(*) Scalar multiplication Rule.

$$[\, s \cdot f(x) \,]' = s \cdot f'(x)$$

(s: scalar)

Example:

$$(x^{10})' = 10\,x^{9} \qquad \text{(power rule)}$$

$$(17\,x^{10})' = 17\,(x^{10})' \qquad \text{(scalar rule)}$$

$$= 17 \cdot 10\,x^{9} = 170\,x^{9}$$

(*) Addition / Subtraction Rule

$$(f(x) + g(x))' = f'(x) + g'(x)$$

$$(f(x) - g(x))' = f'(x) - g'(x)$$

Example: Find $f'(x)$

(1) $f(x) = x^{9} + x^{10}$

$\Rightarrow f'(x) = (x^{9} + x^{10})'$

(addition rule) $= (x^{9})' + (x^{10})'$

(power rule) $= 9x^{8} + 10x^{9}$

Example : Find $f'(x)$

① $f(x) = 6x^7 + 10x^9 - 4x^8 + 20$

$\Rightarrow f'(x) = 6 \cdot 7 x^{7-1} + 10 \cdot 9 \cdot x^{9-1} - 4 \cdot 8 x^{8-1} + (20)'$

$= \boxed{42x^6 + 90x^8 - 32x^7}$

② $f(x) = 2 \cdot x^{2025} + 1000x^{20} + x + 2026$

$f'(x) = 2 \cdot (2025) \cdot x^{2024} + 1000 \cdot 20 x^{19} + 1$

$= \boxed{4050x^{2024} + 20000x^{19} + 1}$

Assignment 13 - Part 1

Find $f'(x)$

①

① $f(x) = 19x^{6} - 40x^{7} + 4x^{3} - 20$

② $f(x) = \frac{1}{2}x^{100} + \frac{1}{3}x^{9} + x + 3$

③ $f(x) = \frac{x^{6}}{3} + \frac{x^{10}}{5} + 6x + 1$

④ $f(x) = 6x^{2} + \frac{x^{3}}{3} + 2x + 1$

Example : Find $f'(x)$

① $f(x) = \frac{1}{x^{2}} + \frac{1}{x^{3}} + 6x + 4$

[rewrite $f(x)$, not taking derivative yet]

$$f(x) = x^{-2} + x^{-3} + 6x + 4$$

[then take derivative]

$$f'(x) = -2 \cdot x^{-2-1} + (-3) \cdot x^{-3-1} + 6$$

$$\Rightarrow f'(x) = \boxed{-2x^{-3} - 3 \cdot x^{-4} + 6}$$

(2) $f(x) = \frac{7}{x} + \frac{10}{x^4} - \frac{6}{x^3} + 2x^{10} + x + 100$

$f(x) = 7 \cdot x^{-1} + 10 \cdot x^{-4} - 6x^{-3} + 2x^{10} + x + 100$

$f'(x) = 7(-1) \cdot x^{-1-1} + 10 \cdot (-4) \cdot x^{-4-1} - 6 \cdot (-3) x^{-3-1} + 2 \cdot 10 x^{9} + 1$

$= -7x^{-2} - 40x^{-5} + 18x^{-4} + 20x^{9} + 1$

(3) $f(x) = \sqrt{x} + \sqrt[3]{x} + 3\sqrt[4]{x} + \frac{\sqrt[6]{x}}{2} + x + 1$

$f(x) = x^{1/2} + x^{1/3} + 3x^{1/4} + \frac{1}{2} x^{1/6} + x + 1$

$f'(x) = \frac{1}{2} x^{1/2-1} + \frac{1}{3} x^{1/3-1} + 3 \cdot \frac{1}{4} x^{1/4-1} + \frac{1}{2} \cdot \frac{1}{6} x^{1/6-1} + 1$

$f'(x) = \frac{1}{2} x^{-1/2} + \frac{1}{3} x^{-2/3} + \frac{3}{4} x^{-3/4} + \frac{1}{12} x^{-5/6} + 1$

Assignment 13 - Part 2. Find $f'(x)$

(1) $f(x) = \frac{1}{x^6} + \frac{1}{x^7} + \frac{10}{x^8} + x^3 + 7x + 2026$

(2) $f(x) = \frac{1}{x^9} - \frac{2}{x^3} + \frac{4}{x^{10}} + 3x + 4$

(3) $f(x) = 10\sqrt{x} + 6\sqrt[3]{x} + 9\sqrt[10]{x} + x^6 + 20$