

Average Rate of Change and Instantaneous Rate of Change

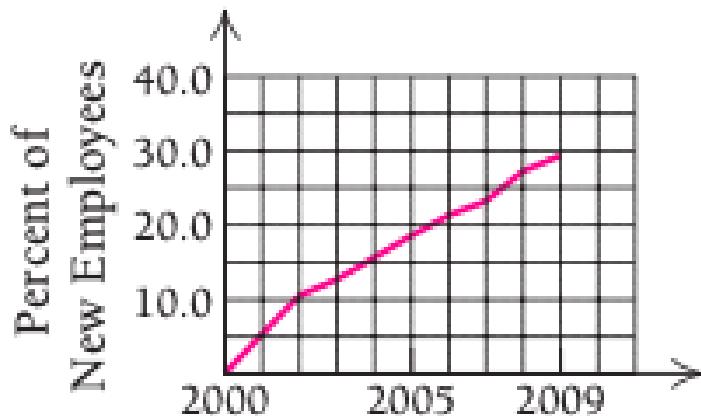
Let's say that a car travels 120 mi in 2 hr. Its *average rate of change (speed)* is $120 \text{ mi}/2 \text{ hr}$, or 60 mi/hr (60 mph). Suppose that you are driving on the highway, and you look down at the speedometer to see that you are traveling at 60 mph . That is your *instantaneous rate of change*.

These are two quite different concepts. The first you are probably familiar with. The second involves Calculus. To understand instantaneous rate of change, we first need to develop a solid understanding of average rate of change.

Average Rate of Change:

Example 1: Use the graph to estimate the average rate of change of the percentage of new employees on the following intervals:

Education

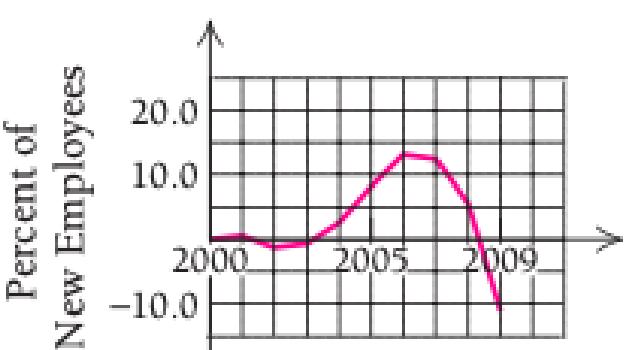


a. 2000 to 2005

b. 2005 to 2009

c. 2000 to 2009

Construction



a. 2000 to 2005

b. 2005 to 2009

c. 2000 to 2009

Example 2: When a balance of \$5000 is owed on a credit card and interest is begin charged at a rate of 22% per year, the total amount owed, $A(t)$, after t years is given by $A(t) = 5000(1.21)^t$. Assume that the credit card holder cannot make any payments on this card.

- a. Find the total amount owed after 2 years
- b. Find the total amount owed after 5 years
- c. What is the average rate of change in the total amount owed from 2 years to 5 years? Include the correct units.

Example 3: A company that manufactures sunglasses has a revenue function of $R(x) = -0.01x^2 + 100x$, where x is the number of pairs of sunglasses sold and $R(x)$ is in dollars.

- a. What is the revenue when 100 pairs of sunglasses are sold?
- b. What is the revenue when 300 pairs of sunglasses are sold.
- c. Find the average rate of change in the company's revenue. Include the correct units.

Example 4: The same company in Example 3, has a cost function of $C(x) = -0.05x^2 + 50x$, where x is the number of pairs of sunglasses produced and $C(x)$ is in dollars.

- a. What is the cost when 100 pairs of sunglasses are produced?
- b. What is the cost when 300 pairs of sunglasses are produced
- c. Find the average rate of change in the company's cost. Include the correct units.

Example 5: On October 14, 2012, Austrian skydiver Felix Baumgartner broke a world record for a high-altitude dive when he ascended 127,850 feet in a helium balloon and then went into a free fall lasting more than 4 minutes.

His elevation (in feet) above sea-level, t seconds after stepping off the balloon can be approximated by $f(t) = 127,850 - 16t^2$

- a. Let's see if we can estimate his velocity exactly 30 seconds after leaving the balloon.

What is his average velocity between $t = 20$ and $t = 30$?

What is his average velocity between $t = 30$ and $t = 40$?

- b. Let's take an interval even closer to 30.

Find the average velocity between $t = 29$ and $t = 30$.

Find the average velocity between $t = 30$ and $t = 31$.

- c. Felix's actual velocity at 30 seconds was -960 ft/sec. Which of the estimates was closest? How could we get an even better estimate?

Derivative:**Derivative Rules****Constant****Power Rule****Example 6:** Find the derivative

a. $y = x^7$	b. $f(x) = -3x$
c. $y = 12$	d. $f(x) = 2x^{15}$
e. $y = -5x^3 + 2x^2 - 3x + 1$	f. $f(x) = 3x^2 - 2x + 3$
g. $y = 2x + 3x^2$	h. $f(x) = 6 - 3x^3 + 6x^4$

Example 7: Simplify and then find $f'(x)$ for each of the following functions.

a. $f(x) = (5x + 1)^2$

b. $f(x) = \frac{3x^4 - 3x^2 - 2x}{x}$

c. $f(x) = (x + 3)(x + 2)$

d. $f(x) = \frac{x^5 - 5x^3}{x^2}$

Example 8: Find the derivative of each function and then evaluate at the given x -value.

a. If $f(x) = x^2 + 4x - 5$, find $f'(10)$

b. If $f(x) = x^3 + 2x - 11$, find $f'(-2)$

Product Rule

Example 9: Find y' given that $y = (2x^2 - 3x)(3x^4 - 3)$

Example 10: Find y' given that $y = (2x^5 + x - 1)(3x - 2)$

Quotient Rule

Example 11: Find $f'(x)$ given that $f(x) = \frac{2x^3+1}{3x^4-x+3}$

Example 12: Given that $y = \frac{4x^2}{x^2-5x}$

Derivative of $f(x) = e^x$

Example 13: Find the following derivatives

a. $y = 3e^x$	b. $y = e^{8x}$
c. $f(x) = e^{-x^2+4x-7}$	d. $f(x) = 4e^{5x} + x^2 - 3x + 7$
e. $y = e^{3x-2}$	f. $f(x) = e^{-x}$
g. $y = (x + 3)e^{x^2-x}$	h. $f(x) = (x^2 + 8x)e^x$

i. $y = \frac{e^{-3x}}{x^3}$	j. $f(x) = \frac{e^{2x}}{1+e^{2x}}$
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Derivative of logarithmic functions

Example 14: Find the following derivatives

a. $y = 3 \ln x$	b. $y = \ln (3x)$
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c. $f(x) = \ln (x^2 - 5)$	d. $f(x) = \ln (4x^3 - 5x^2 + 2x - 6)$
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e. $y = 6 \ln(2x^2 + 1)$

f. $y = \ln(x^3 + 2)$

g. $y = x^5 \ln(2x)$

h. $y = (10x^4 + 3x^3)\ln(7x + 2)$

i. $f(x) = \frac{\ln x}{x^3}$

j. $f(x) = \frac{x+5}{\ln(8x)}$