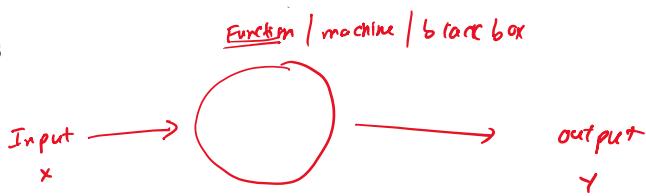
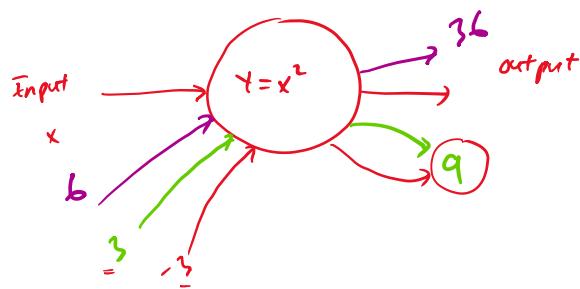


Linear Functions

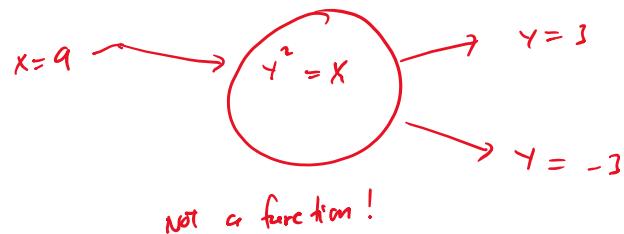
Functions



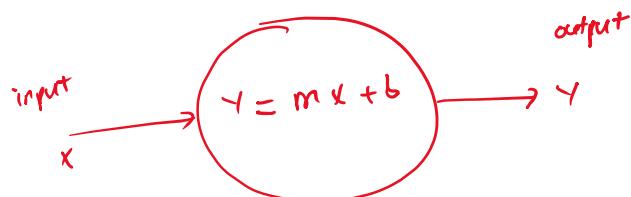
Example :



- If a "machine" gives us two outputs for one given input, that "machine" is not a function
- For a function, two different inputs can produce the same output.



Linear Function



Linear function :

$$y = \underbrace{mx}_{\text{ }} + b$$

where m and b are known constants / numbers .

Example : $y = 3x + 4$ ($m = 3, b = 4$)

