

Linear Inequalities, Systems, and Linear Programming

I. Graphing Linear Inequalities

graph the boundary line using x & y intercepts
 $\geq \leq$ solid $> <$ dashed

Test a point not on the boundary line & then shade appropriately

Example 1: Graph $3x + 2y \geq 6$

x-int: $(y=0)$ \downarrow
solid

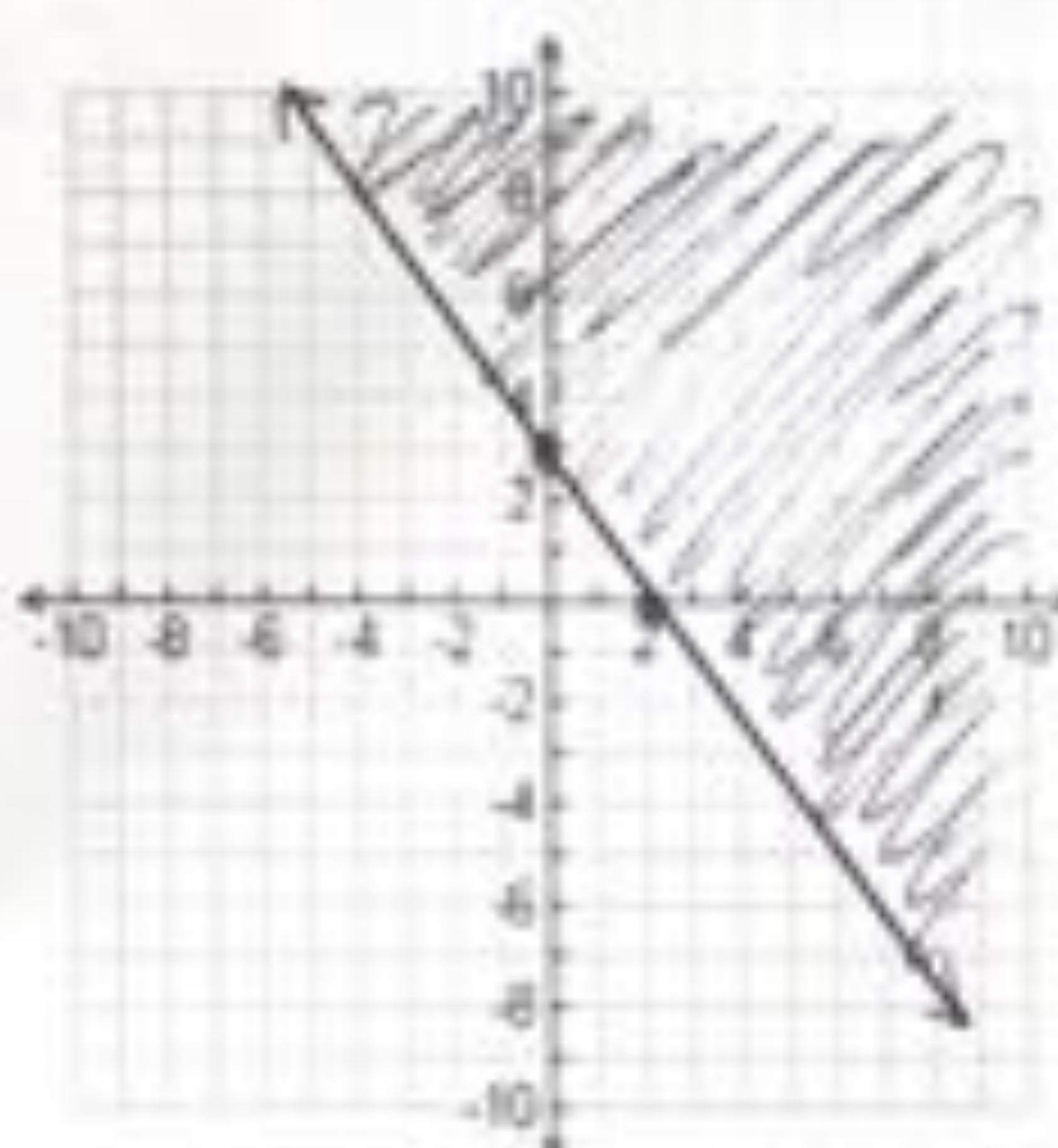
$$3x + 2(0) = 6$$

$$3x = 6 \quad (2, 0)$$

