Exponential Functions:

1. Definition

$$y = a \cdot b^x$$

x : input

y : output

a, b are some constants

b : The base

$\boxed{b > 0}$ and $b \neq 1$

$a \neq 0$

Example : $y = 6 \cdot 4^x$

$y = -7 \cdot (1/3)^x$

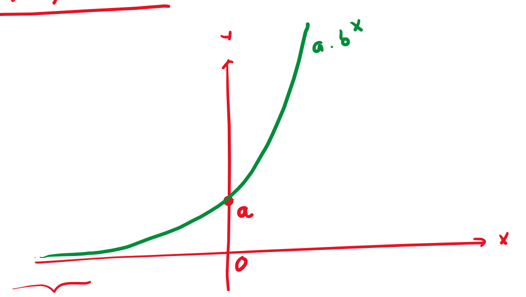
Not valid : $y = -6 \cdot (-4)^x$ not exponential

because $b < 0$

2. Graphs

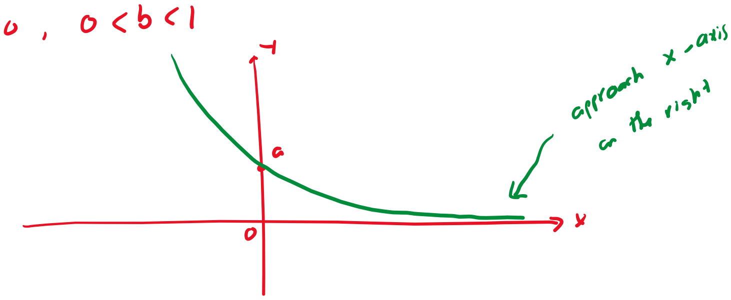
4 cases

① $a > 0$, $b > 1$

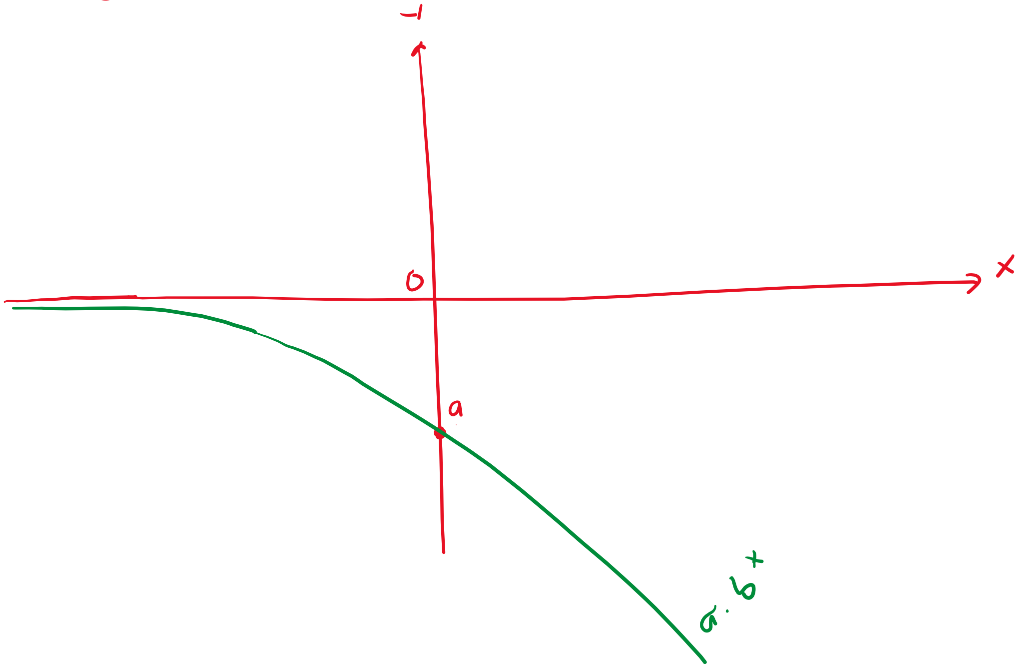


approaching x-axis on the left

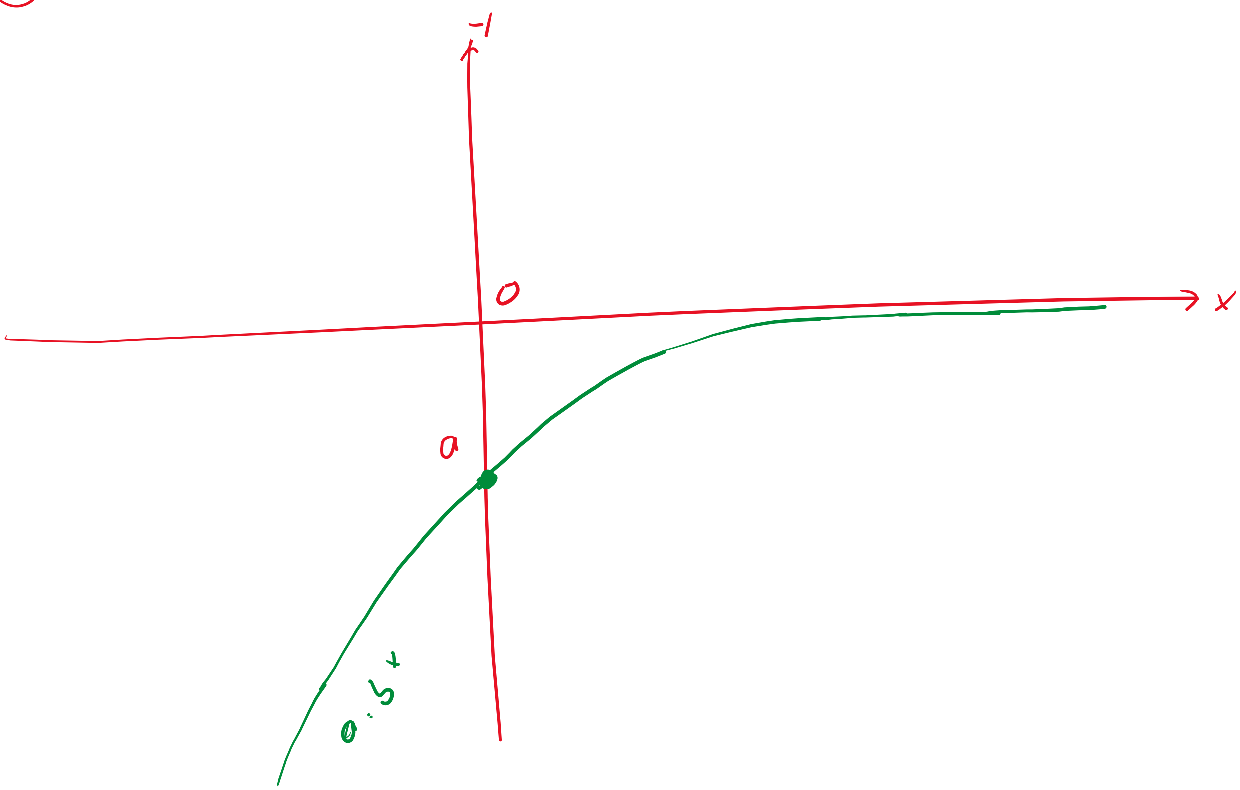
② $a > 0$, $0 < b < 1$



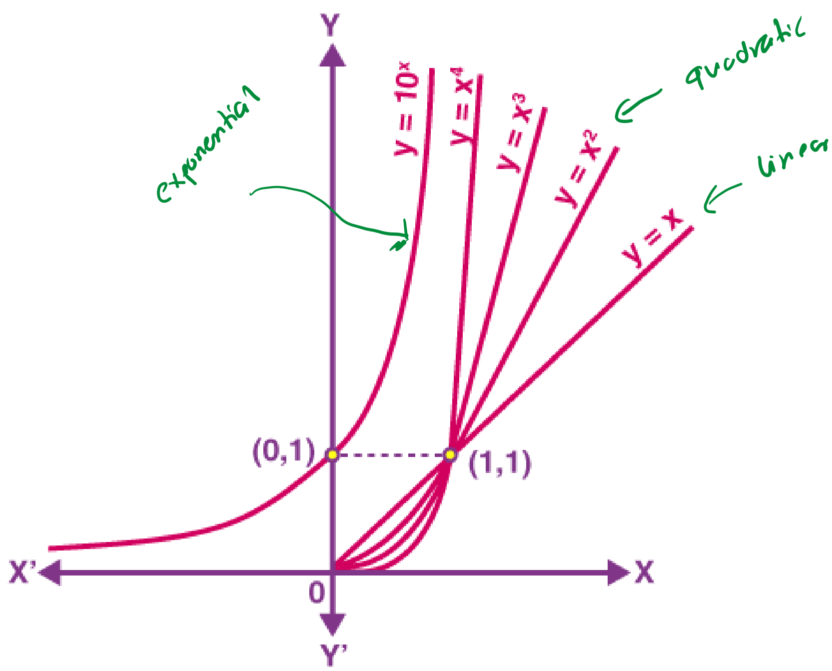
③ $a<0, \quad b>1$



④ $a<0, \quad 0<b<1$



Note: In the long run, exponential growth is faster than any polynomial/power



Paper folding Example

Folding a standard sheet paper a few times. How thick it would be after being folded 42 times?

If we fold a sheet of paper, one time, we double the thickness of the paper.

If we fold a sheet of paper, two time, the thickness of the paper is four times more than the original's thickness.

Every time, we fold a paper, we double its thickness.

Question: How thick would it be after being folded 42 times?
Answer: It would be thicker than the distance from the earth to the moon!

