

## Applications of Linear Functions

### 1. Supply, demand and Equilibrium

#### Supply Functions

A linear supply function is written as:

$$Q_s = -c + dp$$

*↓*  
*out put*      *input*

where:

- $Q_s$  = quantity supplied
- $p$  = price
- $c$  = fixed quantity adjustment
- $d$  = increase in supply for each unit increase in price

[  $Q_s$  is a linear function of  $p$  ]  
(positive number)

#### Demand Functions

A linear demand function is written as:

$$Q_d = a - bp$$

where:

- $Q_d$  = quantity demanded
- $p$  = price
- $a$  = maximum quantity demanded when the price is zero
- $b$  = decrease in demand for each unit increase in price

$Q_d$  is a linear function of  $p$   
(  $b > 0$  )

#### Equilibrium

Equilibrium occurs when quantity demanded equals quantity supplied:

$$\cancel{q_d = q_s} \quad Q_d = Q_s$$

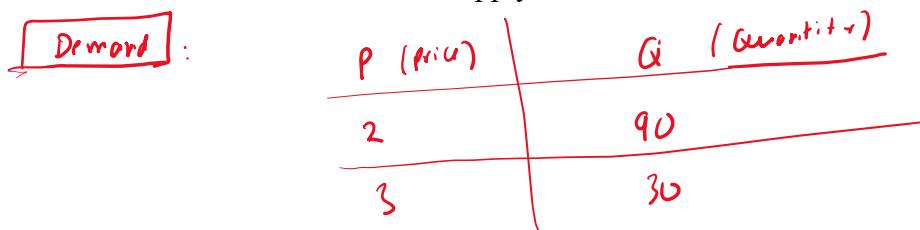
At equilibrium, there is no surplus or shortage in the market.

**Problem:**

(Demand) You own a small convenience store. You can sell 90 packs of gum per week if they are marked at \$2 each, but only 30 each week if they are marked at \$3/pack.

(Supply) Your gum supplier is prepared to sell you 20 packs each week if they are marked at \$1/pack and 100 each week if they are marked at \$3/pack.

- a. Write the demand and supply functions.



we just need to write the equation of linear function. That goes through 2 points  $(2, 90)$  and  $(3, 30)$

Formula:

$$Q = \frac{Q_2 - Q_1}{P_2 - P_1} \cdot (P - P_1) + Q_1$$

$$\Rightarrow Q = \frac{30 - 90}{3 - 2} \cdot (P - 2) + 90$$

$$Q = -60 \overbrace{(P - 2)}^{\leftarrow} + 90$$

$$\Rightarrow Q = -60P + 120 + 90$$

$$\Rightarrow \boxed{Q = -60P + 210} \quad \text{or} \quad \underline{\underline{Q = 210 - 60P}}$$

$$\underline{\underline{Q_d = -60P + 210}} \quad (\text{Demand Equation})$$

## Supply

(Supply) Your gum supplier is prepared to sell you 20 packs each week if they are marked at \$1/pack and 100 each week if they are marked at \$3/pack.

$P$	$Q_s$
1	20
3	100

We need to write the equation of the line passing through

2 points  $(1, 20)$  and  $(3, 100)$

$$\begin{matrix} & \uparrow \\ P_1 & ; & Q_1 \\ & \uparrow \\ P_2 & & Q_2 \end{matrix}$$

Formula :

$$Q_s = \frac{Q_2 - Q_1}{P_2 - P_1} \cdot (P - P_1) + Q_1$$

$$\Rightarrow Q_s = \frac{100 - 20}{3 - 1} \cdot (P - 1) + 20$$

$$\Rightarrow Q_s = 40(P - 1) + 20$$

$$Q_s = 40P - 40 + 20$$

$$Q_s = 40P - 20$$

Supply Equation

- b. Find the equilibrium point. For supply to equal demand, the packs of gum must be priced at how much apiece?

