Quiz 2 Practice

1. In 1990, the tuition at a private college was \$15,000. Tuition has increased by about 5.2% each year.
   a. Write an equation to model the tuition at a private college t years after 1990.

$$y = 15000(1+0.052)^t$$

$$\boxed{y = 15000\,(1.052)^t}$$

   b. Estimate the tuition in 2024.

$2024 - 1990 = 34$

$$y = 15000\,(1.052)^{34} = \boxed{\$84067.17}$$

   c. Sketch a graph. Find the horizontal asymptote, x-intercept, and y-intercept.

$\boxed{y=0}$

$$\frac{15000(1.052)^t}{15000} = \frac{0}{15000}$$

$(1.052)^t = 0$

$\log(1.052)^t = \log 0$

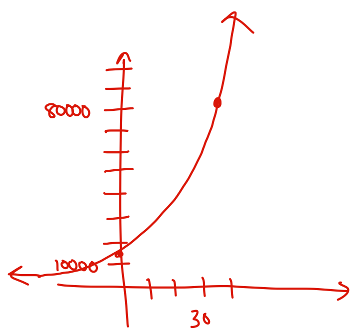
error

$\boxed{\text{no x-intercept}}$

$y = 15000(1.052)^0$

$y = 15000$

$\boxed{(0, 15000)}$



   d. When will the tuition be \$100,000?

$$\frac{100000}{15000} = \frac{15000(1.052)^t}{15000}$$

$$\frac{20}{3} = (1.052)^t$$

$$\log(20/3) = \log(1.052)^t$$

$$\frac{\log(20/3)}{\log(1.052)} = \frac{t\log(1.052)}{\log(1.052)}$$

$t = 37.42$

$\boxed{37.42 \text{ yrs or } 2027}$

2. A house was purchased for \$200,000 in 2005. The value of the home increases by 5% per year.
   a. Write an equation to model the value of the house *t* years after 2005.

$$y = 200000(1+.05)^t \qquad \boxed{y = 200000(1.05)^t}$$

   b. How much is the house worth today (2025)?

$$2025 - 2005 = 20$$

$$y = 200000(1.05)^{20} = \boxed{\$530659.54}$$

   c. Sketch a graph. Find the horizontal asymptote, *x*-intercept, and *y*-intercept.

$y = 0$

$$\frac{200000(1.05)^t}{200000} = \frac{0}{200000}$$

$$(1.05)^t = 0$$

$$\log(1.05)^t = \log 0$$

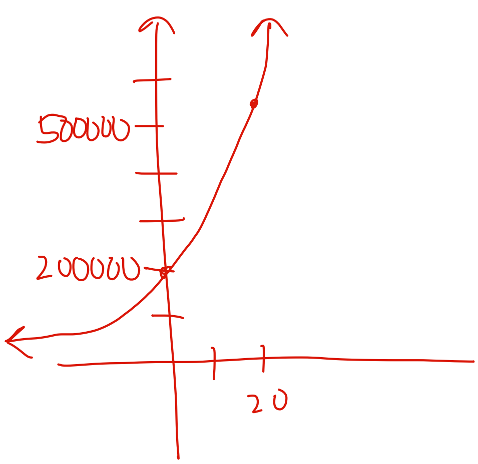
error

no x-intercept

$$y = 200000(1.05)^0$$

$$y = 200000$$

$(0, 200000)$



   d. When will the house be worth \$1 million (1,000,000)?

$$\frac{1000000}{200000} = \frac{200000(1.05)^t}{200000}$$

$$5 = (1.05)^t$$

$$\log(5) = \log(1.05)^t$$

$$\frac{\log(5)}{\log(1.05)} = \frac{t\log(1.05)}{\log(1.05)}$$

$$32.99 = t$$

$$\boxed{\begin{array}{c}32.99 \text{ yrs} \\ \text{or} \\ 2037\end{array}}$$

3. You drink a beverage with 120 mg of caffeine. Each hour, the caffeine in your system decreases by about 12%.

    a. Write an equation to model the amount of caffeine in your system, in mg, t hours after you drink it.

$$y = 120(1-0.12)^t$$

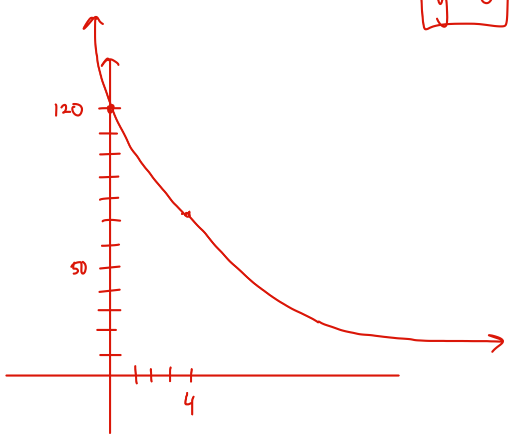
$$\boxed{y = 120(0.88)^t}$$

    b. How much caffeine is in your systems after 4 hours?

$$y = 120(0.88)^4 = \boxed{71.96 \text{ mg}}$$

    c. Sketch a graph. Find the horizontal asymptote, x-intercept, and y-intercept.