y-int: $x=0$

$$3(0) + 2y = 6$$

$$2y = 6 \quad (0, 3)$$



Test point
 $(4, 0)$

$$3x + 2y \geq 6$$

$$3(4) + 2(0) \geq 6$$

$$0 \geq 6$$

False

Don't shade $(4, 0)$

Example 2: Graph $x - 3y > 9$

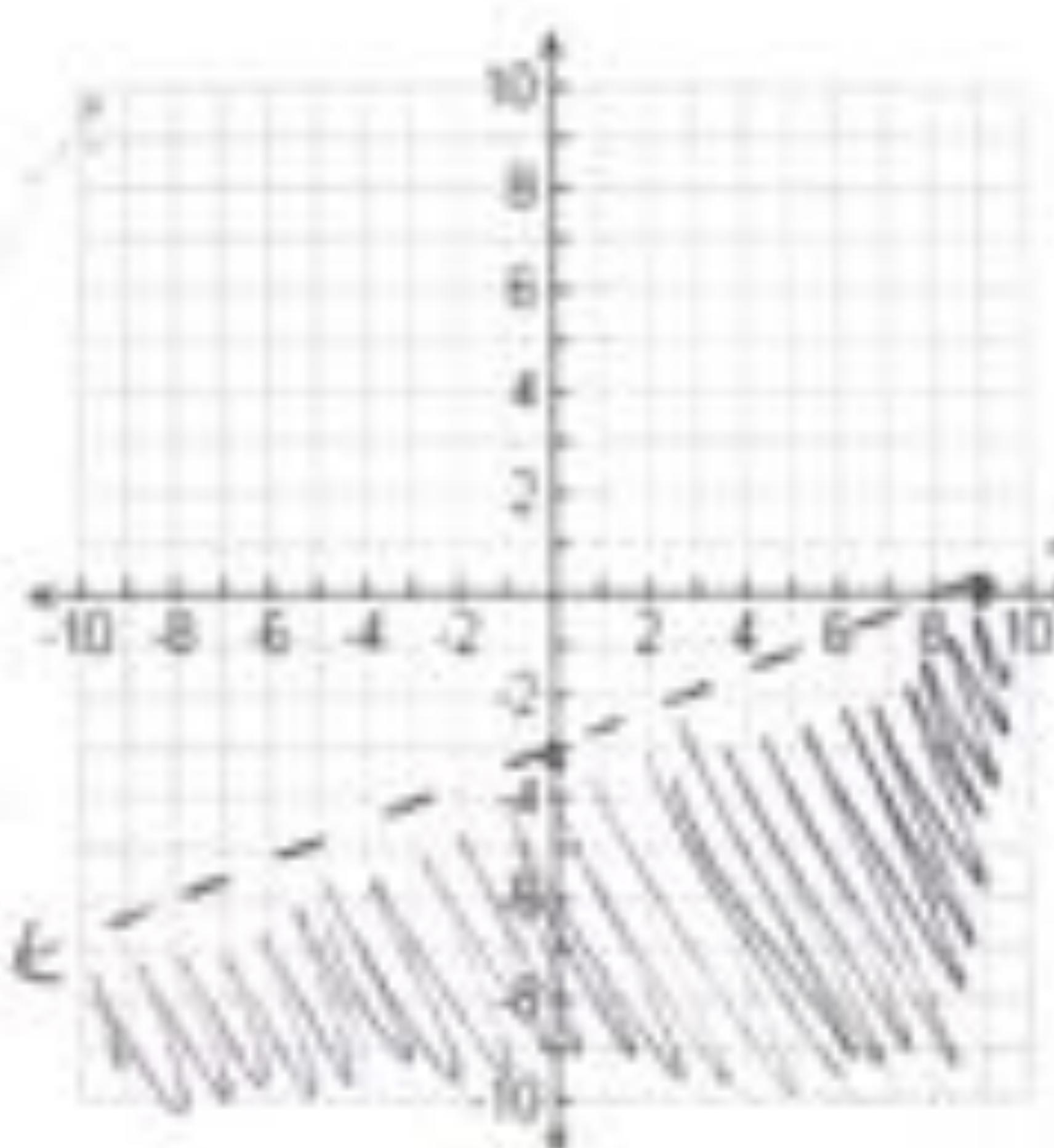
x-int: $(9, 0)$ \downarrow
dashed

y-int: $(0, -3)$

Test point: $(0, 0)$

$$0 - 3(0) > 9$$

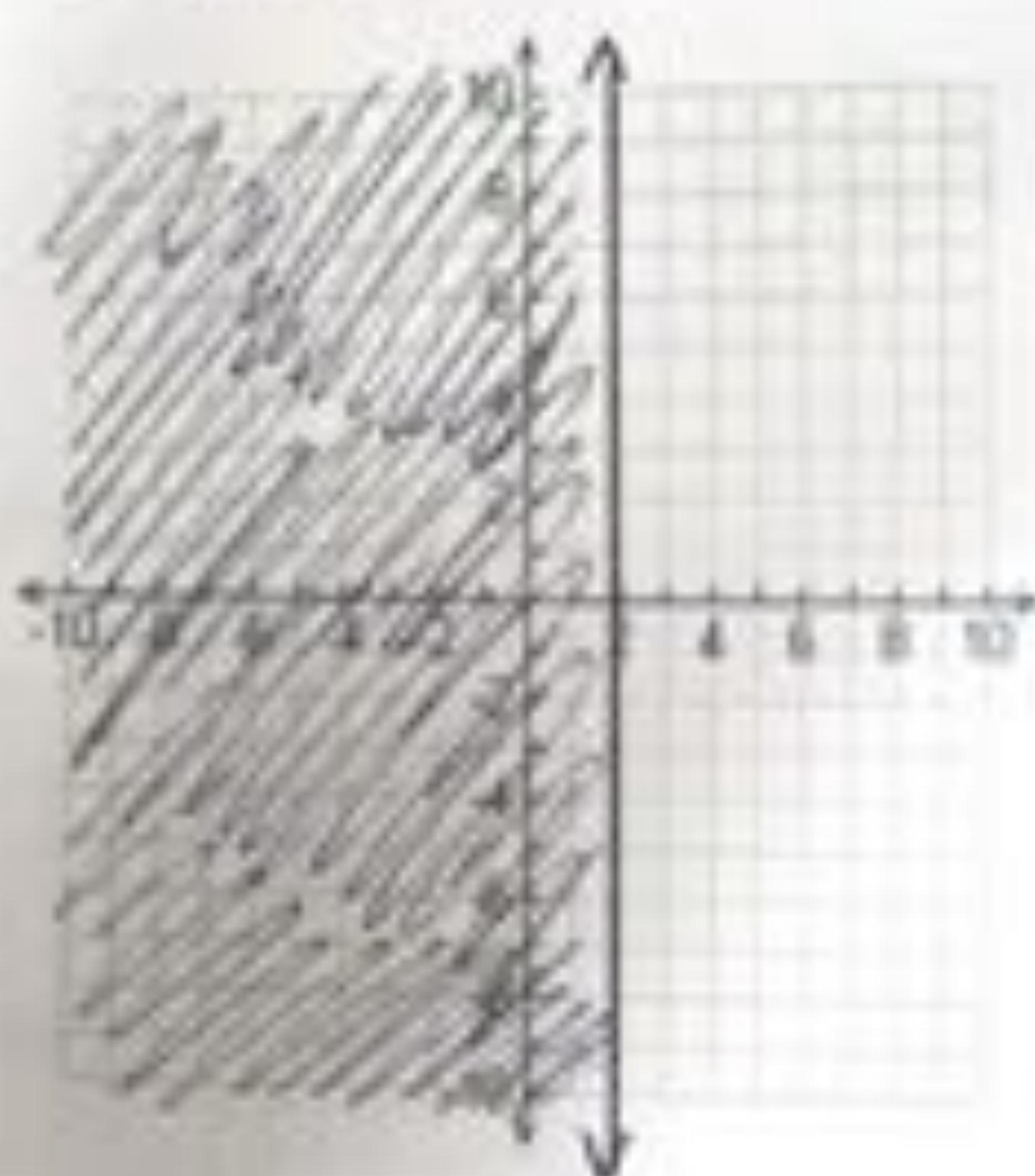
$$0 > 9$$



