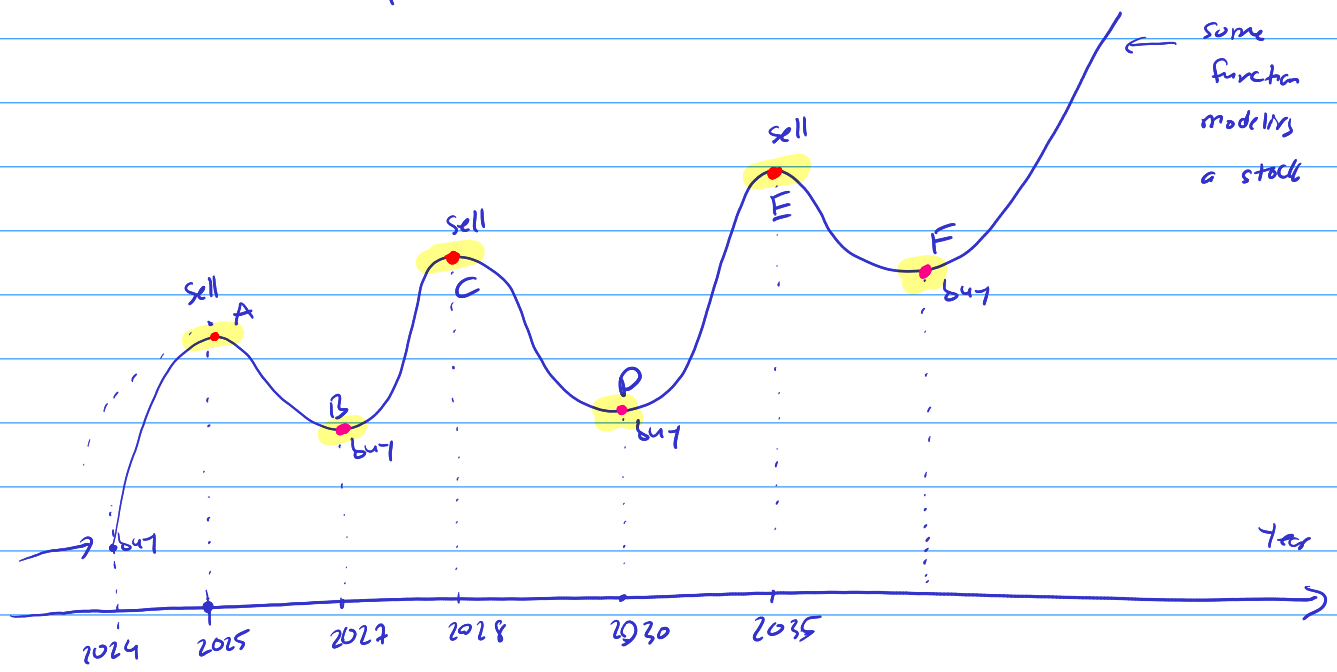
we can use function to model data / real-world issues

why we want to do that ?

The benefit is: - Make future predictions / estimation

- Find some optimal value of the function. For example: Find the number of items that a company should produce to maximize their profit.

Use Derivatives to solve it



A, B, C, D, E, F are called local extrema of the function.

We can use "Derivatives" to find these points for a given function

Points A, C, E: local max where the 1st derivatives are zeros and 2nd derivatives are negative

Points B, D, F: local min 1st derivative are zeros and 2nd ____ are positive.

Derivatives

we will cover the following functions:
- polynomial f.
- Rational f.
- Exponential f.
- Logarithmic f.

Rule 1 (power rule)