$\boxed{y=0}$



$$\frac{120(0.88)^t}{120} = \frac{0}{120}$$

$$(0.88)^t = 0$$

$$\log(0.88)^t = \log(0)$$

error

$\boxed{\text{no } x\text{-intercept}}$

$$y = 120(0.88)^0$$

$$y = 120$$

$\boxed{(0,120)}$

    d. How long until you have 10 mg of caffeine in your system?

$$\frac{10}{120} = \frac{120(0.88)^t}{120}$$

$$\frac{1}{12} = (0.88)^t$$

$$\log(1/12) = \log(0.88)^t$$

$$\frac{\log(1/12)}{\log(0.88)} = \frac{t\log(0.88)}{\log(0.88)}$$

$$19.44 = t$$

$\boxed{19.44 \text{ hrs}}$

4. You buy a new computer for $2100. The computer decreases by 1.2% each month.

a. Write the equation to model the value of the computer *t* months after you buy it.

$$y = 2100(1-0.012)^t \qquad \boxed{y = 2100(0.988)^t}$$

b. What will be the value of the computer after 6 months?

$$y = 2100(0.988)^6 = \boxed{\$1953.26}$$

c. Sketch a graph. Find the horizontal asymptote, *x*-intercept, and *y*-intercept.

$\boxed{y=0}$

$$\frac{2100(0.988)^t}{2100} = \frac{0}{2100}$$

$$(0.988)^t = 0$$

$$\log(0.988)^t = \log(0)$$

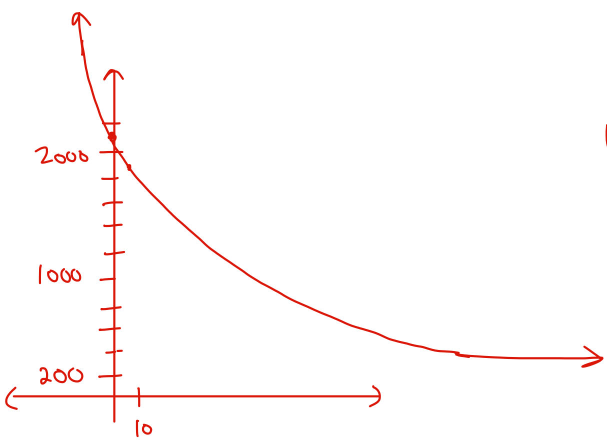
error

$\boxed{\text{no x-intercept}}$

$$y = 2100(0.988)^0$$

$$y = 2100$$

$(0, 2100)$



d. When will the computer have a value of $500?

$$\frac{500}{2100} = \frac{2100(0.988)^t}{2100}$$

$$\frac{5}{21} = (0.988)^t$$

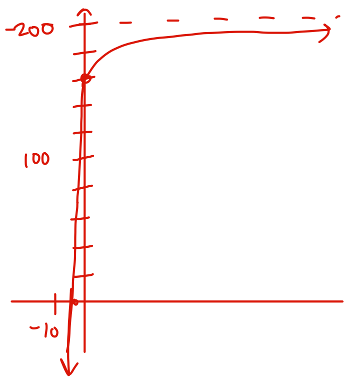
$$\log(5/21) = \log(0.988)^t$$

$$\frac{\log(5/21)}{\log(0.988)} = \frac{t\log(0.988)}{\log(0.988)}$$

$$118.87 = t$$

$\boxed{\text{118.87 months or 9.9 yrs}}$

5. A company's total cost, in millions of dollars, is given by $C(t) = -40e^{-1.3t} + 200$, where *t* is the time in years since the start-up date.
   a. Graph $C(t)$. Find the *x*-intercept, the *y*-intercept and the horizontal asymptote.



$-40e^{-1.3t} + 200 = 0$

$-200 \quad -200$

$\frac{-40e^{-1.3t}}{-40} = \frac{-200}{-40}$

$e^{-1.3t} = 5$

$\ln e^{-1.3t} = \ln(5)$

$\frac{-1.3t}{-1.3} = \frac{\ln(5)}{-1.3}$

$t = -1.24$

$\boxed{(-1.24, 0) \text{ x-int}}$

$-40e^{-1.3(0)} + 200$

$-40 + 200$

160

$\boxed{(0, 160) \text{ y-int}}$

$\boxed{y = 200 \text{ HA}}$

   b. What is the meaning of the *y*-intercept?

