

Quadratic Functions

$$Y = ax^2 + bx + c \quad (a \neq 0)$$

Example : $-1 = 3x^2 + 4x + 1$

$$-1 = \frac{1}{3}x^2 - 9x$$

$$-1 = -9x^2 + 7$$

1. Solving Quadratic Equations

Quadratic Equation : $ax^2 + bx + c = 0$

a. Factoring

Example: $2x^2 = 4x$

$$2x^2 = 4x$$

$$\Leftrightarrow 2x^2 - 4x = 0$$

$$\Leftrightarrow 2x(x - 2) = 0$$

$$\Leftrightarrow \begin{cases} 2x = 0 \\ x - 2 = 0 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ x = 2 \end{cases} \quad (\text{Two solutions})$$

Example: $x^2 + 7x = -10$

$$\begin{aligned} & x^2 + 7x = -10 \\ \Leftrightarrow & x^2 + 7x + 10 = 0 \\ \Leftrightarrow & (x+5) \cdot (x+2) = 0 \end{aligned}$$

[we looked for 2 numbers whose sum is 7 and product is 10 .

5 and 2 are the two numbers]

$$\begin{aligned} \Leftrightarrow & \left[\begin{array}{l} x+5=0 \\ x+2=0 \end{array} \right] \Rightarrow \left[\begin{array}{l} x=-5 \\ x=-2 \end{array} \right] \end{aligned}$$

Example: $x^2 - 4x + 3 = 0$

Two numbers : sum = -4
product = 3

-1, -3

$$\begin{aligned} \Leftrightarrow & (x-1) \cdot (x-3) = 0 \\ \Leftrightarrow & \left[\begin{array}{l} x-1=0 \\ x-3=0 \end{array} \right] \Rightarrow \left[\begin{array}{l} x=1 \\ x=3 \end{array} \right] \end{aligned}$$

Example: $x^2 - 25 = 0$

Formula:

$$x^2 - 25 = 0$$

$$x^2 - a^2 = (x-a) \cdot (x+a)$$

$$\Leftrightarrow x^2 - 5^2 = 0$$

$$\Leftrightarrow (x-5) \cdot (x+5) = 0$$

$$\Leftrightarrow \begin{cases} x-5 = 0 \\ x+5 = 0 \end{cases} \quad \Leftrightarrow \begin{cases} x = 5 \\ x = -5 \end{cases}$$

Example: $x^2 - x = 12$

$$x^2 - x - 12 = 0$$

$$\Leftrightarrow (x-4) \cdot (x+3) = 0$$

$$\Leftrightarrow \begin{cases} x-4 = 0 \\ x+3 = 0 \end{cases} \quad \Leftrightarrow \begin{cases} x = 4 \\ x = -3 \end{cases}$$

b. Quadratic Formula

$$ax^2 + bx + c = 0$$

① If $b^2 < 4ac$: No solution!

② If $b^2 \geq 4ac$:

$$\text{solution: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: $x^2 + 1 = 4x$

$$x^2 + 1 = 4x$$

$$\hookrightarrow 1 \cdot x^2 - 4x + 1 = 0$$

$\uparrow \quad \uparrow \quad \uparrow$
 $a=1 \quad b=-4 \quad c=1$

$$b^2 = (-4)^2 = 16 \quad ; \quad 4ac = 4 \cdot 1 \cdot 1$$

Solution: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\hookrightarrow x = \frac{4 \pm \sqrt{16 - 4}}{2}$$

$$x = \frac{4 + \sqrt{12}}{2}$$

$$\hookrightarrow x = \frac{4 \pm \sqrt{12}}{2}$$

$$x = \frac{4 - \sqrt{12}}{2}$$

Example: $x^2 - 6x + 10 = 0$

$$x^2 - 6x + 10 = 0$$

$\downarrow \quad \downarrow \quad \downarrow$
 $a=1 \quad b=-6 \quad c=10$

$$b^2 = (-6)^2 = 36 \quad ; \quad 4ac = 4 \cdot 1 \cdot 10 = 40$$

$$b^2 < 4ac \quad (36 < 40)$$

\hookrightarrow No solution.

