Rule 8: Logarithmic Rule

$$(\log_b x)' = \frac{1}{x \cdot \ln b}$$

special case: when $b = e = 2.71828\ldots$ then the rule becomes

$$(\ln x)' = \frac{1}{x}$$

Example: Find $f'(x)$

① $f(x) = \log_{11} x$

$\Rightarrow f'(x) = \frac{1}{x \cdot \ln 11}$

② $f(x) = \log_{2024} x$

$f'(x) = \frac{1}{x \cdot \ln 2024}$

③ $f(x) = 6 \cdot \log_9 x$

6 is a scalar

$f'(x) = 6 \cdot \frac{1}{x \ln 9} = \frac{6}{x \cdot \ln 9}$

④ $f(x) = 10 \ln x$

$\Rightarrow f'(x) = 10 \cdot \frac{1}{x} = \frac{10}{x}$

(5) $f(x) = (7\log_9 x + 6\log_3 x - 10\log_4 x - 8\ln x + 1)$

$f'(x) = (7\log_9 x)' + (6\log_3 x)' - (10\log_4 x)' - (8\ln x)' + (1)'$

$f'(x) = 7 \cdot \frac{1}{x \cdot \ln 9} + 6 \cdot \frac{1}{x \ln 3} - 10 \cdot \frac{1}{x \ln 4} - 8 \cdot \frac{1}{x}$

$$f'(x) = \frac{7}{x \ln 9} + \frac{6}{x \ln 3} - \frac{10}{x \ln 4} - \frac{8}{x}$$

(6) $f(x) = (x^2 + x) \cdot \ln x$ — product

$f'(x) = (x^2 + x)' \cdot \ln x + (x^2 + x) \cdot (\ln x)'$

$f'(x) = (2x + 1) \ln x + (x^2 + x) \cdot \frac{1}{x}$

(7) $f(x) = 2^x \cdot \log_3 x$ — product

$f'(x) = (2^x)' \cdot \log_3 x + 2^x \cdot (\log_3 x)'$

$= 2^x \cdot \ln 2 \cdot \log_3 x + 2^x \cdot \frac{1}{x \cdot \ln 3}$

(8) $f(x) = \frac{2^x}{\log_3 x}$ — quotient

$$f'(x) = \frac{(2^x)' \cdot \log_3 x - 2^x \cdot (\log_3 x)'}{(\log_3 x)^2}$$

$$f'(x) = \frac{2^x \cdot \ln 2 \log_2 x - 2^x \cdot \frac{1}{x \ln 3}}{(\log_3 x)^2}$$

Assignment 17 Find $f'(x)$

① $f(x) = \log_8 x$

② $f(x) = \log_9 x$

③ $f(x) = \ln x$

④ $f(x) = 3 \log_4 x$

⑤ $f(x) = 6 \log_7 x + 9 \log_4 x - 3 \ln x - \log_2 x + 1$

⑥ $f(x) = \frac{\log_4 x}{2} + \frac{6 \ln x}{7} - \frac{\log_3 x}{10}$

Solution: $f'(x) = \frac{1}{2} \cdot \frac{1}{x \ln 4} + \frac{6}{7} \cdot \frac{1}{x} - \frac{1}{10} \cdot \frac{1}{x \cdot \ln 3}$

⑦ $f(x) = \frac{2 \ln x}{3} - \frac{6 \log_4 x}{11} + \frac{3 \log_7 x}{10}$

⑧ $f(x) = x \cdot \ln x$

⑨ $f(x) = 2 e^x \ln x$

(10) $f(x) = \log_2 x \cdot \log_3 x$

(11) $f(x) = \dfrac{\ln x}{x}$

(12) $f(x) = \dfrac{\ln x + 1}{\ln x + 2}$ (simplify for extra credit)

(13) $f(x) = \dfrac{3^x + x^2}{6\log_4 x + \ln x}$

We are doing Assignment 17. Please see the assignment on Canvas in note 14.pdf

Rule 9 The chain rule for power function.