The startup costs are \$160 million.

   c. What is the meaning of the horizontal asymptote?

The total costs will never go above \$200 million.

   d. When will the company's cost be \$180 million?

$180 = -40e^{-1.3t} + 200$

$-200 \qquad -200$

$\frac{-20}{-40} = \frac{-40e^{-1.3t}}{-40}$

$0.5 = e^{-1.3t}$

$\ln(0.5) = \ln(e^{-1.3t})$

$\frac{\ln(0.5)}{-1.3} = \frac{-1.3t}{-1.3}$

$t = 0.53$

$\boxed{0.53 \text{ yrs}}$

6. It is reasonable for a manufacturer to expect the daily output of a new worker to start out slow and continue to increase over time, but then tend to level off, never exceeding a certain amount. A firm manufactures 5G smart phones and determines that after working *t* days, the efficiency, in number of phones produced per day, of most workers can be modeled by the function
$N(t) = 80 - 70e^{-0.13t}$

a. Graph $N(t)$. Find the *x*-intercept, the *y*-intercept and the horizontal asymptote.

$80-70e^{-0.13t}=0$

$\frac{-70e^{-0.13t}}{-70}=\frac{-80}{-70}$

$e^{-0.13t}=\frac{8}{7}$

$\ln e^{-0.13t}=\ln(8/7)$

$-0.13t=\ln(8/7)$

$t=\ln(8/7)/-0.13$

$t=-1.03$

$(-1.03, 0)$

x-int

$80-70e^{-0.13(0)}$

$80-70$

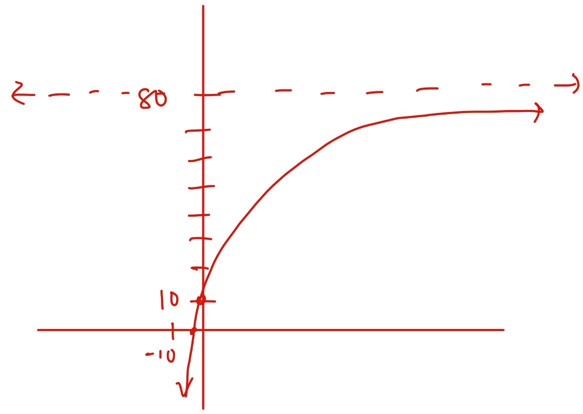
10

$(0, 10)$

y-int

$y=80$

HA



b. What is the meaning of the *y*-intercept?

a worker can start off making 10 phones per day

c. What is the meaning of the horizontal asymptote?

a worker will never make more than 80 phones per day.

d. When will the worker be able to produce 75 smart phones?

$75=80-70e^{-0.13t}$

$-80 \quad -80$

$\frac{-5}{-70}=\frac{-70e^{-0.13t}}{-70}$

$\frac{1}{14}=e^{-0.13t}$

$\ln(1/14)=\ln e^{-0.13t}$

$\frac{\ln(1/14)}{-0.13}=\frac{-0.13t}{-0.13}$

$20.3=t$

$\boxed{20.3 \text{ days}}$

7. A company invests \$30,000 in an account with 3.2% interest compounded monthly.
   a. How much money will be in the account after 8 years?

$$A = 30000\left(1+\frac{0.032}{12}\right)^{12(8)} = \boxed{\$38739.38}$$

   b. How much interest will be earned in 8 years?

$$38739.38 - 30000 = \boxed{\$8739.38}$$

   c. When will the investment be worth triple its original amount?

$$\frac{90000}{30000} = \frac{30000\left(1+\frac{0.032}{12}\right)^{12t}}{30000}$$

$$3 = \left(1+\frac{0.032}{12}\right)^{12t}$$

$$3 = (1.002666667)^{12t}$$

$$\log(3) = \log(1.002666667)^{12t}$$

$$\frac{\log(3)}{\log(1.002666667)} = \frac{12t\log(1.002666667)}{\log(1.002666667)}$$

$$\frac{412.53}{12} = \frac{12t}{12}$$

$$34.38 = t$$

$$\boxed{34.38 \text{ yrs}}$$

8. A family is saving for their child's college education. They invest \$10,000 in an account that pays 2.75% interest compounded quarterly.
   a. How much money will be in the account after 18 years?

$$A = 10000\left(1+\frac{0.0275}{4}\right)^{4(18)} = \boxed{\$16377.22}$$

   b. How much interest will be earned in 18 years?

$$16377.22 - 10000 = \boxed{\$6377.22}$$

   c. When will the account have \$50,000 in it?

