(*) Definite Integral

$$\int_a^b f(x)\, dx$$

Definite integral

$$\int f(x)\, dx$$

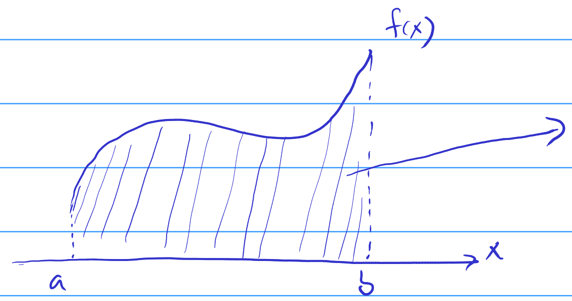
(Indefinite) Integral

upper limit

$$\int_a^b f(x)\, dx$$

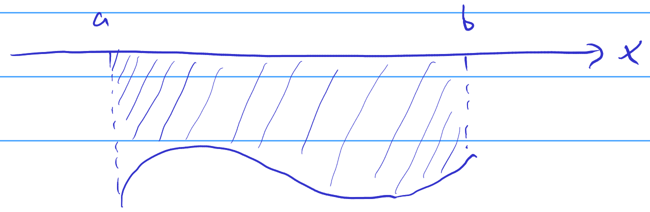
lower limit

① $f(x) > 0$



$\int_a^b f(x)\, dx$ is the area under the curve $f(x)$ from a to b

② $f(x) < 0$



$-\int_a^b f(x)\, dx$ is the area under the curve from a to b.

⊛ Fundamental theorem of calculus

Suppose $\int f(x)\,dx = F(x) + C$

then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Notice: $F(x)\Big|_a^b = F(b) - F(a)$

$$\Rightarrow \quad \int_a^b f(x)\,dx = F(x)\Big|_a^b$$

<u>Example</u>: Calculate:

$$\int_0^1 (x^2 + 3x + 1)\,dx$$

$$= \frac{x^3}{3} + \frac{3x^2}{2} + x \Big|_0^1$$

$$= \left(\frac{1^3}{3} + \frac{3\cdot 1^2}{2} + 1\right) - \left(\frac{0^3}{3} + \frac{3\cdot 0^2}{2} + 0\right)$$

$$= \frac{11}{6}$$

$$\int_1^2 \left(x^3 - x^2 + 2x + 3\right) dx$$

$$= \frac{x^4}{4} - \frac{x^3}{3} + \frac{\cancel{2}x^2}{\cancel{2}} + 3x \Big|_1^2$$

$$= \left(\frac{2^4}{4} - \frac{2^3}{3} + 2^2 + 3.2\right) - \left(\frac{1^4}{4} - \frac{1^3}{3} + 1^2 + 3\right)$$

$$= \left(14 - \frac{8}{3}\right) - \left(4 - \frac{1}{12}\right) = 10 - \frac{8}{3} + \frac{1}{12}$$

$$= 7.25$$

$$\int_1^2 \left(x + \frac{2}{x} + 3^x + 4\sin x + 10\right) dx$$

$$= \frac{x^2}{2} + 2\ln|x| + \frac{3^x}{\ln 3} - 4\cos x + 10x \Big|_1^2$$

$$= \left(\frac{2^2}{2} + 2\ln|2| + \frac{3^2}{\ln 3} - 4\cos 2 + 10.2\right)$$

$$- \left(\frac{1^2}{2} + 2\ln|1| + \frac{3^1}{\ln 3} - 4\cos 1 + 10.1\right)$$

$$= 30.47 - 11.07 = 19.4$$

$$\boxed{\begin{aligned}
&\int x^n\, dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1) \\
&\int x^{-1}\, dx = \int \frac{1}{x}\, dx = \underline{\ln|x|} + C \\
&\int \cos x\, dx = \sin x + C \\
&\int \sin x\, dx = -\cos x + C \\
&\int b^x\, dx = \frac{b^x}{\ln b} + C \\
&\int e^x\, dx = e^x + C
\end{aligned}}$$

Practica :

$$\int_1^2 \left(x^4 - 3x + \frac{4}{x} + 2^x - \cos x + 6\right) dx$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int x^{-1} \, dx = \int \frac{1}{x} \, dx = \underline{\ln|x|} + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int b^x \, dx = \frac{b^x}{\ln b} + C$$