$$Q_d = Q_s$$

$$-60P + 210 = 40P - 20$$

$$-60P - 40P = -20 - 210$$

$$-100P = -230$$

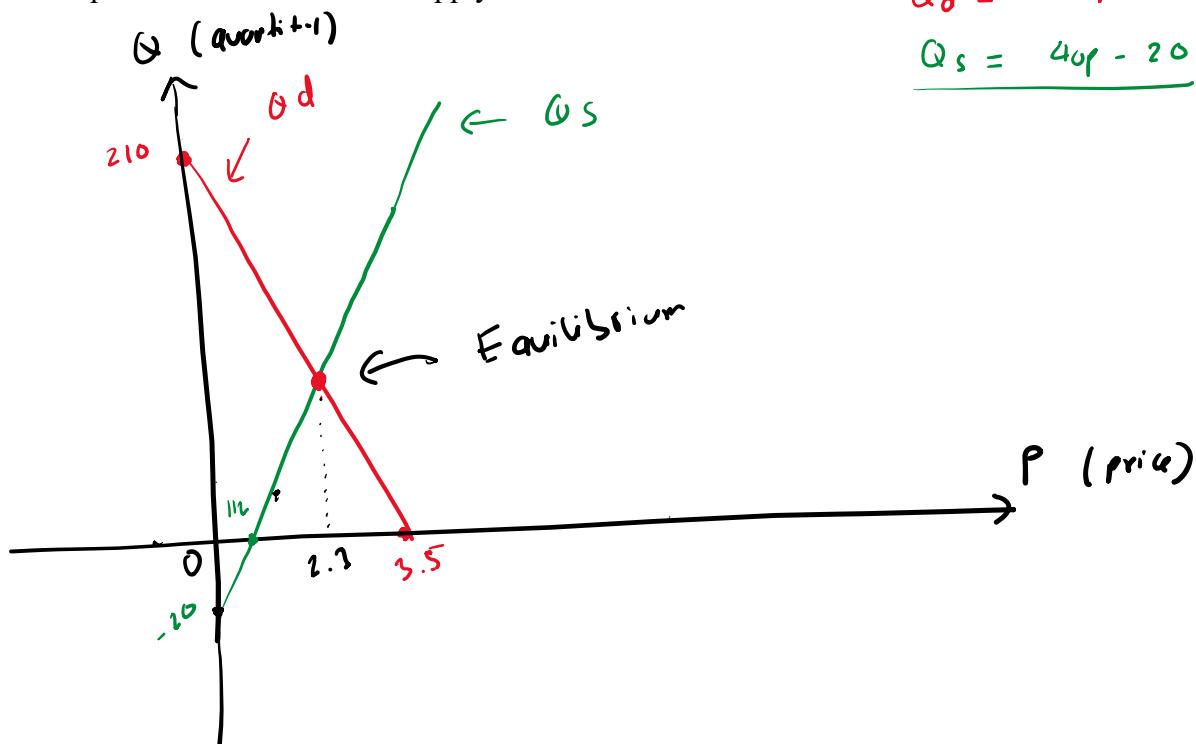
$$P = 2.3$$

- c. Graph both the demand and supply function on the same axis.

$$Q_d = -60P + 210$$


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$$Q_s = 40P - 20$$



$$-60P + 210 = 0 \Rightarrow P = \frac{210}{60}$$

$$40P - 20 = 0 \Rightarrow P = \frac{20}{40} = 1/2$$

**You Try:**

The demand for your college newspaper is 2,000 copies each week if the paper is given away free of charge and drops to 800 each week if the charge is \$1.50 per copy.

However, the university is prepared to supply only 600 copies per week free of charge but will supply 1,500 each week at \$1.50 per copy.

- a. Write the demand and supply functions.
- b. Find the equilibrium point. For supply to equal demand, the newspaper must be priced at how much apiece?
- c. Graph both the demand and supply function on the same axis.

## 2. Cost – Revenue and Break-Even

### Cost Function

The total cost of producing goods is the sum of **fixed costs** and **variable costs**.

A linear **cost function** is written as:

$$C = mq + b$$

where:

- $C$  = total cost
- $q$  = quantity produced
- $m$  = variable cost per unit
- $b$  = fixed cost

*C is a linear function of  
q*

### Revenue

**Revenue** is the total income from selling goods.

Revenue is calculated as:

$$R = pq$$

where:

- $R$  = total revenue
- $p$  = price per unit
- $q$  = quantity sold

*Revenue is a linear function of  
q*

## Profit

Profit represents the financial gain from producing and selling goods. It is calculated as **total revenue minus total cost**.

The profit function is written as:

$$P = R - C$$

*Profit*      *Revenue*  
↓              ↘  
 $P = R - C$       ↙ *Cost*

where:

- $P$  = profit
- $R$  = total revenue
- $C$  = total cost

## Break-even point

The **break-even point** is the production level at which **profit equals zero**. At this point, total revenue is exactly equal to total cost.

Mathematically:

$$P = 0$$

or :  $\underline{\text{Cost}} = \text{Revenue}$ .

