**Quadratic Functions**

$y = ax^2 + bx + c \quad (a \neq 0)$

quadratic term; linear term; constant term; $a$ is not

Example:

- $y = 3x^2 + 6x + 1$
- $y = x^2 + 100$
- $y = 4x^2 + x$

## 1. Solving Quadratics Equations

$ax^2 + bx + c = 0$

### a. Factoring

**Example:** $2x^2 = 4x$

$2x^2 = 4x$

$2x^2 - 4x = 0$

factoring

$2x \cdot (x - 2) = 0$

$\Leftrightarrow \begin{cases} 2x = 0 \\ x - 2 = 0 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ x = 2 \end{cases}$

**Example:** $x^2 + 7x = -10$

$x^2 + 7x = -10$

$x^2 + 7x + 10 = 0$

we look for 2 numbers : sum = 7

product = 10

$\Rightarrow$ 2 ; 5

$(x + 2) \cdot (x + 5) = 0$

$\Leftrightarrow \begin{cases} x + 2 = 0 \\ x + 5 = 0 \end{cases}$

$\Leftrightarrow \begin{cases} x = -2 \\ x = -5 \end{cases}$

**Example**: $x^2 - 4x + 3 = 0$

$x^2 - 4x + 3 = 0$

Look for 2 numbers : sum = $-4$

product = 3

$\rightarrow -1 ; -3$

$(x - 1)(x - 3) = 0$

$$\begin{cases} x-1=0 \\ x-3=0 \end{cases} \Leftrightarrow \begin{cases} x=1 \\ x=3 \end{cases}$$

**Example:** $x^2 - 25 = 0$ Formula:

$$x^2 - 25 = 0$$

$$\Leftrightarrow x^2 = 25$$

$$\Leftrightarrow x = 5 \; ; \; x = -5$$

**Example:** $x^2 - x = 12$

**b. Quadratic Formula**

$$ax^2 + bx + c = 0$$

① Case 1: $b^2 - 4ac < 0$

No solution!

② Case 2: $b^2 - 4ac \geq 0$ (greater or equal to)

Solution(s):

$$\begin{cases} x = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a} \\ x = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a} \end{cases}$$

**Example:** $x^2 + 1 = 4x$

$\Leftrightarrow \quad x^2 - 4x + 1 = 0$

$a = 1, \quad b = -4, \quad c = 1$

$b^2 - 4ac = (-4)^2 - 4 \cdot 1 \cdot 1 = 12$

solutions:

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{4 + \sqrt{12}}{2}$$

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{4 - \sqrt{12}}{2}$$

**Example:** $x^2 - 6x + 10 = 0$

$1 \cdot x^2 - 6x + 10 = 0$

$a = 1, \quad b = -6, \quad c = 10$

$b^2 - 4ac = (-6)^2 - 4 \cdot 1 \cdot 10$

$= -4 < 0$

$\Rightarrow$ No solution.

**Example:** $x^2 - 6x + 9 = 0$

$a = 1, \quad b = -6, \quad c = 9$

$b^2 - 4ac = (-6)^2 - 4 \cdot 1 \cdot 9 = 0$

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{6 + 0}{2} = 3$$

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{6 - 0}{2} = 3$$

$\Rightarrow \boxed{x = 3}$

**Mixed Practice**

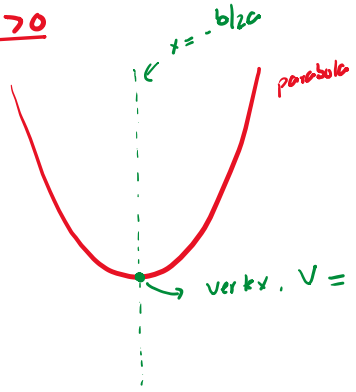
Solve each quadratic by factoring or the quadratic formula.

| | |
|---|---|
| 1. $2x^2 = 4x$ | 2. $x^2 - 4x = 12$ |
| 3. $2x^2 - x + 10 = 0$ | 4. $x^2 - 10x + 24 = 0$ |
| 5. $10x^2 - 25x = 0$ | 6. $x^2 + 5x + 2 = 2x$ |
| 7. $x^2 - 100x + 900 = 0$ | 8. $8x^2 + x - 75 = 0$ |