Mixed Practice

Solve each quadratic by factoring or the quadratic formula.

$$1. \quad 2x^2 = 4x$$

$$2. \quad x^2 - 4x = 12$$

$$3. \quad 2x^2 - x - 10 = 0$$

$$4. \quad x^2 - 10x + 24 = 0$$

$$5. \quad 10x^2 - 25x = 0$$

$$6. \quad x^2 + 5x + 2 = 2x$$

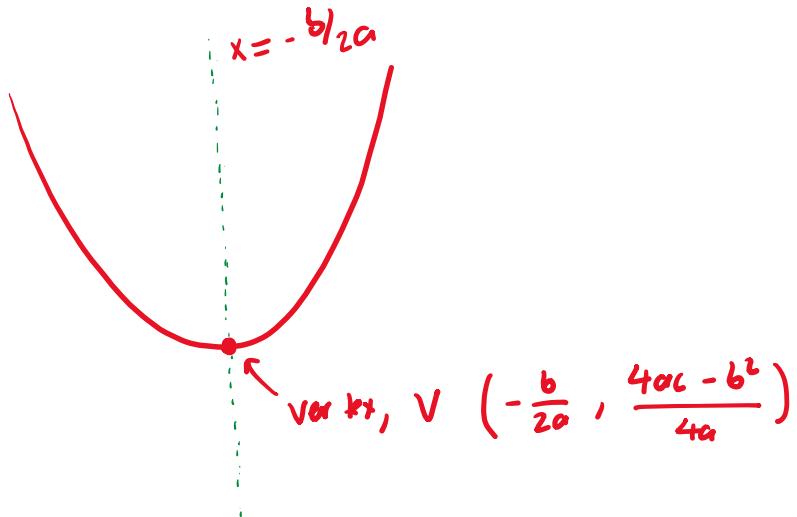
$$7. \quad x^2 - 100x + 900 = 0$$

$$8. \quad 8x^2 + x - 75 = 0$$

2. Graphs of Quadratic Functions

$$y = ax^2 + bx + c \quad (a \neq 0)$$

$a > 0$:



* The graph is a parabola (upward)

ⓐ Symmetry around $x = -\frac{b}{2a}$

ⓑ y -intercept is $(0, c)$

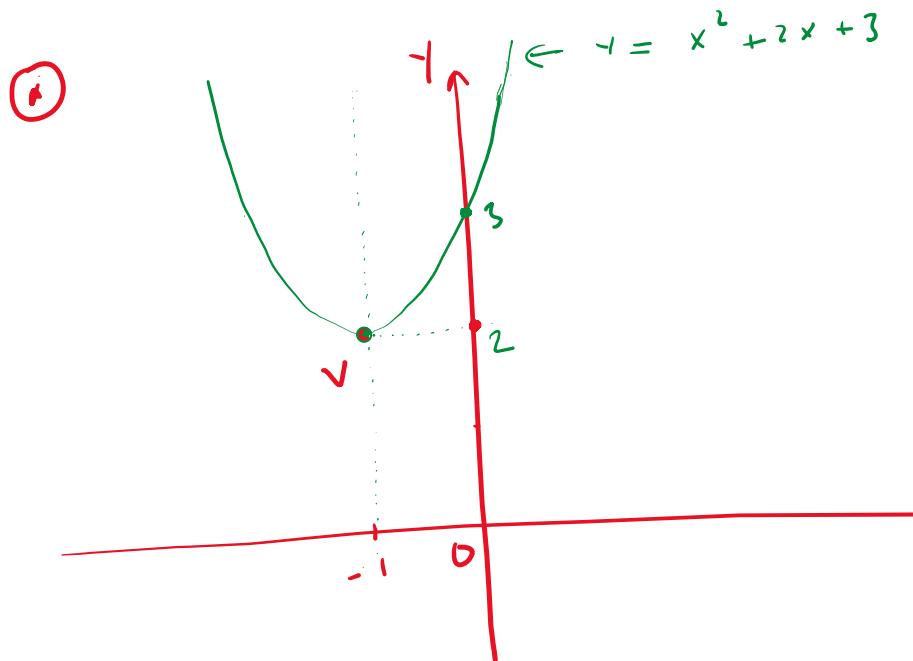
Example: $y = x^2 + 2x + 3$

$$a=1, b=2, c=3$$

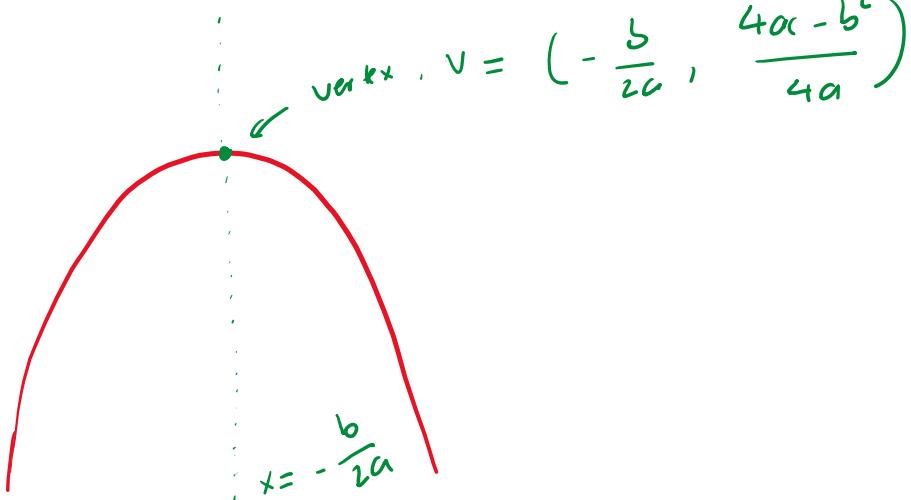
① $v_{\text{vert}} \quad v = \left(-\frac{b}{2a}, \frac{4ac - b^2}{4a} \right)$

