Exam 1 Review

1. Solve by factoring or the quadratic formula

a. $\frac{2x^2 - 6x - 20}{2} = \frac{0}{2}$

$x^2 - 3x - 10 = 0$ multiply -10

$(x-5)(x+2) = 0$ add -3

$\boxed{x=5}$ $\boxed{x=-2}$ $-5, 2$

b. $3x^2 + 4x - 10 = 0$

$a=3$ $b=4$ $c=-10$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(3)(-10)}}{2(3)} = \frac{-4 \pm \sqrt{136}}{6}$$

$x = \frac{-4+\sqrt{136}}{6} = 1.28$ $x = \frac{-4-\sqrt{136}}{6} = -2.61$

$\boxed{x=1.28}$ $\boxed{x=-2.61}$

c. $\frac{5x^2 + 10x - 40}{5} = \frac{0}{5}$

$x^2 + 2x - 8 = 0$ multiplies -8

$(x-2)(x+4) = 0$ adds to 2

$\boxed{x=2}$ $\boxed{x=-4}$ $-2, 4$

d. $6x^2 - x - 3 = 0$

$a = 6$ $b = -1$ $c = -3$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(6)(-3)}}{2(6)}$$

$$x = \frac{1 \pm \sqrt{73}}{12}$$

$x = \frac{1+\sqrt{73}}{12} = 0.80$ $x = \frac{1-\sqrt{73}}{12} = -0.63$

$\boxed{x=0.80}$ $\boxed{x=-0.63}$

Given each of the equations,

a. Find the vertex
b. Find the x-intercepts
c. Find the y-intercept
d. Graph

2. $y = x^2 + 4x - 5$

a) $x = \frac{-b}{2a} = \frac{-4}{2(1)} = -2$

$y = (-2)^2 + 4(-2) - 5$

$y = -9$

$\boxed{(-2, -9)}$

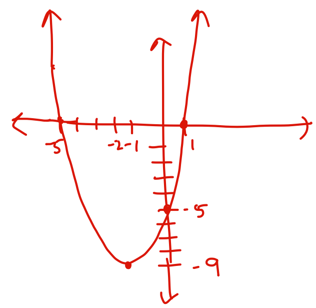
b) $x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$\boxed{x = -5}$ $\boxed{x = 1}$

c) $(0, -5)$

d)



3. $y = x^2 - 6x - 7$

a) $x = \frac{-b}{2a} = \frac{-(-6)}{2(1)} = 3$

$y = 3^2 - 6(3) - 7 = -16$

$(3, -16)$

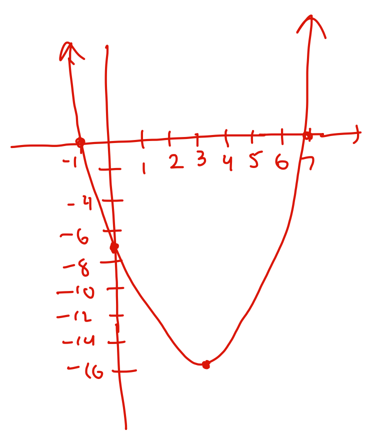
b) $x^2 - 6x - 7 = 0$

$(x-7)(x+1) = 0$

$\boxed{x = 7}$ $\boxed{x = -1}$

c) $(0, -7)$

d)



4. $y = x^2 - 2x - 15$

a) $x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = 1$

$y = 1^2 - 2(1) - 15$

$y = -16$

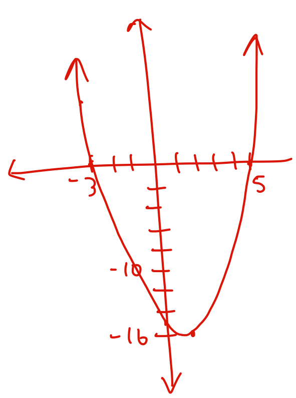
$(1, -16)$

b) $x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$\boxed{x = 5}$ $\boxed{x = -3}$

c) $(0, -15)$



5. In a small apartment complex, all 16 apartments are rented at a monthly rent of \$500. The rental manager has found from experience that each \$50 increase in the monthly rent results in an empty apartment. If x represents the number of \$50 rent increases, then the revenue function is

