Rule 10: Chain Rule for exponential function.

Exponential rule: $\left(b^{x}\right)' = b^{x} \cdot \ln b$

Chain rule for exponential: Replace x by $g(x)$

① $\left[b^{g(x)}\right]' = b^{g(x)} \cdot \ln b \cdot g'(x)$

special case when $b = e = 2.71828\ldots$

② $\left[e^{g(x)}\right]' = e^{g(x)} \cdot g'(x)$

Example: Find $f'(x)$

① $f(x) = 2^{x} \Rightarrow f'(x) = 2^{x} \cdot \ln 2$

② $f(x) = 2^{x^2} \Rightarrow f'(x) = 2^{x^2} \cdot \ln 2 \cdot (x^2)'$

$\Rightarrow f'(x) = 2^{x^2} \cdot \ln 2 \cdot (2x)$

③ $f(x) = 3^{x^3+10x}$

$f'(x) = 3^{x^3+10x} \cdot \ln 3 \cdot (x^3+10x)'$

$= 3^{x^3+10x} \cdot \ln 3 \cdot (3x^2+10)$

④ $f(x) = 10^{x^5 + 2x^2 + 3}$

$f'(x) = 10^{x^5 + 2x^2 + 3} \cdot \ln 10 \cdot (x^5 + 2x^2 + 3)'$

$= 10^{x^5 + 2x^2 + 3} \cdot \ln 10 \cdot (5x^4 + 4x)$

⑤ $f(x) = e^{x^6 + 10x + 1}$

$f'(x) = e^{x^6 + 10x + 1} \cdot \underbrace{\ln e}_{=1} \cdot (x^6 + 10x + 1)'$

$= e^{x^6 + 10x + 1} \cdot (6x^5 + 10)$

⑥ $f(x) = 7^{x \cdot \ln x}$

use product rule for this

$f'(x) = 7^{x \cdot \ln x} \cdot \ln 7 \cdot (x \cdot \ln x)'$

$f'(x) = 7^{x \cdot \ln x} \cdot \ln 7 \cdot \left[\underbrace{(x)'}_{=1} \cdot \ln x + x \cdot \underbrace{(\ln x)'}_{=1/x} \right]$

$f'(x) = 7^{x \cdot \ln x} \cdot \ln 7 \left[\ln x + x \cdot \frac{1}{x} \right]$

$= \boxed{7^{x \ln x} \cdot \ln 7 \cdot (\ln x + 1)}$

⑦ $f(x) = 2024^{\frac{e^x}{x}}$

use quotient rule for this

$$f'(x) = 2024^{e^x/x} \cdot \ln 2024 \cdot \left[\frac{e^x}{x}\right]'$$

$$f'(x) = 2024^{e^x/x} \cdot \ln 2024 \cdot \left[\frac{(e^x)' \cdot x - e^x \cdot (x)'}{x^2}\right]$$

$[(e^x)' = e^x]$

$$\Rightarrow f'(x) = 2024^{e^x/x} \cdot \ln 2024 \cdot \left[\frac{e^x \cdot x - e^x \cdot 1}{x^2}\right]$$

Assignment 19: Find $f'(x)$

① $f(x) = 10^x$

② $f(x) = 3^{x^3 + 2x}$

③ $f(x) = 6^{x^2 + 4x + 10}$

④ $f(x) = e^{3x^2 + 5x + 6}$

⑤ $f(x) = e^{\sqrt{x} + 1}$

⑥ $f(x) = e^{e^x}$

⑦ $f(x) = 8^{(x^2+1)\cdot(x^3+2)}$

(8) $f(x) = 20^{x^2 \cdot 3^x}$

(9) $f(x) = 17^{4^x \cdot \ln x}$

(10) $f(x) = 9^{(x^2+1)\log_7 x}$

(11) $f(x) = 9^{x^2+1 \,/\, \ln x}$

(12) $f(x) = 6^{e^x / x^2 + x}$

Extra credit:

(13) $f(x) = e^{(x^2+1)^{2024}}$