

Linear Inequalities, Systems, and Linear Programming

I. Graphing Linear Inequalities

graph the boundary line using x & y intercepts
 $\geq \leq$ solid $> <$ dashed

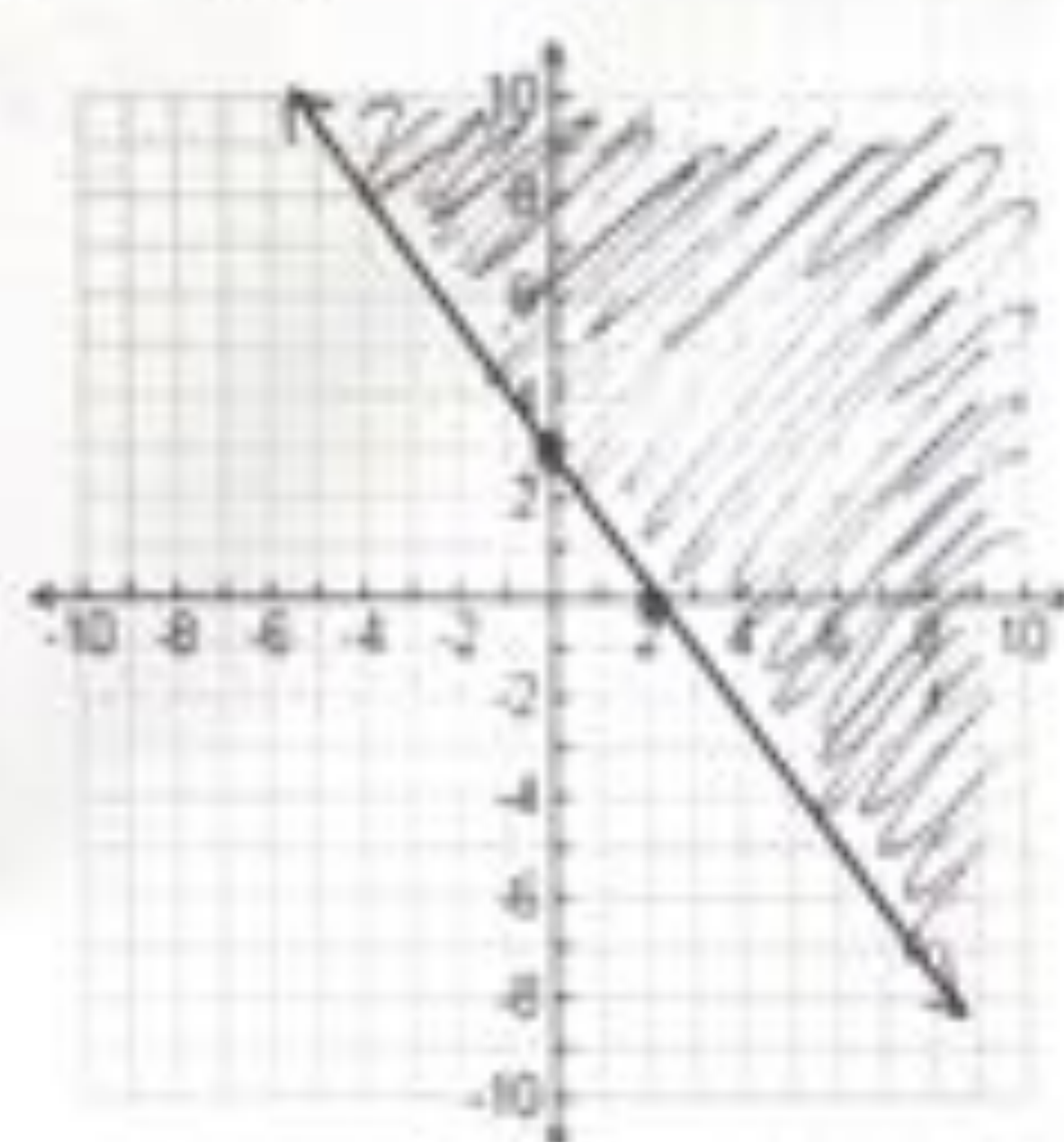
Test a point not on the boundary line & then
shade appropriately

Example 1: Graph $3x + 2y \geq 6$

x -int: ($y=0$)
 $3x + 2(0) = 6$
 $3x = 6$ (2, 0)

y -int: ($x=0$)
 $3(0) + 2y = 6$
 $2y = 6$ (0, 3)

$y = 3$



Test point
(0, 0)

$$3x + 2y \geq 6$$

$$3(0) + 2(0) \geq 6$$

$$0 \geq 6$$

False

Don't shade (0, 0)

Example 2: Graph $x - 3y > 9$

x -int: (9, 0) dashed

y -int: (0, -3)

Test point: (0, 0)

