

## Quadratic Functions

$$y = ax^2 + bx + c \quad (a \neq 0)$$

*a is not zero*

Example:

$$y = \underbrace{2x^2}_{\text{quadratic term}} + \underbrace{4x}_{\text{linear term}} + \underbrace{6}_{\text{constant term}}$$

$$y = 4x^2 + 2026$$

$$y = 6x^2 + x$$

### 1. Solving Quadratic Equations

$$ax^2 + bx + c = 0 \quad (a \neq 0)$$

#### a. Factoring

Example:  $2x^2 = 4x$

$$2x^2 = 4x$$

$$\Leftrightarrow 2x^2 - 4x = 0$$

factoring

$$\Leftrightarrow 2x \cdot (x - 2) = 0$$

$$\Leftrightarrow \begin{cases} 2x = 0 \\ x - 2 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = 0 \\ x = 2 \end{cases} \quad (\text{Two Solutions})$$

Example:  $x^2 + 7x = -10$

$$x^2 + 7x = -10$$

$$\Leftrightarrow x^2 + 7x + 10 = 0$$

$$\Leftrightarrow (x+2)(x+5) = 0$$

Note: we look for 2 numbers: sum = 7  
product = 10  
→ 2 and 5

$$\Leftrightarrow \begin{cases} x+2=0 \\ x+5=0 \end{cases} \quad \Leftrightarrow \begin{cases} x=-2 \\ x=-5 \end{cases}$$

Example:  $x^2 - 4x + 3 = 0$

$$x^2 - 4x + 3 = 0$$

$$\Leftrightarrow (x-1)(x-3) = 0$$

sum = -4  
product = 3  
→ -1; -3

$$\Leftrightarrow \begin{cases} x-1=0 \\ x-3=0 \end{cases} \quad \Leftrightarrow \begin{cases} x=1 \\ x=3 \end{cases}$$

Example:  $x^2 - 25 = 0$

$$x^2 - 25 = 0$$

$$\Leftrightarrow x^2 = 25$$

$$\Leftrightarrow x = \pm \sqrt{25}$$

$$\Leftrightarrow x = \pm 5 \quad \text{or} \quad \begin{cases} x = 5 \\ x = -5 \end{cases}$$

Example:  $x^2 - x = 12$

### b. Quadratic Formula

$$ax^2 + bx + c = 0$$

$$\underline{\text{case 1}}: b^2 - 4ac < 0$$

No solution!

$$\underline{\text{case 2}}: b^2 - 4ac \geq 0$$

$$\text{solutions} : x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example:

$$1 \cdot x^2 + 4x + 1 = 0$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$a=1 \quad b=4 \quad c=1$$

$$b^2 - 4ac = 4^2 - 4 \cdot 1 \cdot 1 = 12$$

$$\text{solution: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{12}}{2}$$

$$x = \frac{-4 + \sqrt{12}}{2} ; \quad x = \frac{-4 - \sqrt{12}}{2}$$

Example:  $x^2 + 10 = 4x$

$$x^2 + 10 = 4x$$

$$\Leftrightarrow x^2 - 4x + 10 = 0$$

$$a=1 ; \quad b = -4 ; \quad c = 10$$

$$\begin{aligned} b^2 - 4ac &= (-4)^2 - 4 \cdot 1 \cdot 10 \\ &= 16 - 40 = -24 < 0 \end{aligned}$$

**NO Solution**

Example:  $x^2 - 6x + 10 = 0$

**You try: Mixed Practice**

Solve each quadratic by factoring or the quadratic formula.

