Exam 3 Review

1. The median annual income for women with *x* years of education can be modeled by the function $W(x) = -1.2x^3 + 367x^2 - 4900x + 26{,}561$.
   a. Find $W(16)$

      $W(16) = 37197.80$

   b. Find $W'(x)$

      $W'(x) = -3.6x^2 + 734x - 4900$

   c. Find $W'(16)$ and interpret the meaning. Include the correct units.

      $W'(16) = \$5922.40$/year of education

      A woman with 16 years of education has a median annual income of \$37,197.80 and the median annual income is increasing at a rate of \$5922.40 per year of education.

2. The monthly sales of Bluetooth headphones can be modeled by $S(x) = 30x - \frac{1}{2}x^2$ where *x* represents the number of months since the headphones were initially sold and $S(x)$ represents the number of units sold in hundreds.
   a. Find $S(6)$

      $S(6) = 162$

   b. Find $S'(x)$

      $S'(x) = 30 - 2x$

   c. Find $S'(6)$ and interpret the meaning. Include the correct units.

      $S'(6) = 18$

      6 months after the initial release of the headphones, the monthly sales are 162 hundred units and they are increasing by 18 hundred units per month.

3. The projected sales of e-books, in millions, can be modeled by the function $S(x) = -17x^3 + 200x^2 - 113x + 44$, where *x* is the number of years since 2000.
   a. Find $S(10)$

      $S(10) = 1914$

   b. Find $S'(x)$

      $S'(x) = -51x^2 + 400x - 113$

   c. Find $S'(10)$ and interpret the meaning. Include the correct units.

      $S'(10) = -1213$

      In 2010, the sales of e-books was \$1914 million and sales were decreased at a rate of \$ -1213 million per year.

4. The average height (in inches) for girls ages 1 to 20 can be modeled by the equation $G(x) = -0.0002x^4 + 0.006x^3 - 0.14x^2 + 3.7x + 26$, where x is the age in years.
   a. Find $G(16)$

      $G(16) = 60.83$

   b. Find $G'(x)$

      $G'(x) = -0.0008x^3 + 0.018x^2 - 0.28x + 3.7$

   c. Find $G'(16)$ and interpret the meaning. Include the correct units.

      $G'(16) = 0.55$ in/yr

      A 16 year old girl is 60.83 inches and their height is increasing at a rate of 0.55 in every year.

5. The median annual income for men with x years of education can be modeled by the function $M(x) = 0.6x^3 + 285x^2 - 2256x + 15{,}112$.
   a. Find the average rate of change in the median annual income for men between 10 years of education and 18 years of education. Make sure to include correct units.

      $M(10) = 21652$

      $M(18) = 70343.2$

      $\frac{70343.2 - 21652}{18-10} = \$6086.4$ /yr of education

   b. Find $M'(x)$

      $M'(x) = 1.8x^2 + 570x - 2256$

   c. Find $M'(14)$ and interpret the meaning. Include the correct units.

      $M'(14) = \$6076.8$ /yr of education

6. The average height (in inches) for boys ages 1 to 20 can be modeled by the equation $B(x) = -0.001x^4 + 0.04x^3 - 0.56x^2 + 5.5x + 25$, where x is the age in years.
   a. Find the average rate of change in a boys height between 12 years and 18 years. Make sure to include correct units.

      $B(12) = 58.74$

      $B(18) = 70.86$

      $\frac{70.86 - 58.74}{18-12} = \boxed{2.02 \text{ inches/yr}}$

   b. Find $B'(t)$

      $B'(t) = -0.004x^3 + 0.12x^2 - 1.12x + 5.5$

   c. Find $B'(10)$ and interpret the meaning. Include the correct units.

      $B'(10) = 2.3$

      When a boy is 10 years old, his height is increasing at a rate of 2.3 inches per year.

7. Evaluate marginal cost and marginal revenue at a production level of 5 items increasing to 6 items. Interpret the meaning of the marginal cost and revenue. Should the company increase production?

$$C(x) = 3x^2 - 5x + 40,\ R(x) = -8x^2 + 10x + 30$$

$C'(x) = 6x - 5$

$C'(5) = 25$

It will cost \$25 to produce the 6th item.

$R'(x) = -16x + 10$

$R'(5) = -70$

The revenue from the sale of the 6th item is $-\$70$.

No, the company should not increased production. $C'(5) > R'(5)$.

