Assignment 14 Find $f'(x)$

① $f(x) = (x^3 + x^2)\cdot(x^4 + x^5)$

$f'(x) = (x^3+x^2)'\cdot(x^4+x^5) + (x^3+x^2)\cdot(x^4+x^5)'$

$= (3x^2+2x)(x^4+x^5) + (x^3+x^2)\cdot(4x^3+5x^4)$

product

② $f(x) = (2x^4+6x+1)\cdot(6x^4+3x^2+2024)$

$f'(x) = (2x^4+6x+1)'\cdot(6x^4+3x^2+2024) + (2x^4+6x+1)\cdot(6x^4+3x^2+2024)'$

$= (8x^3+6)\cdot(6x^4+3x^2+2024) + (2x^4+6x+1)\cdot(24x^3+6x)$

③ $f(x) = (\sqrt{x}+1)\cdot(2x^3+x+1)$

$f'(x) = (x^{1/2}+1)'\cdot(2x^3+x+1) + (\sqrt{x}+1)(2x^4+x+1)'$

$f'(x) = \frac{1}{2}x^{-1/2}\cdot(2x^3+x+1) + (\sqrt{x}+1)(8x^3+1)$

④ Do #1 not using the product rule.

$$f(x) = (x^3 + x^2)\cdot(x^4 + x^5)$$

$$f(x) = x^3\cdot x^4 + x^3\cdot x^5 + x^2\cdot x^4 + x^2\cdot x^5$$

$$f(x) = x^7 + x^8 + x^6 + x^7$$

$$f(x) = x^8 + 2x^7 + x^6$$

$$\boxed{f'(x) = 8x^7 + 14x^6 + 6x^5}$$

Rule 6: Quotient Rule

$$\left[\frac{\text{top}}{\text{bot}}\right]' = \frac{(\text{top})'\cdot \text{bot} - \text{top}\cdot(\text{bot})'}{(\text{bot})^2}$$

Quotient

Example:

① $$f(x) = \frac{3x+1}{7x+2}$$

$$\Rightarrow f'(x) = \frac{(3x+1)'\cdot(7x+2) - (3x+1)\cdot(7x+2)'}{(7x+2)^2}$$

$$f'(x) = \frac{3\cdot(7x+2) - (3x+1)\cdot 7}{(7x+2)^2}$$ (done taking derivative)

$$f'(x) = \frac{21x + 6 - 21x - 7}{(7x+1)^2}$$ simplifying

$$\boxed{f'(x) = \frac{-1}{(7x+1)^2}}$$

② Find $f'(x)$ and simplify $f'(x)$

$$f(x) = \frac{x^2 + 2x}{x^2 + 7}$$

$$f'(x) = \frac{(x^2+2x)'\cdot(x^2+7) - (x^2+2x)\cdot(x^2+7)'}{(x^2+7)^2}$$

$$f'(x) = \frac{(2x+2)(x^2+7) - (x^2+2x)(2x)}{(x^2+7)^2}$$

$$f'(x) = \frac{2x^3 + 14x + 2x^2 + 14 - 2x^3 - 4x^2}{(x^2+7)^2}$$

$$f'(x) = \frac{-2x^2 + 14x + 14}{(x^2+7)^2}$$

Assignment 15:

① Find $f'(x)$ (no need to simplify $f'(x)$)

(a) $f(x) = \dfrac{x^3 + x}{x^2 + 2x}$ (b) $f(x) = \dfrac{x^7 + x^6}{x^6 + 2x^5 + 2024}$

(c) $f(x) = \dfrac{x^3 + 3}{x + 3}$ (d) $f(x) = \dfrac{2x^6 + 2x}{2x^5 + 7}$

② Find $f'(x)$ and simplify $f'(x)$

(a) $f(x) = \dfrac{7x + 2}{6x + 9}$ (b) $f(x) = \dfrac{x^2 + 2}{x^2 + 3}$

(c) $f(x) = \dfrac{x^3 + 1}{x^3 + 2}$

Rule 7: Exponential Rule

① $(b^x)' = b^x \cdot \ln b$ Notice: $\ln b = \log_e b$

$e = 2.71828\ldots$

② $(e^x)' = e^x$

Example: Find $f'(x)$

① $f(x) = 2^x \Rightarrow f'(x) = 2^x \cdot \ln 2$

② $f(x) = 3^x \Rightarrow f'(x) = 3^x \cdot \ln 3$

③ $f(x) = 2024^x \Rightarrow f'(x) = 2024^x \cdot \ln 2024$

④ $f(x) = \dfrac{e^x + 1}{e^x + 2}$

$$\Rightarrow f'(x) = \frac{(e^x+1)' \cdot (e^x+2) - (e^x+1) \cdot (e^x+2)'}{(e^x+2)^2}$$

$$f'(x) = \frac{e^x \cdot (e^x+2) - e^x (e^x)}{(e^x+2)^2}$$

$$f'(x) = \frac{e^x \cdot e^x + 2e^x - e^x \cdot e^x}{(e^x+2)^2}$$

$$\boxed{f'(x) = \frac{2e^x}{(e^x+2)^2}}$$

⑤ $f(x) = x^2 \cdot 7^x$ product

$$f'(x) = (x^2)' \cdot 7^x + x^2 \cdot (7^x)'$$

$$f'(x) = 2x \cdot 7^x + x^2 \cdot 7^x \cdot \ln 7$$

product

⑥ $f(x) = (3^x + 1) \cdot (x^2 + 3)$

$f'(x) = (3^x + 1)' \cdot (x^2 + 3) + (3^x + 1) \cdot (x^2 + 3)'$

$f'(x) = 3^x \cdot \ln 3 \cdot (x^2 + 3) + (3^x + 1)(2x)$

Assignment 16

A. Find $f'(x)$. no need to simplify $f'(x)$.

① $f(x) = 3^x$ ② $f(x) = 20^x$ ③ $f(x) = (\sqrt{3})^x$

④ $f(x) = 6^x + 7^x + 8^x$

⑤ $f(x) = 3 \cdot 9^x + 7 \cdot 6^x + 4 \cdot 2^x$

(3: scalar)

Solution: $f'(x) = 3 \cdot 9^x \cdot \ln 9 + 7 \cdot 6^x \cdot \ln 6 + 4 \cdot 2^x \cdot \ln 2$

⑥ $f(x) = 4 \cdot 20^x - 7 \cdot 4^x + 16 \cdot 15^x + 1$

⑦ $f(x) = (x^2 + 1) \cdot 11^x$

⑧ $f(x) = (x + 9) \cdot (7^x + 6^x)$

⑨ $f(x) = x \cdot e^x$

⑩ $f(x) = \dfrac{x^2 + 2}{ex}$

(11) $f(x) = \dfrac{3^x + 2}{4^x + 1}$

(12) $f(x) = \dfrac{x}{e^x}$

B. Find and simplify $f'(x)$

(1) $f(x) = \dfrac{2^x + 1}{2^x + 9}$

(2) $f(x) = \dfrac{e^x + x + 1}{e^x + x + 2}$

C. For extra credit

$f(x) = \dfrac{x \cdot e^x}{(x+1) \cdot 2^x}$