Elasticity of Demand.

Motivation:

In general: Price ↗ Demand ↘

Milk: Price ↗ by 1% Demand ↘ ½ %

Houses ↗ by 1% Demand ↘ 2%

we use The price elasticity to measure the responsiveness of demand to a change in price

(*) $$E(p) = -\frac{p}{q(p)} \cdot q'(p)$$

p: The price of the item

q: The demand function

$E(p)$: The price elasticity of demand.

(*) $E(p) \approx$ The percentage of decrease in demand produced by 1% increase in price

p ↗ 1% $\Rightarrow$ q ↘ $E(p)$ %

Example: Suppose the demand q and the price p for a certain commodity are given by:

$$q(p) = 240 - 2p \qquad (0 \le p \le 120)$$

(a) Calculate the price elasticity of demand

$$E(p) = -\frac{p}{q(p)} \cdot q'(p)$$

$$\Rightarrow E(p) = -\frac{p}{240-2p} \cdot (240-2p)'$$

$$\Rightarrow E(p) = -\frac{p}{240-2p} \cdot (-2)$$

$$\boxed{E(p) = \frac{2p}{240-2p}}$$

(b) when $p = 100$, $E(100) = \frac{2 \cdot 100}{240 - 2 \cdot 100} = \boxed{5}$

This means

starting at the price 100: If the price increase 1% then the demand decrease 5%.

(a) when $p = 10$: p increased by 1% $(10 \to 10.1)$

Demand q decreased by $E(10) = \frac{2 \cdot 10}{240 - 20}$

$E(10) = .091$

Demand decrease by .091%

Notice:

when $E(p) = 1$: Unitary Demand

$E(p) > 1$: Elastic Demand

$E(p) < 1$: Inelastic Demand

Example:

The manager of a bookstore determines that when a certain new paperback novel is ~~priced at p dollars~~ per copy, the daily demand will be $q = 300 - p^2$ copies, where $0 \le p \le \sqrt{300}$.

① calculate and interpret $E(1)$ and $E(10)$

② Find the price where $E(p) = 1$

$$E(p) = -\frac{p}{q} \cdot q' = -\frac{p}{300 - p^2} \cdot (300 - p^2)'$$

$$= -\frac{p}{300 - p^2} \cdot (-2p)$$

$$= \frac{2p^2}{300 - p^2}$$

$$E(1) = \frac{2}{300 - 1} = \frac{2}{299} \approx .0067$$

Interpret: At $p = 1$\$, 1% increase in price leads to .0067% decrease in demand.

$$E(10) = \frac{2 \cdot 10^2}{300 - 10^2} = \frac{200}{200} = 1$$

At $p = 10$\$, 1% increase in price leads to 1% decrease in demand

② $E(p) = 1$

$$\frac{2p^2}{300 - p^2} = 1 \Rightarrow 2p^2 = 300 - p^2$$

$$\Rightarrow 2p^2 + p^2 = 300$$

$$\Rightarrow 3p^2 = 300$$

$$\Rightarrow p^2 = 100 \Rightarrow \boxed{p = 10}$$

Assignment 24

① Suppose the demand functions are given below

$$q = -1.3p + 10$$

(a) Calculate and interpret $E(1)$ and $E(5)$

(b) at what price $E(p) = 1$

(2) Suppose the demand functions are given below

$$q = \frac{3000}{p} - 100$$

(a) Calculate and interpret $E(1)$ and $E(10)$

(b) at what price $E(p) = 1$ (extra credit)

Assignment 25

(1)

DEMAND FOR ART An art gallery offers 50 prints by a famous artist. If each print in the limited edition is priced at p dollars, it is expected that $q = 500 - 2p$ prints will be sold.

(a) Calculate and interpret $E(1)$ and $E(100)$

(b) at what price $E(p) = 1$

DEMAND FOR AIRLINE TICKETS An airline determines that when a round-trip ticket between Los Angeles and San Francisco costs p dollars ($0 \le p \le 160$), the daily demand for tickets is $q = 256 - 0.01p^2$.

(a) Calculate and interpret $E(1)$ and $E(10)$

(b) at what price $E(p) = 1$