$m$

**Problem:** An anticoagulant drug can be made for \$10 per unit. The total cost to produce 100 units is \$1500.

a. Assuming that the cost function is linear, find its rule.

cost function :  $c = mq + b$

$$m = 10$$

$$\Rightarrow c = 10q + b$$

We know Profit when  $q = 100$ ,  $c = 1500$

This means :

$$1500 = 10 \times 100 + b$$

$$\Rightarrow 1500 = 1000 + b \Rightarrow b = 1500 - 1000$$
$$\Rightarrow b = 500$$

$$\Rightarrow \boxed{c = 10q + 500} \quad \text{cost function}$$

- b. What are the fixed costs?

$$\text{Fixed cost} = 500$$

**You Try:** A product can be made for \$120 per unit. The total cost to provided 100 units is \$15,800.

- a. Assuming that the cost function is linear, find its rule.

- b. What are the fixed costs?

**Problem:**

A bicycle manufacturer experiences fixed monthly costs of \$124,992 and variable costs of \$52 per standard model bicycle produced. The bicycles sell for \$100 each.

- a. Find the cost function

$$\text{cost function : } c = mq + b$$

$$\Rightarrow c = 52q + 124,992$$

- b. Find the revenue function

$$R = p \cdot q \Rightarrow R = 100q$$

- c. Graph and label the cost and revenue functions on the same set of axes. Label the break-even point.

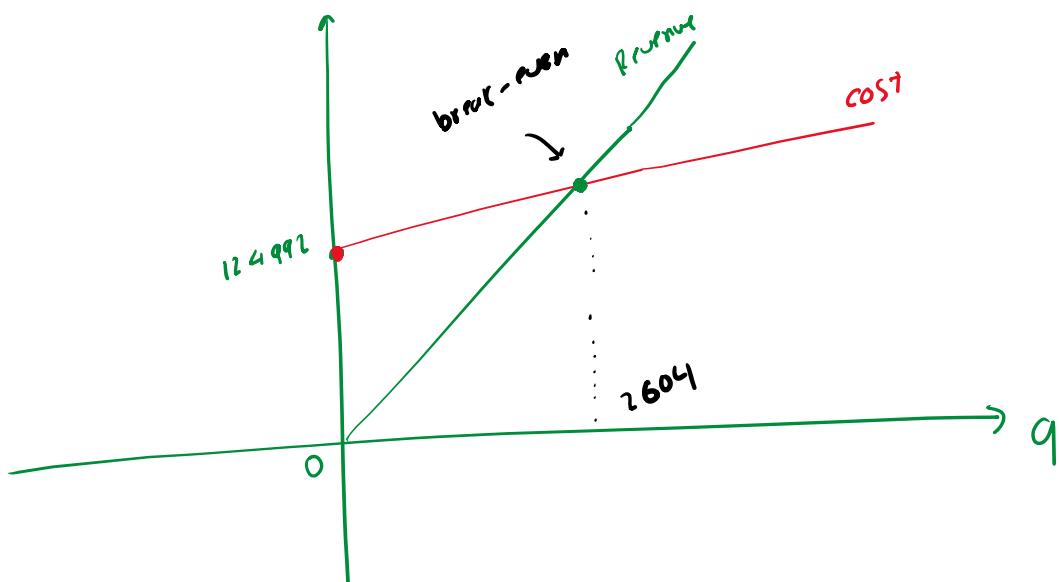
$$\text{Break - even point : } R = C$$

$$100q = 52q + 124,992$$

$$\Leftrightarrow 100q - 52q = 124,992$$

$$\Leftrightarrow 48q = 124,992$$

$$\Leftrightarrow q = \frac{124,992}{48} = 2604$$



- d. Find the profit function

$$P = R - C = 100q - (52q + 124,992)$$

$$P = 48q - 124,992$$

- e. How much profit will they make by producing and selling 2000 bicycles?

$$q = 2000 \Rightarrow P = 48 * 2000 - 124992$$

$$P = -18992$$

- f. How many bicycles must be produced and sold in order to obtain a profit of \$100,000?

$$r = 100,000$$

$$\Rightarrow 48q - 124992 = 100000$$

$$\Rightarrow 48q = 124992 + 100000$$

$$\Rightarrow 48q = 224992 \Rightarrow q = \frac{224992}{48}$$

$$\Rightarrow q = 4687.34$$

**You Try:**

A company manufactures a 65-inch smart TV that sells to retailers for \$550. The costs can be described by a linear cost function with fixed costs of \$213,000 and a variable cost per item of \$250.

- a. Find the cost function

- b. Find the revenue function

- c. Graph and label the cost and revenue functions on the same set of axes. Label the break-even point.

- d. Find the profit function

e. How much profit will they make by producing and selling 5000 TVs?

f. How many units must be produced and sold in order to obtain a profit of \$50,000?

