

Quadratic Functions

$$y = ax^2 + bx + c \quad (a \neq 0)$$

Example : $y = 3x^2 + 4x + 1$

$$y = \frac{1}{3}x^2 - 9x$$

$$y = -9x^2 + 7$$

1. Solving Quadratics Equations

Quadratic Equation : $ax^2 + bx + c = 0$

a. Factoring

Example: $2x^2 = 4x$

$$2x^2 = 4x$$

$$\Leftrightarrow 2x^2 - 4x = 0$$

$$\Leftrightarrow 2x \cdot (x - 2) = 0$$

$$\Leftrightarrow \begin{cases} 2x = 0 \\ x - 2 = 0 \end{cases} \quad \Leftrightarrow \begin{cases} x = 0 \\ x = 2 \end{cases} \quad (\text{Two solutions})$$

Example: $x^2 + 7x = -10$

$$x^2 + 7x = -10$$

$$\Rightarrow x^2 + 7x + 10 = 0$$

$$\Rightarrow (x + 5) \cdot (x + 2) = 0$$

[we looked for 2 numbers whose sum is 7 and product is 10 .

5 and 2 are the two numbers]

$$\Rightarrow \begin{cases} x + 5 = 0 \\ x + 2 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} x = -5 \\ x = -2 \end{cases}$$

Example: $x^2 - 4x + 3 = 0$

Two numbers : sum = -4
product = 3

-1, -3

$$\Rightarrow (x - 1) \cdot (x - 3) = 0$$

$$\Rightarrow \begin{cases} x - 1 = 0 \\ x - 3 = 0 \end{cases} \Rightarrow \begin{cases} x = 1 \\ x = 3 \end{cases}$$

Example: $x^2 - 25 = 0$

Formula:

$$x^2 - a^2 = (x - a) \cdot (x + a)$$

$$x^2 - 25 = 0$$

$$\Rightarrow x^2 - 5^2 = 0$$

$$\Rightarrow (x - 5) \cdot (x + 5) = 0$$

$$\Rightarrow \begin{cases} x - 5 = 0 \\ x + 5 = 0 \end{cases} \Rightarrow \begin{cases} x = 5 \\ x = -5 \end{cases}$$

Example: $x^2 - x = 12$

$$x^2 - x - 12 = 0$$

$$\Rightarrow (x - 4) \cdot (x + 3) = 0$$

$$\Rightarrow \begin{cases} x - 4 = 0 \\ x + 3 = 0 \end{cases} \Rightarrow \begin{cases} x = 4 \\ x = -3 \end{cases}$$

b. Quadratic Formula

$$ax^2 + bx + c = 0$$

① If $b^2 < 4ac$: No solution!

② If $b^2 \geq 4ac$:

solution : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example: $x^2 + 1 = 4x$

$$x^2 + 1 = 4x$$

$$\Rightarrow 1 \cdot x^2 - 4x + 1 = 0$$

$\uparrow \quad \quad \uparrow \quad \quad \uparrow$
 $a=1 \quad b=-4 \quad c=1$

$$b^2 = (-4)^2 = 16 \quad ; \quad 4ac = 4 \cdot 1 \cdot 1$$

Solution : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow x = \frac{4 \pm \sqrt{16 - 4}}{2}$$

$$\Rightarrow x = \frac{4 \pm \sqrt{12}}{2}$$

$$x = \frac{4 + \sqrt{12}}{2}$$

$$x = \frac{4 - \sqrt{12}}{2}$$

Example: $x^2 - 6x + 10 = 0$

$$x^2 - 6x + 10 = 0$$

$\downarrow \quad \quad \downarrow \quad \quad \downarrow$
 $a=1 \quad b=-6 \quad c=10$

$$b^2 = (-6)^2 = 36 \quad ; \quad 4ac = 4 \cdot 1 \cdot 10 = 40$$

$$b^2 < 4ac \quad (36 < 40)$$

\Rightarrow no solution.

Mixed Practice

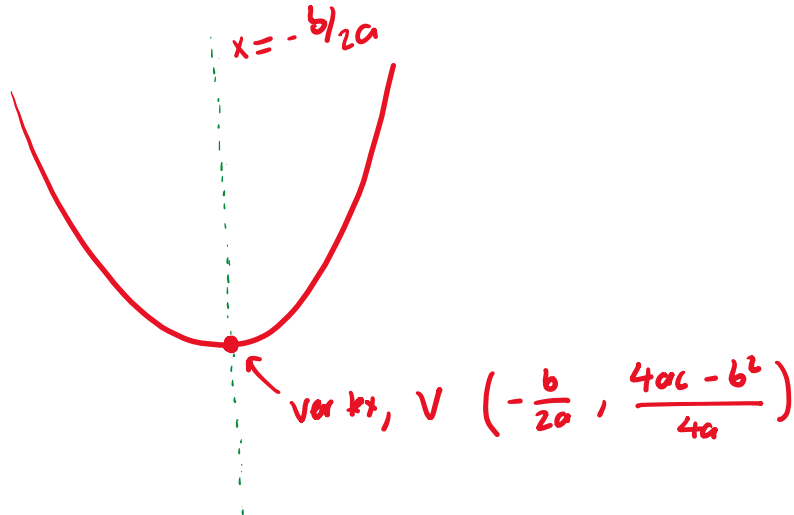
Solve each quadratic by factoring or the quadratic formula.

