Applications of Linear Functions

Total Cost The *total cost* C for a manufacturer during a given time period is a function of the number N of items produced during that period. To determine a formula for the total cost, we need to know two things. The first is the manufacturer's *fixed costs*. This amount covers expenses such as plant maintenance and insurance, and it is the same no matter how many items are produced. The second thing we need to know is the cost for each unit produced, which is called the *variable cost*.

Suppose that a manufacturer of widgets has fixed costs of \$1500 per month and that the variable cost is \$20 per widget (so it costs \$20 to produce 1 widget).

C: Total Cost

Total Cost = Fixed Cost + Variable Cost · x

Total cost = 1500 + 20 · x

where x: the number of widgets.

we can write

$C(x) = 1500 + 20x$

↓

cost function is linear.

Example 2

Another widget manufacturer has a variable cost of \$12 per widget, and the total cost is \$3100 when 150 widgets are produced in a month. What are the fixed costs for this manufacturer?

$x = 150$

$C = 3100$

Total Cost = Fixed Cost + Variable cost

3100 = Fixed cost + $12 \cdot 150$

$\Rightarrow$ $3100 = \text{Fixed cost} + 1800$

$\Rightarrow$ $\text{Fixed cost} = 3100 - 1800 = 1300$

Yet another widget manufacturer has determined the following: The total cost is $2700 when 100 widgets are produced in a month, and the total cost is $3500 when 150 widgets are produced in a month. What are the fixed costs and variable cost for this manufacturer?

C : Total cost ; F : Fixed cost ; w cost per 1 widget / variable cost.

$C = F + 100 \cdot w$

$\Rightarrow \quad 2700 = F + 100 \cdot w$

when 150 widgets are produced we have

$3500 = F + 150 \cdot w$ ←

we need to find F and w

$$\begin{array}{rcl} 3500 & = & F + 150w \\ - \quad 2700 & = & F + 100w \end{array}$$

$(3500 - 2700) = 150w - 100w$

$\Rightarrow \quad 800 = 50w$

$w = \frac{800}{50} = \boxed{16}$ ← variable cost

Plug $w = 16$ to the "green" equation:

$3500 = F + 150 \cdot 16$

$\Rightarrow \quad 3500 = F + 2400$

$\Rightarrow \quad F = 3500 - 2400 = \boxed{1100}$ ← Fixed cost.

Total Revenue and Profit *This is a continuation of* [redacted]. The *total revenue R* for a manufacturer during a given time period is a function of the number N of items produced during that period. In this exercise, we assume that the selling price per unit of the item is a constant, so it does not depend on the number of items produced. The *profit P* for a manufacturer is the total revenue minus the total cost. If the profit is zero, then the manufacturer is at a *break-even point*.

We consider again the manufacturer of widgets in Exercise 13 with fixed costs of \$1500 per month and a variable cost of \$20 per widget. Suppose the manufacturer sells 100 widgets for \$2300 total.

$F = 1500$

$w = 20$

Find the break-even point.

Let x is the number of widgets to be sold.

The cost to produce x widgets is:

Total cost $C(x)$ = Fixed cost + $20x$.

$$\Rightarrow \boxed{C(x) = 1500 + 20x}$$

The revenue obtained when selling x widgets is $\boxed{23x}$ ←

(because 100 widgets = \$2300 $\Rightarrow$ 1 widget = \$23)

The break-even point is when the total cost is the same as the revenue:

$$1500 + 20x = 23x$$

$$\Rightarrow 1500 = 23x - 20x$$

$\Rightarrow \quad 1500 = 3x$

$\Rightarrow \quad x = \frac{1500}{3} = 500.$

How many widgets needed to be sold to make at least $1000

$R(x) - C(x) = 1000$

$\Rightarrow \quad 23x - (1500 + 20x) = 1000$

$23x - 1500 - 20x = 1000$

$3x - 1500 = 1000$

$\Rightarrow \quad 3x = 1500 + 1000 = 2500$

$\Rightarrow \quad x = 2500/3 = 833.33 \Rightarrow$ 834 widgets should be produced.

Assignment (Take photos of the answer and post to Canvas)

widget manufacturer has a variable cost of $12 per widget, and the total cost is $3100 when 150 widgets are produced in a month.

(a) Find the fixed cost of this manufacter

(b) The manufacter sell 100 widgets for $2000. Find the price for each item

(c) Find the break-event point for the manufacter

(d) How many items the manufacter has to sell to

have the profit of $10,000 ?