Exponential Functions

(*) $y = f(x) = P \cdot a^{x}$, y is an exponential function of x

OR

$y = f(t) = P \cdot a^{t}$, y is an exponential function of t

Example 1

$y = f(x) = 2024 \cdot 6^{x}$

$y = f(x) = -20 \cdot \left(\frac{1}{3}\right)^{x}$

$y = f(x) = 100 \cdot x^{6}$ (not exponential!)

$y = f(x) = \frac{1}{x^2+1}$ (not exponential)

(*) $y = f(t) = P \cdot a^{t}$

P : Initial value

a : The base , a is always bigger than 0

$a > 1$: The growing factor

$a < 1$: The decay factor.

Example 2: Linear Growth vs. Exponential Growth

Consider 2 types of interest when deposit \$100 into a bank.

① Simple interest of $r = 5\%$ a year.

The money earns from interest after the first year.

$$5\% \cdot 100 = \$5$$

After 1^{st} year: we have totally $100 + 5 = \$105$

$t = 1 \Rightarrow f(t) = \$105$

$t = 2 \Rightarrow 105 + 5 = \$110$

$t = 3 \Rightarrow 105 + 5 = \$115$

The money growth constantly. The money at year t is

$$f(t) = 100 + 100 \cdot 5\% \cdot t$$

$$\boxed{f(t) = 100 + 5t}$$ (linear function of t)

② Compound Interest at 5% a year.

The growing percentage is a constant every year.

$t = 1$: $\underline{100} + \underbrace{100 \cdot 5\%} = \$105$

$t = 2$: $105 + 105 \times 5\% = \$110.25$

$t = 3$: $110.25 + 110.25 \times 5\% = \$115.76$

After year t, the money is

$$\boxed{100 \cdot (1.05)^t}$$

Formulas :

P : Principal (Initial amount of money)

r : Interest

t : time

$M(t)$: the total money at year t

Simple Interest	Compound Interest
$M(t) = P(1 + rt)$	$M(t) = P(1+r)^t$

Assignment 5.

① Give some examples of exponential functions and non-exponential functions.

② Given: you deposit \$1 to the bank.

In Excel, make a table for the money in 100 year for 2 cases simple interest and compound interest with $r = 7\%$

Logarithmic functions

Example : Solve for x

$3^x = 9$	$2^x = 8 = 2^3$	$4^x = 4^1$
$\Rightarrow 3^x = 3^2$	$2^x = 2^3$	$\boxed{x = 1}$
$\Rightarrow \boxed{x = 2}$	$\boxed{x = 2}$	

$(*)\quad 3^x = 27 = 3^3$

$\Rightarrow x = 3$

$(*)\quad 3^x = 1 = 3^0$

$3^x = 3^0 \Rightarrow \boxed{x = 0}$

Notice : $(\text{anything})^0 = 1$

positive

$(*)\quad 3^x = 3 = 3^1$

$\Rightarrow \boxed{x = 1}$

$(*)\quad 3^x = 2$

$3^0 = 1$

$3^1 = 3$

$3^2 = 9$

$3^3 = 27$

$\vdots$

$3^{.6309} = 1.9999346\ldots$

not 2

In decimal form, it is impossible to write down the exact solution in this case.

$$3^x = 2$$

$$\Rightarrow \boxed{x = \log_3 2}$$

Example:

$$4^x = 29$$

$4^{2} = 16$

$4^{3} = 64$

$$\Rightarrow \boxed{x = \log_4 29}$$

Example:

(1) Solve for x

$$2 \cdot 4^x = 2024$$

$$\Rightarrow 4^x = \frac{2024}{2} = 1012$$

$$\Rightarrow 4^x = 1012$$

$$\Rightarrow \boxed{x = \log_4 1012}$$

② $\frac{6^x}{3} + 10 = 121$

$\Rightarrow \quad \frac{6^x}{3} = 111 \quad \Rightarrow \quad 6^x = 333$

$\Rightarrow \quad x = \log_6 333$

U.S. Investment Abroad In 1980, direct U.S. business investment abroad was about 13.5 billion dollars. From 1980 through 2010, that investment[12] grew at an average annual rate of 11.24%.

compound interest

a. Make an exponential model that shows the U.S. direct investment abroad A, in billions of dollars, t years after 1980.

b. From 1980, how long did it take for U.S. investments abroad to double?