1. $2x^2 = 4x$	2. $x^2 - 4x = 12$
3. $2x^2 - x - 10 = 0$	4. $x^2 - 10x + 24 = 0$
5. $10x^2 - 25x = 0$	6. $x^2 + 5x + 2 = 2x$
7. $x^2 - 100x + 900 = 0$	8. $8x^2 + x - 75 = 0$

2. Graphs of Quadratic Functions

$$y = ax^2 + bx + c \quad (a \neq 0)$$

$a > 0$:



(*) The graph is a parabola (upward)

(*) Symmetry around $x = -\frac{b}{2a}$

(*) y -intercept is $(0, c)$

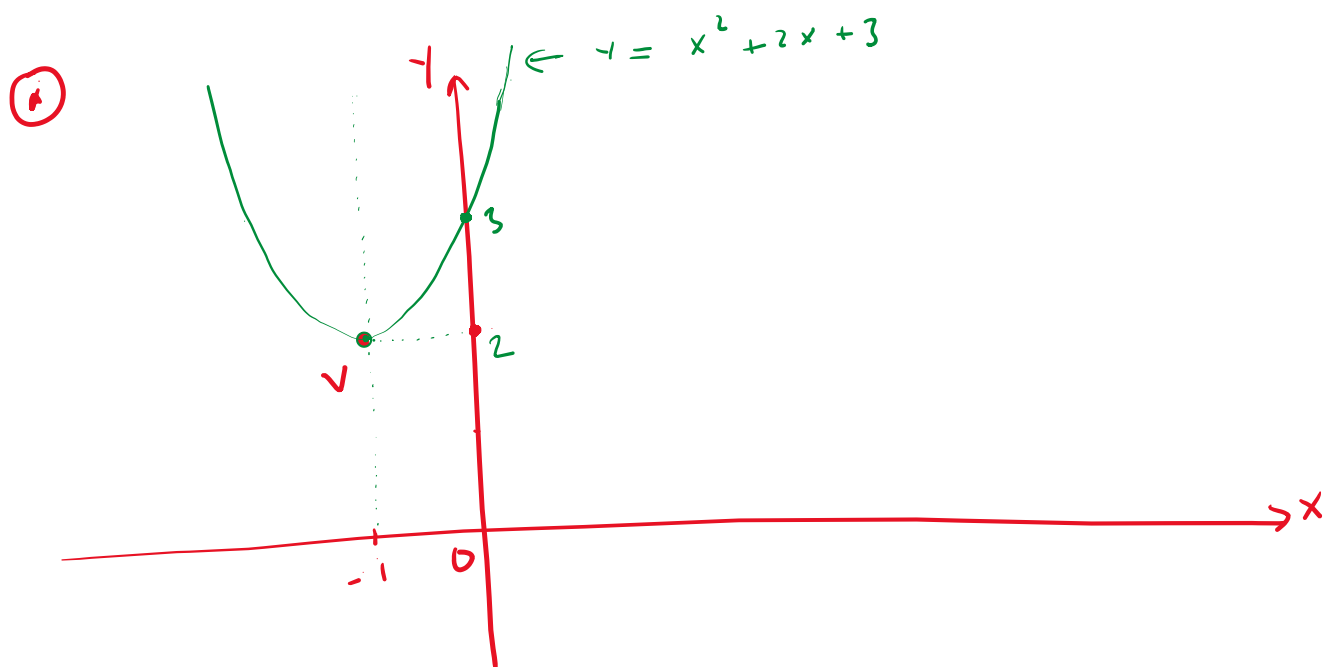
Example: $y = x^2 + 2x + 3$

$$a = 1, \quad b = 2, \quad c = 3$$

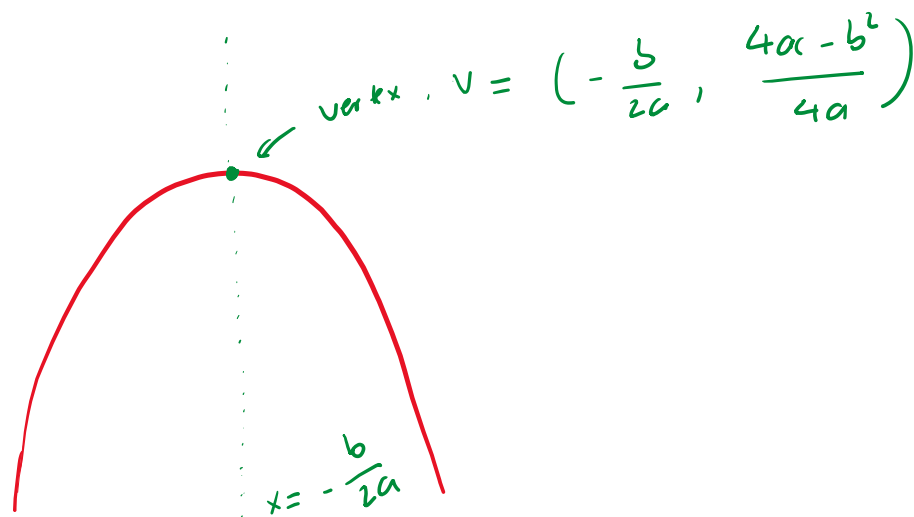
① vertex $V = \left(-\frac{b}{2a}, \frac{4ac - b^2}{4a} \right)$

