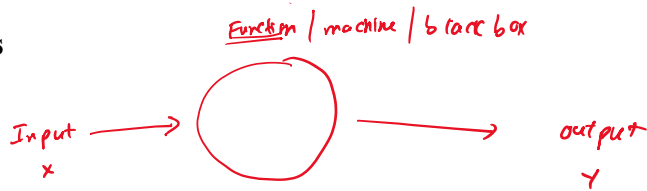
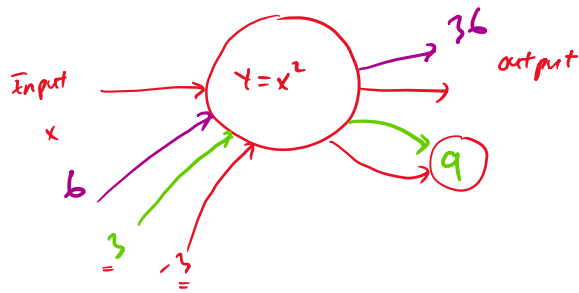


Linear Functions

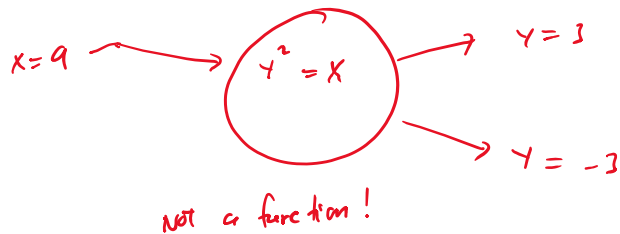
Functions



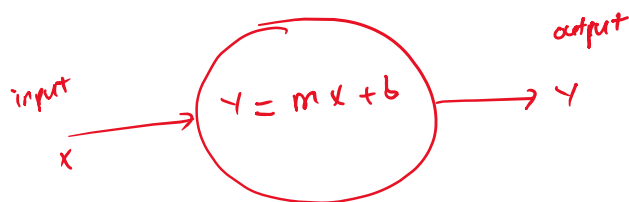
Example :



- If a “machine” gives us two outputs for one given input, that “machine” is not a function
- For a function, two different inputs can produce the same output.



Linear Function



Linear function :

$$y = \underbrace{mx}_{\text{known constants}} + \underbrace{b}_{\text{numbers}}$$

where m and b are known constants / numbers .

Example : $y = 3x + 4$ ($m=3, b=4$)

$$y = -6x + 9 \quad (m=-6, b=9)$$

$$y = 7x \quad (b=0)$$

$$y = 2026 \quad (m=0, b=2026)$$

$$y = \frac{x}{9} + 4$$

$$\Leftrightarrow y = \underset{\substack{\uparrow \\ m}}{\frac{1}{9}} \cdot x + \underset{\substack{\uparrow \\ b}}{4}$$