$$\frac{50000}{10000} = \frac{10000\left(1+\frac{0.0275}{4}\right)^{4t}}{10000}$$

$$5 = (1.006875)^{4t}$$

$$\log(5) = \log(1.006875)^{4t}$$

$$\frac{\log(5)}{\log(1.006875)} = \frac{4t\log(1.006875)}{\log(1.006875)}$$

$$\frac{234.9}{4} = \frac{4t}{4}$$

$$58.73 = t$$

$$\boxed{58.73 \text{ years}}$$

9. A company invests $50,000 in an account with 1.8% interest continuously compounded.
   a. How much money will be in the account after 10 years?

$$A = 50000e^{0.018(10)} = \boxed{\$59860.87}$$

   b. How much interest will be earned in 10 years?

$$59860.87 - 50000 = \boxed{\$9860.87}$$

   c. When will the investment be worth $75,000?

$$\frac{75000}{50000} = \frac{50000e^{0.018t}}{50000}$$

$$1.5 = e^{0.018t}$$

$$\ln(1.5) = \ln e^{0.018t}$$

$$\frac{\ln(1.5)}{0.018} = \frac{0.018t}{0.018}$$

$$22.53 = t$$

$$\boxed{22.53 \text{ years}}$$

10. You have $4000 to invest in an account with 2.3% interest continuously compounded.
    a. How much money will be in the account after 3 years?

$$A = 4000e^{0.023(3)} = \boxed{\$4285.74}$$

    b. How much interest will be earned in 3 years?

$$4285.74 - 4000 = \boxed{\$285.74}$$

    c. When will the account have $5000 in it?

$$\frac{5000}{4000} = \frac{4000e^{0.023t}}{4000}$$

$$1.25 = e^{0.023t}$$

$$\ln(1.25) = \ln e^{0.023t}$$

$$\frac{\ln(1.25)}{0.023} = \frac{0.023t}{0.023}$$

$$\boxed{9.7 \text{ years}}$$

$$9.7 = t$$

11. What interest rate will allow $5300 to grow to $8000 in 5 years if interest is compounded daily?

$$\frac{8000}{5300} = \frac{5300\left(1+\frac{r}{365}\right)^{365(5)}}{5300}$$

$$\sqrt[1825]{\frac{80}{53}} = \sqrt[1825]{\left(1+\frac{r}{365}\right)^{1825}}$$

$$1.000225634 = 1 + \frac{r}{365}$$

$$-1 \qquad -1$$

$$365\left(0.000225634 = \frac{r}{365}\right)365$$

$$0.0824 = r$$

$$\boxed{8.24\%}$$

12. What interest rate will allow $20,000 to double in 12 years if interest is compounded monthly?

$$\frac{40000}{20000} = \frac{20000\left(1+\frac{r}{12}\right)^{12(12)}}{20000}$$

$$\sqrt[144]{2} = \sqrt[144]{\left(1+\frac{r}{12}\right)^{144}}$$

$$1.004825126 = 1 + \frac{r}{12}$$

$$-1 \qquad -1$$

$$12\left(0.004825126 = r/12\right)12$$

$$r = 0.0579$$

5.79%

13. How much money must be initially deposited into an account with 4.6% interest compounded daily if you want to have $10,000 in 5 years?

$$10000 = P\left(1+\frac{0.046}{365}\right)^{365(5)}$$

$$\frac{10000}{1.258581771} = \frac{P(1.258581771)}{1.258581771}$$

$$7945.45 = P$$

$$\boxed{\$7945.45}$$

14. How much money must be initially deposited into an account with 1.9% interest compounded quarterly if you want to have $1,000 in 2 years?

$$1000 = P\left(1 + \frac{0.019}{4}\right)^{4(2)}$$

$$\frac{1000}{1.038637787} = \frac{P(1.038637787)}{1.038637787}$$

$$962.80 = P$$

$$\boxed{\$962.80}$$

15. The number of cell phone subscribers (in millions) in the United States can be modeled by $y = 233(1.058)^t$, where $t = 0$ represents the year 2006.

a. What was the number of cell phone subscribers in 2006?

$\boxed{233 \text{ million}}$

b. Is the rent increasing or decreasing? By what percentage?

$1.058 - 1 = 0.058$ $\boxed{5.8\% \text{ increase}}$

16. A cup of coffee is left out on a countertop. The temperature of the coffee, in degrees Fahrenheit, t minutes after it is left out can be modeled by $y = 169.1(0.971)^t$,. Let t = 0 represent 8 am

a. What was the temperature of the coffee at 8 am?

$\boxed{169.1\ ^\circ F}$

b. Is the temperature increasing or decreasing? By what percentage?

$1 - 0.971 = 0.029$

$\boxed{2.9\% \text{ decrease}}$