Exponential / Logarithmic functions

Example :

initial amount .

U.S. Investment Abroad In 1980, direct U.S. business investment abroad was about 13.5 billion dollars. From 1980 through 2010, that investment[12] grew at an average annual rate of 11.24%.

r

a. Make an exponential model that shows the U.S. direct investment abroad A, in billions of dollars, t years after 1980.

b. From 1980, how long did it take for U.S. investments abroad to double?

a. $A(t) = P * (1 + r)^t$

amount of money t (years) since 1980

initial amount of investment (principal)

interest rate growth rate

we have : $P = 13.5$ (billions)

$r = 11.24\% = .1124$

$$\Rightarrow \boxed{A(t) = 13.5 * (1.1124)^t}$$

(b) we need to find t so that

$A(t) = 13.5 * 2$

$$\Rightarrow 13.5 * (1.1124)^t = 13.5 * 2$$

$$\Rightarrow \quad 1.1124^{t} = 2$$

$$\Rightarrow \quad t = \log_{1.1124} 2 = \frac{\ln 2}{\ln 1.1124}$$

$$\approx 6.507$$

Example

If tuition at a college is increasing by 6.6% each year, how many years will it take for tuition to double?

$$\underbrace{A(t)}_{\text{tuition at the time } t} = \underbrace{P}_{\text{initial tuition}} \cdot (1 + r)^{t} \qquad r \to 6.6\%$$

we need to solve for

$$A(t) = 2P$$

$$\Rightarrow \quad P(1 + .066)^{t} = 2P$$

$$\Rightarrow \quad (1.066)^{t} = 2$$

$$\Rightarrow \quad t = \log_{1.066} 2 = \frac{\ln 2}{\ln 1.066} \approx 10.845$$

* How long does it take for the tuition to triple ?

$$A(t) = 3 \cdot P$$

$$\Rightarrow P \cdot 1.066^{t} = 3P$$

$$\Rightarrow 1.066^{t} = 3$$

$$\Rightarrow t = \log_{1.066} 3 = \frac{\ln 3}{\ln 1.066} \approx 17.189$$

Example :

A freezer maintains a constant temperature of 6 degrees Fahrenheit. An ice tray is filled with tap water and placed in the refrigerator to make ice. The difference between the temperature of the water and that of the freezer was sampled each minute and recorded in the table below.

t	**Time in minutes**	0	1	2	3	4	5
d	**Temperature difference**	69.0	66.3	63.7	61.2	58.8	56.5

Part 1 Test to see that the data are exponential.

Part 2 Find an exponential model for temperature difference.

Part 3 Use your answer in part 2 to make a model for the temperature of the cooling water as a function of time.

Part 4 When will the temperature of the water reach 32 degrees?

Is this a linear growth or decay in temp. difference?

t	d	Change in d
0	69	
1	66.3	$66.3 - 69 = -2.7$
2	63.7	$63.7 - 66.3 = -2.6$
3	61.2	$61.2 - 63.7 = -2.5$
4	58.8	$58.8 - 61.2 = -2.4$
5	56.5	$56.5 - 58.8 = -2.3$

We observe that the differences in d are not the same. This means that the relation between t and d is not linear.

t	d	ratio change in d
0	69	
1	66.3	66.3 / 69 = 0.960869
2	63.7	63.7 / 66.3 = 0.960784
3	61.2	61.2 / 63.7 = 0.96075
4	58.8	58.8 / 61.2 = 0.96078
5	56.5	56.5 / 58.8 = 0.960884

We observe that the ratio changes are "almost" the same.
So we could use exponential model to model the relation between t and d.

Part 2:

$d = \underline{\text{smth}} * \underline{\text{something}}^{t}$

$d = P \cdot b^{t}$

$\begin{cases} \text{when } t = 0 \\ \text{and } d = 69 \end{cases} \Rightarrow d = P * b^0 = P$

$\boxed{69 = P}$

$\begin{cases} t = 1 \\ d = 66.3 \end{cases}$

$P \cdot b^1 = 66.3$

$\Rightarrow 69 \cdot b = 66.3 \Rightarrow b = \dfrac{66.3}{69}$

$$\Rightarrow \boxed{b \approx .9608}$$

$$\Rightarrow \boxed{d = 69 * (.9608)^t}$$

Assignment

① How long does it take for an investment of \$100,000 to be doubled. The annual growth is 7%.

② How long does it take for that investment to be \$1,000,000.