(1) Power function: $(x^n)' = n \cdot x^{n-1}$

(2) Chain rule for power function: x is changed to some function of x, $g(x)$

$x \to g(x)$

$$\left[(g(x))^n\right]' = n \cdot [g(x)]^{n-1} \cdot g'(x)$$

Example:

Find $f'(x)$

power rule

$\left[\left(x^{2024}\right)' = 2024\, x^{2023} \right]$

① $f(x) = \left[x^3 + 2x\right]^{2024}$

$f'(x) = 2024 \cdot \left(x^3 + 2x\right)^{2023} \cdot \left(x^3 + 2x\right)$

$= \boxed{2024 \left(x^3 + 2x\right)^{2023} \cdot \left(3x^2 + 2\right)}$

② $f(x) = \left(x^7 + 6\right)^{100}$

$f'(x) = 100 \cdot \left(x^7 + 6\right)^{99} \cdot \left(x^7 + 6\right)'$

$= 100 \left(x^7 + 6\right)^{99} \left(7x^6\right)$

③ $f(x) = \left(2^x + 3^x\right)^{1000}$

$f'(x) = 1000 \cdot \left(2^x + 3^x\right)^{999} \cdot \left(2^x + 3^x\right)'$

$= 1000 \left(2^x + 3^x\right)^{999} \left(2^x \ln 2 + 3^x \ln 3\right)$

④ $f(x) = \left(\log_3 x + x^2\right)^{20}$

$f'(x) = 20 \left(\log_3 x + x^2\right)^{19} \cdot \left(\log_3 x + x^2\right)'$

$= 20 \left(\log_3 x + x^2\right)^{19} \cdot \left(\frac{1}{x \ln 3} + 2x\right)$

(5) $f(x) = (\ \ )^{30}$

$f'(x) = 30(\ \ )^{29}(\ \ )'$

(6) $f(x) = (x \cdot \ln x)^{30}$

$f'(x) = 30(x\ln x)^{29} \cdot (x \cdot \ln x)'$ ↓ product rule

$= 30(x\ln x)^{29}[(x)' \cdot \ln x + x \cdot (\ln x)']$

$= 30(x\ln x)^{29}(\ln x + x \cdot \frac{1}{x})$

$= 30(x\ln x)^{29}(\ln x + 1)$

(7) $f(x) = \left(\frac{3^x}{x^2}\right)^{45}$

$f'(x) = 45 \cdot \left(\frac{3^x}{x^2}\right)^{44} \cdot \left(\frac{3^x}{x^2}\right)'$

$= 45\left(\frac{3^x}{x^2}\right)^{44} \cdot \left[\frac{(3^x)' \cdot x^2 - 3^x \cdot (x^2)'}{(x^2)^2}\right]$

$= 45\left(\frac{3^x}{x^2}\right)^{44} \cdot \left[\frac{3^x \cdot \ln 3 \cdot x^2 - 3^x \cdot 2x}{x^4}\right]$

Assignment 18 Find $f'(x)$

① $f(x) = (x^3 + x^2)^{50}$

② $f(x) = (x^{10} + 6x + 1)^{10}$

③ $f(x) = (x^4 + 4x^3 + 5x^2 + 6x + 1)^7$

④ $f(x) = (4^x + 6^x)^{19}$

⑤ $f(x) = (e^x + 4 \cdot 3^x)^{100}$

⑥ $f(x) = (6\log_4 x + 10^x + x^7)^{2024}$

⑦ $f(x) = (4e^x - 6\ln x - \sqrt{x} + 4)^{30}$

⑧ $f(x) = (x^2 \cdot 3^x)^{1000}$

⑨ $f(x) = (e^x \ln x)^{60}$

⑩ $f(x) = \left(\frac{2^x + 1}{3^x + 1}\right)^{100}$

⑪ $f(x) = \left(\frac{\ln x + 1}{4\log_3 x + 6}\right)^{10}$

(12) For Extra credits

$$f(x) = \sqrt{x^2 + 2x}$$

We are doing Assignment 18. Please see the assignment on Canvas in note15.pdf