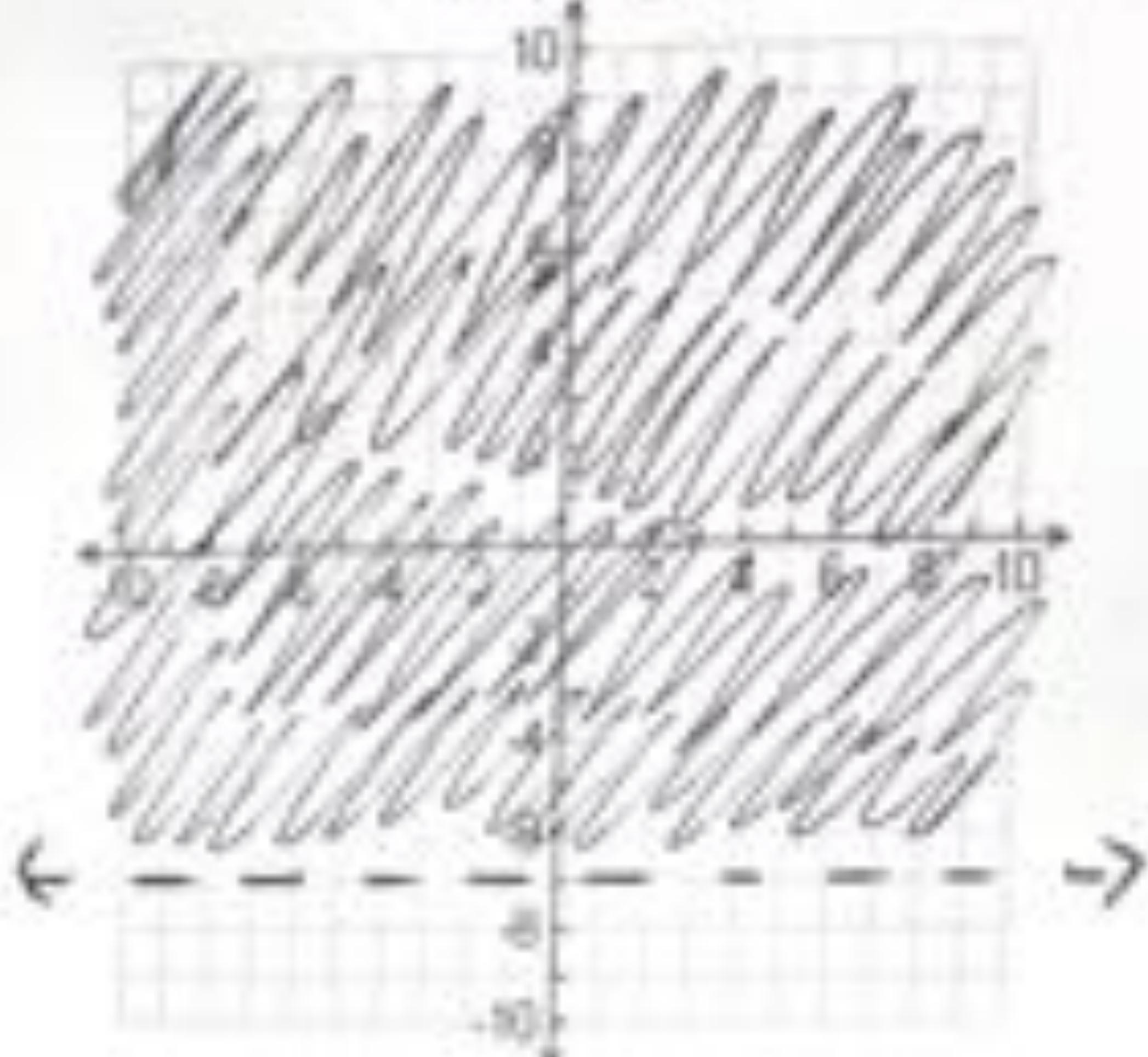
False. Don't shade $(0, 0)$

Example 3:

a. Graph $x \leq 2$



b. Graph $y > -5$



II. Graphing Systems of Linear Inequalities

Solution: where the shaded regions overlap

Graph each inequality and determine where to shade.

Find where all the shaded regions overlap

Example 4: Graph $\begin{aligned} 3x + 2y &\leq 6 \\ 2x - 5y &\geq 10 \end{aligned}$

