Exam 2 - Practice 2

Notice:

- *Calculators are not allowed.*
- *Exam 2 is scheduled for Tuesday, Nov 12.*
- *A page of formula is allowed. Only formulas are allowed on the page. The page will be checked during the exam.*

Problem 1.

Find $f'(x)$.

$$f(x) = -\frac{x}{3} + \frac{3x^4}{2} - \frac{1}{\sqrt[4]{x^3}} + \frac{5}{\sqrt{x}} + 2024x + 1$$

$f(x) = -\frac{1}{3}x + \frac{3}{2}x^4 - x^{-3/4} + 5x^{-1/2} + 2024x + 1$

$\Rightarrow f'(x) = -\frac{1}{3} + \frac{12}{2}x^3 + \frac{3}{4}x^{-7/4} - \frac{5}{2}x^{-3/2} + 2024$

$$f(x) = (\sqrt[3]{x} + 1)(x^3 + x + 1)$$

$f(x) = (x^{1/3} + 1)(x^3 + x + 1)$

$f'(x) = (x^{1/3} + 1)' \cdot (x^3 + x + 1) + (x^{1/3} + 1)(x^3 + x + 1)'$

$= \frac{1}{3}x^{-2/3} \cdot (x^3 + x + 1) + (x^{1/3} + 1)(3x^2 + 1)$

$$f(x) = \frac{x^2 + 2}{x^2 - 2} \text{ (Simplify your answers.)}$$

$f'(x) = \frac{(x^2+2)' \cdot (x^2 - 2) - (x^2 + 2)(x^2 - 2)'}{(x^2 - 2)^2}$

$= \frac{2x(x^2 - 2) - (x^2 + 2)2x}{(x^2 - 2)^2}$

$= \frac{2x^3 - 4x - 2x^3 - 4x}{(x^2 - 2)^2} = \frac{-8x}{(x^2 - 2)^2}$

$$f(x) = 3x \tan x$$

$$f'(x) = (3x)' \cdot \tan x + 3x \, (\tan x)'$$

$$= 3 \tan x + 3x \cdot \sec^2 x$$

$$f(x) = \frac{x^2}{2 \cos x}$$

$$f'(x) = \frac{(x^2)' \cdot 2\cos x - x^2 \cdot (2\cos x)'}{(2\cos x)^2}$$

$$= \frac{2x \cdot 2\cos x + x^2 \cdot 2\sin x}{(2\cos x)^2}$$

$$f(x) = \cos\left(x + \sin x\right)$$

$$f'(x) = \left[-\sin(x + \sin x)\right] \cdot (x + \sin x)'$$

$$= \left[-\sin(x + \sin x)\right] \cdot \left[1 + \cos x\right]$$

$$f(x) = \tan^{2024} x$$

$$f'(x) = 2024 \tan^{2023} x \cdot (\tan x)'$$

$$= 2024 \tan^{2023} x \cdot \sec^2 x$$

$$f(x) = \tan\left(x \sin x\right)$$

$$f'(x) = \left[\sec^2 (x \cdot \sin x)\right] \; (x \sin x)'$$

$$= \left[\sec^2 (x \cdot \sin x)\right] \left[(x)' \sin x + x \, (\sin x)' \right]$$

$$= \left[\sec^2 (x \sin x)\right] \left[\sin x + x \cos x \right]$$

$$f(x) = \left(3\tan x - 2\sin x\right)^{2024}$$

$$f'(x) = 2024 \left(3\tan x - 2\sin x\right)^{2023} \cdot \left(3\tan x - 2\sin x\right)'$$

$$= 2024 \left(3\tan x - 2\sin x\right)^{2023} \left(3\sec^2 x - 2\cos x\right)$$

$$f(x) = e^x + 1007^x - 2\log_4 x + 7\ln x - \frac{3\log_6 x}{5} + \frac{\log_2 x}{3} + 2024x + 1$$

$$f(x) = e^x + 1007^x \cdot \ln 1007 - \frac{1}{2x \cdot \ln 4} + \frac{7}{x} - \frac{3}{5x \ln 6} + \frac{1}{3x \ln 2} + 2024$$

$$f(x) = \ln\left(\sqrt[3]{x} + x^2 + x + 1\right)$$

$$f'(x) = \frac{\left(\sqrt[3]{x} + x^2 + x + 1\right)'}{\sqrt[3]{x} + x^2 + x + 1}$$

