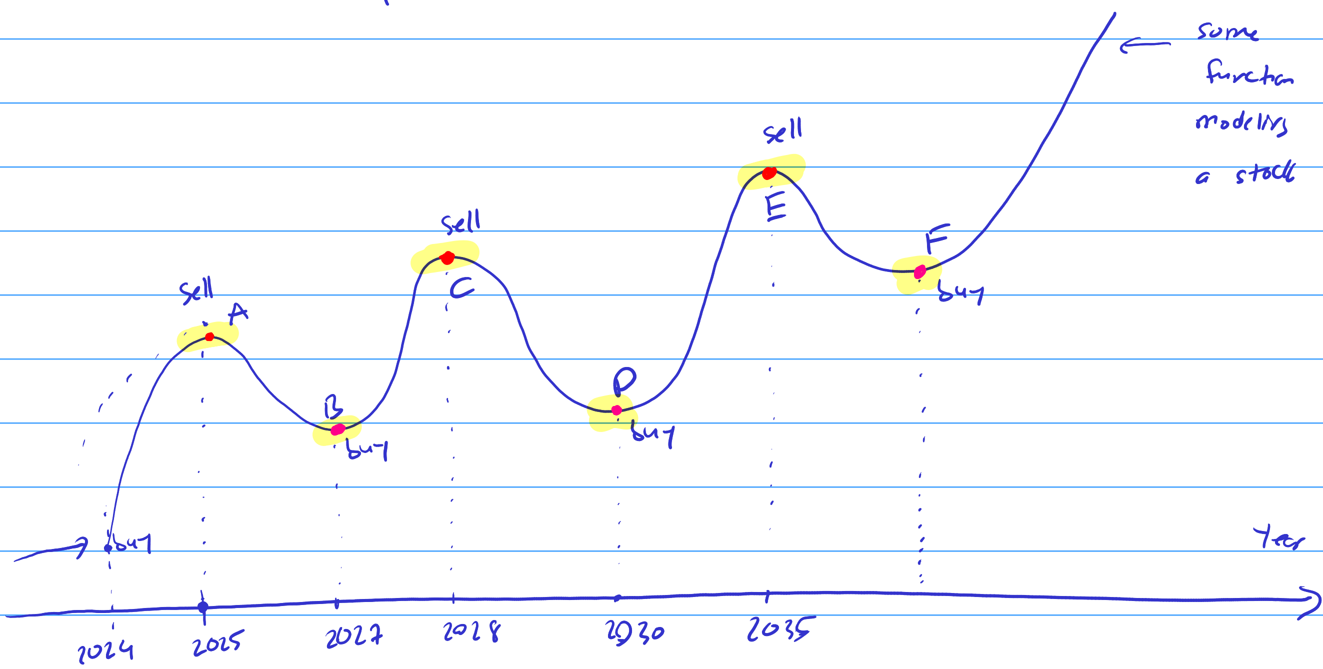
we can use function to model data / real-world issues

why we want to do that ?

The benefit is: – Make future predictions / estimation

– Find some optimal value of the function. For example: Find the number of items that a company should produce to maximize their profit.

use Derivatives to solve it



A, B, C, D, E, F are called local extrema of the function.

we can use "Derivatives" to find these points for a given function

Points A, C, E : local max where the 1^{st} derivatives are zeros and 2^{nd} derivatives are negative

Points B, D, F : local min 1^{st} derivative are zeros and 2^{nd} ______ are positive.

Derivatives

we will cover the following functions:
- polynomial f.
- Rational f.
- Exponential f.
- Logarithmic f.

Rule 1 (power rule)

The derivative of $f(x)$ is denoted by $f'(x)$

$$\boxed{\begin{gathered} f(x) = x^{n} \\ \Rightarrow \quad f'(x) = n \cdot x^{n-1} \end{gathered}}$$

Example : Find $f'(x)$

① $f(x) = x^{4} \Rightarrow f'(x) = 4 \cdot x^{4-1} = 4x^{3}$

② $f(x) = x^{2024} \Rightarrow f'(x) = 2024 \cdot x^{2024-1}$

$= 2024 \cdot x^{2023}$

③ $f(x) = x = x^{1} \Rightarrow f'(x) = \underbrace{1 \cdot x^{1-1}} = x^{0}$

$= 1$

$\Rightarrow f'(x) = 1$

④ $f(x) = \sqrt{x} = x^{1/2}$

$\Rightarrow f'(x) = \frac{1}{2} x^{1/2 - 1} = \frac{1}{2} x^{-1/2}$

⑤ $f(x) = \sqrt[3]{x} = x^{1/3}$

$$f'(x) = \underbrace{\frac{1}{3} x^{1/3 - 1}} = \frac{1}{3} x^{-2/3}$$

In general: $f(x) = \sqrt[n]{x} = x^{1/n}$

$$f'(x) = \frac{1}{n} \cdot x^{1/n - 1}$$

Assignment 12 — Find $f'(x)$

① $f(x) = x^{1000}$

② $f(x) = x^{3}$

③ $f(x) = x^{7}$

④ $f(x) = x$

⑤ $f(x) = \sqrt[4]{x}$

⑥ $f(x) = \sqrt[5]{x}$

⑦ $f(x) = x^{2/3}$

⑧ $f(x) = x^{7/3}$

⑨ $f(x) = x^{-6}$

⑩ $f(x) = \frac{1}{x}$ (extra credit)

Show all the work as in the example.

Do not just give the final answer.

