrt{12}}{2}$$

Example: $x^2 + 10 = 4x$

$$x^2 + 10 = 4x$$

$$\Rightarrow x^2 - 4x + 10 = 0$$

$$a = 1 ; b = -4 ; c = 10$$

$$b^2 - 4ac = (-4)^2 - 4 \cdot 1 \cdot 10$$

$$= 16 - 40 = -24 < 0$$

NO Solution

Example: $x^2 - 6x + 10 = 0$

You try: Mixed Practice

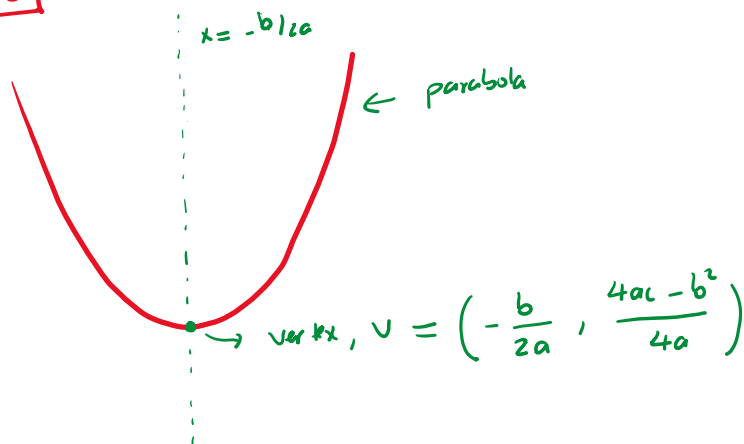
Solve each quadratic by factoring or the quadratic formula.

1. $2x^2 = 4x$	2. $x^2 - 4x = 12$
3. $2x^2 - x - 10 = 0$	4. $x^2 - 10x + 24 = 0$
5. $10x^2 - 25x = 0$	6. $x^2 + 5x + 2 = 2x$
7. $x^2 - 100x + 900 = 0$	8. $8x^2 + x - 75 = 0$

