Recall that

$$[f(x) + g(x)]' = f'(x) + g'(x)$$

$$[f(x) - g(x)]' = f'(x) - g'(x)$$

But :

(1) $$[f(x) \cdot g(x)]' \underbrace{\neq}_{\text{not equal to}} f'(x) \cdot g'(x)$$

(2) $$\left[\frac{f(x)}{g(x)}\right]' \neq \frac{f'(x)}{g'(x)}$$

Instead, we will use the product rule and quotient rule.

$\boxed{\text{The product Rule}}$

$$[f(x) \cdot g(x)]' = f'(x) \cdot g(x) + g'(x) \cdot f(x)$$

Example : Find $h'(x)$

(1) $h(x) = (3x^4 + 4x^5 + 6x + 1) \cdot (7x^2 + 3)$

$h'(x) = (3x^4 + 4x^5 + 6x + 1)' \cdot (7x^2 + 3) + (7x^2 + 3)' \cdot (3x^4 + 4x^5 + 6x + 1)$

$= (3 \cdot 4x^{4-1} + 4 \cdot 5x^{5-1} + 6)(7x^2 + 3) + (7 \cdot 2x^{2-1})(3x^4 + 4x^5 + 6x + 1)$

$= (12x^3 + 20x^4 + 6)(7x^2 + 3) + (14x) \cdot (3x^4 + 4x^5 + 6x + 1)$

② $\left(\sqrt{x}+1\right)\cdot\left(x^{10}+2\cdot\sqrt[3]{x}\right)$

rewrite

$$= \left(x^{1/2}+1\right)\cdot\left(x^{10}+2\,x^{1/3}\right)$$

$$\Rightarrow h'(x) = \left(x^{1/2}+1\right)'\cdot\left(\underline{x^{10}+2x^{1/3}}\right)+\left(x^{10}+2x^{1/3}\right)'\cdot\left(x^{1/2}+1\right)$$

$$= \left(\tfrac{1}{2}x^{1/2-1}\right)\left(x^{10}+2x^{1/3}\right)+\left(10x^{10-1}+2\cdot\tfrac{1}{3}x^{1/3-1}\right)\left(x^{1/2}+1\right)$$

$$= \frac{1}{2}x^{-1/2}\left(x^{10}+2x^{1/3}\right)+\left(10x^{9}+\frac{2}{3}x^{-2/3}\right)\left(x^{1/2}+1\right)$$

③ $h(x) = \left(\dfrac{1}{x^3}-\dfrac{3}{x^4}\right)\cdot\left(3x-\dfrac{6}{x^{10}}\right)$

rewrite in power forms

$$h(x) = \left(x^{-3}-3x^{-4}\right)\cdot\left(3x-6x^{-10}\right)$$

$$\Rightarrow h'(x) = \left(x^{-3}-3x^{-4}\right)'\cdot\left(3x-6x^{-10}\right)+\left(3x-6x^{-10}\right)'\left(x^{-3}-3x^{-4}\right)$$

product

rule

$$= \left(-3x^{-3-1}-\underline{3\cdot(-4)}\cdot x^{-4-1}\right)\left(3x-6x^{-10}\right)+\left(3-6\cdot(-10)\cdot x^{-10-1}\right)\left(x^{-3}-3x^{-4}\right)$$

$$= \left(-3x^{-4}+12x^{-5}\right)\left(3x-6x^{-10}\right)+\left(3+60x^{-11}\right)\cdot\left(x^{-3}-3x^{-4}\right)$$

④

$$h(x) = \frac{x+1}{x} = (x+1)\cdot\frac{1}{x}$$

$$= (x+1)\cdot x^{-1}$$

$$h'(x) = (x+1)'\cdot x^{-1}+\left(x^{-1}\right)'\cdot(x+1)$$

product rule

$$= (1)\cdot x^{-1} + (-1\cdot x^{-1-1})(x+1)$$

$$= x^{-1} - x^{-2}\cdot(x+1)$$

Assignment: Find $h'(x)$

1. $h(x) = (x^3 + 2x + 1)\cdot(x^4 + 2x^3 + 4)$

2. $h(x) = (x^6 + 3)(x^7 + 4)$

3. $h(x) = (3x^5 - 4x)(2x^2 + 3x + 4)$

4. $h(x) = (\sqrt{x} + 1)(\sqrt[3]{x} + 2)$

5. $h(x) = (x^2 + 2\sqrt[6]{x})(x^2 + x)$

6. $h(x) = \left(\frac{2}{x} + \frac{3}{x^2}\right)\left(\frac{1}{x^3} + x\right)$

7. $h(x) = \left(\frac{3}{x} - \frac{1}{x^4}\right)(\sqrt{x} + 2)$

8. $h(x) = \dfrac{x^2 + 1}{x^3}$

9. $h(x) = \dfrac{\sqrt{x} + 1}{x^2}$