Example:

① $f(x) = \frac{1}{x} = x^{-1}$

$$f'(x) = -1 \cdot x^{-1-1} = \boxed{-x^{-2}}$$

2) $f(x) = \frac{1}{x^3} = x^{-3}$

$f'(x) = -3 \cdot x^{-3-1} = -3x^{-4}$ □

$= -\frac{3}{x^4}$

3) $f(x) = \frac{1}{x^{2024}} = x^{-2024}$

$f'(x) = -2024 \cdot x^{-2024-1}$

$= -2024\, x^{-2025} = -\frac{2024}{x^{2025}}$

4) $f(x) = \frac{x^3}{x^9} = x^{3-9} = x^{-6}$

$f'(x) = -6 \cdot x^{-6-1} = -6x^{-7} = -\frac{6}{x^7}$

5) $f(x) = \frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}} = x^{-1/2}$

$f'(x) = -\frac{1}{2} \cdot x^{-1/2-1} = \boxed{-\frac{1}{2} x^{-3/2}} = -\frac{1}{2x^{3/2}}$

$\boxed{\text{Rule 2 : Scalar Rule}}$

$$[K \cdot f(x)]' = K \cdot f'(x)$$

scalar

$\boxed{\text{Rule 3 : Addition Rule}}$

$$[f(x) + g(x)]' = f'(x) + g'(x)$$

$$[f(x) - g(x)]' = f'(x) - g'(x)$$

$\boxed{\text{Rule 4 : constant rule}}$

$$(c)' = 0$$

Example : $(2019)' = 0$

<u>Example</u> : Find $f'(x)$

① $f(x) = 9 \cdot x^6$ $\qquad [9 + x^6]$

scalar $\qquad$ not a scalar

$$f'(x) = (9 \cdot x^6)' = 9\,(x^6)' = 9 \cdot 6 \cdot x^{6-1}$$
$$= 54 \cdot x^5$$

scalar

(2) $f(x) = 7 \cdot x^3$

$f'(x) = (7x^3)' = 7(x^3)' = 7 \cdot 3 x^{3-1} = 21x^2$

addition

(3) $f(x) = 7 + x^3$

$f'(x) = (7 + x^3)'$

$f'(x) = (7)' + (x^3)' = 0 + 3x^{3-1}$

$= 3x^2$

(4) $f(x) = 6x^3 + 5x^2 - 7x + 2024$

$f'(x) = (6x^3 + 5x^2 - 7x + 2024)'$

$= \underbrace{(6x^3)'} + \underbrace{(5x^2)'} - \underbrace{(7x)'} + (2024)'$

$= 6 \cdot 3 \cdot x^{3-1} + 5 \cdot 2 x^{2-1} - 7 + 0$

$= \boxed{18x^2 + 10x - 7}$

Show all the work as in the example.

Do not just give the final answer.

$\boxed{\text{Assignment 13}}$ Find $f'(x)$

① $f(x) = \dfrac{1}{x^7}$

② $f(x) = \dfrac{x^6}{x^{10}}$

③ $f(x) = \dfrac{x^6}{\sqrt{x}}$

④ $f(x) = 7x^3$

⑤ $f(x) = 10x^9$

⑥ $f(x) = -5x^2$

⑦ $f(x) = 9x^6 + 6x^2 + 7$

⑧ $f(x) = 6x^3 + 4x + 9$

⑨ $f(x) = 6\sqrt{x} + x^3$

⑩ $f(x) = \dfrac{6}{x^2} - \dfrac{7}{x^3} + 1$

⑪ $f(x) = \dfrac{7}{\sqrt{x}} + x + 1$

⑫ $f(x) = \dfrac{9}{3\sqrt{x}} + \dfrac{x}{7}$

⑩ $f(x) = 6x^{-2} - 7x^{-3} + 1$

$f'(x) = 6 \cdot (-2) \cdot x^{-2-1} - 7 \cdot (-3) \cdot x^{-3-1}$

$f'(x) = -12x^{-3} + 21x^{-4}$

$= \dfrac{-12}{x^3} + \dfrac{21}{x^4}$

⑪ $f(x) = \dfrac{7}{\sqrt{x}} + x + 1 = 7x^{-1/2} + x + 1$

$\Rightarrow f'(x) = 7 \cdot \left(-\frac{1}{2}\right) \cdot x^{-1/2-1} + 1 = -\dfrac{7}{2} x^{-3/2} + 1$

$= -\dfrac{7}{2x^{3/2}} + 1$

(12) $f(x) = \frac{9}{\sqrt[3]{x}} + \frac{x}{7} = 9 \cdot x^{-1/3} + \frac{1}{7} \cdot x$

$f'(x) = 9 \cdot \left(-\frac{1}{3}\right) x^{-1/3 - 1} + \frac{1}{7}$

$= \boxed{-3\, x^{-4/3} + 1/7}$ ← it is OK to stop here!

$= -3 \frac{1}{x^{4/3}} + \frac{1}{7}$

Rule 5: Product Rule

$$[f(x) \cdot g(x)]' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

a product

Example: Find $f'(x)$

$f(x) = (2x^2 + x) \cdot (4x^3 + 2x + 1)$

a product

$f'(x) = (2x^2 + x)' \cdot (4x^3 + 2x + 1) + (2x^2 + x) \cdot (4x^3 + 2x + 1)'$

$f'(x) = \left[(2x^2)' + (x)'\right] \cdot (4x^3 + 2x + 1) + (2x^2 + x)\left[(4x^3)' + (2x)' + (1)'\right]$

$$= (4x+1)(4x^3+2x+1) + (2x^2+x)(12x^2+2)$$

$\boxed{\text{Assignment 14}}$ Find $f'(x)$

① $f(x) = (x^3+x^2)\cdot(x^4+x^5)$

② $f(x) = (2x^4+6x+1)\cdot(6x^4+3x^2+2024)$

③ $f(x) = (\sqrt{x}+1)\cdot(2x^3+x+1)$

④ Do #1 not using the product rule.