$$3x + 2y \leq 6$$

X-int: $(2, 0)$

Y-int: $(0, 3)$

$$2x - 5y \geq 10$$

X-int: $(5, 0)$

Y-int: $(0, -2)$

Test: $(0, 0)$

$$3(0) + 2(0) \leq 6$$

$0 \leq 6$ TRUE

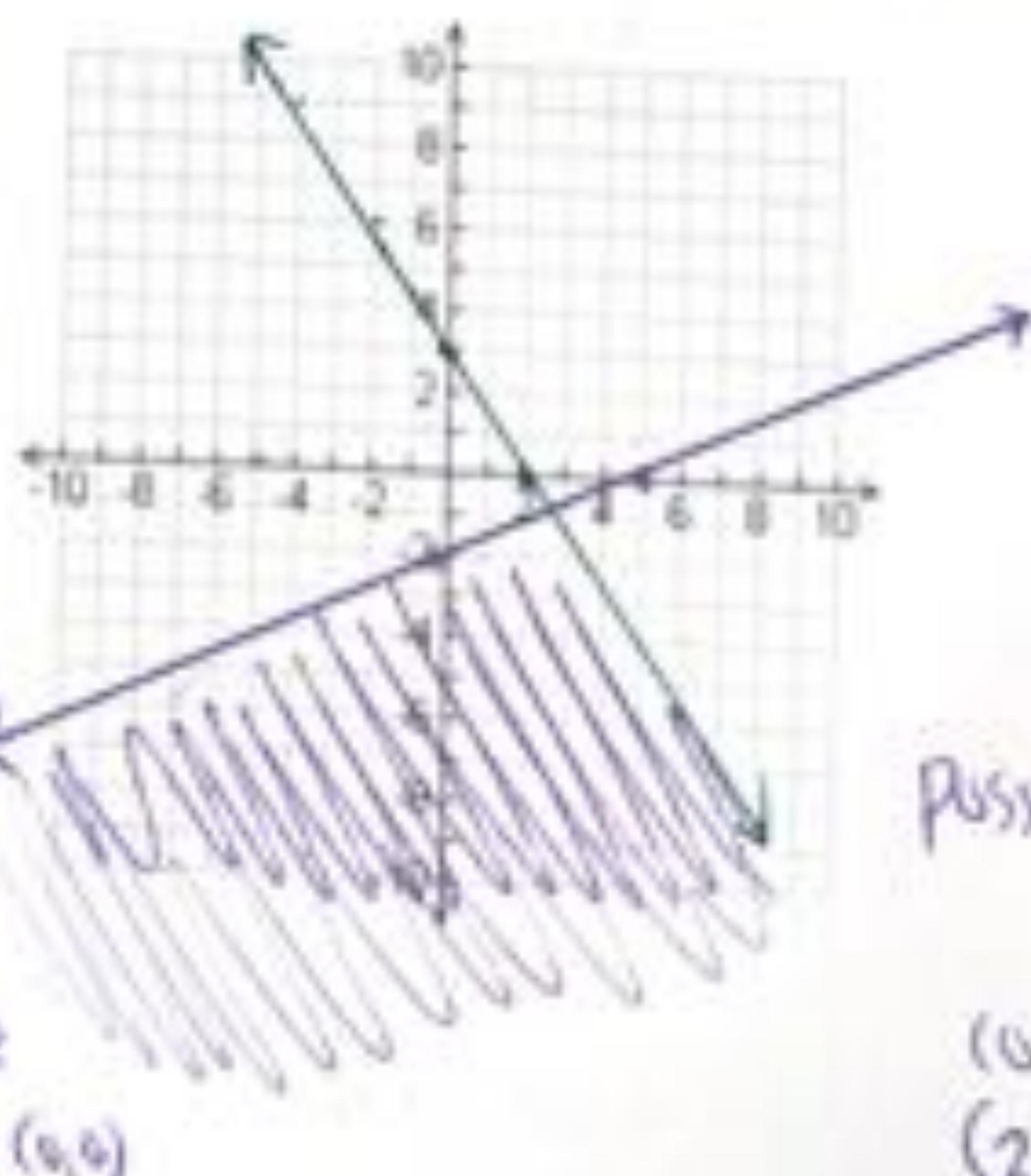
Shade $(0, 0)$

Test $(0, 0)$

$$2(0) - 5(0) \geq 10$$

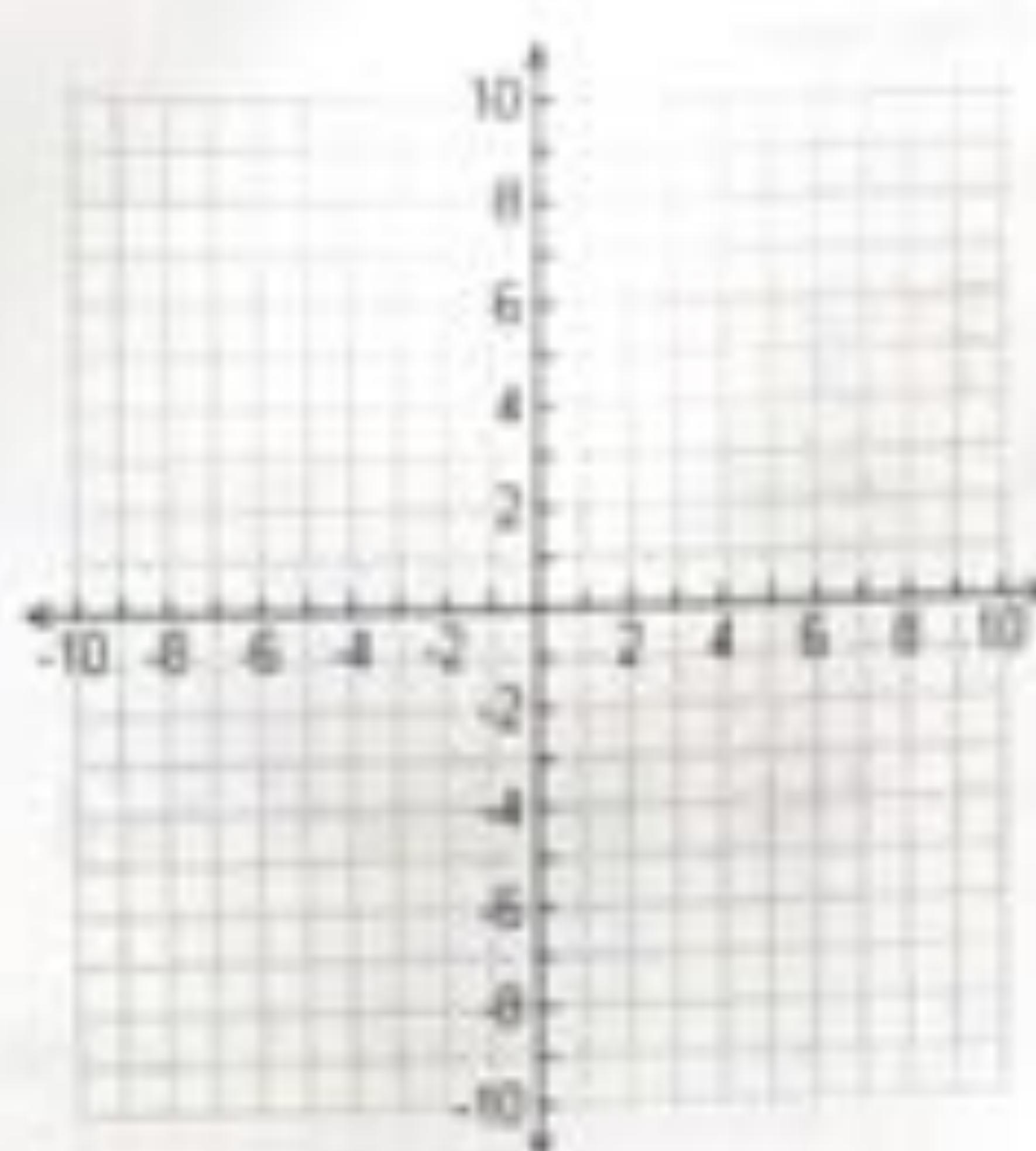
$0 \geq 10$ False

don't shade $(0, 0)$

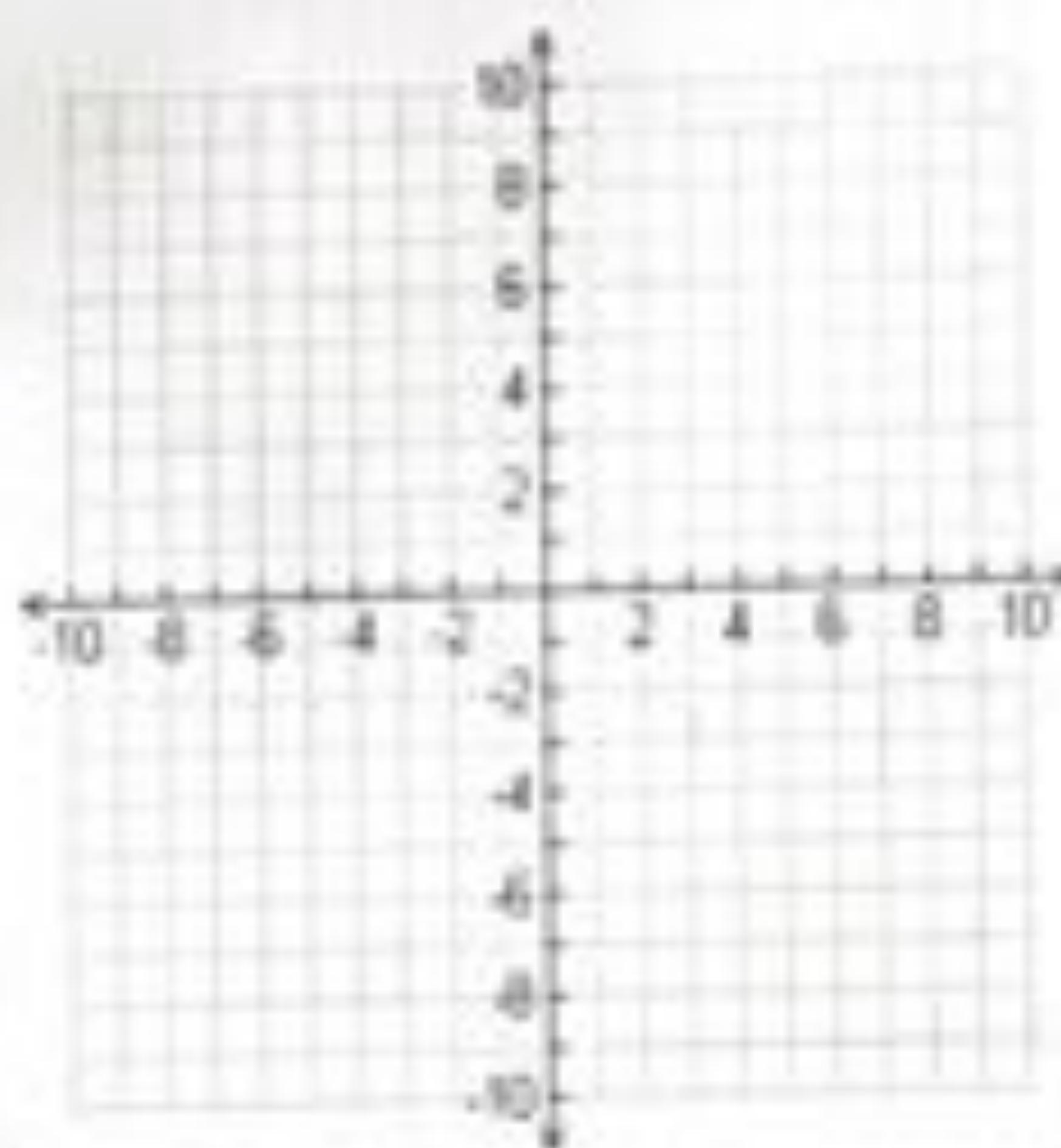


Possible solutions:
 $(0, -4)$
 $(2, -4)$

