

Derivatives of Exponential and Logarithmic Functions.

① Exponential Rule

$$(b^x)' = b^x \cdot \ln b$$

Special case: $b = e = 2.71828 \dots$

$$(e^x)' = e^x \cdot \underbrace{\ln e}_{=1}$$

$$(e^x)' = e^x$$

Example: Find $f'(x)$

$$① \quad f(x) = 3^x \Rightarrow f'(x) = 3^x \cdot \ln 3$$

$$② \quad f(x) = 2000^x \Rightarrow f'(x) = 2000^x \cdot \ln 2000$$

$$③ \quad f(x) = \left(\frac{1}{3}\right)^x \Rightarrow f'(x) = \left(\frac{1}{3}\right)^x \cdot \ln\left(\frac{1}{3}\right)$$

$$④ \quad f(x) = 7^x + 6e^x - 9^x$$

$$f'(x) = 7^x \cdot \ln 7 + 6e^x - 9^x \cdot \ln 9$$

$$(5) \quad f(x) = x + e^x$$

$$\Rightarrow f'(x) = 1 + e^x$$

$$(6) \quad f(x) = x e^x$$

$$f'(x) = \underset{\substack{\uparrow \\ \text{product} \\ \text{rule}}}{(x)' \cdot e^x + (e^x)' \cdot x}$$

$$= 1 \cdot e^x + e^x \cdot x$$

$$(7) \quad f(x) = \frac{x^2}{2^x}$$

$$f'(x) = \frac{(x^2)' \cdot 2^x - (2^x)' \cdot x^2}{(2^x)^2}$$

$$f'(x) = \frac{2x \cdot 2^x - 2^x \cdot (\ln 2) \cdot x^2}{(2^x)^2}$$

2. Logarithmic Rule

$$(\log_b x)' = \frac{1}{x \cdot \ln b}$$

Special case : $b = e = 2.71828 \dots$

$$(\log_e x)' = \frac{1}{x \cdot \ln e}$$

OR

$$(\ln x)' = \frac{1}{x}$$

Example: Find $f'(x)$

$$(1) \quad f(x) = \log_3 x \Rightarrow f'(x) = \frac{1}{x \cdot \ln 3}$$

$$(2) \quad f(x) = \log_7 x \Rightarrow f'(x) = \frac{1}{x \ln 7}$$

$$(3) \quad f(x) = 4 \log_2 x - 6 \ln x + 20 \log_{100} x$$

$$\Rightarrow f'(x) = 4 \cdot \frac{1}{x \ln 2} - 6 \frac{1}{x} + 20 \frac{1}{x \ln 100}$$

$$= \frac{4}{x \ln 2} - \frac{6}{x} + \frac{20}{x \ln 100}$$

$$(4) \quad f(x) = x \ln x$$

$$\begin{aligned} \Rightarrow f'(x) &= (x)' \cdot \ln x + (\ln x)' \cdot x \\ &= 1 \cdot \ln x + \frac{1}{x} \cdot x \end{aligned}$$

$$= \ln x + 1$$

$$\textcircled{5} \quad f(x) = \frac{2^x + x^2}{\log_4 x}$$

$$\Rightarrow f'(x) = \frac{(2^x + x^2)' \cdot \log_4 x - (\log_4 x)' \cdot (2^x + x^2)}{(\log_4 x)^2}$$

$$= \frac{(2^x \cdot \ln 2 + 2x) \log_4 x - \frac{1}{x \ln 4} \cdot (2^x + x^2)}{(\log_4 x)^2}$$

Assignment: Find $f'(x)$

(1) $f(x) = 11^x$

(3) $f(x) = \left(\frac{1}{20}\right)^x$

(5) $f(x) = 6^x - 7^x + 8^x$

(7) $f(x) = x e^x$

(9) $f(x) = (x^2 + 3x) \cdot (3^x + 4^x)$

(11) $f(x) = \log_{17} x$

(13) $f(x) = 6 \ln x - 3 \log_2 x$

(15) $f(x) = \frac{3x}{e^x + 1}$

(17) $f(x) = \frac{2^x + e^x}{\log_2 x}$

(2) $f(x) = 200^x$

(4) $f(x) = e^x$

(6) $f(x) = 9^x + x^9$

(8) $f(x) = x^9 \cdot 9^x$

(10) $f(x) = x^3 \cdot (e^x + x)$

(12) $f(x) = \log_{19} x$

(14) $f(x) = \frac{\ln x}{x}$

(16) $f(x) = \frac{e^x}{4 \ln x}$

(18) $f(x) = \frac{3x}{x^3 + \log_6 x}$