$$y = -6x + 9 \quad (m = -6, b = 9)$$

$$y = 7x \quad (b = 0)$$

$$y = 2026 \quad (m = 0, b = 2026)$$

$$y = \frac{x}{9} + 4$$

$$\Leftrightarrow y = \frac{1}{9} \cdot x + 4$$

\uparrow \uparrow
 m b

equivalent to $\Rightarrow y = \frac{7x}{8} - \frac{1}{2}$

$$y = \frac{7}{8} \cdot x - \frac{1}{2}$$

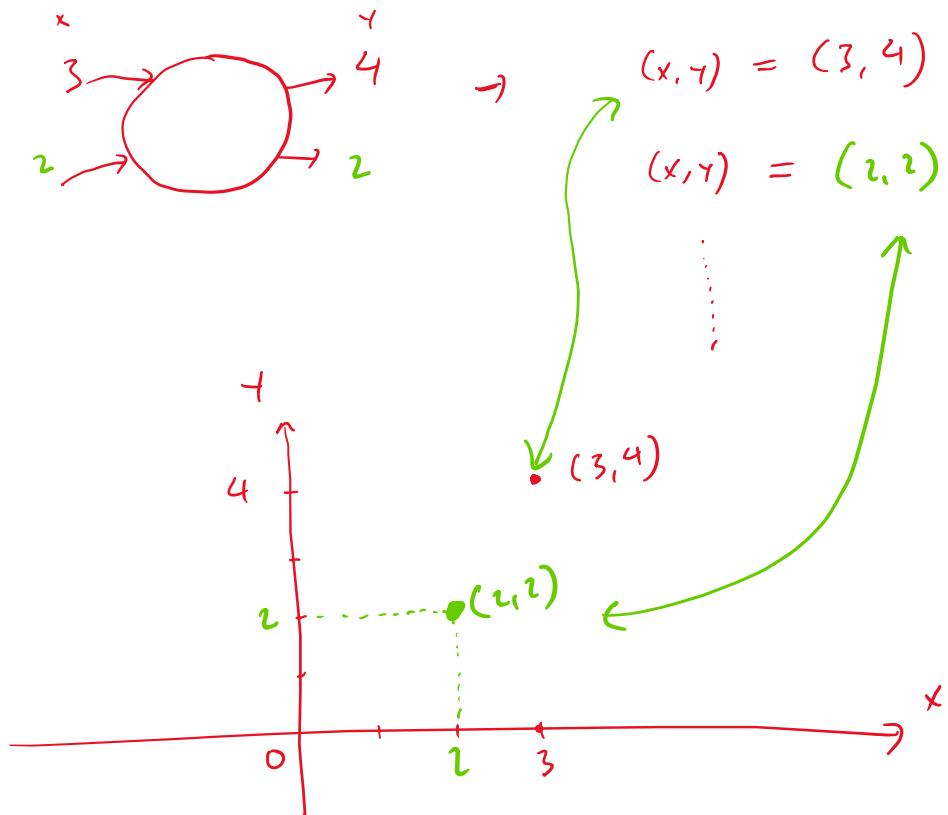
$$(m = \frac{7}{8}, b = -\frac{1}{2})$$

(non-linear) $\rightarrow y = \frac{6}{x} + 4$

$$\Rightarrow y = 2^x$$

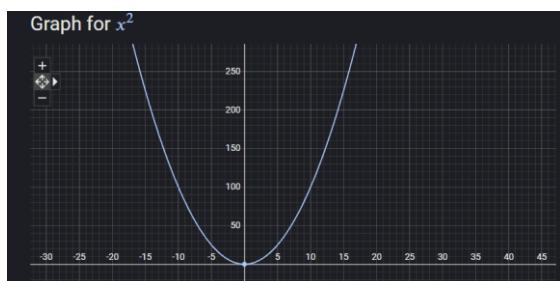
The graph of a function

The graph of a function is the collection of ALL the pairs of input and output the function produced that presented on the xy -plane.