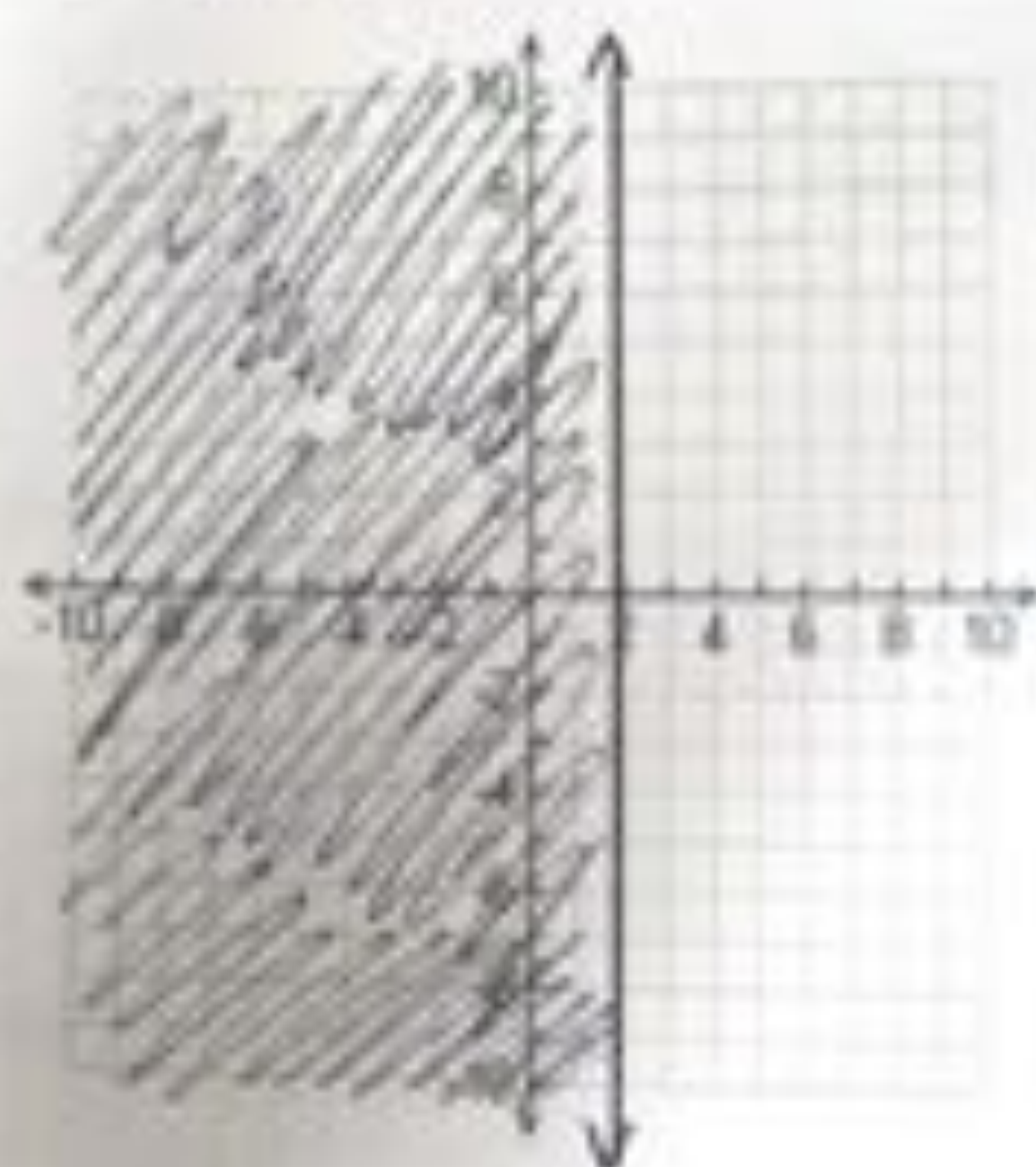
$$0 - 3(0) > 9$$

$$0 > 9$$

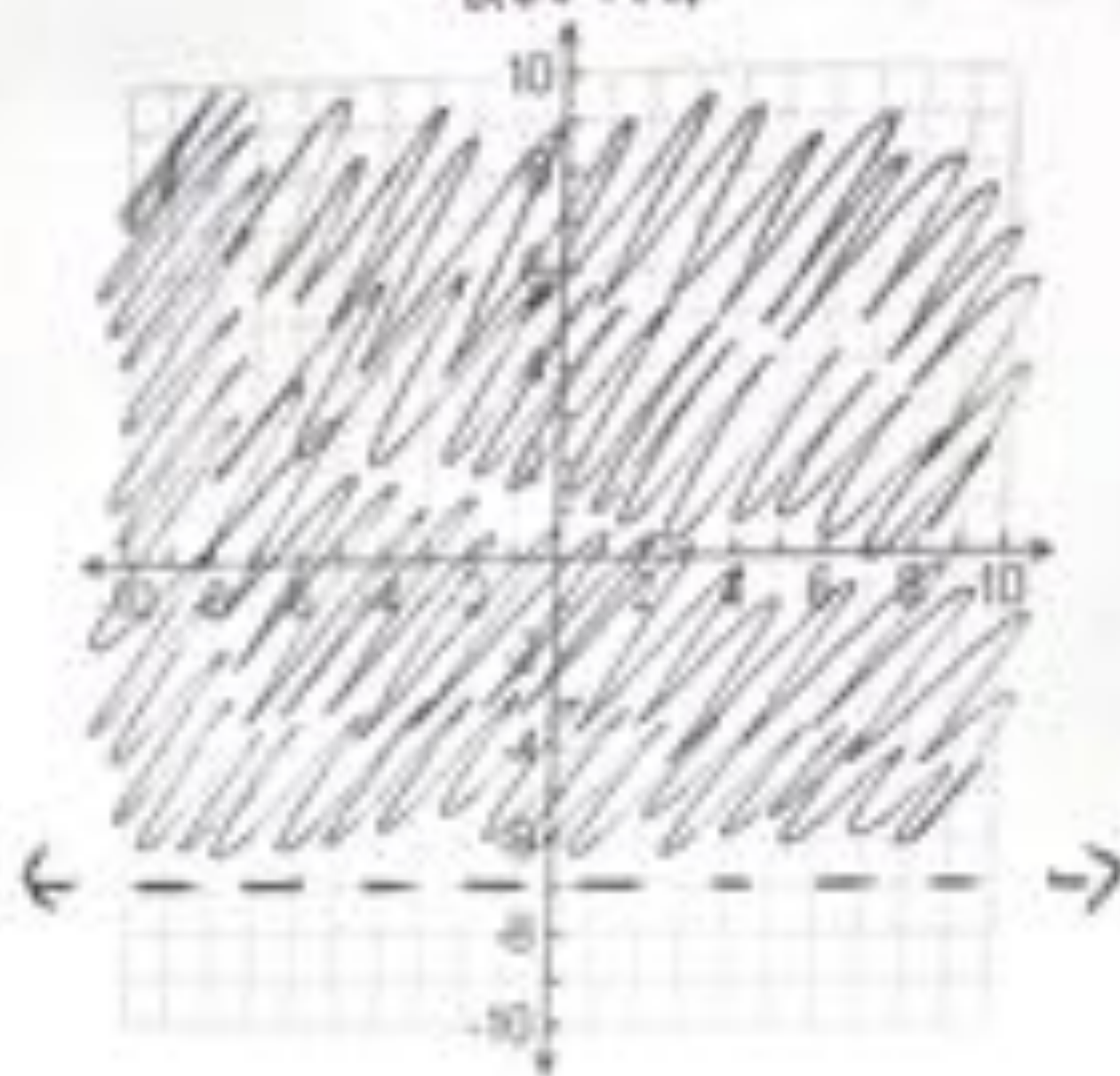
False. Don't shade (0, 0)



