Exam 1 - Practice 2

Name:

Notice: Calculators are not allowed.

Some formulas:

- The derivative of $f(x)$ is defined by the formula:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- An equation of the tangent line at $x = a$ is

$$y - f(a) = f'(a)(x - a)$$

Problem 1.

Find the following limits.

$$\lim_{x \to 1} \frac{x^2 - 6x + 5}{x^2 - 3x + 2}$$

$$\lim_{x \to 2024} \frac{x^2 - 6x + 5}{x^2 - 3x + 2}$$

$$\lim_{x \to 0} \frac{2x^6 + 4x^2 + 3}{4x^6 + 5x + 6}$$

$$\lim_{x \to \infty} \frac{2x^6 + 4x^2 + 3}{4x^6 + 5x + 6}$$

$$\lim_{x \to \infty} \frac{2x^7 + 4x^2 + 3}{4x^6 + 5x + 6}$$

$$\lim_{x \to \infty} \frac{2x^6 + 4x^9 + 3}{4x^7 + 5x + 6}$$

$$\lim_{x \to 1} \frac{\sin x}{\sin 7x}$$

$$\lim_{x \to 0} \frac{\sin 30x}{\sin 15x}$$

$$\lim_{x \to 0} \frac{x^3 + \sin 3x}{2x + 4 \sin 5x}$$

Problem 2

Find values of x, if any, at which the function is not continuous.

a. $f(x) = 10x^2 + \frac{x-1}{3} + 2024$

b. $f(x) = x^2 + \frac{3(x-2)}{(x-1)(x-3)} + 2024$

c. $f(x) = \frac{3}{2x+5} + \frac{x-1}{x^2-8x+12}$

Problem 3.

Find a value of the constant k, if possible, that will make the function continuous everywhere.

a.

$$f(x) = \begin{cases} 2x - 2, & x \leq 1 \\ kx^2 + k, & x > 1 \end{cases}$$

b.

$$f(x) = \begin{cases} x^2 + x + 4, & x \leq 3 \\ x^2 + kx + k, & x > 3 \end{cases}$$

Problem 4.

a. Use the definition of derivatives to find $f'(x)$, and then find the tangent line to the graph of $y = f(x)$ at $x = 1$

$$f(x) = -2x^2 + 4x + 1$$

b. Use the definition of derivatives to find $f'(x)$, and then find the tangent line to the graph of $y = f(x)$ at $x = 0$

$$f(x) = \frac{3}{2x+1}$$