2.5.

Formulas:

① $(\sin x)' = \cos x$

② $(\cos x)' = -\sin x$

③ $(\tan x)' = \frac{1}{\cos^2 x}$

Notice: $\frac{1}{\cos x} = \sec x$. Thus,

$(\tan x)' = \sec^2 x = (\sec x)^2$

④ $\cot x = \frac{\cos x}{\sin x}$

$(\cot x)' = -\frac{1}{\sin^2 x}$ OR

$(\cot x)' = -\csc^2 x$

⑤ $(\sec x)' =$

⑥ $(\csc x)' =$

$$(\sec x)' = \left(\frac{1}{\cos x}\right)' \qquad \left[\text{notice}: \sec x = \frac{1}{\cos x}\right]$$

quotient rule

$$= \frac{(1)' \cdot \cos x - 1 \cdot (\cos x)'}{\cos^2 x}$$

$$= \frac{\sin x}{(\cos x)^2}$$

$$\Rightarrow \quad (\sec x)' = \frac{\sin x}{(\cos x)^2} = \sin x \cdot \frac{1}{(\cos x)^2}$$

$$= \sin x \cdot \left(\frac{1}{\cos x}\right)^2$$

$$= \sin x \cdot \sec^2 x$$

5 $\quad (\sec x)' = \sin x \cdot \sec^2 x$

Also :

$$(\sec x)' = \frac{\sin x}{(\cos x)^2} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$\Rightarrow \quad (\sec x)' = \tan x \cdot \sec x$$

⑥ $(\csc x)' = \ ?$ Note: $\csc x = \dfrac{1}{\sin x}$

$$= \left(\frac{1}{\sin x}\right)'$$

$$= \frac{(1)' \cdot \sin x - 1 \cdot (\sin x)'}{\sin^2 x}$$

$$= \frac{-\cos x}{\sin^2 x}$$

$$\Rightarrow \boxed{(\csc x)' = \frac{-\cos x}{\sin^2 x}}$$

Example: Find $f'(x)$

① $f(x) = \dfrac{\sec x}{1 + \tan x}$

$$\Rightarrow f'(x) = \frac{(\sec x)' \cdot (1 + \tan x) - \sec x \cdot (1 + \tan x)'}{(1 + \tan x)^2}$$

quotient rule

$$= \frac{\tan x \cdot \sec x \, (1 + \tan x) - \sec x \cdot (\sec x)^2}{(1 + \tan x)^2}$$

(2) $$f(x) = (x^2+1)\sec x$$

(3) $$f(x) = \frac{\sec x + \sin x}{x^2 + 2\tan x}$$

(*) Product Rule

$$[f(x)\cdot g(x)]' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

2.6. The chain Rule

(*) Another way to write derivatives

$$y = f(x) = x^3$$

$$f'(x) = 3x^2$$

OR

$$(x^3)' = 3x^2$$

OR: $$\frac{df}{dx} = 3x^2$$

OR: $$\frac{dy}{dx} = 3x^2$$

Example: Find $\frac{dy}{dx}$

(1) $y = \sin x \Rightarrow \frac{dy}{dx} = (\sin x)' = \cos x$

(2) $y = x^2 + 2 \Rightarrow \frac{dy}{dx} = (x^2+2)' = 2x$

We can also write: $\dfrac{d}{dx}(\sin x) = \cos x$

(*) Chain Rule:

1. Let $g(x) = h(k(x)) = (h \circ k)(x)$

Then, we can find $g'(x)$ as follows.

$$g'(x) = \frac{dg}{dx} = h'(k(x)) \cdot k'(x)$$