Example 3:
a. Graph $x \leq 2$



b. Graph $y > -5$
dashed horizontal line



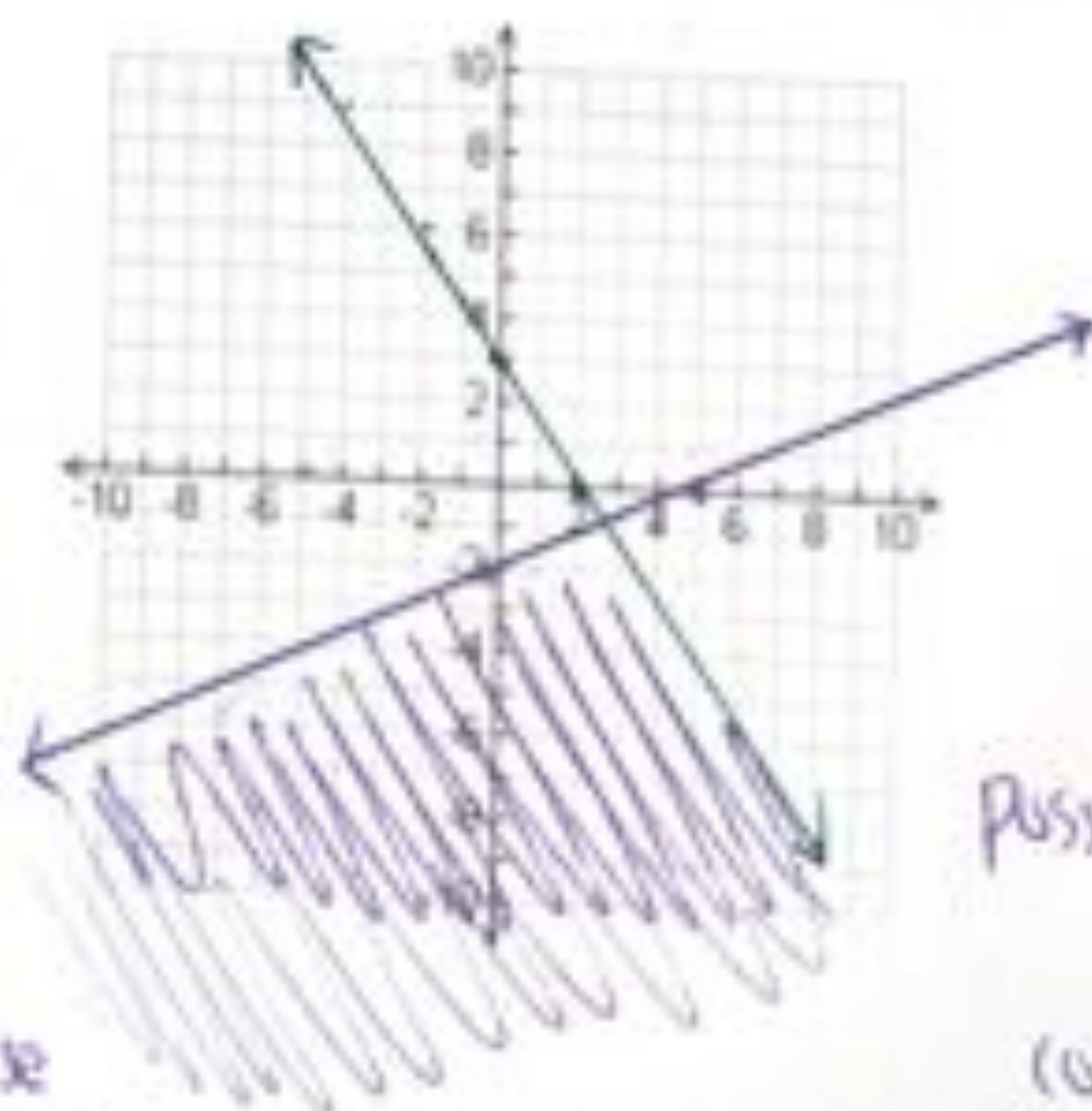
II. Graphing Systems of Linear Inequalities

Solution: where the shaded regions overlap

Graph each inequality and determine where to shade

Find where all the shaded regions overlap

Example 4: Graph $3x + 2y \leq 6$
 $2x - 5y \geq 10$



$$3x + 2y \leq 6$$

$$x\text{-int: } (2, 0)$$

$$y\text{-int: } (0, 3)$$

$$\text{Test: } (0, 0)$$

$$3(0) + 2(0) \leq 6$$

$$0 \leq 6 \text{ True}$$

Shade (true)

$$2x - 5y \geq 10$$

$$x\text{-int: } (5, 0)$$

$$y\text{-int: } (0, -2)$$

$$\text{Test: } (0, 0)$$

$$2(0) - 5(0) \geq 10$$

$$0 \geq 10 \text{ False}$$

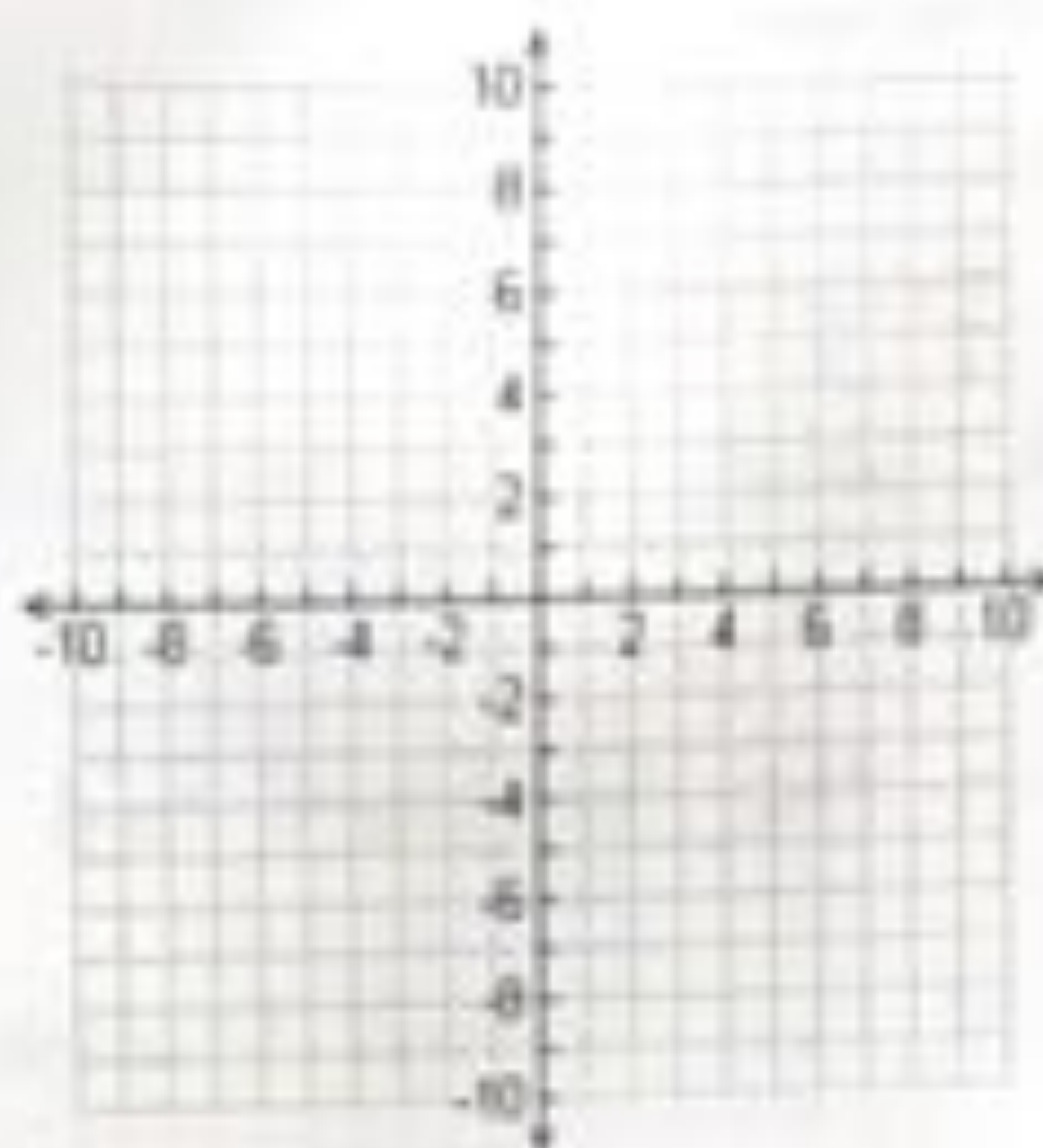
don't shade (0, 0)