$$R(x) = (16 - x)(500 + 50x).$$

a. Multiply out $R(x)$

$R(x) = 8000 + 800x - 500x - 50x^2$

$\boxed{R(x) = -50x^2 + 300x + 8000}$

b. Find the number of \$50 rent increases needed to maximize revenue

$x = \frac{-b}{2a} = \frac{-300}{2(-50)} = 3$ $\boxed{x = 3}$

c. Find the new rent amount

$500 + 50(3) = \boxed{\$650}$

d. Find the number of apartments rented

$16 - 3 = \boxed{13}$

e. Find the maximum revenue

$650(13) = \boxed{\$8450}$

6. The manager of a peach orchard is trying to decide when to arrange for picking the peaches. If they are picked now, the average yield per tree will be 100 pounds, which can be sold for \$0.40 per pound. Past experience shows that the yield per tree will increase about 6 pounds per week, while the price will decrease about \$0.02 per pound per week. If x represents the number of weeks that the manager should wait, then the revenue function is $R(x) = (100 + 6x)(0.40 - 0.02x)$.

a. Multiply out $R(x)$

$R(x) = 40 - 2x + 2.4x - 0.12x^2$

$R(x) = -0.12x^2 + 0.4x + 40$

b. Find the number of weeks needed to maximize revenue.

$x = \frac{-0.4}{2(-0.12)} = 1.67$ round up to 2

c. Find the number of pounds per tree.

$100 + 6(2) = \boxed{112}$

d. Find the new price per pound.

$0.40 - 0.02(6) = \boxed{\$0.36}$

e. Find the maximum revenue

$112(0.36) = \boxed{\$40.32}$

7. Find the maximum revenue Given $R(x) = -3x^2 + 1800x$

a. Find the number of units to achieve maximum revenue

$x = \frac{-b}{2a} = \frac{-1800}{2(-3)} = 300$ $\boxed{300}$

b. Find the maximum sales revenue

$-3(300)^2 + 1800(300) = \boxed{\$270,000}$

c. Find the selling price for the following revenue function

$\frac{270,000}{300} = \boxed{\$900/\text{unit}}$

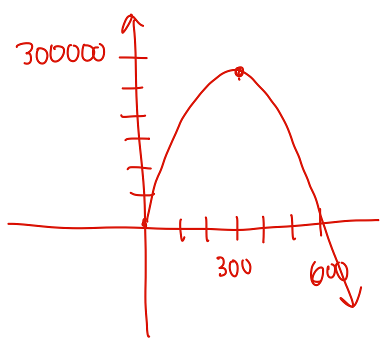
d. Find the x-intercepts of $R(x)$

$-3x^2 + 1800x = 0$

$-3x(x - 600) = 0$

$\boxed{x = 0}$ $\boxed{x = 600}$

e. Graph the revenue function showing the vertex and the x-intercepts.



f. After how many units does the company stop making money?

$\boxed{600}$

8. Find the maximum revenue Given $R(x) = -5x^2 + 800x$

a. Find the number of units to achieve maximum revenue

$$x = \frac{-b}{2a} = \frac{-800}{2(-5)} = \boxed{80}$$

b. Find the maximum sales revenue

$$-5(80)^2 + 800(80) = \boxed{32000}$$

c. Find the selling price ~~for the following revenue function~~

$$\frac{32000}{80} = \boxed{\$400\text{ /unit}}$$

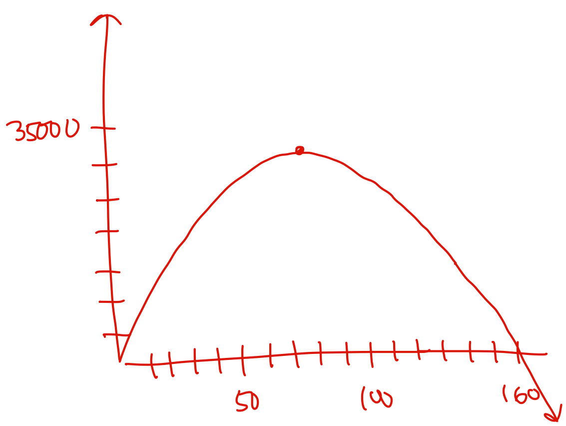
d. Find the x-intercepts of $R(x)$

$$-5x^2 + 800x = 0$$

$$-5x(x - 160) = 0$$

$$\boxed{x = 0} \quad \boxed{x = 160}$$

e. Graph the revenue function showing the vertex and the x-intercepts.



f. After how many units does the company stop making money?

$$\boxed{160}$$

9. A furniture company making desks has a profit function given by $P(x) = -2.5x^2 + 500x - 5000$.

   a. What is the *y*-intercept? Interpret the meaning in the context of the problem.