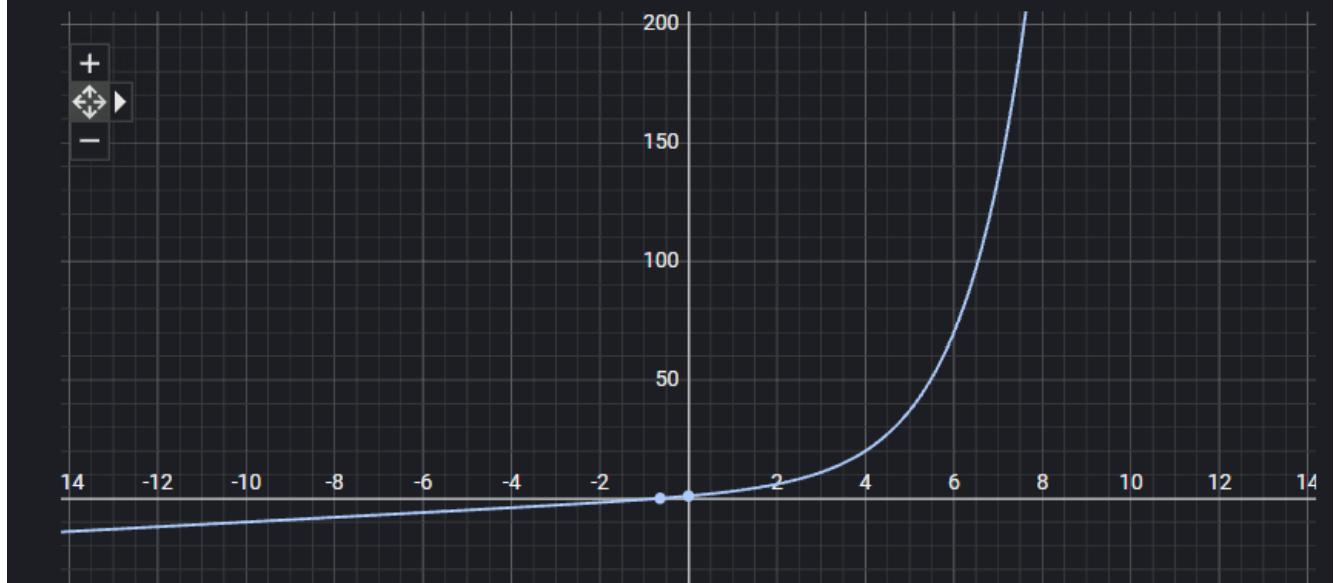


Some examples

$$\textcircled{1} \quad y = x^2$$



Graph for $2^x + x$



The graph of linear functions

The graph of a linear function is a line.

so to graph a linear function , we just need

2 pairs of input and output then connect

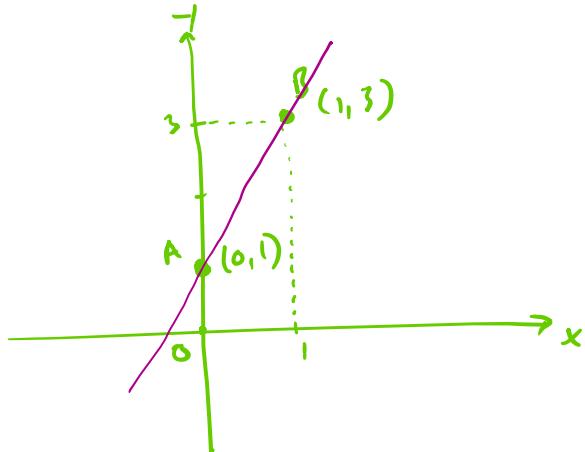
the two pairs to make the line.

Example:

Graph $y = 2x + 1$

① pick $x = 0$, $y = 2 \cdot 0 + 1 = 1$
 $\Rightarrow (x, y) = (0, 1)$

② pick $x = 1$, $y = 2 \cdot 1 + 1 = 3$
 $\Rightarrow (x, y) = (1, 3)$



Practice Problems

Graph the following linear functions. Make sure to show the calculations for the pairs as in the example.

1. $y = 3x + 2$

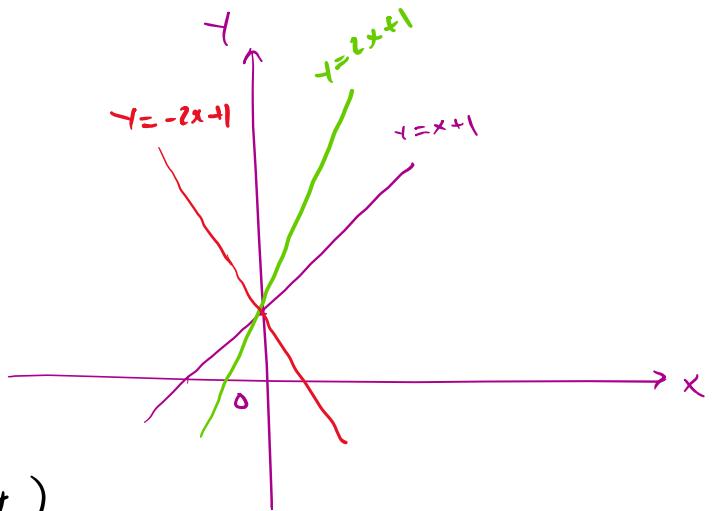
2. $y = -2x + 1$

Slope

① $y = 1 \cdot x + 1$ (slope = 1)