2. **Graphs of Quadratic Functions**

$y = ax^2 + bx + c$

case 1: $a > 0$




To graph we need 2 points : ① The vertex

② Another point

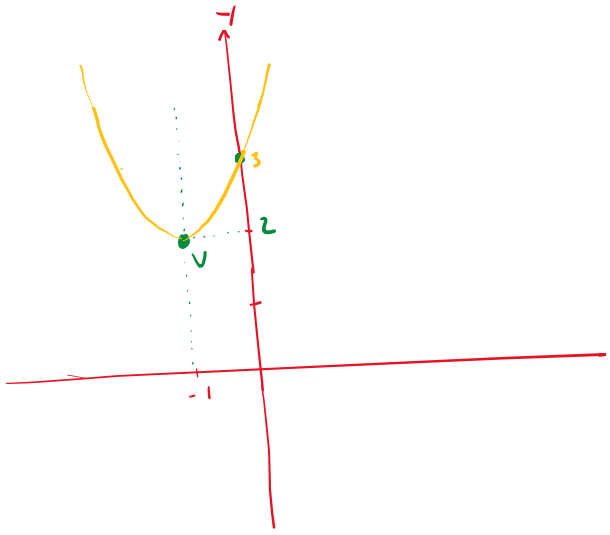
**Example:** $y = x^2 + 2x + 3$

$a = 1, \quad b = 2, \quad c = 3$

① $V = \left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$

$= \left(\frac{-2}{2 \cdot 1}, \frac{4 \cdot 1 \cdot 3 - 2^2}{4 \cdot 1}\right)$

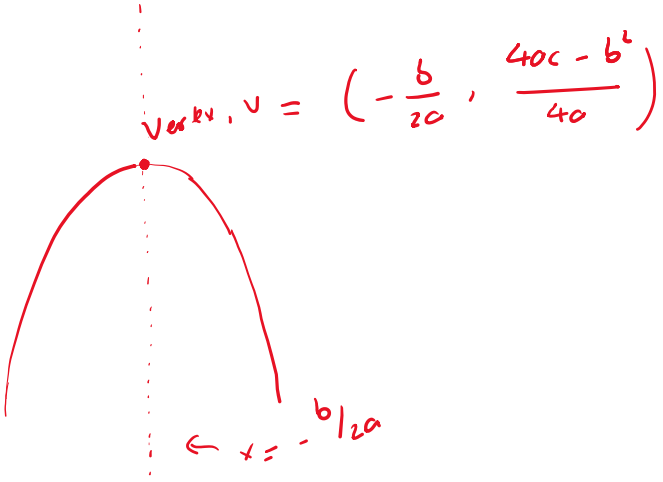
$= (-1, 2)$




② Another point :

pick $x = 0 \Rightarrow y = 3$

Case 2 : $a < 0$




To graph we need 2 points : ① Vertex

② Another point.

**Example:** $y = -2x^2 + 4x + 4$

$a = -2$, $b = 4$, $c = 4$

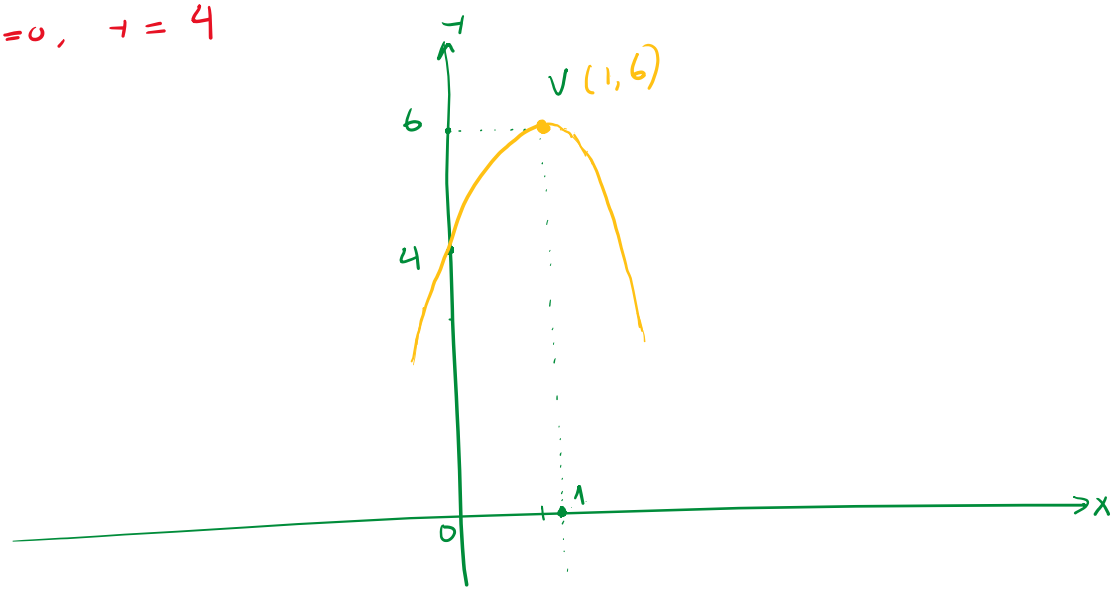
Vertex : $V = \left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$

$= \left(\frac{-4}{2\cdot(-2)}, \frac{4\cdot(-2)\cdot 4 - 4^2}{4\cdot(-2)}\right)$

$= (1, 6)$

Another point :

pick $x = 0$, $y = 4$



**You Try**

Graph the following quadratic functions. Label the vertex and a point in the graph.

1. $y = 2x^2 - 4x - 1$

2. $y = x^2 - 1$

3. $y = -x^2 + 4$

4. $y = -3x^2 - 12x + 1$

**Example**: $y = -x^2 + 6x - 9$