Exponential / Logarithmic Functions

U.S. Investment Abroad In 1980, direct U.S. business investment abroad was about 13.5 billion dollars. From 1980 through 2010, that investment[12] grew at an average annual rate of 11.24%.

a. Make an exponential model that shows the U.S. direct investment abroad A, in billions of dollars, t years after 1980.

b. From 1980, how long did it take for U.S. investments abroad to double?

(a) $A(t) = P \cdot (1 + r)^t$ — growth rate

$A(t)$: the amount of money at the time t (since 1980)

P: initial amount of money (Principal)

$$A(t) = 13.5 \cdot (1 + 11.24\%)^t$$

(b) we need to find t so that

$$A(t) = 2 \cdot 13.5$$

$$13.5 \cdot (1.1124)^t = 2 \cdot 13.5$$

$$1.1124^t = 2$$

$$t = \log_{1.1124} 2 = \frac{\ln 2}{\ln 1.1124}$$

$t \simeq 6.507$ (years)

(c) In what year, the investment will be 100 (billions).

we need to find t so that

$$A(t) = 100$$

$$13.5 * (1.1124)^t = 100$$

$$1.1124^t = \frac{100}{13.5} \simeq 7.407$$

$$\Rightarrow \quad t = \log_{1.1124} 7.407 = \frac{\ln 7.407}{\ln 1.1124}$$

$t \simeq 18.8$ (years)

Example

If tuition at a college is increasing by 6.6% each year, how many years will it take for tuition to double?

r

$$A(t) = P(1+r)^t$$

$A(t)$: tuition at the time t

P: Initial tuition ($t=0$)

r: growth rate

we need to find t so that

$$A(t) = 2P$$

$$P(1+.066)^t = 2P$$

$$\Rightarrow \quad 1.066^t = 2$$

$$\Rightarrow \quad t = \log_{1.066} 2 = \frac{\ln 2}{\ln 1.066} \approx 10.845$$

Assignment 9

Suppose you invest $\$1000$ to an SP500 index ETF.

(a) How long does it take for your investment to be $\$100,000$.

(b) How long does it tall to double the investment.

(c) How long does it take to triple the investment.

Use Google to find the average annual growth of SP 500.

Example :

A freezer maintains a constant temperature of 6 degrees Fahrenheit. An ice tray is filled with tap water and placed in the refrigerator to make ice. The difference between the temperature of the water and that of the freezer was sampled each minute and recorded in the table below.

t	Time in minutes	0	1	2	3	4	5
d	Temperature difference	69.0	66.3	63.7	61.2	58.8	56.5

Part 1 Test to see that the data are exponential.

Part 2 Find an exponential model for temperature difference.

Part 1 :

t	d	difference in d
0	69	
1	66.3	$66.3 - 69 = -2.7$
2	63.7	$63.7 - 66.3 = -2.6$
3	61.2	$61.2 - 63.7 = -2.5$
4	58.8	$58.8 - 61.2 = -2.4$
5	56.5	$56.5 - 58.8 = -2.3$

Since the difference in d changes by minutes, this is not a linear relation.

t	d	Ratio changes in d
0	69	
1	66.3	66.3 / 69 = 0.9608
2	63.7	63.7 / 66.3 = 0.96078
3	61.2	61.2 / 63.7 = 0.96075
4	58.8	58.8 / 61.2 = 0.96078
5	56.5	56.5 / 58.8 = 0.96088

(b)

Since the ratio changes of d are "almost" a constant, we can use exponential to model the data.

Part 2 : write the model equation.

General form of equation :

$$D = a \cdot b^t$$

we need to find a and b to specify the equation.

t	Time in minutes	0	1	2	3	4	5
d	Temperature difference	69.0	66.3	63.7	61.2	58.8	56.5

(a) when $t = 0$, $D = 69$. Plug in the equation :

$$69 = a \cdot b^0$$

$$\Rightarrow \quad 69 = a \cdot 1 \quad \Rightarrow \quad \boxed{a = 69}$$

(*) when $t = 1$, $D = 66.3$.

Plug in the equation:

$$D = a \cdot b^t$$

$$\Rightarrow \quad 66.3 = a \cdot b^1$$

$$\Rightarrow \quad 66.3 = 69 \cdot b$$

$$\Rightarrow \quad b = \frac{66.3}{69} \approx .9608$$

So the equation is

$$D = a \cdot b^t \quad \text{or}$$

$$\boxed{D = 69 * (.9608)^t}$$

Part 3 Rewrite the model equation using base e. ($e = 2.71828\ldots$)

Formula:

$$a \cdot b^t = a \cdot e^{(\ln b) \cdot t}$$

(base b) (base e)

Apply the formular, we have:

$$69 * (.9608)^{t} = 69 * e^{(\ln .9608)\cdot t}$$

$$\boxed{D = 69 \cdot e^{-0.04t}}$$

Part 4 when the difference in temp. reaches 10 (F)

we need to solve:

$$D = 10$$

$$69 \cdot e^{-0.04t} = 10$$

$$e^{-0.04t} = \frac{10}{69} \quad \text{(exp. form)}$$

$$-0.04t = \log_e (10/69) \quad \text{(log form)}$$

$$\left[b^x = y \iff x = \log_b y \right]$$

exponential form

log form

$$-0.04\,t = \ln(10/69)$$

$$t = \frac{\ln(10/69)}{-0.04} \approx 48.288.$$

Assignment 10 : Given the data.

t	D
0	32
1	25.6
2	20.48
3	16.38
4	13.11

① verify that D is an exponential function of t.

② write the model equation.

③ Rewrite the equation using base e.

④ Find t so that D = 1

⑤ verify the equation in ③ using excel.