# Applications of Derivatives

## Instantaneous Rate of Change

**Example 1:** A company determines that the value of an investment is $V$, in millions of dollars, after time $t$, in years, where $V$ is given by $V(t) = 5t^3 - 30t^2 + 45t + 5t^{\frac{1}{2}}$

a. Find the average rate of change in the investment between year 1 and year 5. Make sure to include the correct units.

b. Find the instantaneous rate of change at year 3. Explain the meaning.

**Example 2:** The median weight of a boy whose age is between 0 and 36 months can be approximated by the function $w(t) = 8.15 + 1.82t - 0.0596t^2 + 0.000758t^3$, where t is measured in months and w is measured in pounds.

a. Find the average rate of change in the weight of a boy between 6 months and 12 months. Make sure to include the correct units.

b. Find the instantaneous rate of change at 9 months. Explain the meaning.

**Example 3:** The population of a city grows from an initial size of 100,000 to a size $P$ given by $P(t) = 100{,}000 + 2000t^2$, where $t$ is in years.

a. Find $P(10)$

b. Find $P'(t)$

c. Find $P'(10)$. Explain the meaning to your answer.

**Example 4:** The U.S. gross domestic product (in billions of dollars) can be approximated using the function $P(t) = 36t^{1.6} - 104t + 576$, where $t$ is the number of the years since 1960.

a. Find $P(45)$

b. Find $P'(t)$

c. Find $P'(45)$ and interpret the meaning

**Example 5:** The population P, in thousands, of a small city is given by $P(t) = \frac{500t}{2t^2+9}$ where $t$ is the time, in years.

a. Find $P(12)$

b. Find $P'(t)$

c. Find $P'(12)$. Explain the meaning to your answer.

**Example 6:** The temperature T of a person during an illness is given by $T(x) = \frac{4x}{x^2+1} + 98.6$, where T is the temperature in degrees Fahrenheit, at time $x$ hours.

a. Find $T(2)$

b. Find $T'(x)$

c. Find $T'(2)$. Explain the meaning to your answer.

**Example 7:** In a psychological experiment, students were shown a set of nonsense syllables, such as POK, RIZ, DEQ, and so on, and asked to recall them every minute thereafter. The percentage $R(t)$ who retained the syllables after *t* minutes was found to be given by the logarithmic learning model

$$R(t) = 80 - 27 \ln t$$

a. Find $R'(2)$

b. Find $R'(t)$

c. Find $R'(2)$ and explain what it represents.

<u>**Marginal Revenue, Cost, and Profit**</u>

The **marginal revenue** function models the revenue generated by selling one more unit, the **marginal cost** function models the cost of making one more unit, and the **marginal profit** function models the profit made by selling one more unit.

Companies will produce **up to** the point where marginal cost equals marginal revenue, after this point the company would be losing money with every product created.

**Example 8:** Suppose that the monthly cost, in dollars, of producing *x* chair is $C(x) = 0.001x^3 + 0.07x^2 + 19x + 700$, and currently 25 chairs are produced monthly.

a. Find the marginal cost function.

b. What is the exact rate of change when 25 chairs are produced? Interpret the meaning.

**Example 9:** Pierce Manufacturing determines that the daily revenue, in dollars, from the sale of $x$ lawn chairs is $R(x) = 0.005x^3 + 0.01x^2 + 0.5x$. Currently, Pierce sells 70 lawn chairs daily.

a. Find the marginal revenue function.

b. What is the exact rate of change when 70 lawn chairs are sold? Interpret the meaning.

**Example 10:** Crawford Computing finds that its weekly profit, in dollars, from the production and sale of $x$ laptop computers is $P(x) = -0.004x^3 - 0.3x^2 + 600x - 800.$ Currently Crawford builds and sells 9 laptops weekly.

a. Find the marginal profit function.

b. What is the exact rate of change when 9 laptops are built and sold? Interpret the meaning.

**Example 11:** Evaluate marginal cost and marginal revenue at a production level of 5 items increasing to 6 items. Should the company increase production?

a. $C(x) = 0.01x^2 + x + 250$
   $R(x) = -x^2 + 12x$

b. $C(x) = 3x^2 + 100$
   $R(x) = -2x^2 + 40x$

**Example 12:** The revenue for a new video game can be modeled by $R(x) = 40x - 6x \ln x$, where $R(x)$ is in thousands of dollars and $x$ is the number of games sold, in thousands.

a. Find $R'(x)$

b. Find $R'(5)$ and interpret the results.

**Example 13:** A company's total cost, in millions of dollars, is given by $C(t) = 150 - 30e^{-t}, (t > 0)$, where $t$ is the time in years since the start-up date.

a. Find the marginal cost $C'(t)$

b. Find and interpret the results $C'(0)$

c. Find and interpret the results $C'(5)$