Example 5: Graph $x + y < 1$
 $2x - y < 4$



Example 6: Graph $x + 2y \leq 10$
 $x > 3$



Example 7: Graph
 $2x + 3y \geq 12$
 $7x + 4y \geq 28$
 $y \leq 6$ horizontal
 $x \leq 5$ vertical

$$2x + 3y \geq 12$$

x-int: (6, 0)

y-int: (0, 4)

Test: (0, 0)

$$7x + 4y \geq 28$$

x-int: (4, 0)

y-int: (0, 7)

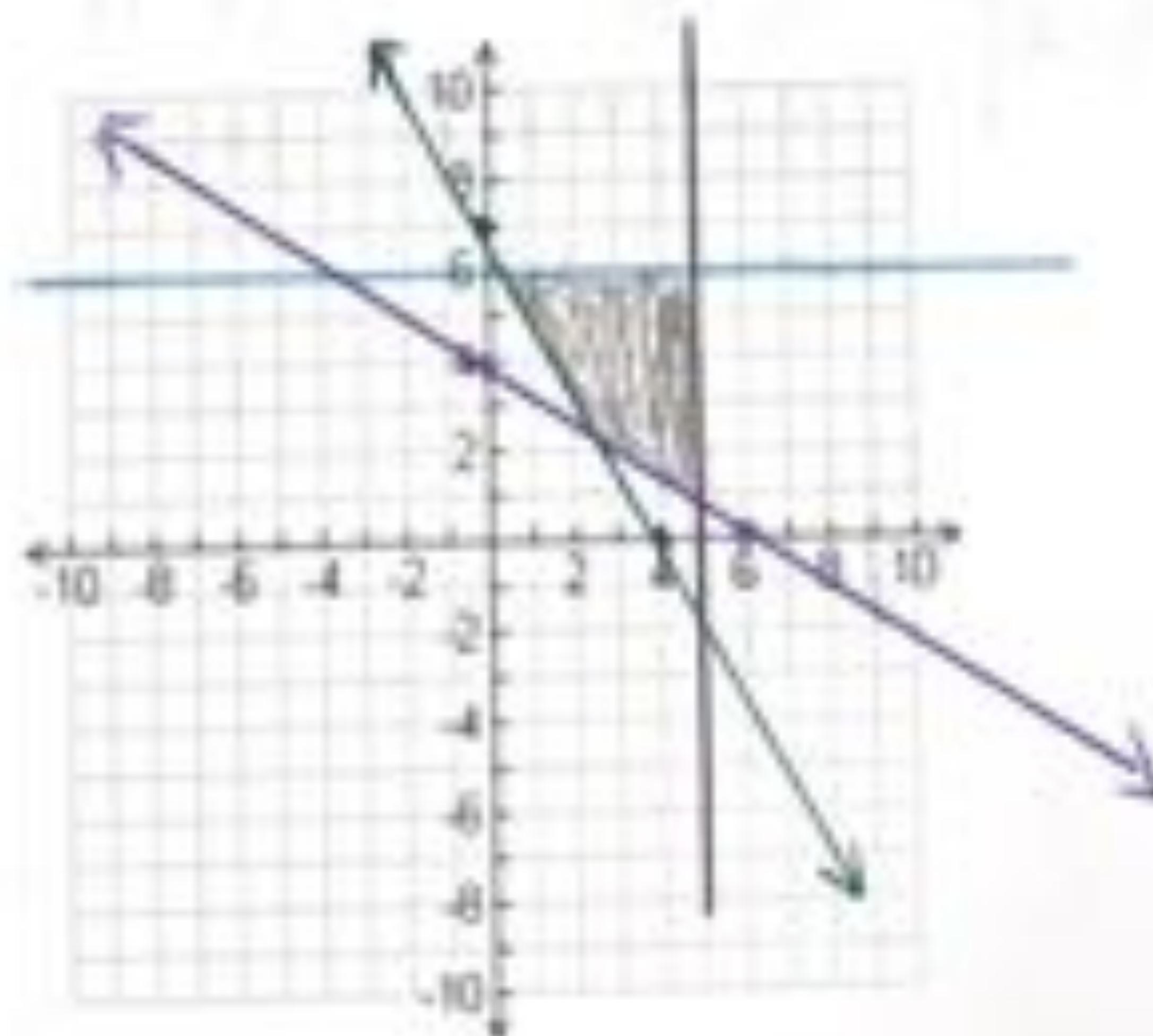
Test: (0, 0)

$$0 \geq 12 \quad x$$

Don't shade (0, 0)

$$0 \geq 28 \quad x$$

Don't shade (0, 0)



