Exam 2 – Practice 2 Math 110.

Exam Guidelines This is an in-class, written exam with a 75-minute time limit.

- **Permitted Materials:** You may use a basic calculator and formula sheets.
- **Formula Sheet Restrictions:** Your sheets must contain formulas only; no examples or worked problems are permitted. All sheets will be inspected at the start of the exam.
- **Prohibited Items:** Phones and all other smart devices are strictly forbidden.
- **Academic Integrity:** The use of AI is prohibited. Any AI usage will result in an automatic F for the exam and may lead to failing the entire course.
- Show **ALL** your work for credits.

1. Solve each quadratic by factoring or the quadratic formula.

   a. $x^2 = 4x - 4$

   $x^2 - 4x + 4 = 0$

   $\Leftrightarrow$ $(x-2)(x-2) = 0$

   Solution: x = 2

   b. $x^2 - 6x + 5 = 0$

   $(x-1)(x-5) = 0$

   Solution: x = 5, x = 1

   c. $x^2 - 2x + 9 = 0$

   $a=1$; $b=-2$; $c=9$

   $b^2 - 4ac < 0$

   $\Rightarrow$ No Solution

d. $x^2 + 7x = 2x - 6$

$x^2 + 7x - 2x + 6 = 0$

$\Leftrightarrow \quad x^2 + 5x + 6 = 0$

$\Leftrightarrow \quad (x+2)(x+3) = 0$

Solution: x = -2, x = -3

2. Graph of the quadratic functions. Label the vertex and another point.

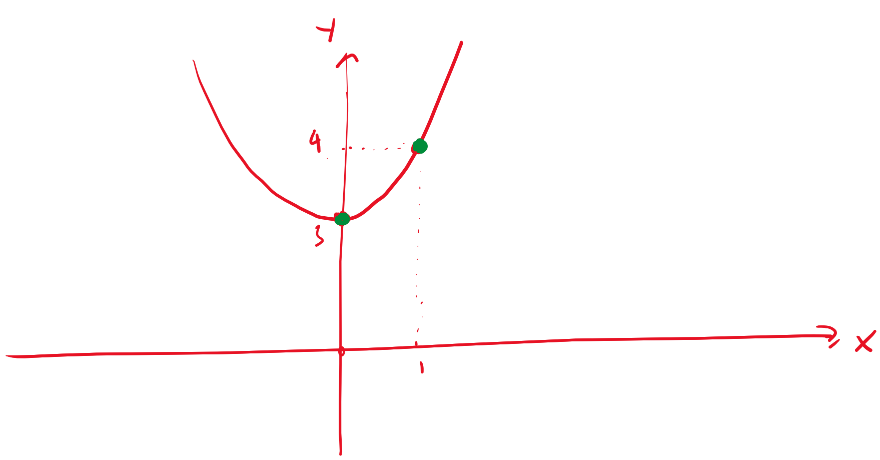
a. $y = x^2 + 3$

$a = 1$; $b = 0$; $c = 3$

(*) vertex : $x = -\frac{b}{2a} = 0$

$y = \frac{4ac - b^2}{4a} = \frac{12}{4} = 3$

(*) Another point : $x = 1$; $y = 4$



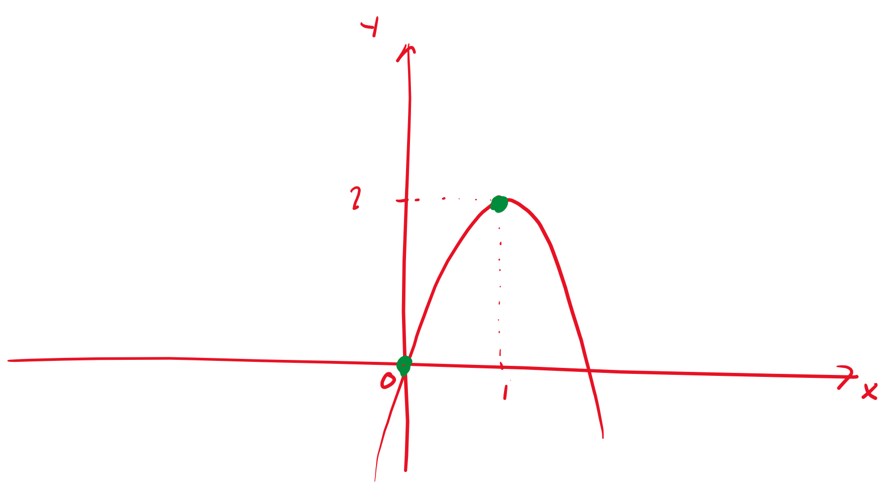
b. $y = -2x^2 + 4x$

$a = -2$; $b = 4$; $c = 0$

(*) Vertex: $x = -\frac{b}{2a} = -\frac{4}{(-2)\cdot 2} = 1$

$$y = \frac{4ac - b^2}{4a} = \frac{-4^2}{-8} = 2$$

(*) Another point: $x = 0$; $y = 0$



3. Suppose that in a monopoly market the total cost per week of producing a high-tech product is given by $C = 200 + 100 + 2q^2$. Suppose further that the sales price function for this product is
$p = 300 - 2q$.