1.  $2x^2 = 4x$

2.  $x^2 - 4x = 12$

3.  $2x^2 - x - 10 = 0$

4.  $x^2 - 10x + 24 = 0$

5.  $10x^2 - 25x = 0$

6.  $x^2 + 5x + 2 = 2x$

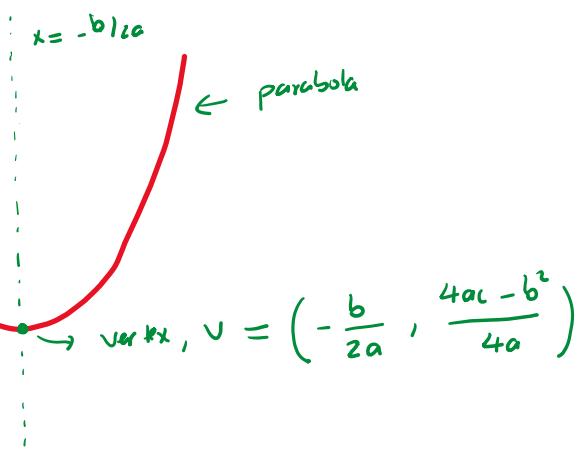
7.  $x^2 - 100x + 900 = 0$

8.  $8x^2 + x - 75 = 0$

## 2. Graphs of Quadratic Functions

$$y = ax^2 + bx + c \quad (a \neq 0)$$

case 1 :  $a > 0$



To graph we need ① vertex

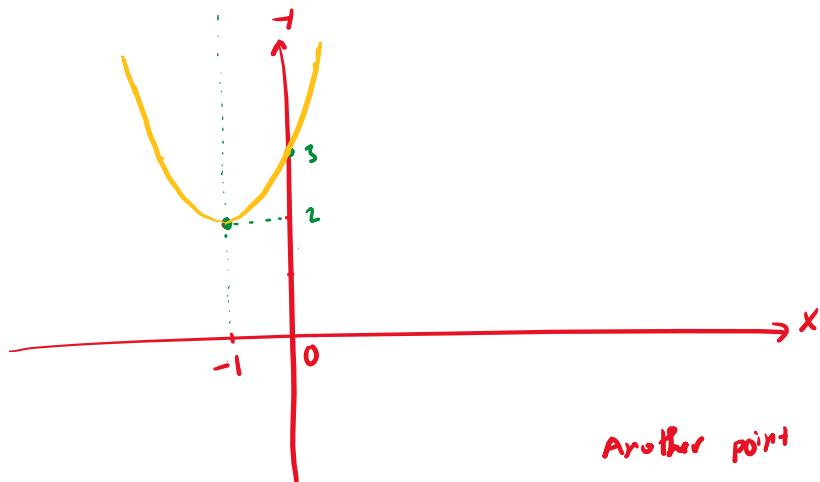
② Another point

Example:  $y = x^2 + 2x + 3$

$$a = 1, b = 2, c = 3$$

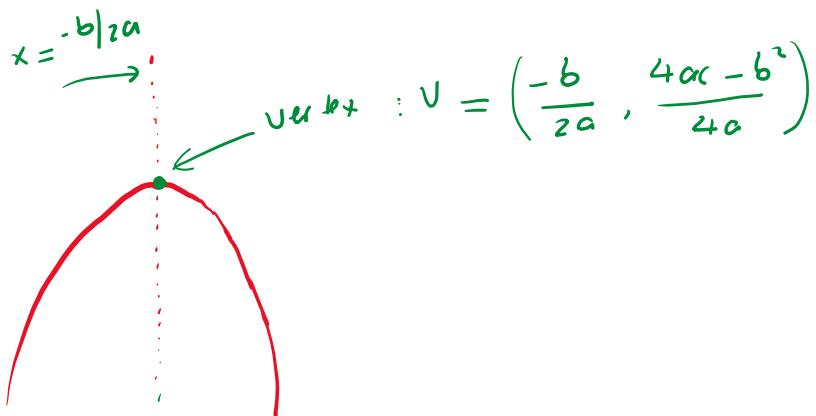
$$\text{vertex } V = \left( -\frac{b}{2a}, \frac{4ac - b^2}{4a} \right)$$

$$V = \left( -\frac{2}{2}, \frac{4 \cdot 1 \cdot 3 - 2^2}{4} \right) = (-1, 2)$$



Another point  $x = 0, y = 3$

**Case 2 :  $a < 0$**



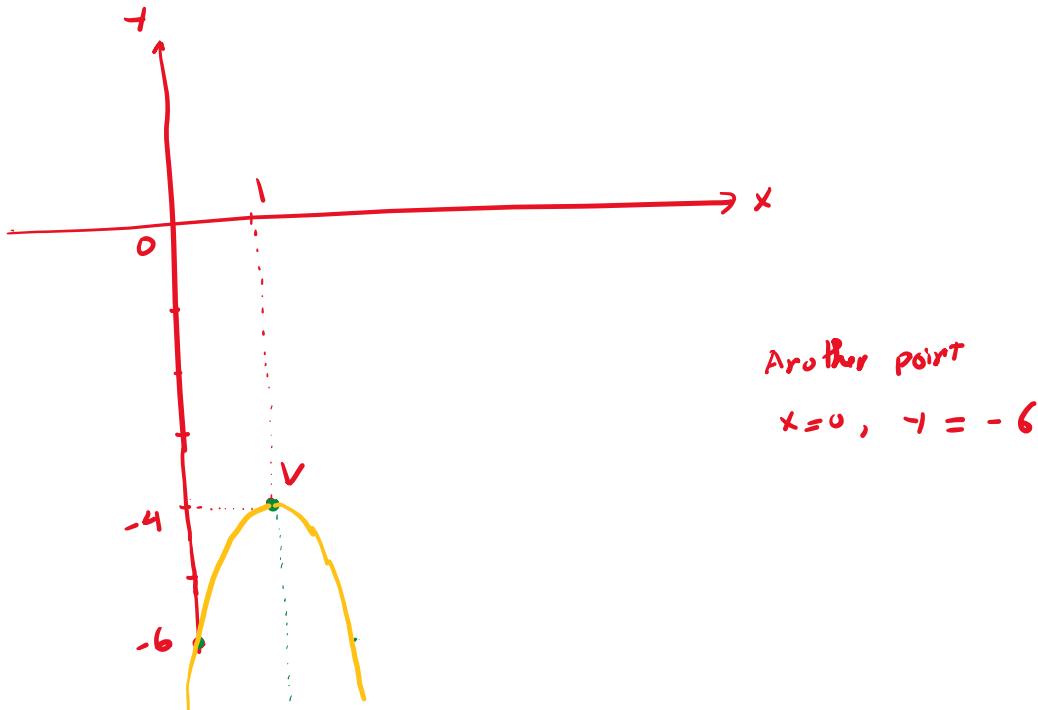
To graph we need ① Vertex

② Another point.

Example:  $y = -2x^2 + 4x - 6$

$$a = -2, b = 4, c = -6$$

$$\begin{aligned}\text{Vertex: } V &= \left( -\frac{b}{2a}, \frac{4ac - b^2}{4a} \right) \\ &= \left( -\frac{4}{2 \cdot (-2)}, \frac{4 \cdot (-2) \cdot (-6) - 4^2}{4 \cdot (-2)} \right) \\ &= (1, -4)\end{aligned}$$



## You Try

Graph the following quadratic functions. Label the vertex and a point in the graph.

1.  $y = 2x^2 - 4x - 1$
2.  $y = x^2 - 1$
3.  $y = -x^2 + 1$
4.  $y = -3x^2 - 12x + 1$

**Example:**  $y = -x^2 + 6x - 9$