### 3. Simple Interest

#### General Simple Interest Model

The total amount under simple interest is given by:

$$A = P + Prt$$

where

A is a linear function  
of t

- $A$  = total amount after  $t$  years
- $P$  = principal (initial amount)
- $r$  = annual interest rate (decimal)
- $t$  = time in years

👉 This is a linear equation in  $t$ .

- Intercept:  $P$  (amount when  $t = 0$ )
- Slope:  $Pr$  (interest earned per year)

#### Problem

You deposit \$2,000 in a savings account that earns 4% simple interest per year.

1. Write a linear equation that gives the total amount  $A$  after  $t$  years.
2. How much money will be in the account after 5 years?

①  $P = 2000, r = 4\%$

$$\Rightarrow A = P + Prt$$

$$\Rightarrow A = 2000 + 2000 * 4\% \cdot t$$

$$A = 2000 + 80t$$

②  $t = 5 \Rightarrow A = 2000 + 80 * 5$   
 $A = 2400$

## You try

You deposit \$3,500 in a savings account that earns 3% simple interest per year.

1. Write a linear equation that gives the total amount  $A$  after  $t$  years.
2. How much money will be in the account after 6 years?

## Problem

You invest \$1,800 at 5% simple interest.

1. Write a linear equation for the total amount  $A$  after  $t$  years.
2. How long will it take for the investment to reach \$2,070?

$$\textcircled{1} \quad P = 1800 \quad r = 5\%$$

$$\Rightarrow A = P + Prt \quad <$$

$$\Rightarrow A = 1800 + 1800 * 5\% * t$$

$$A = 1800 + 90t \quad \leftarrow$$

$$\textcircled{2} \quad A = 2070$$

$$\rightarrow 2070 = 1800 + 90t$$

$$\rightarrow 2070 - 1800 = 90t$$

$$\Rightarrow 270 = 90t \rightarrow t = \frac{270}{90}$$

$$\boxed{t = 3}$$

## You try

You invest \$2,400 at **4% simple interest**.

1. Write a linear equation for the total amount  $A$  after  $t$  years.
2. How long will it take for the investment to reach \$2,880?

## Problem

Two simple-interest investments are available:

- Investment A: \$1,000 at 5% simple interest
  - Investment B: \$1,500 at 3% simple interest
1. Write a linear equation for the total amount  $A$  for each investment.
  2. Which investment grows faster? Explain using slopes.
  3. Will the two investments ever have the same value? If so, when?

## You try

Two simple-interest investments are available:

- Investment A: \$1,200 at **4% simple interest**
  - Investment B: \$2,000 at **2% simple interest**
1. Write a linear equation for the total amount  $A$  for each investment.
  2. Which investment grows faster? Explain using slopes.
  3. Will the two investments ever have the same value? If so, when?

## 4. Depreciation and Appreciation

### Depreciation (Linear Decrease)

A value **depreciates linearly** if it decreases by the **same dollar amount each year**.

#### Model

$$V = b - mt \quad (m > 0)$$

#### Interpretation

- **Intercept  $b$ :** initial value
- **Slope  $-m$ :** loss in value per year

### Appreciation (Linear Increase)

A value **appreciates linearly** if it increases by the **same dollar amount each year**.

#### Model

$$V = b + mt \quad (m > 0)$$

#### Interpretation

- **Intercept  $b$ :** starting value
- **Slope  $m$ :** amount of increase per year

**Problem: (Depreciation)** Suppose that the manufacturer's suggested retail price (MSRP) on a 2021 Ford F150 XL pickup truck is \$38,345 and that in three years it is worth \$26,120.

- a. Assuming linear depreciation, find the depreciation function for this vehicle.
- b. At what rate is the car depreciating?
- c. What will the truck be worth in 6 years?
- d. When will the vehicle be worth half of its original value?

## You try

A company purchases a laptop system for \$18,000. After 2 years, the system is worth \$13,200.

- a. Assuming linear depreciation, find the value function.
- b. Find the depreciation rate per year.
- c. What will the system be worth after 5 years?
- d. When will the system be worth half of its original value?

## Problem: (appreciation)

A house increases in value in an approximately linear fashion from \$222,000 to \$300,000 in 6 years.

- a. Find the appreciation function that gives the value of the house in year  $x$ .
- b. At what rate is the house appreciating?
- c. If the house continues to appreciate at this rate, what will it be worth 12 years from now?
- d. When will the house be worth double its original value?

## You try

A piece of artwork increases in value from \$8,500 to \$11,500 over 5 years.

- a. Find the appreciation function that gives the value of the artwork in year  $x$ .
- b. At what rate is the artwork appreciating?
- c. What will the artwork be worth after 10 years?
- d. When will the artwork be worth triple its original value?

## 5. Line of best fit (Excel)