② $y = 2x + 1$ (slope = 2)

③ $y = -2x + 1$ (slope = -2)



we observe that (it is in fact correct)

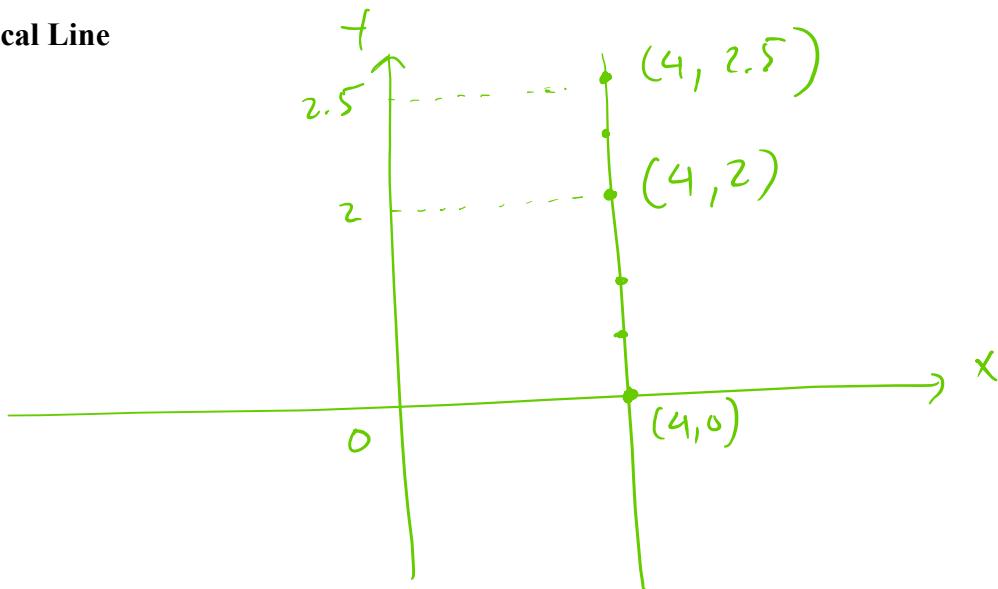
① Lines with positive slope go up. (from left to right)

② Lines with negative slope go down

③ Lines with greater positive slope go up faster.

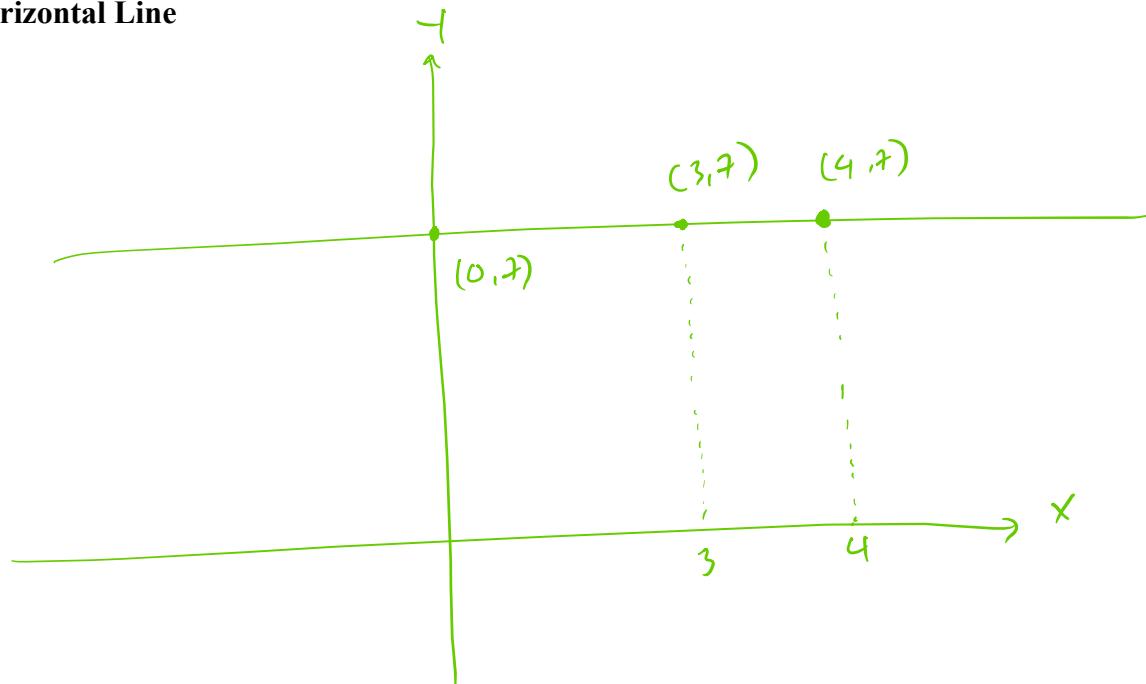
Also : ④ lines with greater negative slope go down faster.