   a. Find the revenue function in term of q.

$$R = p \cdot q = (300 - 2q)\cdot q = -2q^2 + 300q$$

b. Find the number of units that will **maximize the revenue.**

$R = -2q^2 + 300q$

$a = -2;\ b = 300$

Maximized at $q = -\frac{b}{2a} = -\frac{300}{2\cdot(-2)} = 75$

c. Find the profit function

$\text{Profit} = R - C$

$= -2q^2 + 300q - (200 + 100q + 2q^2)$

$= -4q^2 + 200q - 200$

d. Find the number of units that will give **break-even** for the product

Profit $= 0$

$\Leftrightarrow \quad -4q^2 + 200q - 200 = 0$

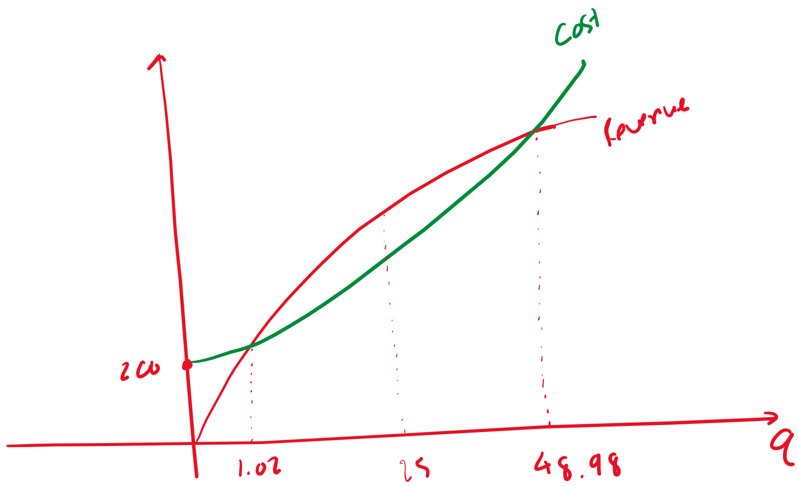
$a = -4;\ b = 200;\ c = -200$

$q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \begin{cases} q \approx 1.02 \\ q = 48.98 \end{cases}$

e. Find **the maximum profit** and the number of products needed to maximize the profit.

maximized at $q = -\frac{b}{2a} = -\frac{200}{-8} = 25$

f. Graph the revenue function and the cost function label the break-even points, fixed cost, and the maximized profit point.



4. On a certain route, an airline carries 4000 passengers per month, each paying \$25. A market survey indicates that for each \$1 increase in the ticket price, the airline will lose 50 passengers.

   a. What is the airline's current revenue?

   Revenue $= 4000 \times 25 = 100{,}000$

   b. Create an income (revenue) function if "x" is defined as the number of \$1 price increases

	Before	After
price	25	$25 + x$
quantity	4000	$4000 - 50x$

$$R = (25 + x) \cdot (4000 - 50x)$$

$$R = -50x^2 + 2750x + 100,000$$

c. Find the number of \$1 price increases that will maximize the revenue.

$$R = -50x^2 + 2750x + 100,000$$

$$a = -50, \quad b = 2750 \ ; \ c = 100,000$$

R is maximized when $x = -\dfrac{b}{2a} = -\dfrac{2750}{2 \times (-50)}$

$$x = 27.5$$

d. Find the new ticket price (that will maximize the revenue)

New price $= 25 + x = 25 + 27.5 = 52.5$

e. Find the number of passengers at that price in d.

New number of passengers : $4000 - 50x$

$$= 4000 - 50 \times 27.5$$

$$= 2625$$

f. Find the new maximum income (income at that price in d)

New maximum income/revenue $= 52.50 \times 2650$

$$= 137\ 812.50$$

5. If the supply function for a commodity is given by $p = 2q^2 + 6q$ and the demand function is given by $p = 180 - 3q^2$, find the point of market equilibrium (Supply equals Demands).

$$2q^2 + 6q = 180 - 3q^2$$

$$\Leftrightarrow \quad 2q^2 + 6q + 3q^2 - 180 = 0$$

$$\Leftrightarrow \quad 5q^2 + 6q - 180 = 0$$

$$\Leftrightarrow \quad q = \frac{-6 \pm \sqrt{6^2 - 4 * 5 * (-180)}}{2 * 5}$$

$q \simeq 5.43$ (ignore the negative solution)

$p = 180 - 3q^2 \simeq 91.54.$