

## Exam 2 – Practice 2 Math 110.

Exam Guidelines This is an in-class, written exam with a 75-minute time limit.

- **Permitted Materials:** You may use a basic calculator and formula sheets.
- **Formula Sheet Restrictions:** Your sheets must contain formulas only; no examples or worked problems are permitted. All sheets will be inspected at the start of the exam.
- **Prohibited Items:** Phones and all other smart devices are strictly forbidden.
- **Academic Integrity:** The use of AI is prohibited. Any AI usage will result in an automatic F for the exam and may lead to failing the entire course.
- Show **ALL** your work for credits.

1. Solve each quadratic by factoring or the quadratic formula.

a.  $x^2 = 4x - 4$

$$x^2 - 4x + 4 = 0$$
$$\Rightarrow (x - 2)(x - 2) = 0$$

Solution:  $x = 2$

b.  $x^2 - 6x + 5 = 0$

$$(x - 1)(x - 5) = 0$$

Solution:  $x = 5, x = 1$

c.  $x^2 - 2x + 9 = 0$

$$a = 1 ; b = -2 ; c = 9$$

$$b^2 - 4ac < 0$$

$\Rightarrow$  No Solution

d.  $x^2 + 7x = 2x - 6$

$$x^2 + 7x - 2x + 6 = 0$$

$$\Rightarrow x^2 + 5x + 6 = 0$$

$$\Rightarrow (x+2)(x+3) = 0$$

Solution:  $x = -2, x = -3$

2. Graph of the quadratic functions. Label the vertex and another point.

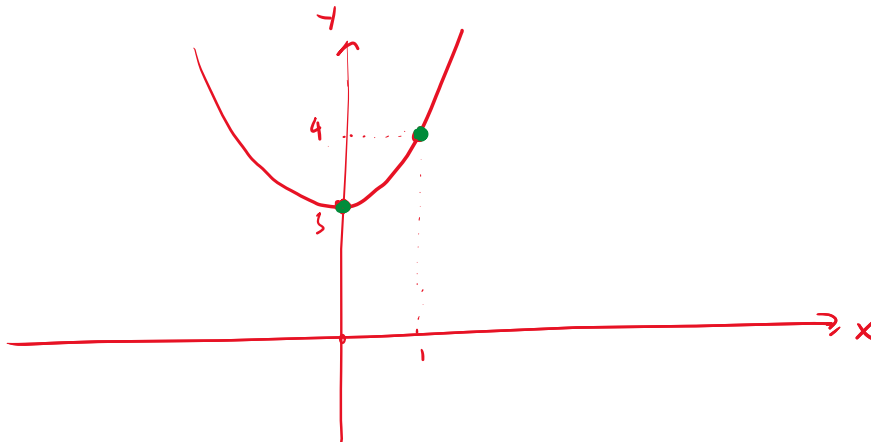
a.  $y = x^2 + 3$

$$a = 1 ; b = 0 ; c = 3$$

(\*) Vertex :  $x = -\frac{b}{2a} = 0$

$$y = \frac{4ac - b^2}{4a} = \frac{12}{4} = 3$$

(\*) Another point :  $x = 1 ; y = 4$



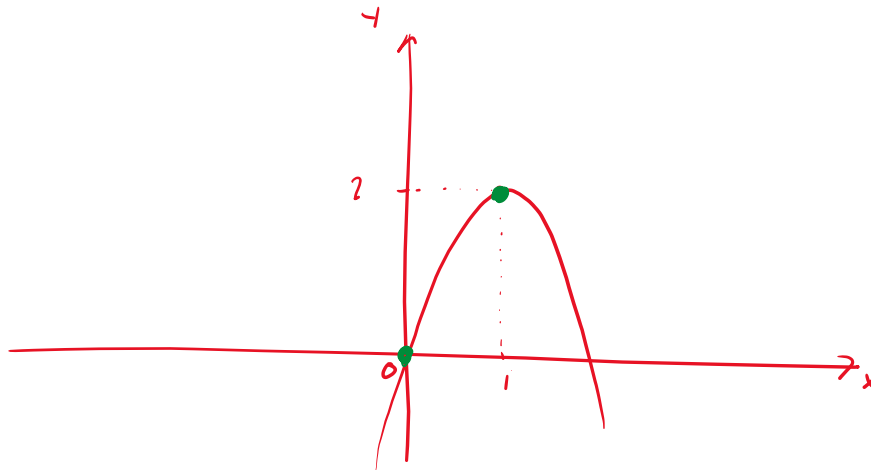
b.  $y = -2x^2 + 4x$

$a = -2$  ;  $b = 4$  ;  $c = 0$

② Vertex :  $x = -\frac{b}{2a} = -\frac{4}{(-2) \cdot 2} = 1$

$y = \frac{4ac - b^2}{4a} = \frac{-4^2}{-8} = 2$

③ Another point :  $x = 0$  ;  $y = 0$



3. Suppose that in a monopoly market the total cost per week of producing a high-tech product is given by  $C = 200 + 100 + 2q^2$ . Suppose further that the sales price function for this product is  $p = 300 - 2q$ .

