Exponential Functions

$f(x)$ is an exponential function if

$$f(x) = a \cdot b^x$$

b is the base of the exponential function.

$$b > 0 \quad \text{and} \quad b \neq 1$$

$$a \neq 0.$$

[NOTE: If $b = 1$, $f(x) = a \cdot 1^x = a$ (constant)]

Example:

① $f(x) = 3^x$

② $f(x) = \left(\frac{1}{2}\right)^x$

③ $f(x) = 2024 \cdot (8)^x$

④ $f(x) = -7 \cdot 6^x$

（★） Graphs of exponential functions.

$$f(x) = a \cdot b^x$$

Case 1 : $b > 1$ and $a > 0$

$x = 0 \Rightarrow Y = a$
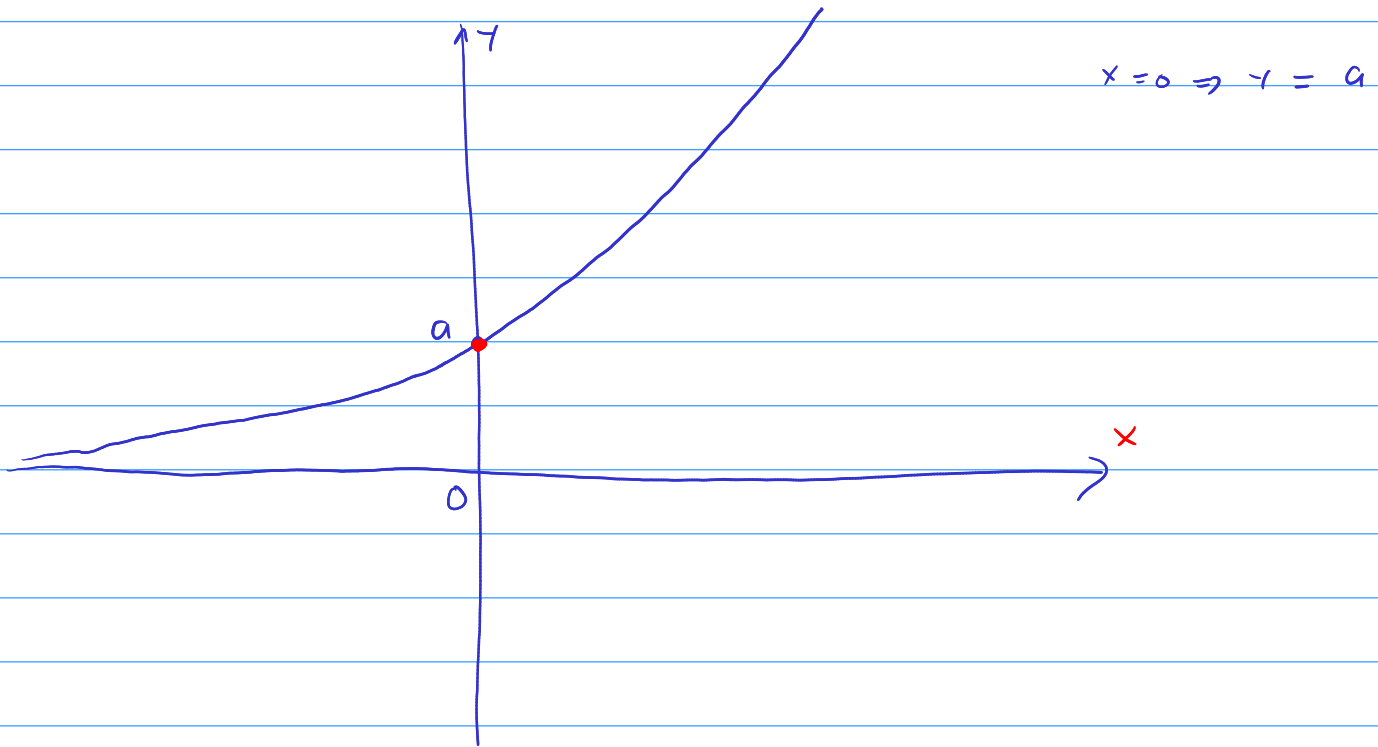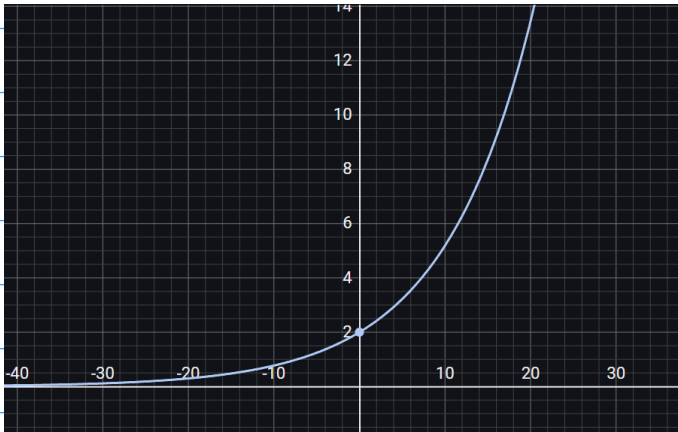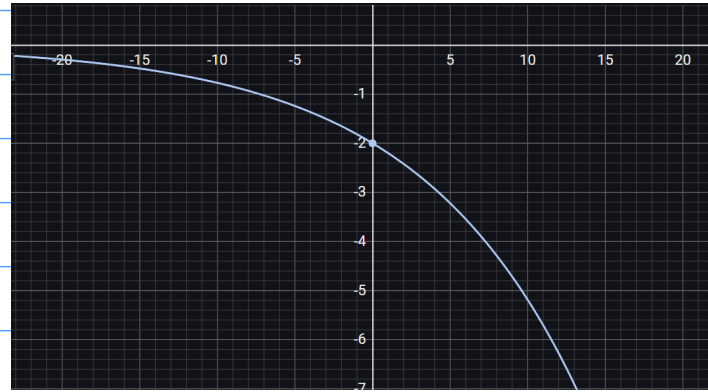
Case 2 : $b > 1$ and $a < 0$
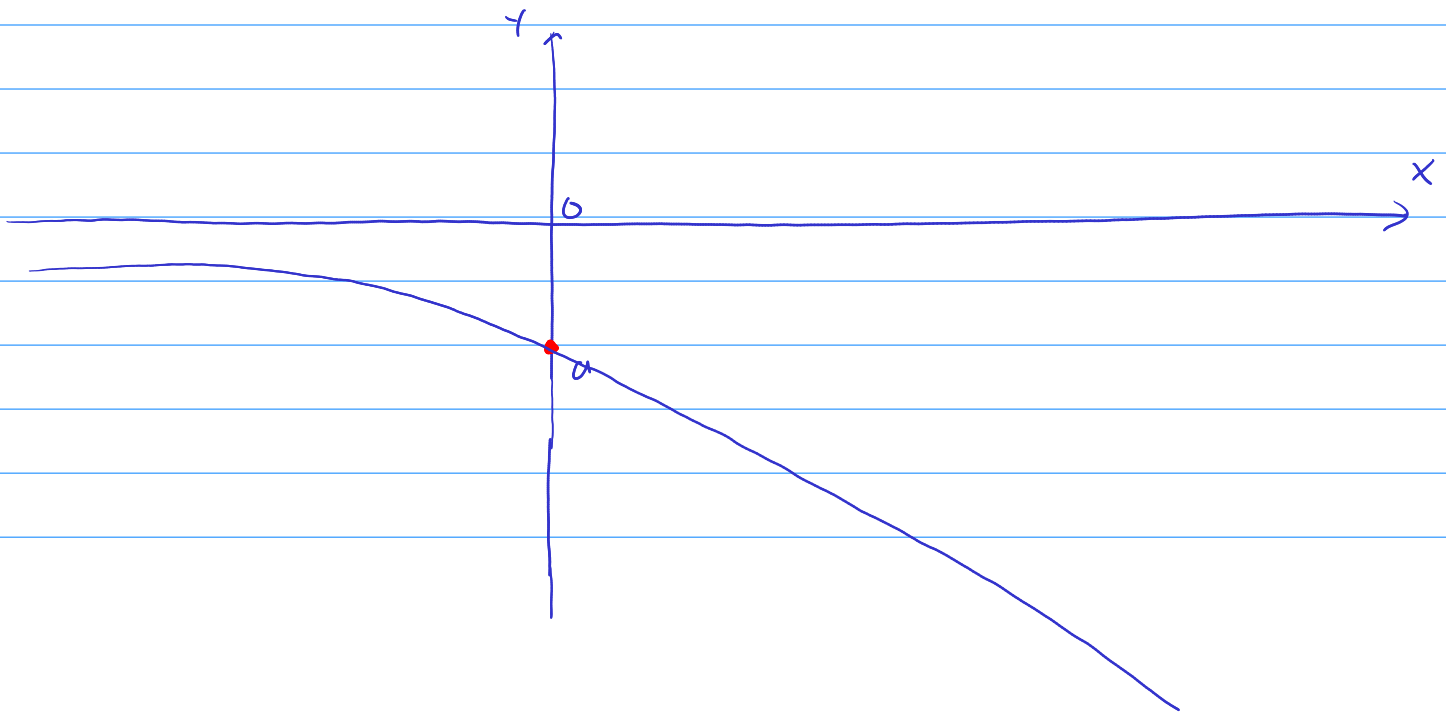
Example :