8. Evaluate marginal cost and marginal revenue at a production level of 5 items increasing to 6 items. Interpret the meaning of the marginal cost and revenue. Should the company increase production?

$$C(x) = 0.05x^2 + 2x + 80,\ R(x) = -2x^2 + 100x$$

$C'(x) = 0.1x + 2$

$C'(5) = 2.5$

It costs \$2.50 to produce the 6th item

$R'(x) = -4x + 100$

$R'(5) = 80$

The revenue from the sale of the 6th item is \$80.

Yes the company should increase production. $R'(5) > C'(5)$

9. During the course of an illness, a patient's temperature (in degrees Fahrenheit) x hours after the start of the illness is given by $T(x) = \frac{98.6x^2+530}{x^2+5}$

a. Find $T(3)$ $T(3) = 101.24$

b. Find $T'(x)$

$f(x) = 98.6x^2 + 530$ $g(x) = x^2 + 5$

$f'(x) = 197.2x$ $g'(x) = 2x$

$(-98.6x^2 - 530)$

$$T'(x) = \frac{(x^2+5)(197.2x) + (98.6x^2+530)(2x)}{(x^2+5)^2}$$

$$T'(x) = \frac{197.2x^3 + 986x - 197.2x^3 - 1060x}{(x^2+5)^2}$$

$$T'(x) = \frac{-74}{(x^2+5)^2}$$

c. Find $T'(3)$ and interpret the meaning. Use correct units.

$T'(3) = -0.38\ ^\circ F/hr$

3 hours after the start of the illness, the patient's temperature is 101.24°F and decreasing at a rate of .38°F every hour.

10. The percent of concentration of a drug in the bloodstream *x* hours after the drug is administered is given by $K(x) = \frac{400x^2+100}{3x^2+27}$.

a. Find $K(2)$

$K(2) = 43.59$

b. Find $K'(x)$

$f(x) = 400x^2 + 100$ $\quad$ $g(x) = 3x^2 + 27$

$f'(x) = 800x$ $\quad$ $g'(x) = 6x$

$(-400x^2 - 100)$

$$K'(x) = \frac{(3x^2+27)(800x) + (400x^2+100)(6x)}{(3x^2+27)^2}$$

$$K'(x) = \frac{2400x^3 + 21600x - 2400x^3 - 600x}{(3x^2+27)^2}$$

$$K'(x) = \frac{21000x}{(3x^2+27)^2}$$

c. Find $K'(2)$ and interpret the meaning. Use correct units.

$K'(2) = 27.61\% / \text{hours}$

2 hours after the drug is administered, the percent concentration is 43.59% and increasing at a rate of 27.61% per hour.

11. The temperature of a certain patient (in degrees Fahrenheit) *t* hours after the onset of an infection can be approximated by $P(t) = 98.6 + te^{-0.2t}$.

a. Find $P(12)$

$P(12) = 99.69$

b. Find $P'(t)$

$f(x) = t$ $\quad$ $g(x) = e^{-0.2t}$

$f'(x) = 1$ $\quad$ $g'(x) = -0.2e^{-0.2t}$

$$P'(t) = t(-0.2e^{-0.2t}) + e^{-0.2t}(1)$$

c. Find $P'(12)$ and interpret the meaning. Include correct units.

$P'(12) = -0.13\,°F / hr$

12 hours after the onset of an infection a patient's temperature was 99.69°F and decreasing at a rate of 0.13°F/hr.

12. Sales at a fireworks outlet (in thousands of dollars) on day x can be approximated by $S(x) = 1 + 5xe^{-0.3x}$, where $x = 1$ corresponds to July 1.

   a. Find $S(2)$

      $S(2) = 6.49$

   b. Find $S'(x)$

   c. Find $S'(2)$ and interpret the meaning. Include correct units.

13. A business manager estimates that when x thousand people are employed at her firm, the profit will be $P(x)$ million dollars, where $P(x) = \ln(x) + 1.3x + 5$

   a. Find $P(2)$

      $P(2) = 8.29$

   b. Find $P'(x)$

      $P'(x) = \frac{1}{x} + 1.3$

   c. Find $P'(2)$ and interpret the meaning. Include correct units.

      $P'(2) = \$1.8$ million / thousand people

      When 2000 people are employed, the firm's profit is \$8.29 million and it is increasing by \$1.8 million per 1000 people employed.