$$= \left(-\frac{2}{2}, \frac{4 \cdot 1 \cdot 3 - 2^2}{4 \cdot 1} \right)$$

$$= (-1, 2)$$



$a < 0$



Example: $y = 2x^2 - 4x + 4$

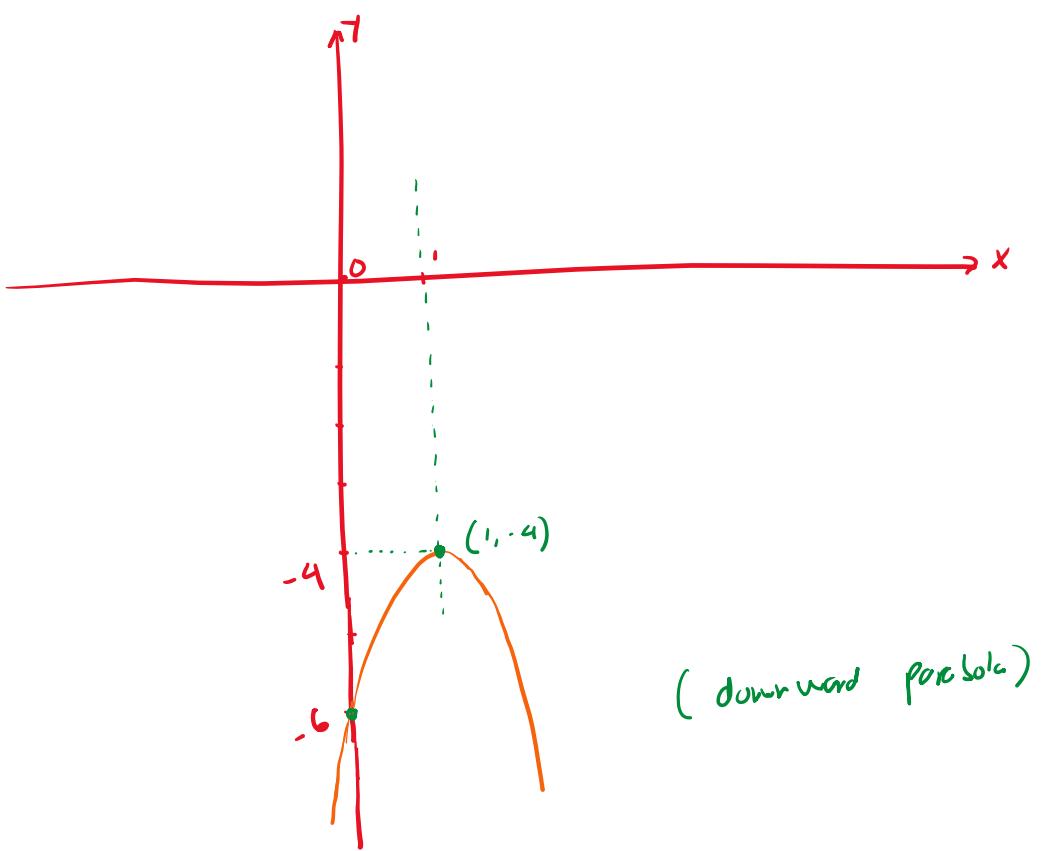
Example: $y = -2x^2 + 4x - 6$

$$\begin{array}{ccc} \downarrow & \downarrow & \rightarrow \\ a = -2 & b = 4 & c = -6 \end{array}$$

$$\text{vertex, } V = \left(-\frac{b}{2a}, \frac{4ac - b^2}{4a} \right)$$

$$= \left(-\frac{4}{-2 \cdot 2}, \frac{4 \cdot (-2) \cdot (-6) - 4^2}{4 \cdot (-2)} \right)$$

$$= (1, -4)$$



You Try

Graph the following quadratic functions. Label the vertex and a point in the graph.

$$1. y = 2x^2 - 4x - 1$$

$$2. y = -3x^2 - 12x + 1$$