How thick is a standard sheet of paper? 0.004 inches

The thickness of the paper after being folded x times is:

$$Y = .004 \cdot 2^x$$

$x = 42$ (fold 42 times)

$$Y = .004 * 2^{42}$$

Y = 17592186044.4 (inches) = 277,654 (miles)

What is the distance between the earth and the moon: 238,855 miles

3. Two forms: base e and regular base, b

$e = 2.71828$, Euler's number.

$$Y = \underbrace{a \cdot b^x}_{\text{standard form}} = \underbrace{a \cdot e^{(\ln b)\cdot x}}_{\text{e-based form}}$$

Notice: $b = e^{\ln b}$

Example: convert to based-e form:

① $Y = 6 \cdot 7^x$ (regular base)

$$= 6 \cdot e^{(\ln 7)\cdot x}$$

$$\approx 6 \cdot e^{1.9459x}$$

② $Y = 9 \cdot \left(\frac{1}{4}\right)^x$

$$= 9 \cdot e^{(\ln 1/4)\cdot x}$$

$$\approx 9 \cdot e^{-1.3863x}$$

Example: convert to a regular base

① $y = 7e^{-4x}$ (base e)

$= 7 \cdot (e^{-4})^x$

$\approx 7 \cdot (.0183)^x$

② $y = 8 \cdot e^{2x}$

$= 8 \cdot (e^2)^x$

$= 8 \cdot (7.389)^x$

Mini-Break.

Class will be back in 20-25 minutes.

We will have an assignment at the end of the next session.

4. Solving Exponential Equations.

Log operator/function

Example :

$\circledast \quad 10^x = 100$

$\Leftrightarrow \quad x = 2$

$\circledast \quad 10^x = 1000$

$\Leftrightarrow \quad x = 3$

$\circledast \quad 10^x = 10$

$\Leftrightarrow \quad x = 1$

$\circledast \quad 10^x = 11$

$x = \log 11 \approx 1.0414$

$\circledast \quad 10^x = 2026$

$x = \log 2026 \simeq 3.3066$

In general, $\log K$ is the solution of the equation : $10^x = K$

OR : $10^{\log K} = K$

Properties of Log

1. $10^{\log K} = K$

2. $\log 10^{K} = K$

3. $\log b^{x} = x \cdot \log b$

4. $\log (a \cdot b) = \log a + \log b$

5. $\log \left(\frac{a}{b}\right) = \log a - \log b$

How to solve exponential equations using log:

Solving steps:

- Isolate the base
- Take the log of both sides
- Use logarithmic properties to simplify the expression and solve for *x*
- Round all answers to 4 decimal places

Example:

Solve:

1. $4^{x} = 16$

$\Leftrightarrow x = 2$

2. $4^{x} = 17$

Take log of both sides.

$\log 4^{x} = \log 17$

$\Leftrightarrow x \cdot \log 4 = \log 17$

$\Leftrightarrow \quad x = \frac{\log 17}{\log 4} \approx 2.0437$

③ $9^x = 7$

$\Leftrightarrow \quad \log 9^x = \log 7$

$\Leftrightarrow \quad x \cdot \log 9 = \log 7$

$\Leftrightarrow \quad x = \frac{\log 7}{\log 9} \approx .8856$

④ $6 \cdot 7^{4x+1} - 11 = 5$

$\Leftrightarrow \quad 6 \cdot 7^{4x+1} = 5 + 11$

$\Leftrightarrow \quad 6 \cdot 7^{4x+1} = 16$

$\Leftrightarrow \quad 7^{4x+1} = \frac{16}{6}$

$\Leftrightarrow \quad \log 7^{4x+1} = \log\left(\frac{16}{6}\right)$

$$\Leftrightarrow \quad (4x+1)\cdot\log 7 = \log\left(\frac{16}{6}\right)$$

$$\Leftrightarrow \quad 4x+1 = \frac{\log\left(\frac{16}{6}\right)}{\log 7} \approx .5040$$

$$\Leftrightarrow \quad 4x = .5040 - 1$$

$$\Leftrightarrow \quad x = \frac{.5040-1}{4} = -.124$$

⑤ $$7^{4x+1} = 9^{2x+3}$$

$$\log\left(7^{4x+1}\right) = \log\left(9^{2x+3}\right)$$

$$\Leftrightarrow \quad (4x+1)\cdot\log 7 = (2x+3)\cdot\log 9$$

$$\Leftrightarrow \quad (4x+1) * .845 = (2x+3) * .954$$

$$\Leftrightarrow \quad 4 * .845\,x + .845 = 2 * .954\,x + 3 * .954$$

$$\Leftrightarrow \quad 3.38x + .845 = 1.908x + 2.862$$

$$\Leftrightarrow \quad 3.38x - 1.908x = 2.862 - .845$$

$$\Leftrightarrow \quad 1.472x = 2.017$$

$$\Leftrightarrow \quad x = \frac{2.017}{1.472} \approx \boxed{1.37}$$

Practice Problem (Assignment_Feb_25)

Solve for x

1. $5^x = 4$
2. $6^x = 10$
3. $5 \cdot 7^x = 1$
4. $8 \cdot 3^{4x+3} = 2$
5. $8^{x+1} = 7^{1-x}$

Take photos/screenshots of your answer and submit it to Canvas (Assignment_Feb_25).