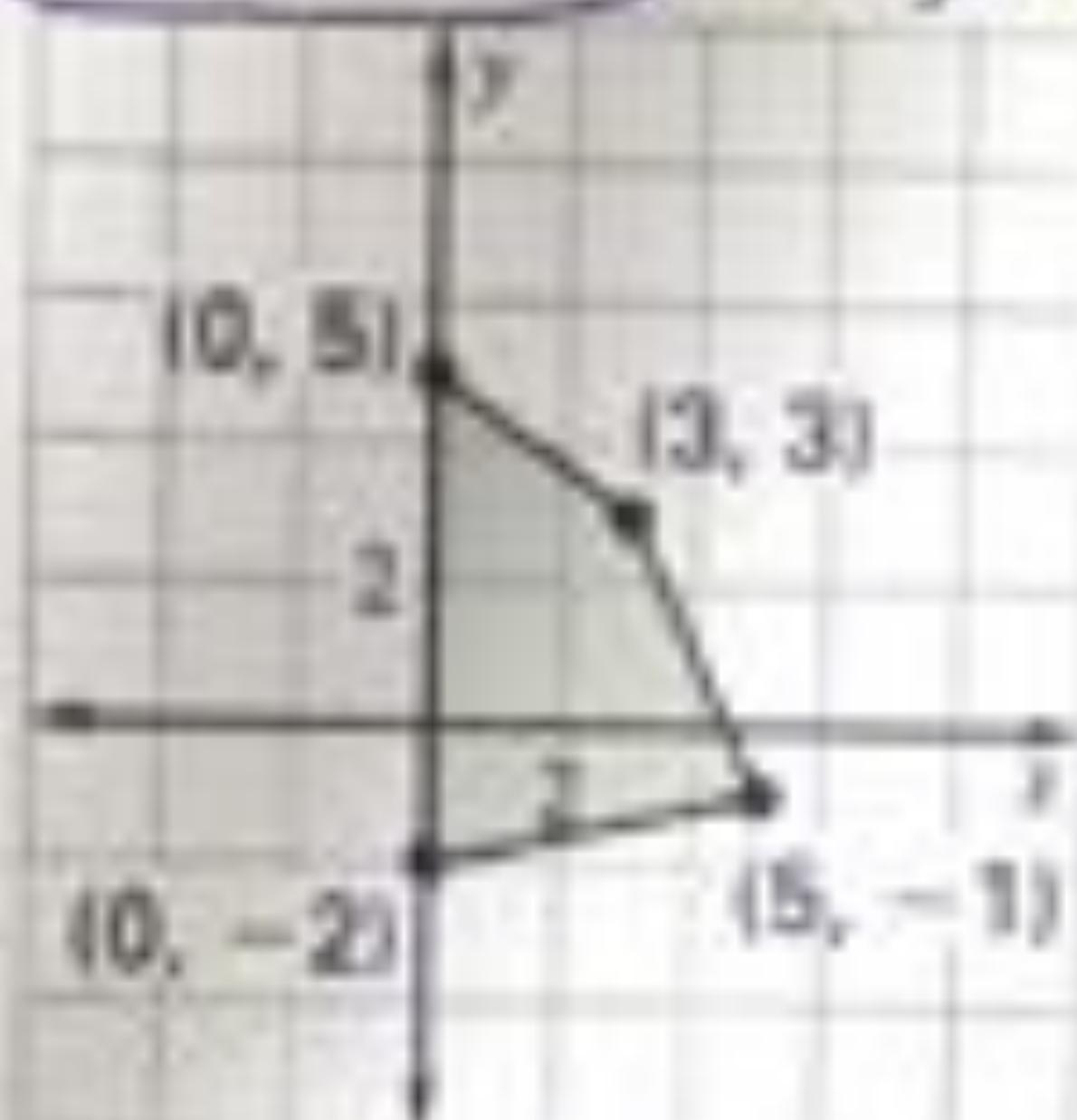
III. Linear Programming

Linear programming allows us to find the optimal value when faced with constraints (inequalities)

- ① Graph the constraints and find the feasible region
- ② Locate the vertices (corner points) of the feasible region and test them in the objective function
vertices \rightarrow only possible max/minimum
- ③ Determine maximum/minimum values

Example 8: The feasible region determined by a system of constraints is given. Find the minimum and maximum values of the objective function for the given feasible region

$C = x + 2y$ objective function



(0, 5)	$C = 0 + 2(5) = \underline{10}$
(3, 3)	$C = 3 + 2(3) = \underline{9}$
(5, -1)	$C = 5 + 2(-1) = \underline{3}$
(0, -2)	$C = 0 + 2(-2) = \underline{-4}$

maximum:

10 @ (0, 5)

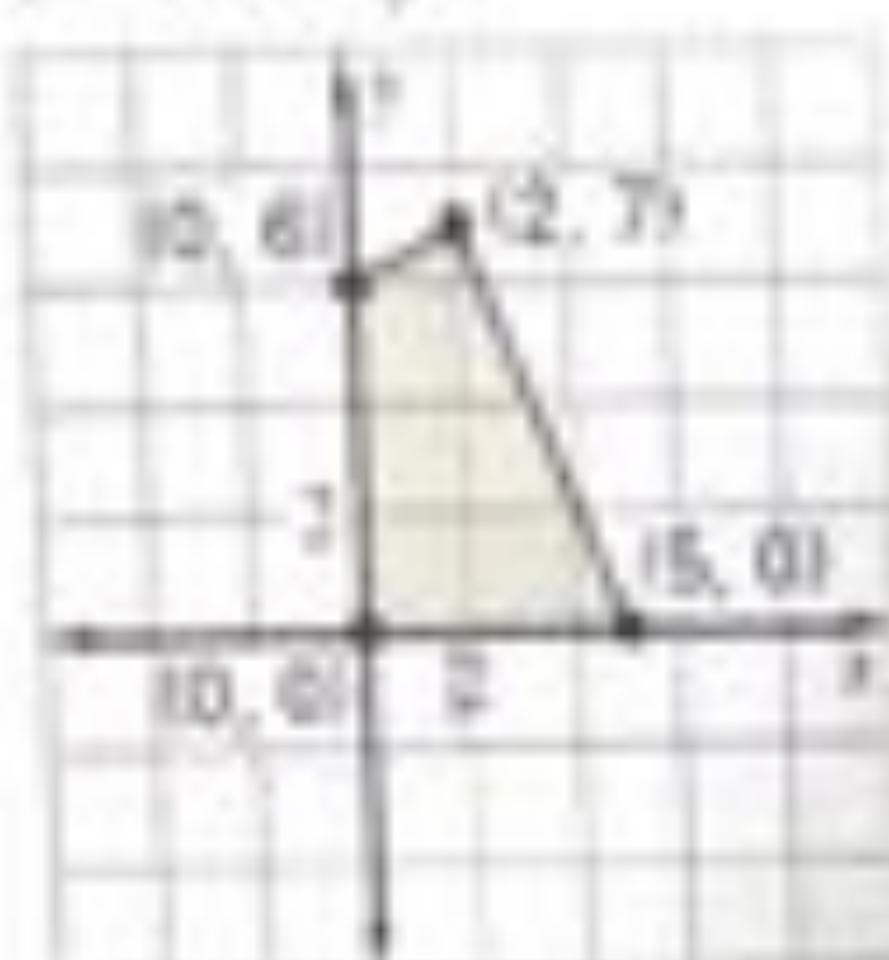
minimum

-4 @ (0, -2)

Example 9:

The feasible region determined by a system of constraints is given. Find the minimum and maximum values of the objective function for the given feasible region.

$$C = x - y$$



Minimum:

$$-6 \text{ @ } (0, 6)$$

Maximum:

$$5 \text{ @ } (5, 0)$$

Example 10:

Graph the constraints:

$$x \geq 0$$

$$x \leq 2$$

$$y \geq 0$$

$$(3, 0)$$

$$(0, 6)$$

$$y \leq -2x + 6$$

Find the vertices of the feasibility region:

$$(0, 0) \quad (2, 0)$$

$$(0, 4) \quad (2, 2)$$

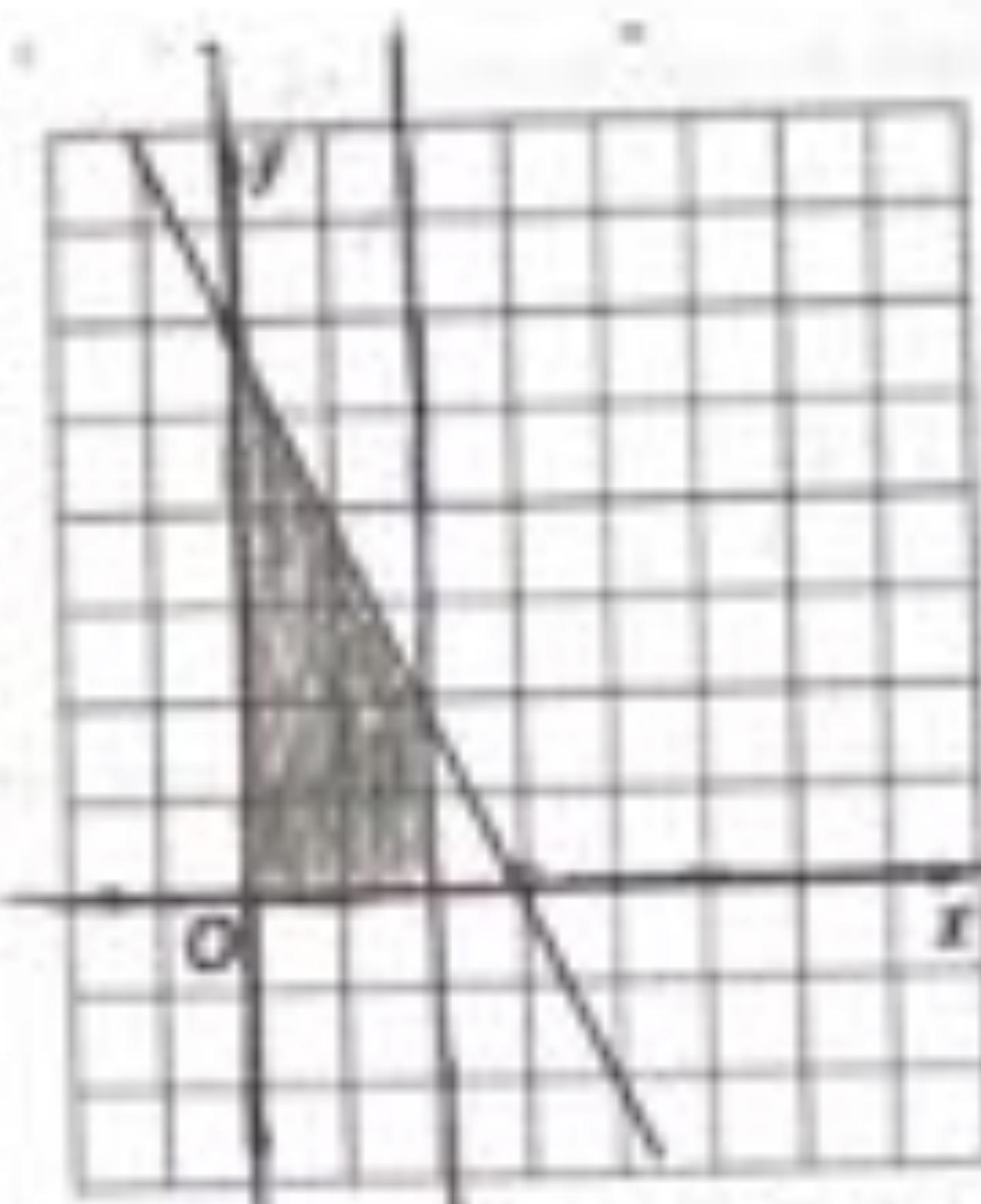
Test each vertex in the objective function $C = -x + 3y$:

$$(0, 0) \quad C = 0$$

$$(0, 4) \quad C = 12$$

$$(2, 0) \quad C = -2$$

Minimum: $-2 \text{ @ } (2, 0)$



$$\text{Maximum: } 12 \text{ @ } (0, 4)$$

Example 11:

Graph the constraints.

1st quadrant

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x + y \leq 5 \\ -3x + 6y \leq 12 \end{cases}$$

Find the vertices of the feasibility region

$$(0,0) \quad (0,2) \quad (2,3) \quad (5,0)$$

Test each vertex in the objective function $C = 5x + 6y$.

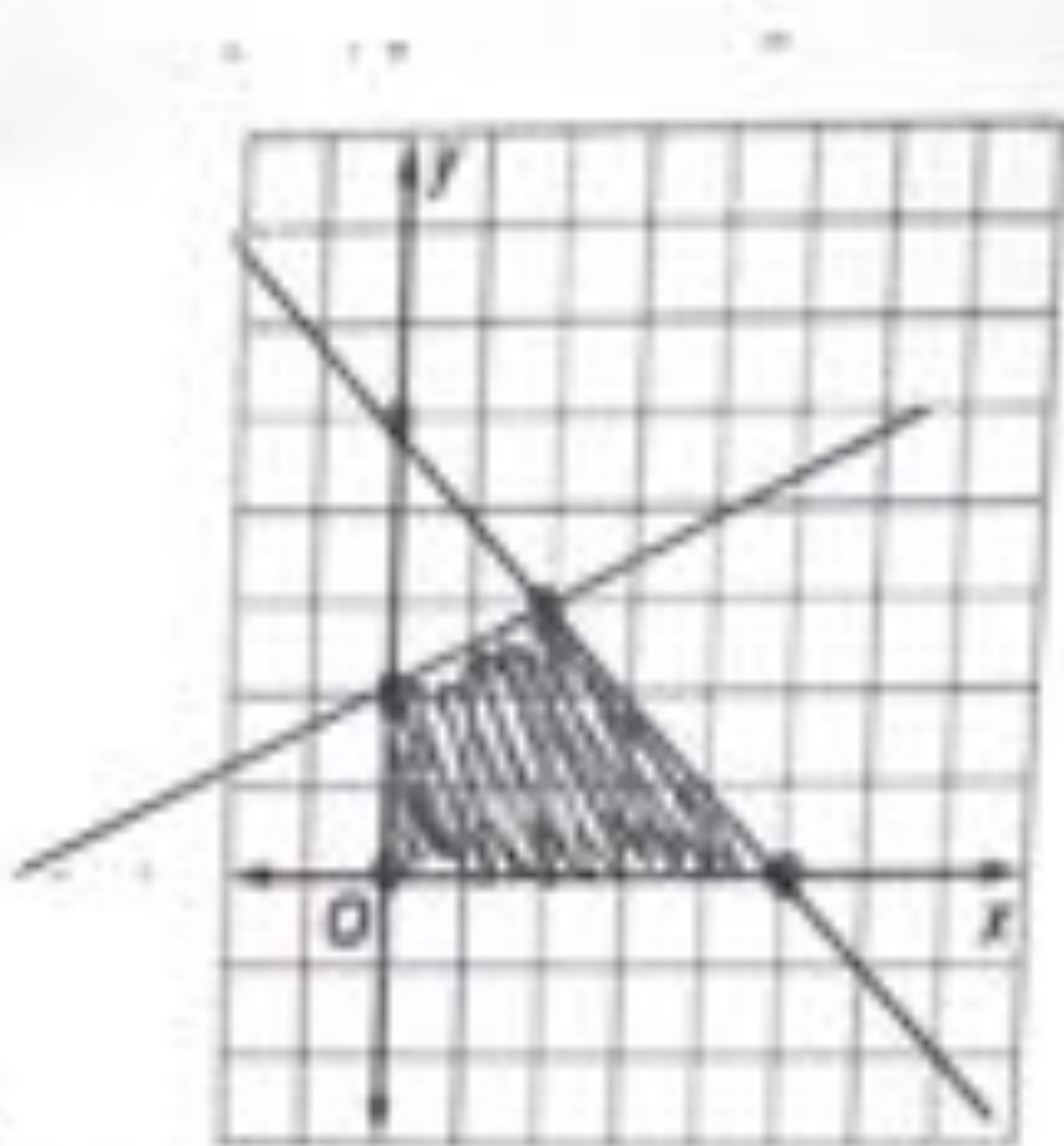
$(0,0)$	$C = 0$	$(2,3)$	$C = 28$
$(0,2)$	$C = 12$	$(5,0)$	$C = 25$