$$= \left(-\frac{2}{2}, \frac{4 \cdot 1 \cdot 3 - 2^2}{4 \cdot 1} \right)$$

$$= (-1, 2)$$



$a < 0$



Example: $y = 2x^2 - 4x + 4$

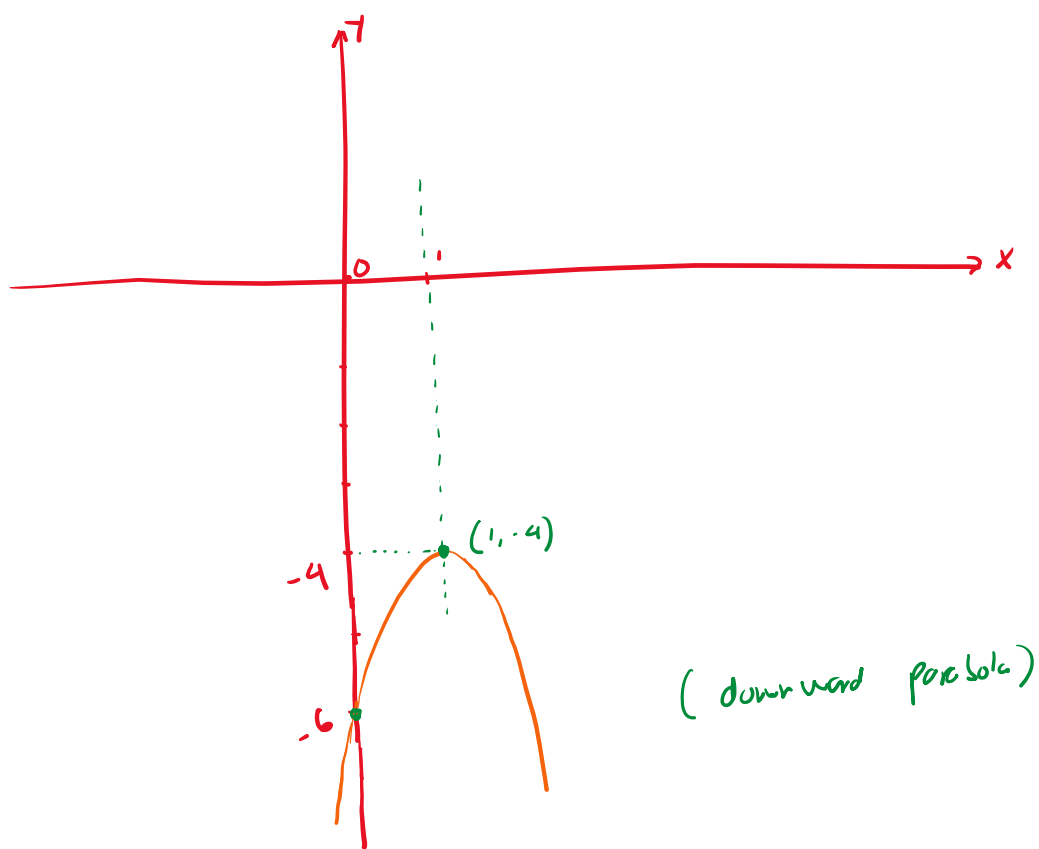
Example: $y = -2x^2 + 4x - 6$

$a = -2$ $b = 4$ $c = -6$

Vertex, $V = \left(-\frac{b}{2a}, \frac{4ac - b^2}{4a} \right)$

$$= \left(\frac{-4}{-2 \cdot 2}, \frac{4 \cdot (-2) \cdot (-6) - 4^2}{4 \cdot (-2)} \right)$$

$$= (1, -4)$$



You Try

Graph the following quadratic functions. Label the vertex and a point in the graph.

1. $y = 2x^2 - 4x - 1$

2. $y = -3x^2 - 12x + 1$