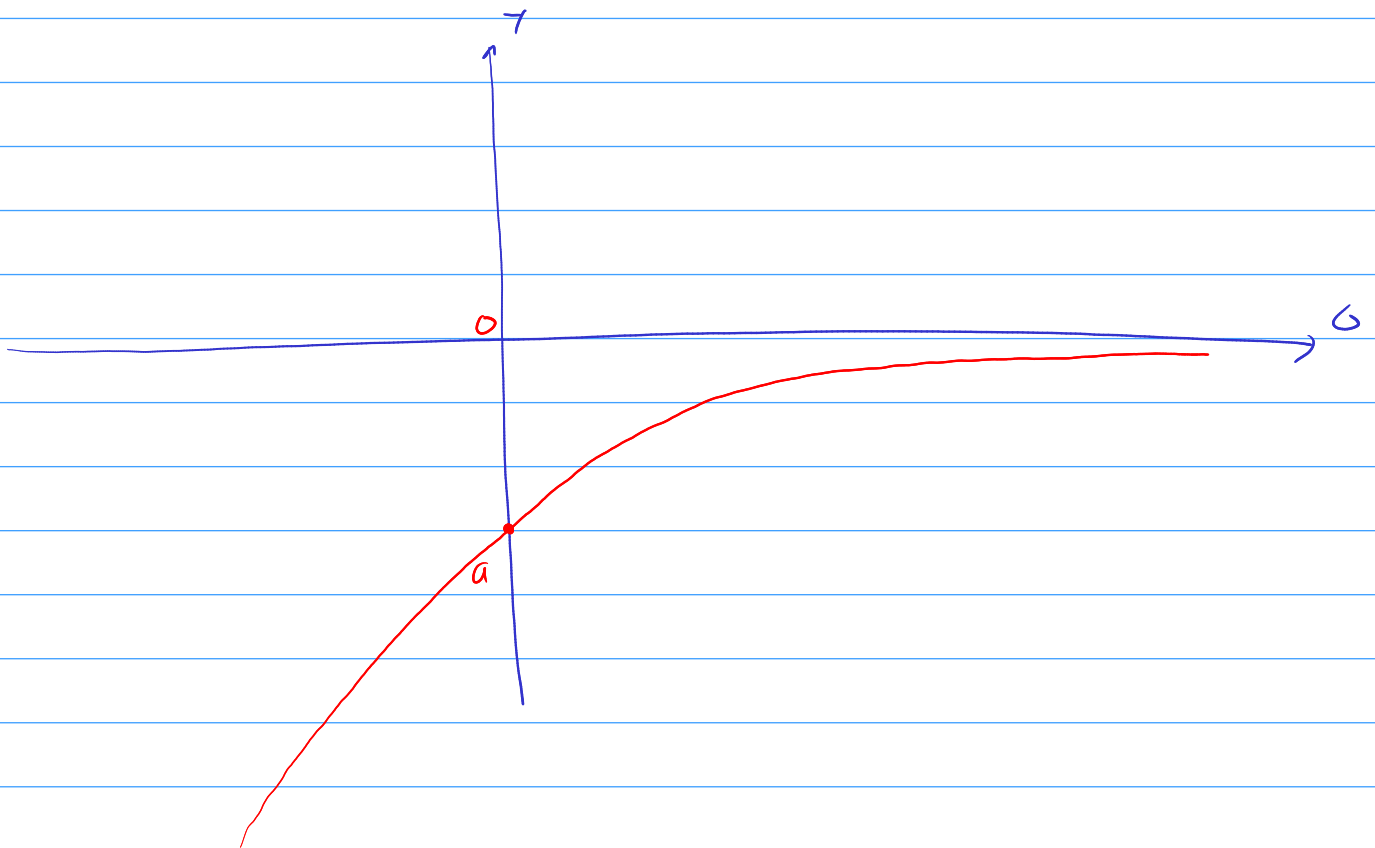$y = 2 \times (1.1)^x$ $y = -2 \times (1.1)^x$

Case 3 : $0 < b < 1$ and $a > 0$
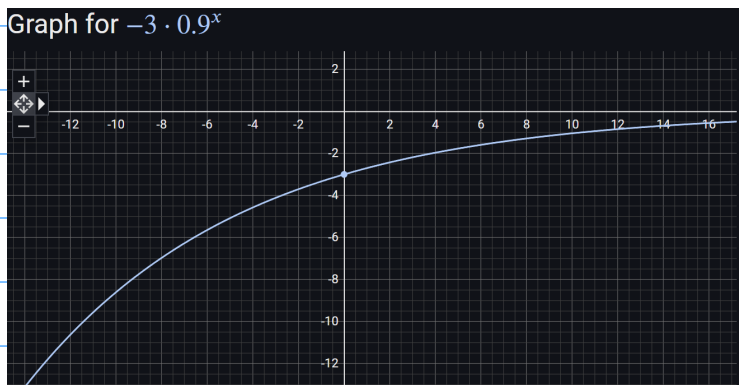
Case 4 :   0 < b < 1   and   a < 0