Possible Solutions:

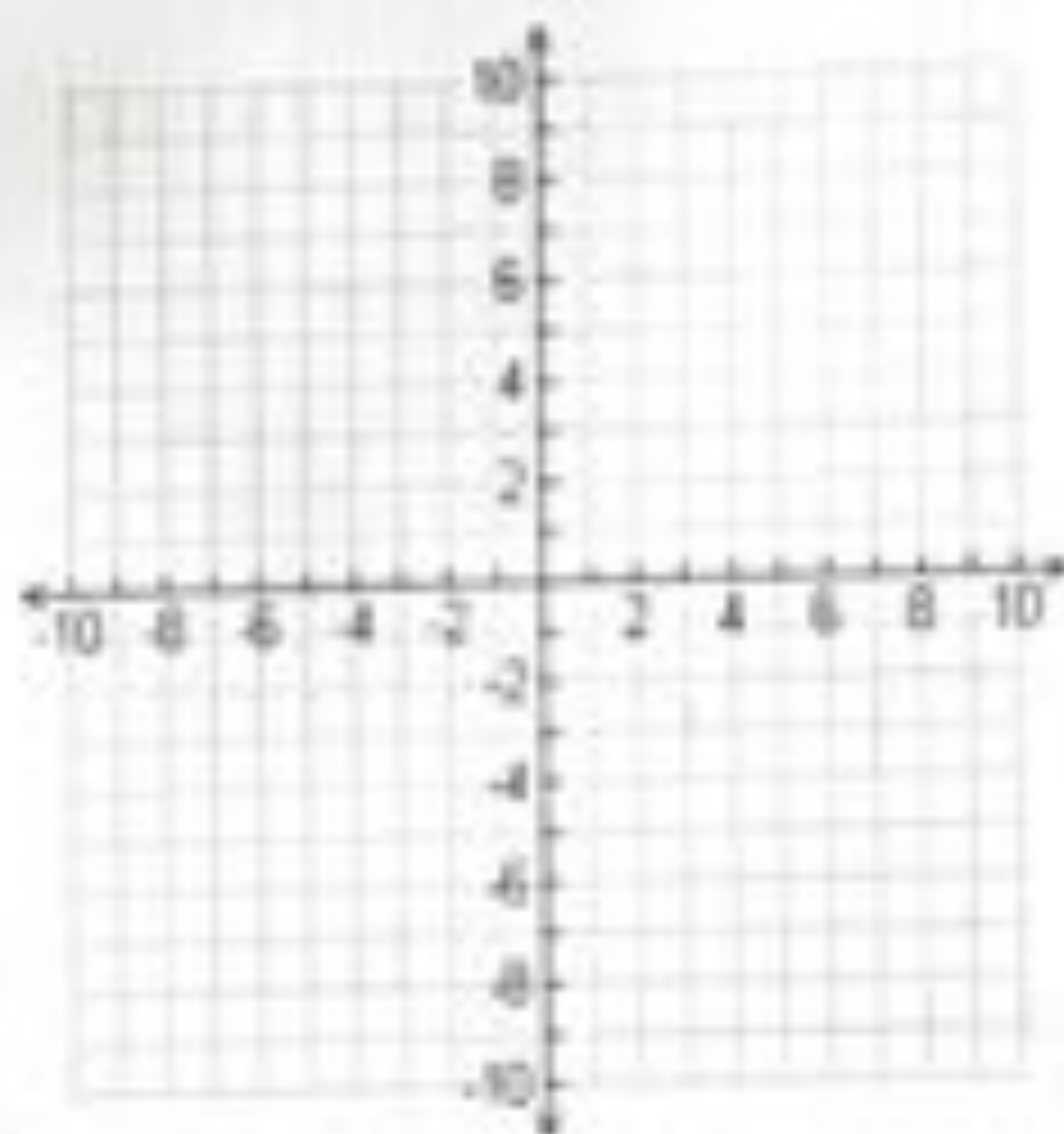
$$(0, -4)$$

$$(2, -4)$$

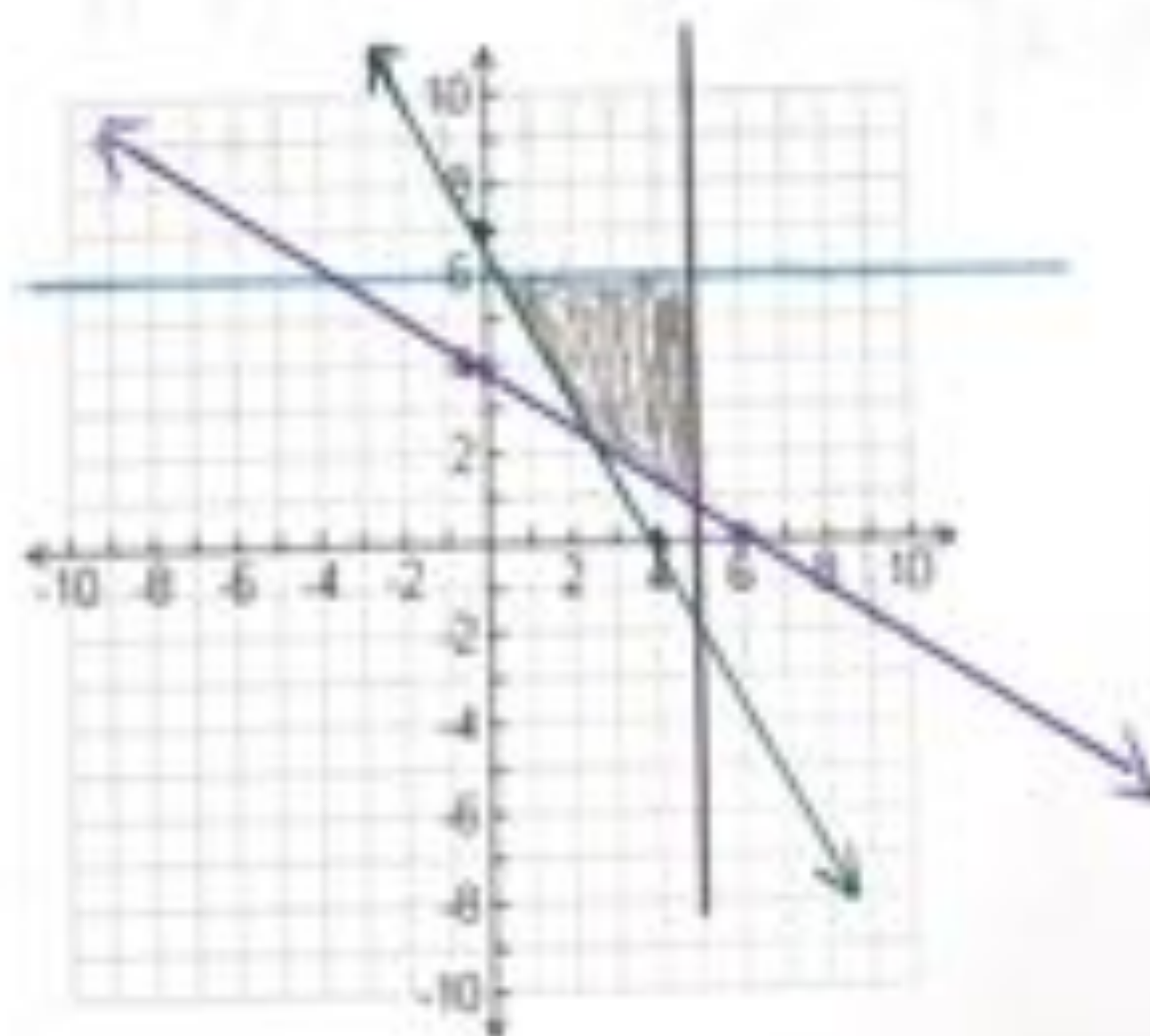
Example 5: Graph $x + y < 1$
 $2x - y < 4$



Example 6: Graph $x + 2y \leq 10$
 $x > 3$



Example 7: Graph $2x + 3y \geq 12$
 $7x + 4y \geq 28$
 $y \leq 6$ horizontal
 $x \leq 5$ vertical



$$2x + 3y \geq 12$$

$$x\text{-int: } (6, 0)$$

$$y\text{-int: } (0, 4)$$

$$\text{Test } (0, 0)$$

$$0 \geq 12 \quad \times$$

don't shade (0,0)

$$7x + 4y \geq 28$$

$$x\text{-int: } (4, 0)$$

$$y\text{-int: } (0, 7)$$

$$\text{Test } (0, 0)$$

$$0 \geq 28 \quad \times$$

don't shade (0,0)