Minimum:

$$0 @ (0,0)$$

Maximum:

$$28 @ (2,3)$$



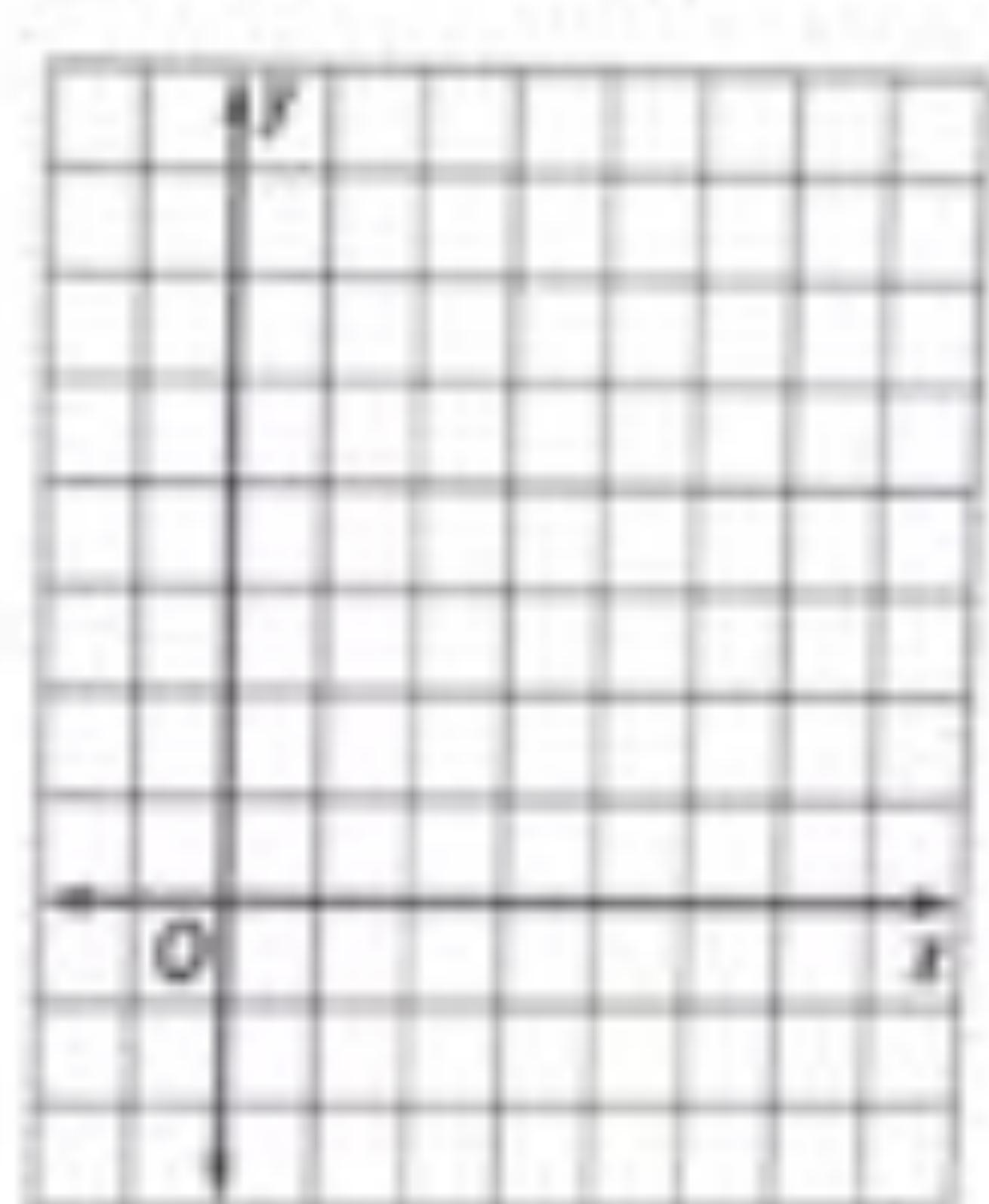
Example 12:

Graph the constraints:

$$\begin{array}{l} x \geq 0 \\ y \geq 0 \\ x - 2y \geq -6 \\ y \leq -2x + 8 \end{array}$$

Find the vertices of the feasibility region

Test each vertex in the objective function $C = 3x - y$.



Minimum:

Maximum:

Beta value > 1 Perform higher than market

Beta value = 1 Perform same as the market

Beta value < 1 Perform less than market \rightarrow risk adverse

Example 13: Nike Inc. stock sells for \$167 a share and has a 3-year average annual return of 5.2% a share. The beta value is .93. Walt Disney Co. sells for \$169 a share and has a 3-year average annual return of 5.24 a share. The beta value is 1.17. Roselyn wants to spend no more than \$8,000 investing in these two stocks, but she wants to earn at least \$1200 in annual revenue. Roselyn also wants to minimize the risk. Determine the number of shares of each stock that Roselyn should buy.

- Define the variables

$$x = \# \text{ of Nike shares}$$

$$y = \# \text{ of Disney shares}$$

- Write the constraints

$$x \geq 0$$

$$167x + 169y \leq 8000$$

$$y \geq 0$$

$$52x + 24y \geq 1200$$

- Write the objective function $\text{Minimize } C \rightarrow \text{lower beta value}$

$$C = 0.93x + 1.17y$$

Example 14: As part of your weight training regimen, you want to consume lean sources of protein.

You want to consume at least 300 Calories a day from at least 48 grams of protein. One ounce of chicken provides 35 Calories and 8.5 g of protein. One ounce of tofu provides 20 Calories and 2.5 g of protein. Your local supermarket charges \$0.31 an ounce for chicken and \$0.16 an ounce for tofu. How much of each food should you eat each day if you want to meet your requirements with the lowest cost? What is this daily cost?

- Define the variables

- Write the constraints

- Write the objective function