

## Logarithmic Functions

Examp.

Solve :

$$x + 4 = 6$$

$$x = 6 - 4$$

$$x = 2$$

To solve an equation with addition, we need "Subtraction"

$$x + 123 = 999$$

$$x = 999 - 123$$

$$x = 876$$

⑧ Solve :  $4x = 7$  ←

$$x = \frac{7}{4}$$
 ←

To solve equations with multiplication, we need division.

⑧ Solve :  $4^x = 16$  ←

$$x = 2$$

Solve :  $4^x = 15$

$$\Rightarrow x = \log_4 15$$

Solve :

(\*)  $2^x = 3$

$$\Rightarrow x = \log_2 3$$

(\*)  $4^x = 2025$

$$\Rightarrow x = \log_4 2025$$

4: base of the log.

(\*) Definition:

$$\text{If } x = b^y \text{ then}$$

$$y = \log_b x$$

$b$  is called the base of the log.

(\*) Some properties of  $\log$ .

①  $\log_b 1 = 0$  (because:  $b^0 = 1$ )

$$(2) \log_b b = 1$$

$$\textcircled{3} \quad \log_b a^x = x \log_b a$$

(4)  $\log_b (x \cdot y) = \log_b x + \log_b y$

$\uparrow$   
product
 $\uparrow$   
sum

(5)  $\log_b \left( \frac{x}{y} \right) = \log_b x - \log_b y$

↑                                  ↑  
division                              difference

$$(6) \quad \log_b x = \frac{\log_c x}{\log_c b}$$

Notice : Some special base :

⑦ when the base is  $e$  ( $2.71828 \dots$ )

we write  $\ln$  instead of  $\log_e$

② when the base is 10

we write  $\log$  instead  $\log_{10}$

we have the following property:

$$\log_b a = \frac{\ln a}{\ln b} \quad (\ln a = \log_e a)$$

$$\log_b a = \frac{\log a}{\log b} \quad (\log a = \log_{10} a)$$

Example: Solve:

$$① \quad 7^x = 8 \quad \Rightarrow \quad x = \log_7 8$$

$$② \quad 9^x = 20 \quad \Rightarrow \quad x = \log_9 20$$

$$③ \quad e^x = 10 \quad \Rightarrow \quad \underline{x} = \log_e 10 = \ln 10$$

$$④ \quad e^{(3x+1)} = 10$$

$$\Rightarrow \quad 3x + 1 = \ln 10$$

$$\Rightarrow \quad 3x = \ln 10 - 1$$

$$\Rightarrow \quad x = \frac{\ln 10 - 1}{3} \approx .434 \dots$$

$$(5) \quad 6 \cdot 3^x + 1 = 20$$

$$\Rightarrow \quad \underline{6} \cdot \underline{3^x} = 19$$

$$\Rightarrow \quad 3^x = \frac{19}{6}$$

$$\Rightarrow \quad x = \log_3(19/6) \approx 1.049...$$

$$(6) \quad 3 \cdot e^{4x+7} = 5$$

$$e^{4x+7} = 5/3$$

$$\Rightarrow \quad 4x + 7 = \ln(5/3)$$

$$\Rightarrow \quad 4x = \ln(5/3) - 7$$

$$\Rightarrow \quad x = \frac{\ln(5/3) - 7}{4} \approx -1.622...$$

Assignment 8 Solve for x

$$(1) \quad 2^x = 9$$

$$(2) \quad 3^x = 4$$

$$(3) \quad 2025^x = 2024$$

$$(4) \quad 10^x = e$$

$$(5) \quad e^x = 20$$

$$(6) \quad e^{4x-6} = 2$$

$$(7) \quad 6 \cdot 9^{2-x} = 7$$

$$(8) \quad 7 \cdot e^{4x+1} = 8$$

