Rule 8: Logarithmic Rule

$$\left(\log_b x\right)' = \frac{1}{x \cdot \ln b}$$

special case: when $b = e = 2.71828\ldots$ then the rule becomes

$$\left(\ln x\right)' = \frac{1}{x}$$

Example: Find $f'(x)$

① $f(x) = \log_{11} x$

$\Rightarrow \quad f'(x) = \dfrac{1}{x \cdot \ln 11}$

② $f(x) = \log_{2024} x$

$f'(x) = \dfrac{1}{x \cdot \ln 2024}$

③ $f(x) = 6 \cdot \log_9 x$

scalar

$$f'(x) = 6 \cdot \frac{1}{x \ln 9} = \frac{6}{x \cdot \ln 9}$$

④ $f(x) = 10 \ln x$

$$\Rightarrow \quad f'(x) = 10 \cdot \frac{1}{x} = \frac{10}{x}$$

⑤ $f(x) = \left(7\log_9 x + 6\log_3 x - 10\log_4 x - 8\ln x + 1 \right)$

$f'(x) = \left(7\log_9 x \right)' + \left(6\log_3 x \right)' - \left(10\log_4 x \right)' - \left(8\ln x \right)' + (1)'$

$f'(x) = 7 \cdot \dfrac{1}{x \cdot \ln 9} + 6 \cdot \dfrac{1}{x \ln 3} - 10 \cdot \dfrac{1}{x \ln 4} - 8 \cdot \dfrac{1}{x}$

$$f'(x) = \dfrac{7}{x \ln 9} + \dfrac{6}{x \ln 3} - \dfrac{10}{x \ln 4} - \dfrac{8}{x}$$

product

⑥ $f(x) = \left(x^2 + x \right) \cdot \ln x$

$f'(x) = \left(x^2 + x \right)' \cdot \ln x + \left(x^2 + x \right) \cdot \left(\ln x \right)'$

$f'(x) = (2x + 1)\ln x + \left(x^2 + x \right) \cdot \dfrac{1}{x}$

⑦ $f(x) = 2^x \cdot \log_3 x$

product

$f'(x) = \left(2^x \right)' \cdot \log_3 x + 2^x \cdot \left(\log_3 x \right)'$

$= 2^x \cdot \ln 2 \cdot \log_3 x + 2^x \cdot \dfrac{1}{x \cdot \ln 3}$

⑧ $f(x) = \dfrac{2^x}{\log_3 x}$ quotient

$$f'(x) = \frac{(2^x)' \cdot \log_3 x - 2^x \cdot (\log_3 x)'}{(\log_3 x)^2}$$

$$f'(x) = \frac{2^x \cdot \ln 2 \, \log_2 x - 2^x \cdot \frac{1}{x \ln 3}}{(\log_3 x)^2}$$

Assignment 17   Find $f'(x)$

① $f(x) = \log_8 x$

② $f(x) = \log_9 x$

③ $f(x) = \ln x$

④ $f(x) = 3 \log_4 x$

⑤ $f(x) = 6 \log_7 x + 9 \log_4 x - 3 \ln x - \log_2 x + 1$

⑥ $f(x) = \dfrac{\log_4 x}{2} + \dfrac{6 \ln x}{7} - \dfrac{\log_3 x}{10}$

Solution: $f'(x) = \dfrac{1}{2} \cdot \dfrac{1}{x \ln 4} + \dfrac{6}{7} \cdot \dfrac{1}{x} - \dfrac{1}{10} \cdot \dfrac{1}{x \cdot \ln 3}$

⑦ $f(x) = \dfrac{2 \ln x}{3} - \dfrac{6 \log_4 x}{11} + \dfrac{3 \log_7 x}{10}$

⑧ $f(x) = x \cdot \ln x$

⑨ $f(x) = 2 e^x \ln x$

10. $f(x) = \log_2 x \cdot \log_3 x$

11. $f(x) = \dfrac{\ln x}{x}$

12. $f(x) = \dfrac{\ln x + 1}{\ln x + 2}$ (simplify for extra credit)

13. $f(x) = \dfrac{3^x + x^2}{6 \log_4 x + \ln x}$

we are doing Assignment 17. Please see the assignment on

Canvas in note 13. pdf