Exam 1

Name:

Notice: Calculators are not allowed.

Some formulas:

- The derivative of $f(x)$ is defined by the formula:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- An equation of the tangent line at $x = a$ is

$$y - f(a) = f'(a)(x - a)$$

Problem 1. (5 points each)

Find the following limits.

$$\lim_{x \to 2} \frac{x^2 + 4x + 7}{x^2 + 5x + 1}$$

$$\lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 - 7x + 10}$$

$$\lim_{x \to \infty} \frac{2x^5 + 4x^4 + 3x^3}{7x^5 + 2024}$$

$$\lim_{x \to \infty} \frac{-2x^8 + 4x + 3}{3x^3 + 5x + 6}$$

$$\lim_{x \to \infty} \frac{7x + 3}{x^3 + 5x + 6}$$

$$\lim_{x \to 2} \frac{\sin x}{\sin 7x}$$

$$\lim_{x \to 0} \frac{\sin 10x}{\sin 5x}$$

$$\lim_{x \to 0} \frac{2x^2 + x + \sin 3x}{3x^2 + 3\sin 5x}$$

Problem 2 (5 points each)

Find values of x, if any, at which the function is not continuous.

a. $f(x) = 3x^2 + \frac{x}{x-3} + 2024x + 1$

b. $f(x) = x^2 + \frac{3}{(x-1)(x-2)(x-3)} + 2024$

c. $f(x) = \frac{3}{x+1} + \frac{x-1}{x^2-7x+6}$

Problem 3. (5 points each)

Find a value of the constant k, if possible, that will make the function continuous everywhere.

a.

$$f(x) = \begin{cases} x^2, & x \leq 1 \\ x^2 - 3kx + 1, & x > 1 \end{cases}$$

b.

$$f(x) = \begin{cases} 3x^2 + 4x + 1, & x \leq 0 \\ -9x + k^2, & x > 0 \end{cases}$$

Problem 4. (17.5 points each)

a. Use the definition of derivatives to find $f'(x)$, and then find the tangent line to the graph of $y = f(x)$ at $x = 3$

$$f(x) = 5x^2 - 6x + 1$$

b. Use the definition of derivatives to find $f'(x)$, and then find the tangent line to the graph of $y = f(x)$ at $x = 4$

$$f(x) = \frac{5}{3x+1}$$

c (Optional - 5 Points Extra Credits). Use the definition of derivatives to find $f'(x)$.

$$f(x) = x^4$$