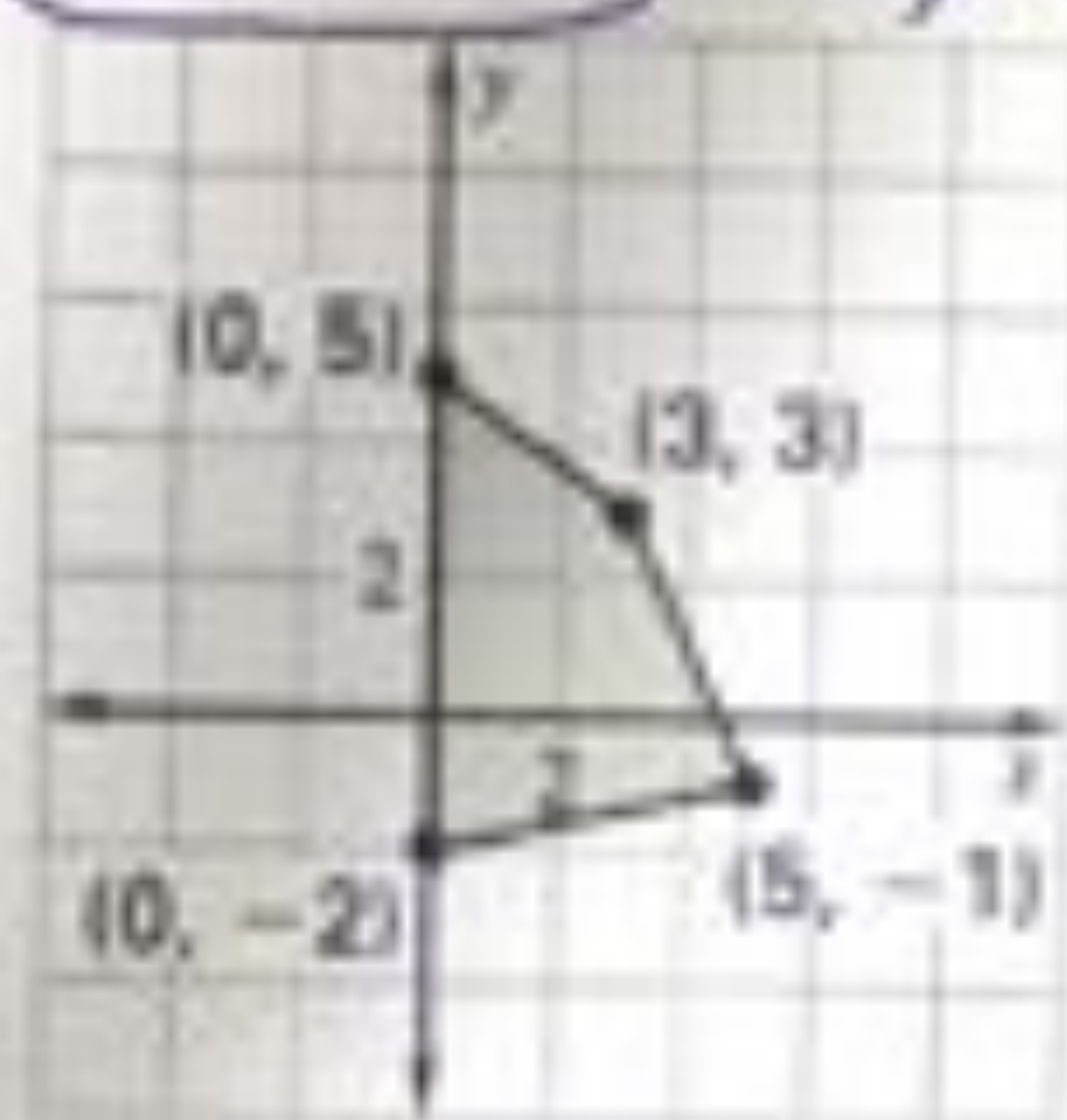
III. Linear Programming

Linear programming allows us to find the optimal value when faced with constraints (inequalities).

- ① Graph the constraints and find the feasible region
- ② Locate the vertices (corner points) of the feasible region and test them in the objective function
vertices \rightarrow only possible max/minimums
- ③ Determine maximum/minimum values

Example 8: The feasible region determined by a system of constraints is given. Find the minimum and maximum values of the objective function for the given feasible region.

$C = x + 2y$ objective function



$(0, 5)$	$C = 0 + 2(5) = \underline{10}$
$(3, 3)$	$C = 3 + 2(3) = \underline{9}$
$(5, -1)$	$C = 5 + 2(-1) = \underline{3}$
$(0, -2)$	$C = 0 + 2(-2) = \underline{-4}$

maximum:

10 @ $(0, 5)$

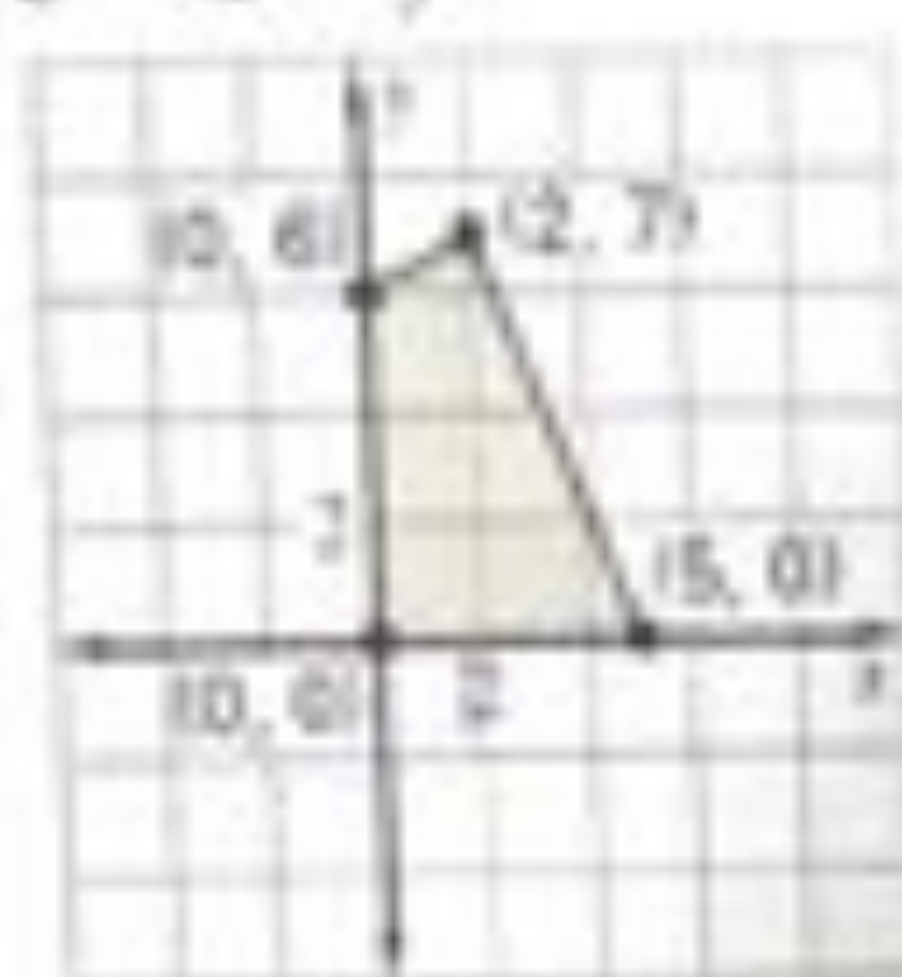
minimum:

-4 @ $(0, -2)$

Example 9:

The feasible region determined by a system of constraints is given. Find the minimum and maximum values of the objective function for the given feasible region.

$$C = x - y$$



Minimum:

$$-6 \text{ @ } (0, 6)$$

Maximum:

$$5 \text{ @ } (5, 0)$$

Example 10:

Graph the constraints:

$$x \geq 0$$

$$x \leq 2$$

$$y \geq 0$$

$$y \leq -2x + 6$$

$$(3, 4) \quad (0, 6)$$

Find the vertices of the feasibility region.