- a. Find the revenue function in term of  $q$ .

$R = p \cdot q = (300 - 2q) \cdot q = -2q^2 + 300q$

- b. Find the number of units that will **maximize the revenue**.

$$R = -2q^2 + 300q$$

$$a = -2; b = 300$$

$$\text{Maximized at } q = -\frac{b}{2a} = -\frac{300}{2 \cdot (-2)} = 75$$

- c. Find the profit function

$$\text{Profit} = R - C$$

$$= -2q^2 + 300q - (200 + 100q + 2q^2)$$

$$= -4q^2 + 200q - 200$$

- d. Find the number of units that will give **break-even** for the product

$$\text{Profit} = 0$$

$$\Rightarrow -4q^2 + 200q - 200 = 0$$

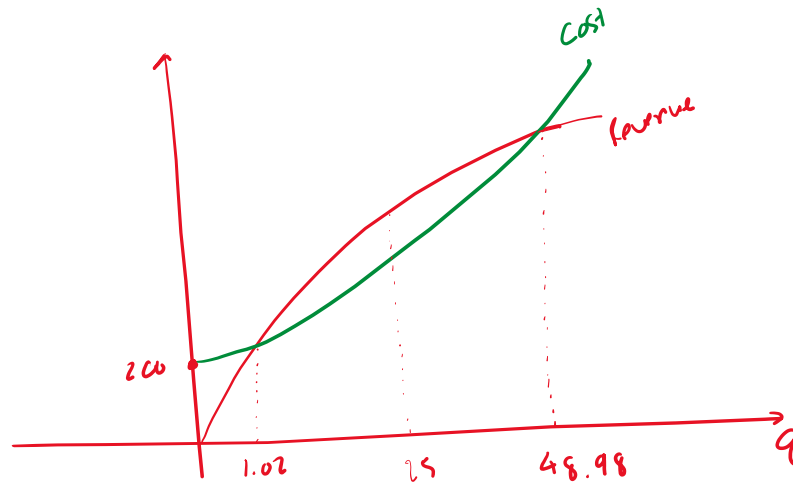
$$a = -4; b = 200; c = -200$$

$$q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \begin{cases} q \approx 1.02 \\ q = 48.98 \end{cases}$$

- e. Find **the maximum profit** and the number of products needed to maximize the profit.

$$\text{maximized at } q = -\frac{b}{2a} = -\frac{200}{-8} = 25$$

- f. Graph the revenue function and the cost function label the break-even points, fixed cost, and the maximized profit point.



4. On a certain route, an airline carries 4000 passengers per month, each paying \$25. A market survey indicates that for each \$1 increase in the ticket price, the airline will lose 50 passengers.
- What is the airline's current revenue?
  - Create an income (revenue) function if " $x$ " is defined as the number of \$1 price increases
  - Find the number of \$1 price increases that will maximize the revenue.

- d. Find the new ticket price (that will maximize the revenue)
  - e. Find the number of passengers at that price in d.
  - f. Find the new maximum income (income at that price in d)
5. If the supply function for a commodity is given by  $p = 2q^2 + 6q$  and the demand function is given by  $p = 180 - 3q^2$ , find the point of market equilibrium (Supply equals Demands).