ⓐ $M(t) = P \cdot (1 + r)^t$

$= 13.5\,(1 + .1124)^t$

$M(t) = 13.5\,(1.1124)^t$

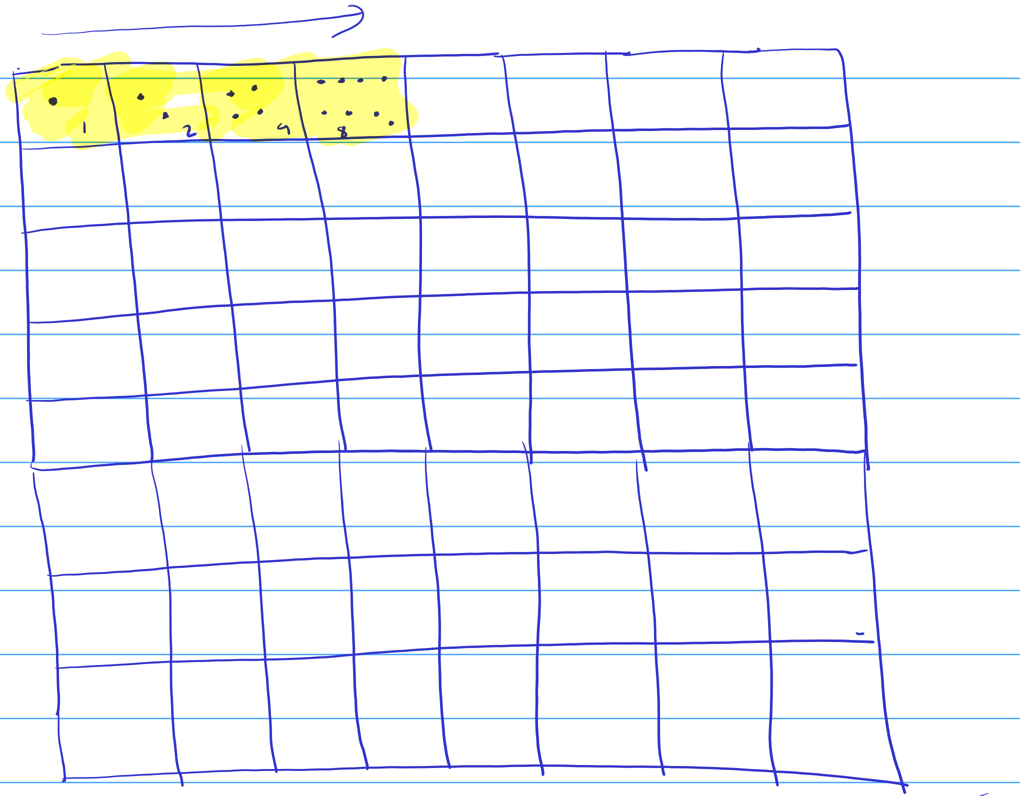
ⓑ

$13.5\,(1.1124)^t = 2 * 13.5$

$\Rightarrow \quad (1.1124)^t = 2$

$\Rightarrow \quad \boxed{t = \log_{1.1124} 2} = \boxed{6.507}$

In year 7th the money will be (more than) double



Assignment 6

① Solve for x

ⓐ $3^x = 10$

ⓑ $4^x = 1$

ⓒ $6 \cdot 10^x = 3$

ⓓ $7 \cdot 6^x + 1 = 20$

ⓔ $\dfrac{8^x + 1}{2} = 2024$

② How does it take for the investment of \$100 with interest rate 6% compound annually to

ⓐ double

ⓑ Triple

③ Starting from 1 cent (\$.01), doubling it every day this means

day 1 : .01

day 2 : .01 x 2 = .02

day 3 : .02 x 2 = .04

day 4 : .04 x 2 = .08

⋮

now log: does it take to take to be the richest person in the world ?