$$(0, 6) \quad (2, 2)$$

$$(0, 4) \quad (2, 0)$$

Test each vertex in the objective function $C = -x + 3y$

$$(0, 6) \quad C = 18$$

$$(0, 4) \quad C = 0$$

$$(2, 2) \quad C = 4$$

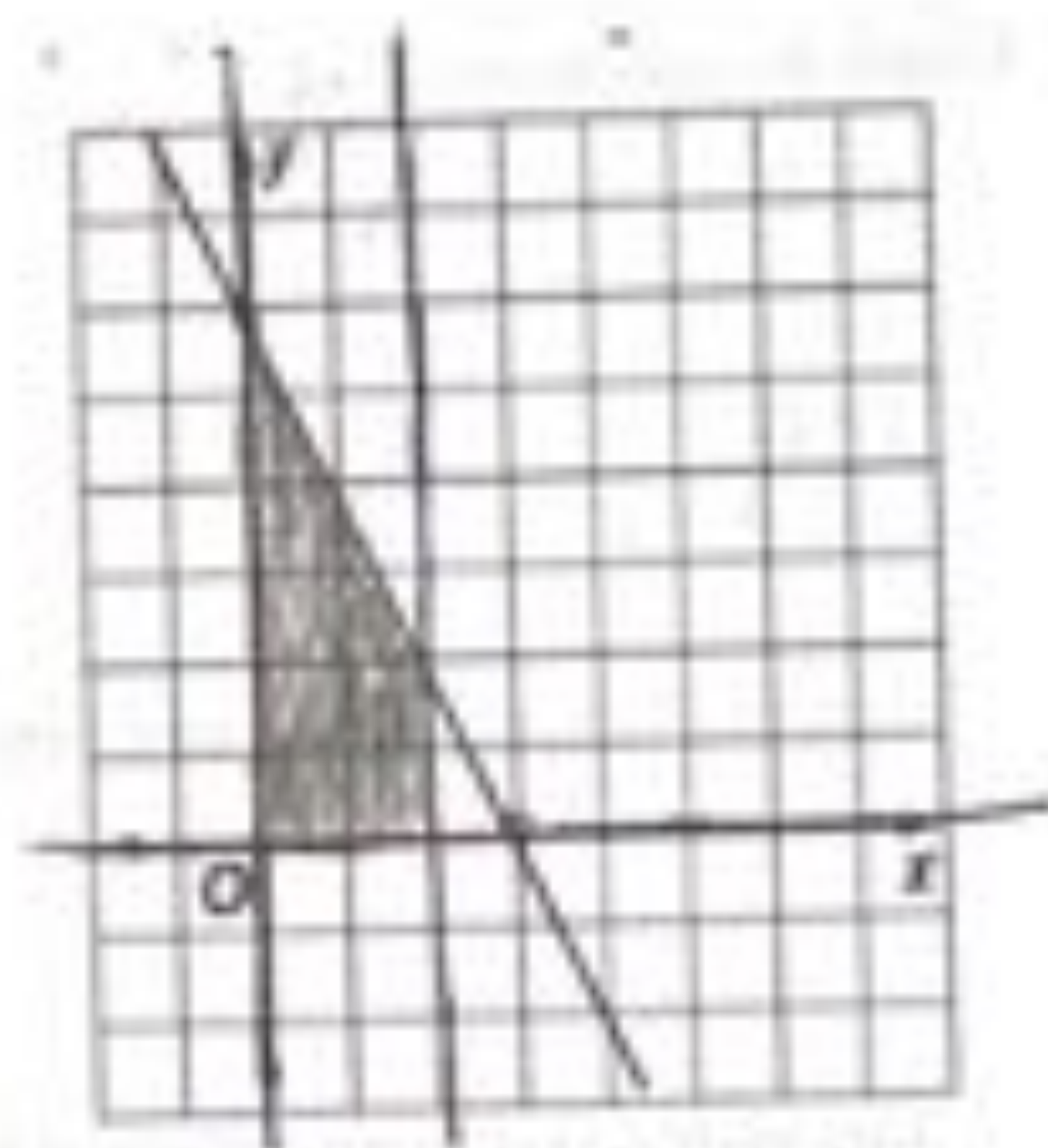
$$(2, 0) \quad C = -2$$

Minimum:

$$-2 \text{ @ } (2, 0)$$

Maximum:

$$18 \text{ @ } (0, 6)$$



Example 11:

Graph the constraints:

1st quadrant $\begin{cases} x \geq 0 \\ y \geq 0 \\ x + y \leq 5 \\ -3x + 6y \leq 12 \end{cases}$

Find the vertices of the feasibility region.

$$(0,0) \quad (0,2) \quad (2,3) \quad (5,0)$$

Test each vertex in the objective function $C = 5x + 6y$.

$$(0,0) \quad C = 0 \quad (2,3) \quad C = 28$$

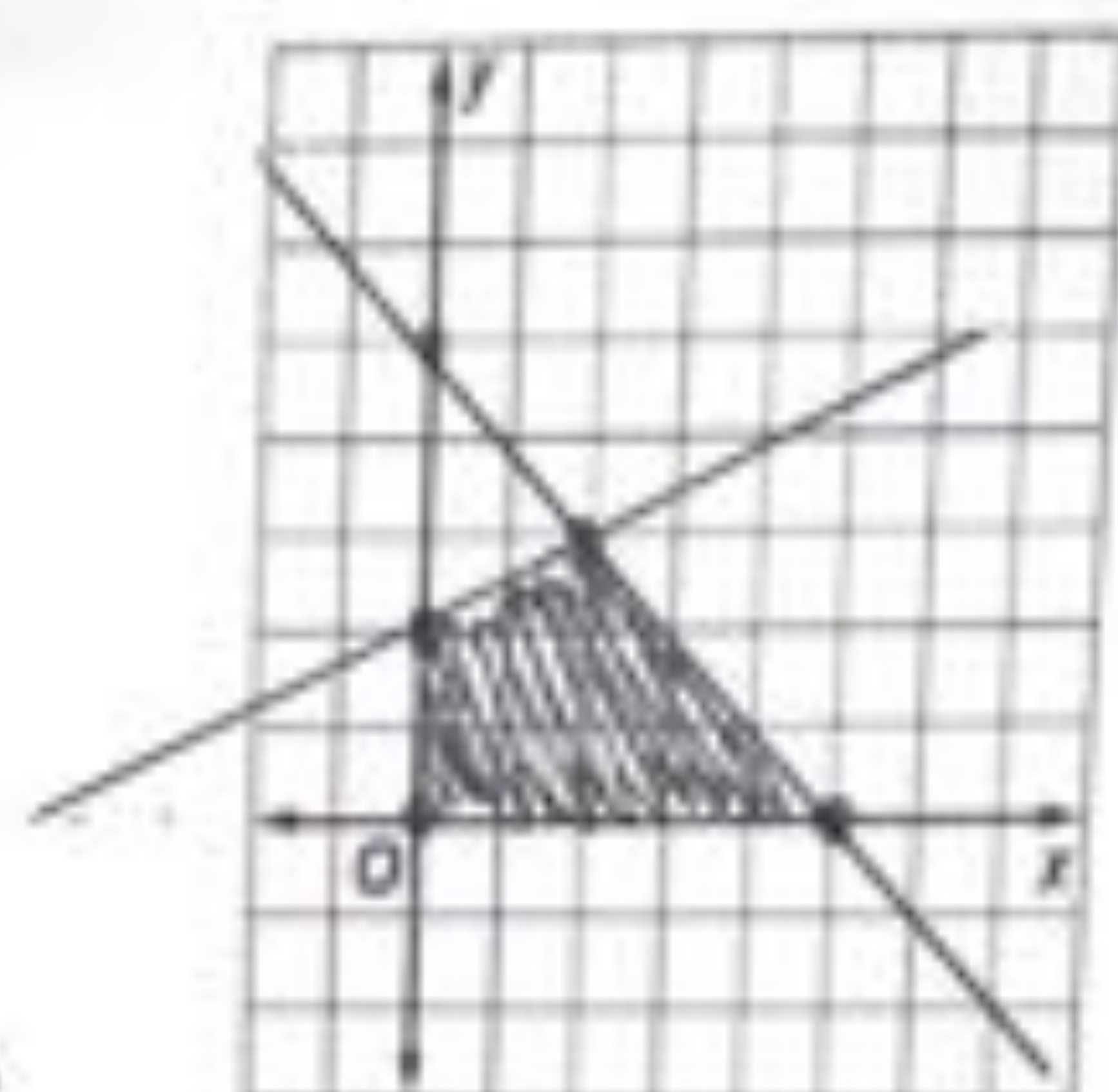
$$(0,2) \quad C = 12 \quad (5,0) \quad C = 25$$