equivalent to



$$y = \frac{7x}{8} - \frac{1}{2}$$

$$\Leftrightarrow y = \frac{7}{8} \cdot x - \frac{1}{2}$$

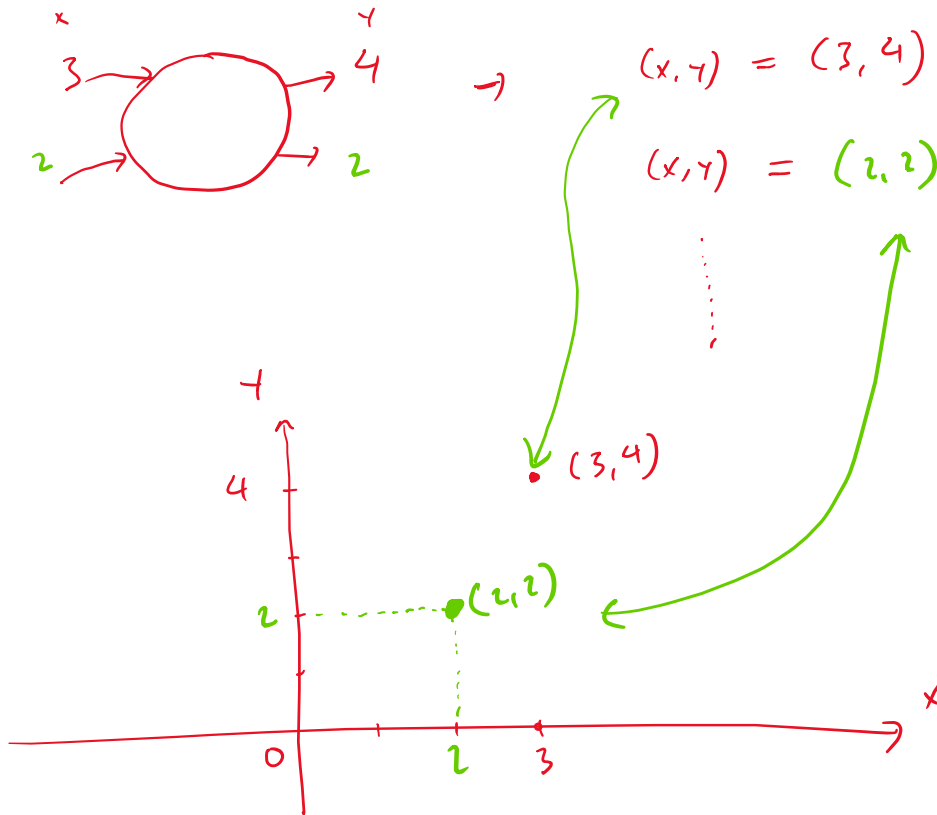
$$(m=7/8, b=-1/2)$$

(non-linear) $\rightarrow y = \frac{6}{x} + 4$

$$\rightarrow y = 2^x$$

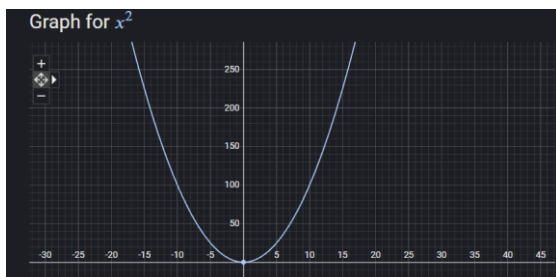
The graph of a function

The graph of a function is the collection of ALL the pairs of input and output the function produced that presented on the xy -plane.

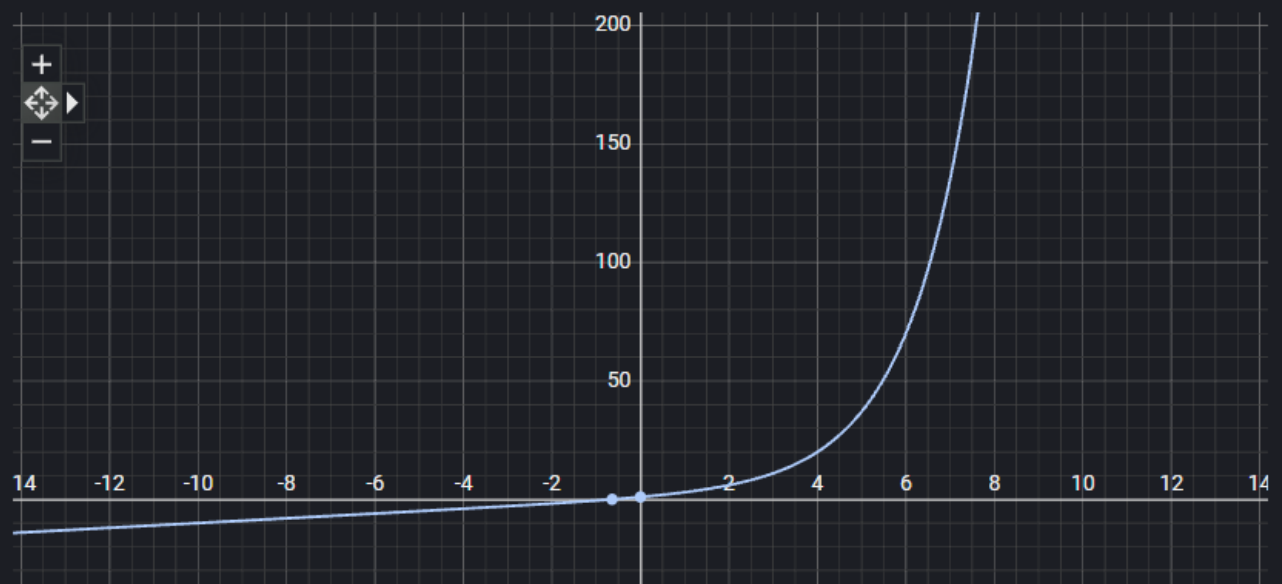


Some examples

① $y = x^2$



Graph for $2^x + x$



The graph of linear functions

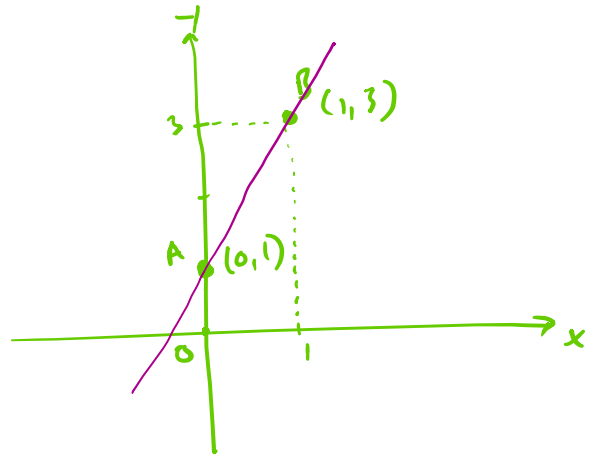
The graph of a linear function is a line.
So to graph a linear function, we just need
2 pairs of input and output then connect
the two pairs to make the line.