   $(0, -5000)$ When no desks are made, the profit is $-\$5000$.

   b. Find the number of desks that should be produced and sold to maximize profit.

   $$x = \frac{-b}{2a} = \frac{-500}{2(-2.5)} = 100 \qquad \boxed{100}$$

   c. What is the maximum profit?

   $$-2.5(100)^2 + 500(100) - 5000 = \boxed{\$20{,}000}$$

   d. When profit is zero, the company is said to break even. Determine the number of desks (to the nearest whole number) that must be manufactured and sold so that the company breaks even.

   $$\frac{-2.5x^2 + 500x - 5000}{-2.5} = \frac{0}{-2.5}$$

   $$x^2 - 200x + 2000 = 0$$

   $$x = \frac{-(-200) \pm \sqrt{(-200)^2 - 4(1)(2000)}}{2(1)}$$

   $$x = \frac{200 \pm \sqrt{32000}}{2} < \begin{matrix} 189.4 \\ 10.6 \end{matrix} \qquad \boxed{190 \text{ or } 11}$$

10. The profit made by a small business on the production and sale of *x* items is

$$P(x) = -2x^2 + 340x - 1600$$

   a. What is the *y*-intercept? Interpret the meaning in the context of the problem.

   $(0, -1600)$ When no desks are made, the profit is $-\$1600$

   b. Find the number of items that should be produced and sold to maximize profit. \

   $$x = \frac{-b}{2a} = \frac{-340}{2(-2)} = \boxed{85}$$

   c. What is the maximum profit?

   $$-2(85)^2 + 340(85) - 1600 = \boxed{\$12850}$$

   d. When profit is zero, the company is said to break even. Determine the number of items (to the nearest whole number) that must be manufactured and sold so that the company breaks even.

   $$\frac{-2x^2 + 340x - 1600}{-2} = \frac{0}{-2}$$

   $$x^2 - 170x + 800 = 0$$

   $$x = \frac{-(-170) \pm \sqrt{(-170)^2 - 4(1)(800)}}{2(1)} = \frac{170 \pm \sqrt{25700}}{2} < \begin{matrix} 165.2 \\ 4.8 \end{matrix} \qquad \boxed{166 \text{ or } 4}$$

11. The population of Rhode Island can be modeled by $y = 5500x + 1057000$, where $x = 0$ corresponds to 2014.

a. What is the slope?

5500

b. Interpret the meaning of the slope in the context of the problem

Every year the population of RI increases by 5500 people.

c. What is the y-intercept?

1057000

d. Interpret the meaning of the y-intercept in the context of the problem.

In 2014, the population of RI was 1057000.

12. The average lifespan of American women, in years, has been tracked, and the model for the data is $y = 0.2x + 73$, where $x = 0$ corresponds to 1960.

a. What is the slope?

0.2

b. Interpret the meaning of the slope in the context of the problem

Every year, the average lifespan of American women increases by 0.2 years.

c. What is the y-intercept?

73

d. Interpret the meaning of the y-intercept in the context of the problem.

In 1960 the average lifespan of American women was 73.

13. A company that produces laptops has fixed costs of \$750,000 and a variable cost per laptop of \$160. Each laptop sells for \$785. If x is the number of laptops sold,

a. Find the cost function.

$$\boxed{C(x) = 160x + 750000}$$

b. Find the revenue cost.

$$\boxed{R(x) = 785x}$$

c. Find the break-even point.

$$160x + 750000 = 785x$$
$$-160x \qquad\qquad -160x$$
$$\frac{750000}{625} = \frac{625x}{625}$$

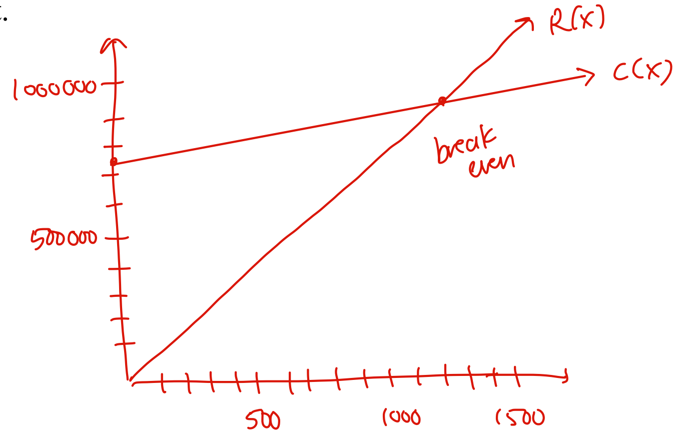
$x = 1200$

$785(1200)$

$942{,}000$

$$\boxed{(1200, 942000)}$$

d. Graph the cost and revenue functions on the same graph. Label both functions and the break-even point.