Minimum:

$$0 \text{ @ } (0,0)$$

Maximum:

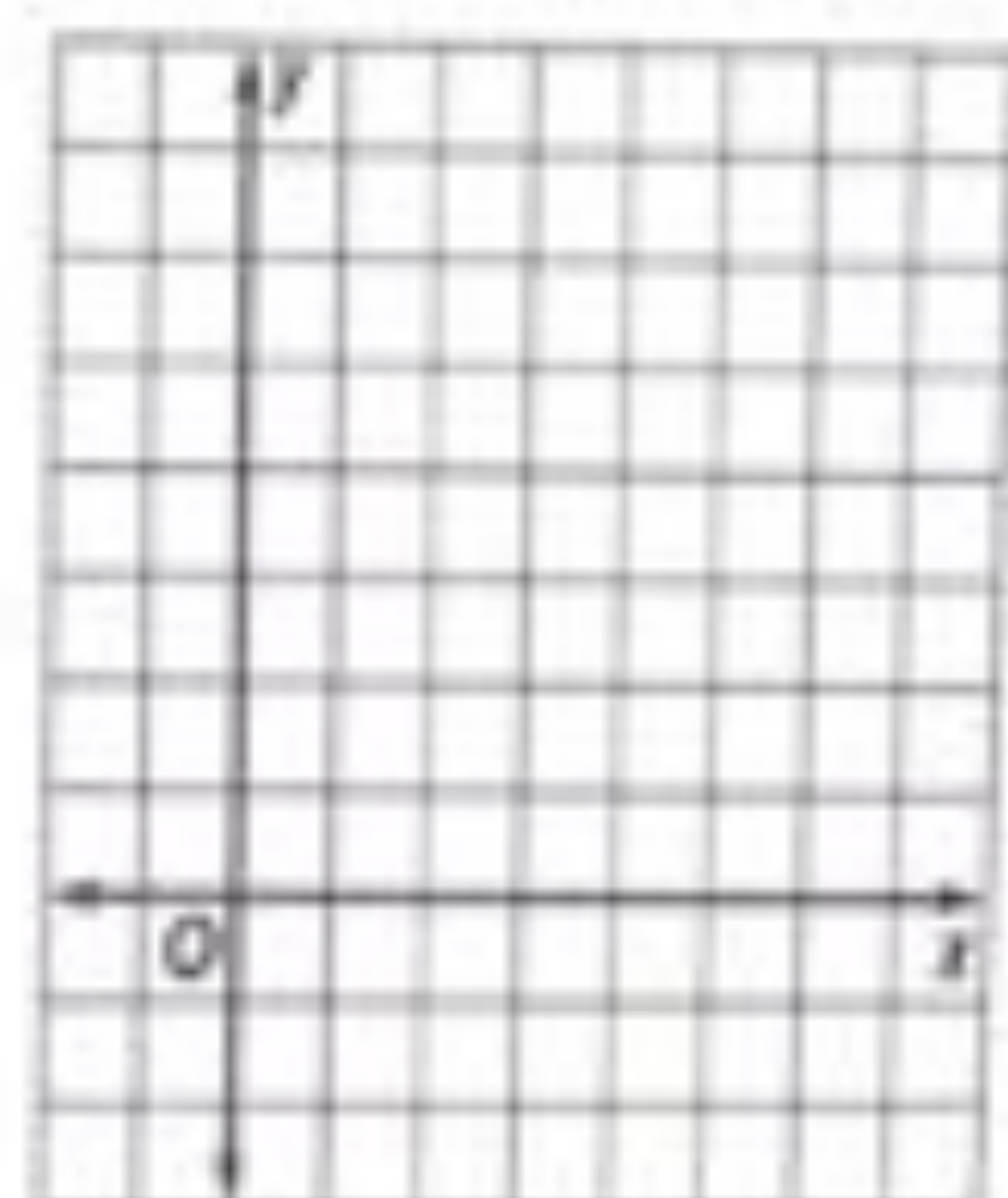
$$28 \text{ @ } (2,3)$$

**Example 12:**

Graph the constraints:

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x - 2y \geq -6 \\ y \leq -2x + 8 \end{cases}$$

Find the vertices of the feasibility region.

Test each vertex in the objective function $C = 3x - y$.

Minimum:

Maximum:

Beta value > 1 Performs higher than market

Beta value $= 1$ Perform same as the market

Beta value < 1 Performs less than the market \rightarrow risk adverse

Example 13: Nike Inc. stock sells for \$167 a share and has a 3-year average annual return of \$52 a share. The beta value is .93. Walt Disney Co. sells for \$169 a share and has a 3-year average annual return of \$24 a share. The beta value is 1.17. Roselyn wants to spend no more than \$8,000 investing in these two stocks, but she wants to earn at least \$1200 in annual revenue. Roselyn also wants to minimize the risk. Determine the number of shares of each stock that Roselyn should buy.

a. Define the variables

$x = \# \text{ of Nike shares}$

$y = \# \text{ of Disney shares}$

b. Write the constraints

$$x \geq 0$$

$$y \geq 0$$

$$167x + 169y \leq 8000$$

$$52x + 24y \geq 1200$$

c. Write the objective function minimize risk \rightarrow lower beta value

$$C = 0.93x + 1.17y$$

Example 14: As part of your weight training regimen, you want to consume lean sources of protein. You want to consume at least 300 Calories a day from at least 48 grams of protein. One ounce of chicken provides 35 Calories and 8.5 g of protein. One ounce of tofu provides 20 Calories and 2.5 g of protein. Your local supermarket charges \$0.31 an ounce for chicken and \$0.16 an ounce for tofu. How much of each food should you eat each day if you want to meet your requirements with the lowest cost? What is this daily cost?

a. Define the variables

b. Write the constraints

c. Write the objective function