$$= \frac{\frac{1}{3}x^{-2/3} + 2x + 1}{\sqrt[3]{x} + x^2 + x + 1}$$

$$f(x) = 100^{\csc x - \tan x + 2x^3}$$

$$\left[b^{g(x)}\right]' = b^{g(x)} \cdot g'(x) \cdot \ln b$$

$$(\csc x)' = -\cot x \cdot \csc x$$

$$f'(x) = 100^{\csc x - \tan x + 2x^3} \cdot [\ln 100] \cdot (\csc x - \tan x + 2x^3)'$$

$$= 100^{\csc x - \tan x + 2x^3} \cdot [\ln 100] \cdot [-\cot x \cdot \csc x - \sec^2 x + 6x^2]$$

$$f(x) = 3^{x^2 \cos x}$$

$$f'(x) = 3^{x^2 \cos x} \cdot [\ln 3] \cdot (x^2 \cos x)'$$

$$f'(x) = 3^{x^2 \cos x} (\ln 3) \cdot [2x \cos x - x^2 \sin x]$$

Problem 2

$$y + x^2 y + 2x^3 y + x^2 = 1$$

(a) Find dy/dx or y' by differentiating implicitly.

$$(y + x^2 y + 2x^3 y + x^2)' = (1)'$$

$$\Rightarrow y' + (x^2 y)' + (2x^3 y)' + 2x = 0$$

$$\Rightarrow y' + (2xy + x^2 y') + (6x^2 y + 2x^3 y') + 2x = 0$$

$$\Rightarrow y' + x^2 y' + 2x^3 y' = -2x - 2xy - 6x^2 y$$

$$\Rightarrow y'(1 + x^2 + 2x^3) = -2x - 2xy - 6x^2 y$$

$$\Rightarrow y' = \frac{-2x - 2xy - 6x^2 y}{1 + x^2 + 2x^3}$$

(b) Solve the equation for y as a function of x, and find dy/dx from that equation.

$$y + x^2 y + 2x^3 y + x^2 = 1$$

$$\Rightarrow y(1 + x^2 + 2x^3) = 1 - x^2$$

$$\Rightarrow y = \frac{1 - x^2}{1 + x^2 + 2x^3}$$

$$\Rightarrow y' = \frac{(1-x^2)'(1 + x^2 + 2x^3) - (1-x^2)(1 + x + 2x^3)'}{(1 + x^2 + 2x^3)^2}$$

$$y' = \frac{-2x(1 + x^2 + 2x^3) - (1-x^2)(1 + 6x^2)}{(1 + x^2 + 2x^3)^2}$$

Problem 3

Given the equation

$$x^3y + y^3x + 1 = 3xy$$

1. Use implicit differentiation to find dy/dx or y'

$$(x^3y + y^3x + 1)' = (3xy)'$$

$$\Rightarrow \quad 3x^2y + x^3y' + 3y^2y' \cdot x + y^3 = 3y + 3xy'$$

$$\Rightarrow \quad x^3y' + 3y^2xy' - 3xy' = 3y - y^3 - 3x^2y$$

$$\Rightarrow \quad y'[x^3 + 3y^2x - 3x] = 3y - y^3 - 3x^2y$$

$$\Rightarrow \quad y' = \frac{3y - y^3 - 3x^2y}{x^3 + 3y^2x - 3x}$$

2. Find an equation for the tangent line at the point (1, 1)

the slope at (1, 1) is

$$y' = \frac{3 - 1 - 3}{1 + 3 - 3} = -1$$

the tangent line is

$$y - 1 = -1(x - 1)$$

or

$$y = -x + 2$$

Problem 4

(a) Find the local linear approximation of $f(x) = \sqrt[3]{x}$ at $x_0 = 1$

we have $f'(x) = \frac{1}{3}x^{-2/3}$

and

$$f(x) \approx f'(x_0)(x - x_0) + f(x_0)$$

$$= f'(1)(x - 1) + f(1)$$

$$= \frac{1}{3}(-1)^{-2/3} \cdot (x - 1) + \sqrt[3]{1}$$

$$= \frac{1}{3}(x - 1) + 1$$

(b) Use the local linear approximation obtained in part (a) to approximate $\sqrt[3]{.9}$

From part (a)

$$f(x) \approx \frac{1}{3}(x-1) + 1$$

$$\Rightarrow \quad f(.9) \approx \frac{1}{3}(.9-1) + 1$$