Example :

Graph $y = 2x + 1$

① pick $x = 0$, $y = 2 \times 0 + 1 = 1$
 $\Rightarrow (x, y) = (0, 1)$ ← A

② pick $x = 1$, $y = 2 \times 1 + 1 = 3$
 $(x, y) = (1, 3)$ ← B



Practice Problems

Graph the following linear functions. Make sure to show the calculations for the pairs as in the example.

1. $y = 3x + 2$

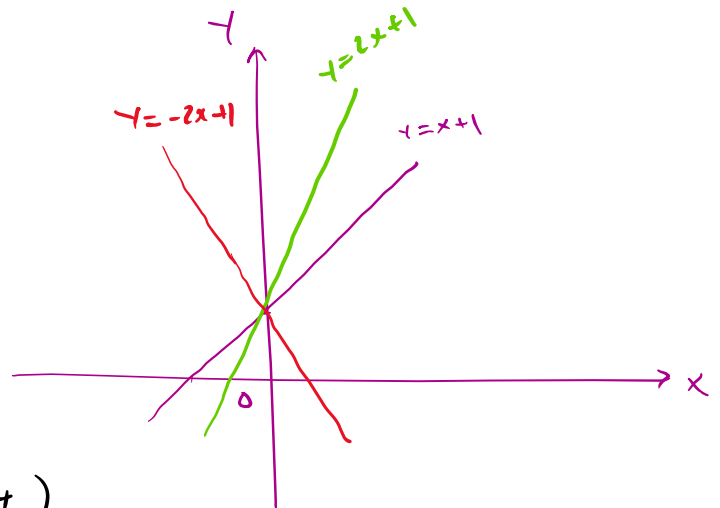
2. $y = -2x + 1$

Slope

① $y = 1 \cdot x + 1$ (slope = 1)

② $y = 2x + 1$ (slope = 2)
↑ slope

③ $y = -2x + 1$ (slope = -2)
↑ slope



we observe that (it is in fact correct)

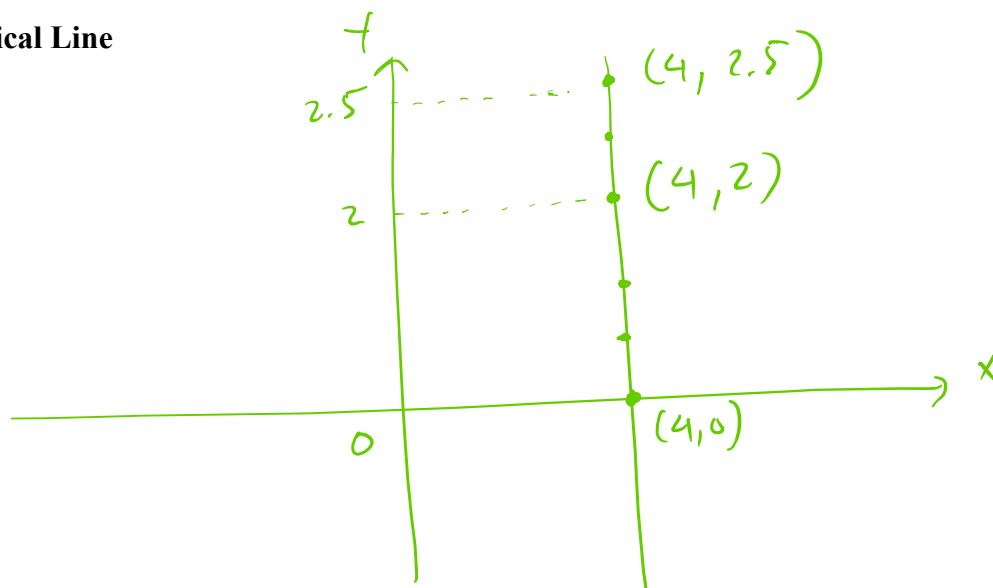
① Lines with positive slope go up. (from left to right)

② Lines with negative slope go down

③ Lines with greater positive slope go up faster.

Also : (4) lines with greater negative slope go down faster.