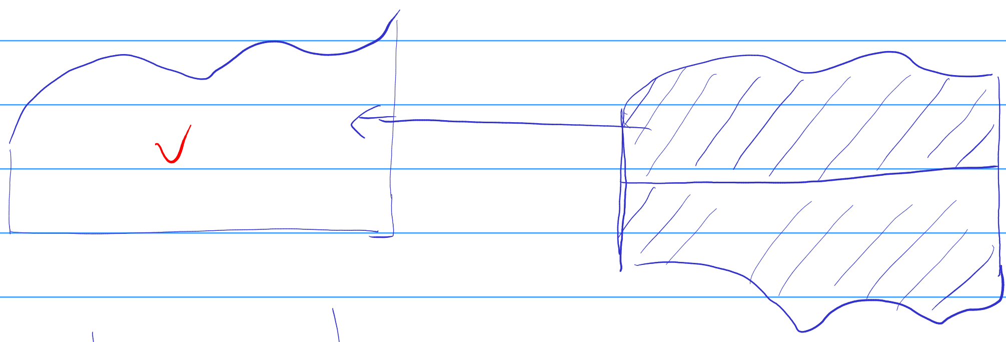
$$\int e^x \, dx = e^x + C$$

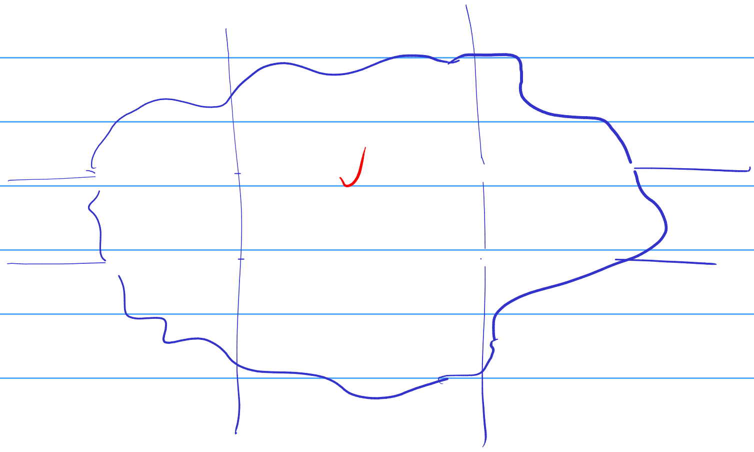
Formulas :

$$\int dx = x + C$$

$$\int k \, dx = kx + C$$

$$\int x \, dx = \frac{x^2}{2} + C$$

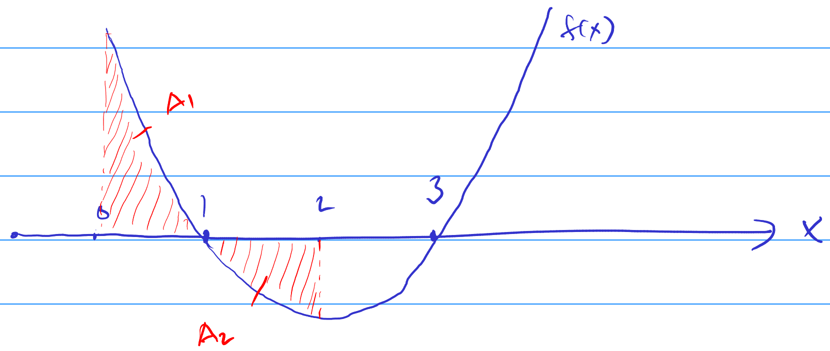




<u>Example</u>: Calculate the area between f(x) and x − axis bounded by $x = 0$ and $x = 2$

$$f(x) = x^2 - 4x + 3$$

$$= (x-1)(x-3)$$



$$A_1 = \int_0^1 f(x)\,dx$$

The area $= A_1 + A_2$

$$A_2 = -\int_1^2 f(x)\,dx$$

$$A_1 = \int_0^1 (x^2 - 4x + 3)\,dx = \frac{x^3}{3} - 2x^2 + 3x \Big|_0^1 = 4/3$$

$$A_2 = -\int_1^2 (x^2 - 4x + 3)\,dx = \int_2^1 (x^2 - 4x + 3)\,dx$$

$$= \frac{x^3}{3} - 2x^2 + 3x \Big|_2^1 = 4/3 - \left(\frac{2^3}{3} - 2.2^2 + 3.2\right)$$

$$= \frac{4}{3} - \frac{8}{3} + 2 = \frac{6}{3} - \frac{4}{3} = 2/3$$

$\Rightarrow$ Area $= A_1 + A_2 = 4/3 + 2/3 = 2$

Practice :

Calculate the area between f(x) and x - axis bounded by $x = 1$ and $x = 3$

$f(x) = x^2 - 6x + 8$