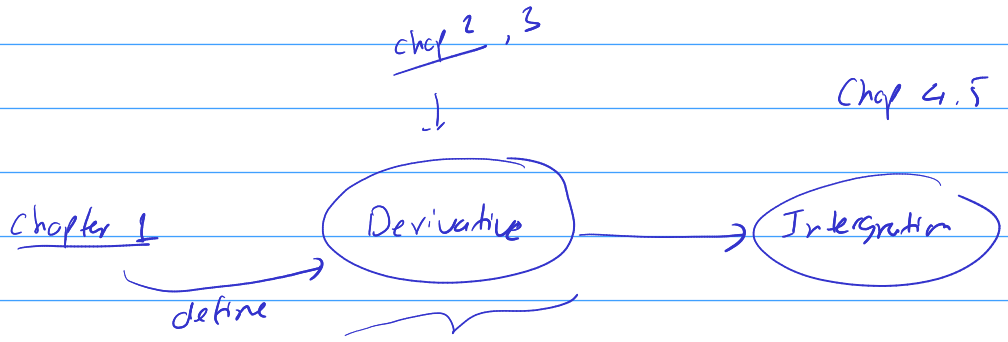



Chapter 1: Limits and continuity

1.1: Limits

Example: $y = f(x) = 2x^2 + x + 1$

$x = 1.9 \Rightarrow y = f(1.9) = 2 \cdot 1.9^2 + 1.9 + 1 = 10.12$

$x = 1.99 \Rightarrow y = f(1.99) = 2 \cdot 1.99^2 + 1.99 + 1 = 10.9102$

$x = 1.999 \Rightarrow y = f(1.999) = 2 \cdot 1.999^2 + 1.999 + 1 = 10.991$

$x = 1.9999 \Rightarrow y = f(1.9999) = \cdots = 10.9991$

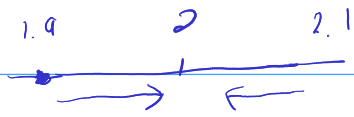
$x = 1.99999 \Rightarrow y = f(1.99999) = \cdots = 10.99991$

(from the left)

It looks like: when $x$ goes to 2, $y$ goes to 11

we can write: when $x \to 2^-$, $y \to 11$ OR

$$\lim_{x \to 2^-} y = 11$$




Let $x$ goes to 2 from the right and observe the values of $y$.

$x = 2.1 \Rightarrow y = f(2.1) = 11.92$

$x = 2.01 \Rightarrow y = f(2.01) = 11.0902$

$x = 2.001 \Rightarrow y = f(2.001) = 11.009$

$x = 2.0001 \Rightarrow y = \ldots = 11.0009$

$x = 2.00001 \Rightarrow y = = 11.00009$

we observe that when $x$ goes to 2 from the right

$y$ goes to 11 as well

we write: $x \to 2^+$, $y \to 11$ or

$$\lim_{x \to 2^+} y = 11$$

we call $\lim_{x \to 2^-} y$ and $\lim_{x \to 2^+} y$ one-sided limits

If $\lim_{x \to 2^-} y = \lim_{x \to 2^+} y$ then we say

$\lim_{x \to 2} -1$ exists and $\lim_{x \to 2} 4 = 11$

Example: $4 = f(x) = 2x^2 + x + 1$

Guess $\lim_{x \to 3} 4$

Approach 1: plug in $x = 2.9, 2.99, 2.999, 2.9999$

and observe 4 values

plug in $x = 3.1, 3.01, 3.001, 3.0001 \ldots$

and observe 4

we observe: 4 goes to 22 when $x$ goes to 3

Approach 2: plug in $x = 3$

$f(3) = 2 \cdot 3^2 + 3 + 1 = 22$

Example: $4 = f(x) = \dfrac{x^2 - 9}{x - 3}$

Guess $\lim_{x \to 2024} 4$

Approach 1

| $x$ | $y$ |
|---|---|
| 2024.01 | 2027.01 |
| 2024.001 | 2027.001 |
| ⋮ | ⋮ |
| 2024.00001 | 2027.00001 |

Guess: $\lim_{x \to 2024} y = 2027$

Approach 2 : plug $x = 2024$ in

$$f(2024) = \frac{2024^2 - 9}{2027 - 3} = 2027$$

Exam 6: : $y = f(x) = \frac{x^2 - 9}{x - 3}$

Guess $\lim_{x \to 3} y$

Approach 2 does not work because we cannot plug $x = 3$ in, or $f(3)$ does not exist.

Approach 1

| $x$ | $y$ |
|---|---|
| 3.01 | 6.01 |
| 3.001 | 6.001 |
| 3.0001 | 6.0001 |
| | ↓ |
| | 6 |

$$\lim_{x \to 3} 4 = 6$$