Exponential Functions

(*)
$$y = f(x) = P \cdot a^x$$
, y is an exponential function of x

or
$$y = f(t) = P \cdot a^t$$
, y is an exponential function of t

Example 1

$$y = f(x) = \underline{2014} \cdot 6^x$$

$$y = f(x) = \underline{-20} \cdot \left(\frac{1}{3}\right)^x$$

$$y = f(x) = 100 \cdot x^6 \qquad \text{(not exponential!)}$$

$$y = f(x) = \frac{1}{x^2 + 1} \qquad \text{(not exponential)}$$

(*) $\qquad y = f(t) = P \cdot a^t$

P : Initial value

a : The base , a is always bigger than 0

$\underline{a > 1}$: The growing factor

a < 1 : The decay factor.

Example 2 : Linear Growth vs. Exponential Growth

Consider 2 types of interest when deposit $100 into a bank.

① Simple interest of $r = 5\%$ a year.

The money earns from interest after the first year.

$$5\% \cdot 100 = \$5$$

After 1st year : we have totally $100 + 5 = \$105$

$t = 1 \implies f(t) = \$105$

$t = 2 \implies 105 + 5 = \$110$

$t = 3 \implies 105 + 5 = \$115$

The money growth constantly. The money at year t is

$$f(t) = 100 + 100 \cdot 5\% \cdot t$$

$$\boxed{f(t) = 100 + 5t}$$ (linear function of t)

② Compound Interest at 5% a year.

The growing percentage is a constant every year.

$t = 1 : \quad 100 + 100 \cdot 5\% \cdot \quad = \$105$

$t = 2:$ $105 + 105 * 5\% \cdot = \boxed{\$110.25}$

$t = 3:$ $110.25 + 110.25 * 5\% = \boxed{\$115.76}$

After year t, the money is

$$\boxed{100 \cdot (1.05)^t}$$

Formulas :

P : Principal (Initial amount of money)

r : Interest

t : time.

$M(t)$: the total money at year t

Simple Interest	Compound Interest
$M(t) = P(1 + rt)$	$M(t) = P(1 + r)^t$

Assignment 5.

① Give some examples of exponential functions and non-exponential functions.

② Given: you deposit $\$1$ to the bank.

In Excel, make a table for the money in 100 year

for 2 cases simple interest and compound interest