14. A model for consumers' response to advertising is given by $N(x) = 2000 + 58.5x^2 + \ln x$ where $N(x)$ is the number of units sold and x is the amount spent on advertising, in thousands of dollars.

a. Find $N(10)$

7852.3

b. Find $N'(x)$

$N'(x) = 117x + \frac{1}{x}$

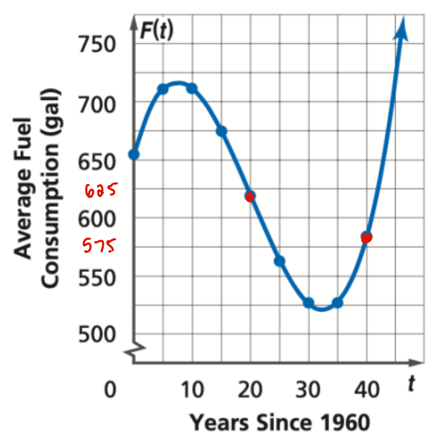
c. Find $N'(10)$ and interpret the meaning. Include correct units.

$N'(10) = 1170.10$ units / \$1000 spent on advertising

When \$10000 is spent on advertising, 7852.3 units are sold and the number of units sold is increasing at a rate of 1170.10 units per \$1000 spent on advertising.

15. The average fuel (in gallons) consumed by individual vehicles in the United States from 1960 to 2000 is shown in the graph below.

Find the average rate of change in the fuel consumption of individual vehicles in the United States between 1920 and 1940. Make sure to include correct units.



(1980, 620)

(2000, 580)

$\frac{620-580}{1980-2000} = \frac{40}{-20} = -2$

$\boxed{-2 \text{ gallons/yr}}$

For the following problems:

a. Find the first derivative
b. Find the critical values
c. Find the second derivative
d. Evaluate the second derivative for all critical values. Find the coordinates of the maximums/minimums.

16. $f(x) = 2x^3 + 3x^2 - 12x$	17. $f(x) = -x^3 + 6x^2 - 9x - 1$
a) $f'(x) = 6x^2 + 6x - 12$ b) $6x^2 + 6x - 12 = 0$ $x = 1$ $x = -2$ c) $f''(x) = 12x + 6$ d) $f''(1) = 18$ concave up minimum @ $x=1$ $f''(-2) = -18$ concave down maximum @ $x=-2$ minimum: $(1, -7)$ maximum: $(-2, 20)$	a) $f'(x) = -3x^2 + 12x - 9$ b) $-3x^2 + 12x - 9 = 0$ $x=1$ $x=3$ c) $f''(x) = -6x + 12$ d) $f''(1) = 6$ concave up minimum @ $x=1$ $f''(3) = -6$ concave down maximum @ $x=3$ minimum: $(1, -5)$ maximum: $(3, -1)$

18. $f(x) = -x^3 + 6x^2 - 15$

a) $f'(x) = -3x^2 + 12x$

b) $-3x^2 + 12x = 0$

$x = 0 \quad x = 4$

c) $f''(x) = -6x + 12$

$f''(0) = 12$

concave up

minimum @ $x = 0$

$f''(4) = -12$

concave down

maximum @ $x = 4$

minimum: $(0, -15)$

maximum: $(4, 17)$

19. $f(x) = x^3 - 3x^2 + 10$

a) $f'(x) = 3x^2 - 6x$

b) $3x^2 - 6x = 0$

$x = 0 \quad x = 2$

c) $f''(x) = 6x - 6$

d) $f''(0) = -6$

concave down

maximum @ $x = 0$

$f''(2) = 6$

concave up

minimum @ $x = 2$

maximum: $(0, 10)$

minimum. $(2, 6)$

20. $f(x) = x^2 + 6x + 10$ a) $f'(x) = 2x + 6$ b) $2x + 6 = 0$ $2x = -6$ $x = -3$ c) $f''(x) = 2$ d) $f''(-3) = 2$ concave up minimum @ $x = -3$ minimum: $(-3, 1)$	21. $f(x) = -2x^2 + 4x + 3$ a) $f'(x) = -4x + 4$ b) $-4x + 4 = 0$ $-4x = -4$ $x = 1$ c) $f''(x) = -4$ d) $f''(1) = -4$ concave down maximum @ $x = 1$ maximum: $(1, 5)$