Problem 1.

Find f'(x) (Power Rule)

$$f(x) = x^3 - 3x^2 + 3x + 3\sqrt{x} + 2\sqrt[3]{x} + \frac{2}{x} - \frac{3}{x^4} + 2025$$

(Rewrite) $f(x) = x^3 - 3x^2 + 3x + 3x^{1/2} + 2x^{1/3} + 2x^{-1} - 3x^{-4} + 2025$

$\Rightarrow f'(x) = 3x^{3-1} - 6x^{2-1} + 3 + 3.\frac{1}{2}.x^{1/2-1} + 2.\frac{1}{3}x^{1/3-1} + 2.(-1).x^{-1-1}$

$- 3.(-4)x^{-4-1} + 0$

$\Rightarrow f'(x) = 3x^2 - 6x + 3 + \frac{3}{2}x^{-1/2} + \frac{2}{3}x^{-2/3} - 2x^{-2} + 12x^{-5}$

Formulas $(x^n)' = n \cdot x^{n-1}$

$\frac{1}{x} = x^{-1}$

$\sqrt{x} = x^{1/2}$

$\frac{1}{x^k} = x^{-k}$

$\sqrt[k]{x} = x^{1/k}$

$$f(x) = 3\log_2 x + 4\ln x + 5\log_7 x + 2020$$

$f'(x) = 3.\frac{1}{x.\ln 2} + 4.\frac{1}{x} + 5.\frac{1}{x.\ln 5} + 0$

$= \frac{3}{x\ln 2} + \frac{4}{x} + \frac{5}{x\ln 5}$

Formulas: $(\log_b x)' = \frac{1}{x \cdot \ln b}$

$(\ln x)' = \frac{1}{x}$

Problem 3

Find f'(x) (Exponential Rule)

$$f(x) = 4e^x + 3^x + \frac{4^x}{3} + 1$$

$f'(x) = 4 \cdot e^x + 3^x \cdot \ln 3 + \frac{4^x \cdot \ln 4}{3}$

Formulas: $(b^x)' = b^x \cdot \ln b$

$(e^x)' = e^x$

Problem 4

Find f'(x) (Product Rule)

$$f(x) = (2^x + x^2)(\ln x + 3e^x)$$

$\Rightarrow f'(x) = (2^x + x^2)' \cdot (\ln x + 3e^x) + (\ln x + 3e^x)' \cdot (2^x + x^2)$

$\Rightarrow f'(x) = (2^x \cdot \ln 2 + 2x)(\ln x + 3e^x) + (\frac{1}{x} + 3e^x)(2^x + x^2)$

Problem 5

Find f'(x) and simplify (Quotient Rule)

$$\frac{x^3+1}{x^3-1}$$

$$f'(x) = \frac{(x^3+1)'\cdot(x^3-1) - (x^3-1)'(x^3+1)}{(x^3-1)^2}$$

$$= \frac{3x^2\cdot(x^3-1) - (3x^2)(x^3+1)}{(x^3-1)^2}$$

(Simplifying)

$$= \frac{3x^5 - 3x^2 - 3x^5 - 3x^2}{(x^3-1)^2}$$

$$= \frac{-6x^2}{(x^3-1)^2}$$