Vertical Line

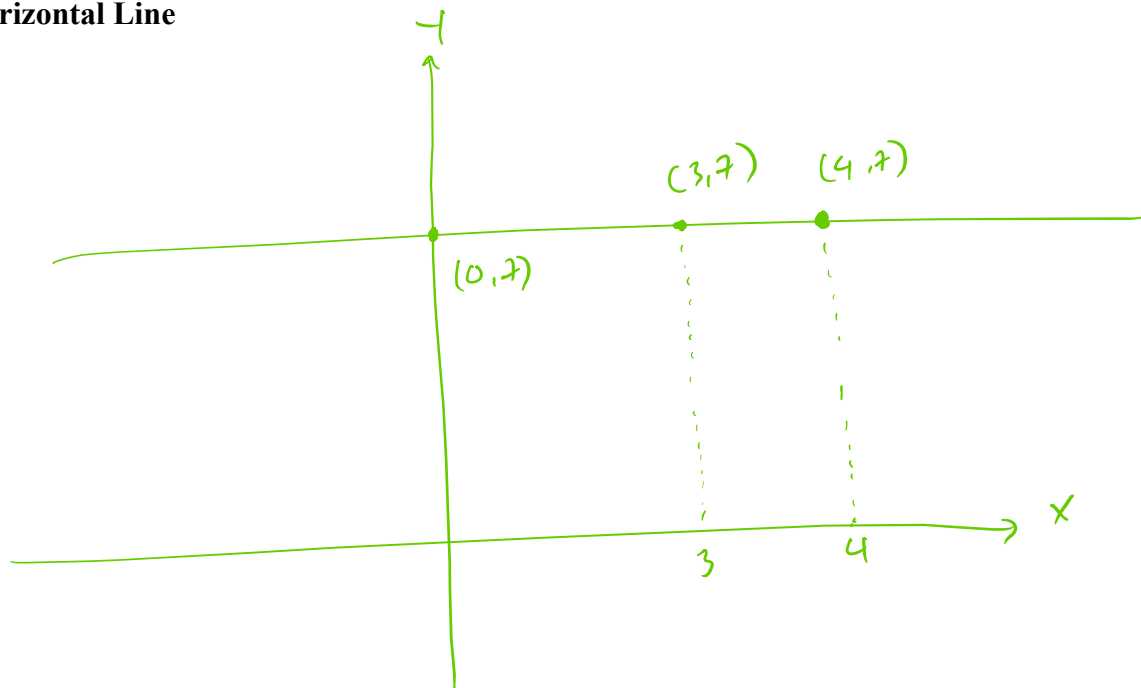


(1) This does not present a function.

(2) The equation of this line is $x = 4$

(3) This line has undefined slope
infinity

Horizontal Line



① The equation of this line: $y = 7$

② the slope of this line is : 0

Write the equation of a line

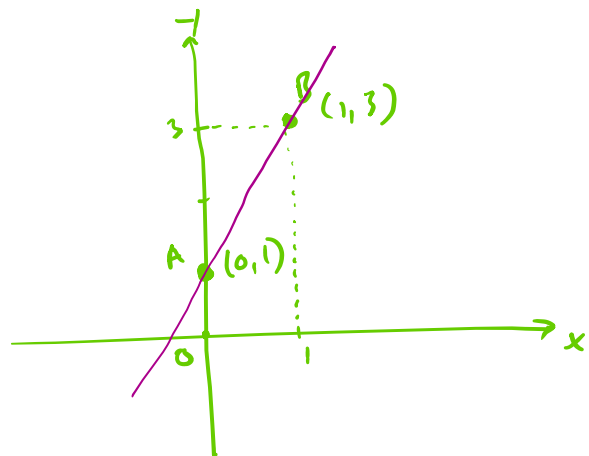
Example :

Graph

$$y = 2x + 1$$

⊛ pick $x = 0$, $y = 2 \times 0 + 1 = 1$
 \Rightarrow $(x, y) = (0, 1)$ ← A

⊛ pick $x = 1$, $y = 2 \times 1 + 1 = 3$
 $(x, y) = (1, 3)$ ← B



Can we recover the equation of the line knowing that the line passes through the points (0,1) and (1,3)?

Yes!

we will use the following result. (Result 1)

The equation of the line passing through the two points (x_1, y_1) and (x_2, y_2) is

$$y = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1) + y_1$$

Example: Write the equation of the line passing through two points $(0,1)$ and $(1,3)$.

using the formula with $(0,1)$ $(1,3)$
 x_1 y_1 x_2 y_2

$$y = \frac{3-1}{1-0} \cdot (x - 0) + 1$$

$$\Leftrightarrow y = 2x + 1$$

Result 2

The equation of the line with slope m and passing through the point (x_1, y_1) is

$$y = m(x - x_1) + y_1$$

Example: Write the equation of the line with slope 2 and passing through $(1, 3)$

x_1 y_1

$$m = 2$$

using the formula:

$$y = 2(x - 1) + 3$$

$$\Rightarrow y = 2x - 2 + 3$$

$$\Rightarrow \boxed{y = 2x + 1}$$

Assignment 2

1. Write the equation of the line passing through $(1, 3)$ and $(0, 2)$
2. Write the equation of the line with slope -3 and passing through $(2, 4)$