Example :

Graph for $-3 \cdot 0.9^x$

Graph for $3 \cdot 0.9^x$
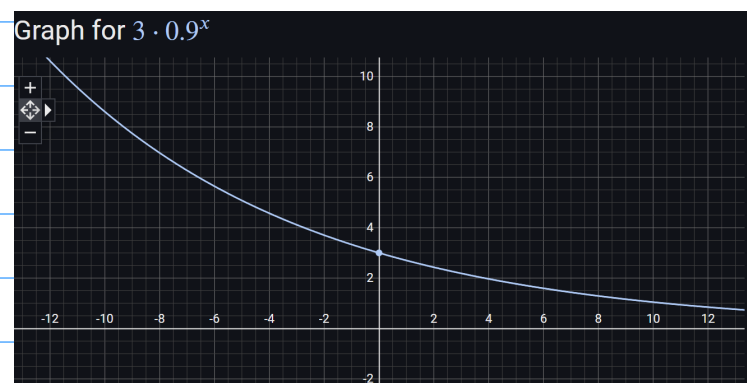
Linear vs. Exponential Growth.

Linear Growth

exponential growth
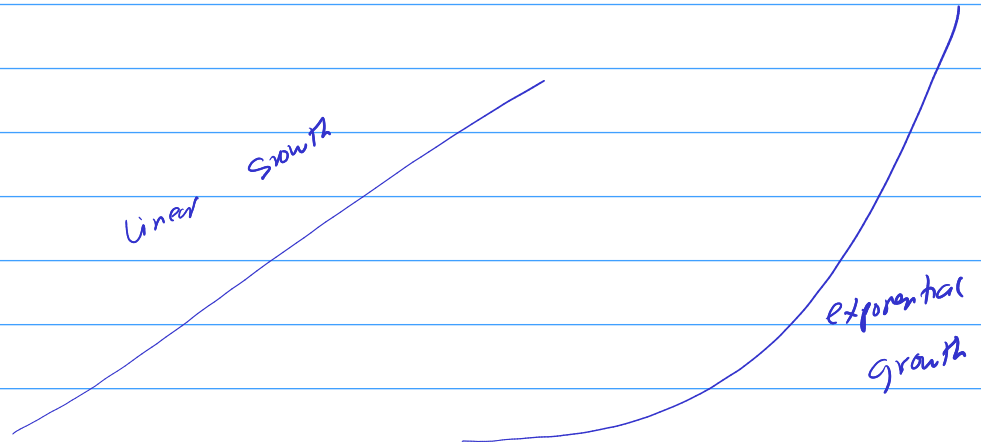
Simple Interest vs. Compound Interest.

(*) |Simple Interest| : The interest is only apply to

the principal

Suppose we deposit $ 100 to a bank with

simple interest of 5% yearly.

The principal : $P = 100$

The interest rate : $r = 5\%$

Year $\textcircled{1}$:   You earn: $P \cdot r$ = 100 × 5% = $^{\$}$5

$\Rightarrow$ Your total money: 100 + 5 = $^{\$}$105

$\underbrace{\qquad\qquad\qquad}_{A}$

$A = 100 + \underbrace{100 \times 5\%}_{} = 100 \left(1 + 1.5\% \right) = \underline{105}$

Year $\textcircled{2}$:   You earn the same amount $^{\$}$5

$\rightarrow$ total money: 105 + 5 = $^{\$}$110

$A = \underbrace{100}_{} \left(1 + \underline{5\%} \right) + \underbrace{100}_{} \cdot \underline{5\%}_{}$

$= 100 \left(1 + 2 \times 5\% \right)$

Year 3 :

$A = 100 \left(1 + 3 \times 5\% \right)$

Year t :

$$\boxed{A = 100 \left(1 + t \times 5\% \right)}$$ (linear functn)

This is a linear Growth

(*) Compound Interest : The interest is applied to the current amount of money.

Year 1 : $100 + 100 \cdot 5\% = 100(1 + .05)$

$= 100 \times 1.05$

Year 2 :

$100 \times 1.05 \times 1.05 = 100 \times (1.05)^2$

Year 3 :

$A = 100 \times (1.05)^3$

Year t :

$A = 100(1.05)^t$ (Exponential Growth)

Assignment :

① Plot $y = 3 + (1.02)^x$

② Plot $y = 2 + (.8)^x$

(3) Find the total amount of money you have if you invest $2000 after 5 years if.

(a) the interest rate is 6% and it is a simple interest.

(b) the interest rate is 6% and it is a compound interest.