Vertical Line



- ① This does not present a function.
- ② The equation of this line is $x = 4$
- ③ This line has undefined slope
infinity

Horizontal Line



① The equation of this line: $y = 7$

② The slope of this line is: 0

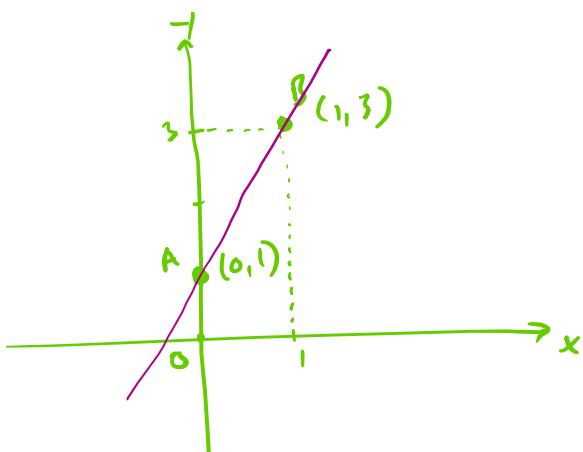
Write the equation of a line

Example:

Graph $y = 2x + 1$

$$\text{④ pick } x = 0, y = 2 \cdot 0 + 1 = 1 \\ \Rightarrow (x, y) = (0, 1) \quad \text{A}$$

$$\text{⑤ pick } x = 1, y = 2 \cdot 1 + 1 = 3 \\ \boxed{(x, y) = (1, 3)} \quad \text{B}$$



Can we recover the equation of the line knowing that the line passes through the points $(\underline{0}, \underline{1})$ and $(\underline{1}, \underline{3})$?

Yes!

We will use the following result. (Result 1)

The equation of the line passing through the two points (x_1, y_1) and (x_2, y_2) is

$$y = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1) + y_1$$

Example: Write the equation of the line passing through two points $(0, 1)$ and $(1, 3)$.

using the formula with

$$(0, 1) \quad (1, 3)$$
$$\begin{matrix} y_1 \\ x_1 \end{matrix} \quad \begin{matrix} y_2 \\ x_2 \end{matrix}$$

$$y = \frac{3-1}{1-0} \cdot (x - 0) + 1$$

$$\Leftrightarrow y = 2x + 1$$

Result 2.

The equation of the line with slope m and passing through the point (x_1, y_1) is

$$y = m(x - x_1) + y_1$$

Example: Write the equation of the line with slope 2

and passing through $(1, 3)$

$$\underline{m = 2}$$

using the formula:

$$y = 2(x - 1) + 3$$

$$\Leftrightarrow y = 2x - 2 + 3$$

$$\Leftrightarrow \boxed{y = 2x + 1}$$

Assignment 2

1. Write the equation of the line passing through $(1, 3)$ and $(0, 2)$
2. Write the equation of the line with slope -3 and passing through $(2, 4)$