The derivative of $f(x)$ is denoted by $f'(x)$

$$f(x) = x^n$$

$$\Rightarrow f'(x) = n \cdot x^{n-1}$$

Example: Find $f'(x)$

① $f(x) = x^4 \Rightarrow f'(x) = 4 \cdot x^{4-1} = 4x^3$

② $f(x) = x^{2024} \Rightarrow f'(x) = 2024 \cdot x^{2024-1}$

$= 2024 \cdot x^{2023}$

③ $f(x) = x = x^1 \Rightarrow f'(x) = 1 \cdot x^{1-1} = x^0$

$= 1$

$\Rightarrow f'(x) = 1$

④ $f(x) = \sqrt{x} = x^{1/2}$

$\Rightarrow f'(x) = \frac{1}{2} x^{1/2-1} = \frac{1}{2} x^{-1/2}$

(5) $f(x) = \sqrt[3]{x} = x^{1/3}$

$f'(x) = \frac{1}{3} x^{1/3-1} = \frac{1}{3} x^{-2/3}$

In general: $f(x) = \sqrt[n]{x} = x^{1/n}$

$f'(x) = \frac{1}{n} \cdot x^{1/n-1}$

Assignment 12 Find $f'(x)$

(1) $f(x) = x^{1000}$

(2) $f(x) = x^3$

(3) $f(x) = x^7$

(4) $f(x) = x$

(5) $f(x) = \sqrt[4]{x}$

(6) $f(x) = \sqrt[5]{x}$

(7) $f(x) = x^{2/3}$

(8) $f(x) = x^{7/3}$

(9) $f(x) = x^{-6}$

(10) $f(x) = \frac{1}{x}$ (extra credit)

Show all the work as in the example.

Do not just give the final answer.

Example:

(1) $f(x) = \frac{1}{x} = x^{-1}$

$f'(x) = -1 \cdot x^{-1-1} = \boxed{-x^{-2}}$

(2) $f(x) = \frac{1}{x^3} = x^{-3}$

$f'(x) = -3 \cdot x^{-3-1} = -3x^{-4}$ □

$= -\frac{3}{x^4}$

(3) $f(x) = \frac{1}{x^{2024}} = x^{-2024}$

$f'(x) = -2024 \cdot x^{-2024-1}$

$= -2024\, x^{-2025} = -\frac{2024}{x^{2025}}$

(4) $f(x) = \frac{x^3}{x^9} = x^{3-9} = x^{-6}$

$f'(x) = -6 \cdot x^{-6-1} = -6x^{-7} = -\frac{6}{x^7}$

(5) $f(x) = \frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}} = x^{-1/2}$

$f'(x) = -\frac{1}{2} \cdot x^{-1/2-1} = \boxed{-\frac{1}{2} x^{-3/2}} = -\frac{1}{2x^{3/2}}$

Rule 2: Scalar Rule

$$\left[K \cdot f(x)\right]' = K \cdot f'(x)$$

K: scalar

Rule 3: Addition Rule

$$\left[f(x) + g(x)\right]' = f'(x) + g'(x)$$

$$\left[f(x) - g(x)\right]' = f'(x) - g'(x)$$

Rule 4: constant rule

$$(C)' = 0$$

Example: $(2019)' = 0$

Example: Find $f'(x)$

(1) $f(x) = 9 \cdot x^6$ $\quad\left[9 + x^6\right]$

9: scalar; $9 + x^6$: not a scalar

$$f'(x) = (9 \cdot x^6)' = 9\,(x^6)' = 9 \cdot 6 \cdot x^{6-1}$$

$$= 54 \cdot x^5$$

Scalar

② $f(x) = 7 \cdot x^3$

$f'(x) = (7x^3)' = 7(x^3)' = 7 \cdot 3x^{3-1} = 21x^2$

addition

③ $f(x) = 7 + x^3$

$f'(x) = (7 + x^3)'$

$f'(x) = (7)' + (x^3)' = 0 + 3x^{3-1}$

$= 3x^2$

④ $f(x) = 6x^3 + 5x^2 - 7x + 2024$

$f'(x) = (6x^3 + 5x^2 - 7x + 2024)'$

$= (6x^3)' + (5x^2)' - (7x)' + (2024)'$

$= 6 \cdot 3 \cdot x^{3-1} + 5 \cdot 2x^{2-1} - 7 + 0$

$= \boxed{18x^2 + 10x - 7}$

Show all the work as in the example.

Do not just give the final answer.

Assignment 13

Find $f'(x)$

① $f(x) = \frac{1}{x^7}$

② $f(x) = \frac{x^6}{x^{10}}$

③ $f(x) = \frac{x^6}{\sqrt{x}}$

④ $f(x) = 7x^3$

⑤ $f(x) = 10x^9$

⑥ $f(x) = -5x^2$

⑦ $f(x) = 9x^6 + 6x^2 + 7$

⑧ $f(x) = 6x^3 + 4x + 9$

⑨ $f(x) = 6\sqrt{x} + x^3$

⑩ $f(x) = \frac{6}{x^2} - \frac{7}{x^3} + 1$

⑪ $f(x) = \frac{7}{\sqrt{x}} + x + 1$

⑫ $f(x) = \frac{9}{\sqrt[3]{x}} + \frac{x}{7}$