2. Graphs of Quadratic Functions

$$y = ax^2 + bx + c \quad (a \neq 0)$$

case 1 : $a > 0$



To graph we need

① Vertex

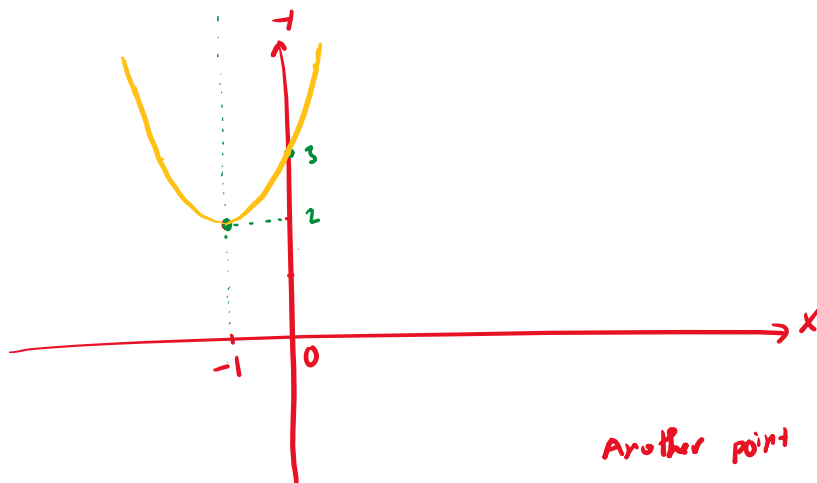
② Another point

Example: $y = x^2 + 2x + 3$

$$a = 1, b = 2, c = 3$$

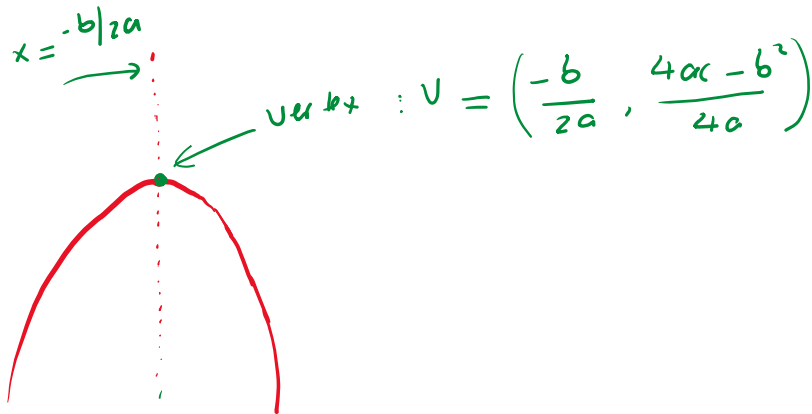
$$\text{Vertex } V = \left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$$

$$V = \left(-\frac{2}{2}, \frac{4 \cdot 1 \cdot 3 - 2^2}{4}\right) = (-1, 2)$$



Another point $x = 0, y = 3$

Case 2 : $a < 0$



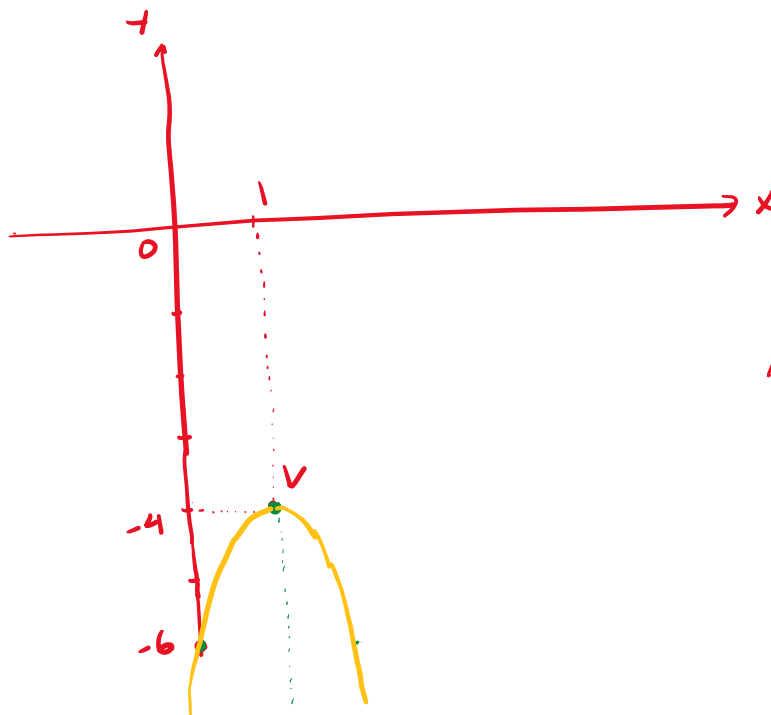
To graph we need

- ① Vertex
- ② Another point.

Example: $y = -2x^2 + 4x - 6$

$$a = -2, b = 4, c = -6$$

$$\begin{aligned} \text{Vertex : } V &= \left(-\frac{b}{2a}, \frac{4ac - b^2}{4a} \right) \\ &= \left(-\frac{4}{2 \cdot (-2)}, \frac{4 \cdot (-2) \cdot (-6) - 4^2}{4 \cdot (-2)} \right) \\ &= (1, -4) \end{aligned}$$



Another point

$$x = 0, y = -6$$

You Try

Graph the following quadratic functions. Label the vertex and a point in the graph.

1. $y = 2x^2 - 4x - 1$

2. $y = x^2 - 1$

3. $y = -x^2 + 1$

4. $y = -3x^2 - 12x + 1$

Example: $y = -x^2 + 6x - 9$