

# Data Modeling with Exponential Functions

Linear data

x	y	y diff
1	7	
2	14	7
3	21	7
4	28	7
5	35	7

slope = 7

Exponential data

x	y	y ratio
1	7	
2	14	$14/7 = 2$
3	28	$28/14 = 2$
4	56	$56/28 = 2$
5	112	$112/56 = 2$

base = 2

$$y = a \cdot 2^x$$

when  $x=1$ ,  $y=7$

$$7 = a \cdot 2 \Rightarrow a = 7/2$$

$$\Rightarrow y = \left(\frac{7}{2}\right) \cdot 2^x$$

$$x=2 \Rightarrow y = \frac{7}{2} \cdot 2^2 = 14$$

Example :

x	Time in months	0	1	2	3	4	5
y	Contaminant level difference	64	45.44	32.26	22.91	16.26	11.55

(a) Use exponential function to model the data (y is an exp. function of x)

(b) Use the model to estimate y when x is 7

Exponential Model:

$$y = a \cdot b^x$$

$$\Rightarrow y = a \cdot (.71)^x$$

To find a set

$$x = 0 \text{ and } y = 64$$

$$64 = a \cdot (.71)^0$$

$$\Rightarrow 64 = a \Rightarrow \boxed{a = 64}$$

The model is

$$\boxed{y = 64 \cdot (.71)^x}$$

x	y	ratio (b)
0	64	
1	45.44	$45.44 / 64 = .71$
2	32.26	$32.26 / 45.44 = .71$
3	22.91	$22.91 / 32.26 = .71$
4	16.26	$16.26 / 22.91 = .71$
5	11.55	$11.55 / 16.26 = .71$

$$(b) \quad x = 7 \Rightarrow y = 64 \cdot (.71)^7 = \boxed{5.82}$$

## Assignment 7

The salinity of a saltwater aquarium is being raised to a target value of 32 parts per thousand of dissolved solids in the tank. The table below shows the difference  $D$  between the target salinity and the current salinity  $S$  after  $t$  hours. Both  $D$  and  $S$  are measured in parts per thousand.

$t = \text{time in hours}$	0	1	2	3	4
$D = \text{salinity difference}$	32	25.60	20.48	16.38	13.11

- (\*) Make a model of  $D$  as an exponential function of  $t$ . Find a formula that gives the salinity  $S$  as a function of  $t$ . ■

(\*) Use the model to estimate  $D$  when  $t = 9$

Notice: Show all your work to get to the answer. (set up the equation to find  $a$ ,  $b$ .)