e. Find the profit function.

$$P(x) = R(x) - C(x) = 785x - (160x + 750000) = 625x - 750{,}000$$

$$\boxed{P(x) = 625x - 750000}$$

f. How much profit will they make if they produce and sell 5000 laptops?

$$625(5000) - 750{,}000 = \boxed{2375000}$$

g. How many laptops must be produced and sold to obtain a profit of \$500,000?

$$500000 = 625x - 750000$$
$$+750000 \qquad\qquad +750000$$
$$\frac{1250000}{625} = \frac{625x}{625}$$

$$\boxed{2000}$$

14. A company that manufactures HD TV's has fixed costs of \$132,660 and a variable cost of \$98 per TV. Each TV sells for \$230. If *x* is the number of TV's sold,

a. Find the cost function.

$$\boxed{C(x) = 98x + 132660}$$

b. Find the revenue cost.

$$\boxed{R(x) = 230x}$$

c. Find the break-even point.

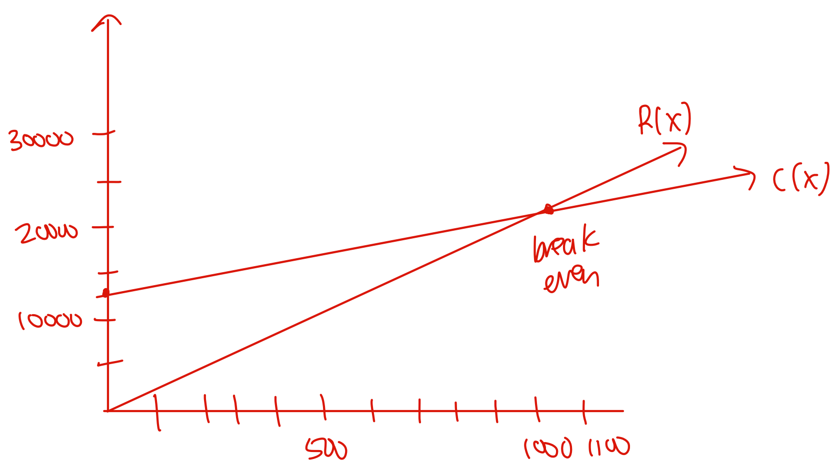
$$98x + 132660 = 230x$$
$$-98 \qquad\qquad -98x$$
$$\frac{132660}{132} = \frac{132x}{132}$$

$$x = 1005$$

$$230(1005)$$
$$231150$$

$$\boxed{(1005,\ 231150)}$$

d. Graph the cost and revenue functions on the same graph. Label both functions and the break-even point.



e. Find the profit function.

$$P(x) = 230x - (98x + 132660) = 132x - 132660$$

$$\boxed{P(x) = 132x - 132660}$$

f. How much profit will they make if they produce and sell 2000 TVs?

$$132(2000) - 132660 = \boxed{\$131340}$$

g. How many TVs must be produced and sold to obtain a profit of \$50,000?

$$50000 = 132x - 132660$$
$$+132660 \qquad\qquad +132660$$
$$\frac{182660}{132} = \frac{132x}{132}$$

$$1383.79 = x$$

$$\boxed{1384}$$

15. A 2025 SUV costs about \$46,200. In five years, the SUV will be worth \$27,720.

a. Find the linear model to represent the situation.

$(0, 46200) \quad (5, 27720)$

* y-int

$$m = \frac{27720 - 46200}{5 - 0} = \frac{-18480}{5} = -3696$$

$$\boxed{y = -3696x + 46200}$$

b. What is the value of the SUV after 2 years?

$$-3696(2) + 46200 = \boxed{\$38,808}$$

c. When was the SUV worth half of its original value?

$$\frac{46200}{2} = 23100$$

$$23100 = -3696x + 46200$$
$$-46200 \qquad\qquad -46200$$
$$-23100 = -3696x$$
$$x = 6.25$$

$$\boxed{6.25 \text{ yrs}}$$

16. A new iPhone costs \$799. In three years, it will be worth ~~\$440~~.
\$445

a. Find the linear model to represent the situation.

y-int

$(0, 799) \quad (3, 440)$

$$m = \frac{445 - 799}{3 - 0} = -118$$

$$\boxed{y = -118x + 799}$$

b. How much is the iPhone worth after 5 years?

$$-118(5) + 799 = \boxed{\$209}$$

c. When will the iPhone be worth 100?

$$100 = -118x + 799$$

$$-699 = -118x$$

$$x = 5.92$$

$$\boxed{5.92 \text{ years}}$$