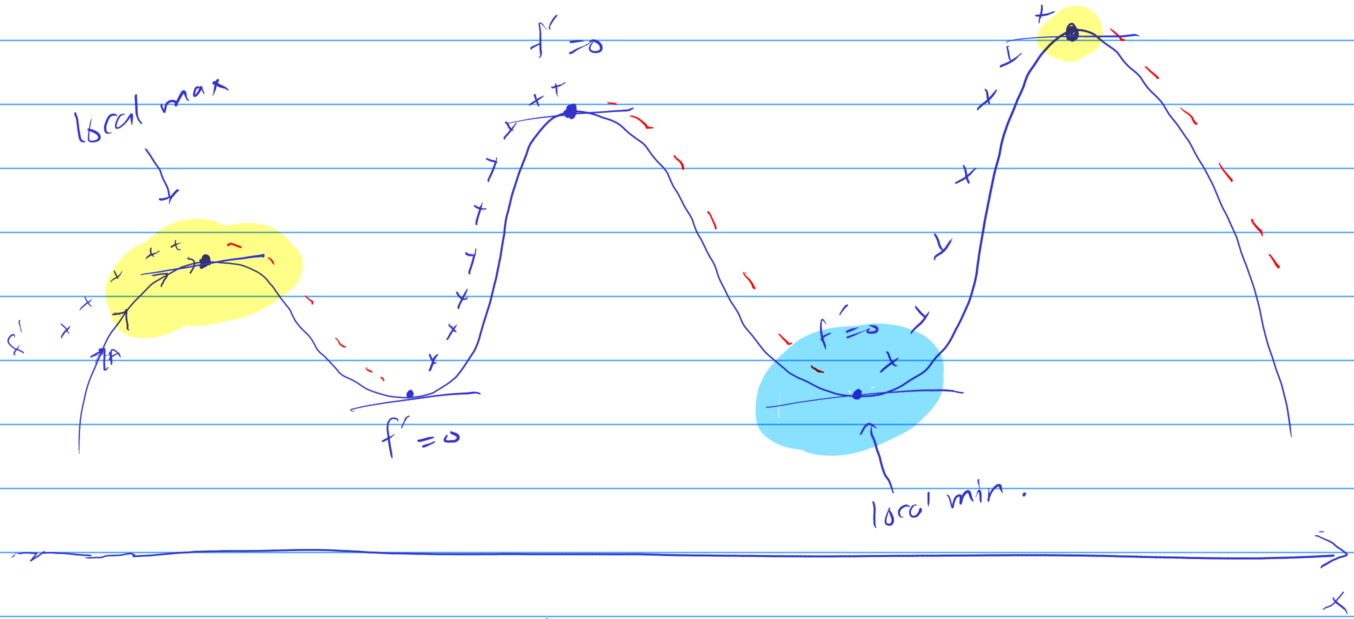
⊛ Applications of Derivatives.



If $f'(a) > 0$ then the function is increasing at $x = a$.

If $f'(a) < 0$ then $f(x)$ is decreasing at $x = a$.

If $f'(x)$ changes the sign from (+) to (−) at $x = a$ then $f(a)$ is a local maximum.

If $f'(x)$ changes the sign from (−) to (+) at $x = a$, then $f(a)$ is a local minimum.

In order to find all local max / min, we just need to find all the points where the derivative changes the sign.

Example

The per-day cost function for the manufacture of portable MP3 players is given by

$$C(x) = 128,000 + 30x + x^3,$$

where x is the number of MP3 players manufactured per day. Assume that the company cannot manufacture more than 100 MP3 players per day. How many MP3 players should be manufactured in order to minimize average cost?

x: number of items manufactured.

$C(x)$: the cost to manufacture x items.

$x \leq 100$

Average cost to manufacture 1 item: $\frac{C(x)}{x}$

Find x that minimizes $\frac{C(x)}{x}$

Let $f(x) = \frac{C(x)}{x}$

we will use derivatives to solve the problem.

(*) Step 1: Find $f'(x)$ and simplify.

$$f(x) = \frac{128,000 + 30x + x^3}{x}$$

$$f'(x) = \frac{(128{,}000 + 30x + x^3)' \cdot x - (x)' \cdot (128\,000 + 30x + x^3)}{x^2}$$

$$f'(x) = \frac{(30 + 3x^2)\, x - (128{,}000 + 30x + x^3)}{x^2}$$

$$f'(x) = \frac{\cancel{30x} + 3x^3 - 128{,}000 - \cancel{30x} - x^3}{x^2}$$

$$f'(x) = \frac{2x^3 - 128{,}000}{x^2}$$

Step 2 : Obtain a sign chart of $f'(x)$

① Solve $f'(x) = 0$;

$$\frac{2x^3 - 128{,}000}{x^2} = 0$$

$$\Leftrightarrow \quad 2x^3 - 128{,}000 = 0$$

$$\Leftrightarrow \quad 2x^3 = 128{,}000$$

$$x^3 = \frac{128{,}000}{2} = 64{,}000$$

$$x = \sqrt[3]{64000} = 40$$

② Sign chart:

x	0		40		100
$f'(x)$		$-$	0	$+$	
$f(x)$		$\searrow$		$\nearrow$	

① $f'(x) = \dfrac{2x^3 - 128,000}{x^2}$: pick x from 0 to 40, say $x = 1$

$$\Rightarrow f'(1) = \frac{2\cdot 1^3 - 128,000}{1^2} = (-)$$

② Pick $x = 60$, $f'(60) = \dfrac{2\cdot 60^3 - 128000}{60^2} = \dfrac{304000}{60^2} = (+)$

Since $f'(x)$ changes the sign from $(-)$ to $(+)$ at $x = 40$, $f(x)$ is minimized at $x = 40$.

Therefore, the company should manufacture 40 items to minimized the average cost.

Assignment.

The per-day cost function for the manufacture of portable MP3 players is given by

$$C(x) = 686,000 + 30x + x^3,$$

where x is the number of MP3 players manufactured per day. Assume that the company cannot manufacture more than 100 MP3 players per day. How many MP3 players should be manufactured in order to minimize average cost?

Note: we just need to solve for $f'(x) = 0$

where $f(x) = \frac{C(x)}{x}$.