Home work

7. $\displaystyle \lim_{x \to 1^+} \frac{x^4 - 1}{x - 1}$

8. $\displaystyle \lim_{t \to -2} \frac{t^3 + 8}{t + 2}$

Hint :

(7) $x^4 - 1 = (x^2)^2 - 1^2$

$= (x^2 - 1)(x^2 + 1)$

$= (x - 1)(x + 1)\ (x^2 + 1)$

(8) notice :

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$t^3 + 8 = t^3 + 2^3$

$= (t + 2)(t^2 - 2t + 4)$

Find value of x where f(x) is not cont.

① $f(x) = \dfrac{x^2+1}{x^2-9}$

② $f(x) = \sqrt[3]{x+1}$

Solutions:

① f(x) is not cont where the denominator is zero

$\Rightarrow x^2 - 9 = 0$

$\Leftrightarrow x^2 = 9 \Leftrightarrow x = \pm\sqrt{9} = \pm 3$

② f(x) is cont. everywhere

29–30 Find a value of the constant k, if possible, that will make the function continuous everywhere.

29. (a) $f(x) = \begin{cases} 7x - 2, & x \le 1 \\ kx^2, & x > 1 \end{cases}$

we notice that f(x) is cont. everywhere except where $x = 1$. (we don't know if f(x) is cont. at $x = 1$).

we just need to find k so that $f(x)$ is cont. at $x=1$. This means we need to make sure the 3 conditions are satisfied at $x=1$.

① $f(1)$ exists

② $\lim_{x\to 1} f(x)$ exists

③ $\lim_{x\to 1} f(x) = f(1)$

we have: $f(1) = 7\cdot 1 - 2 = 5 \Rightarrow$ ① is satisfied ✓

$$f(x) = \begin{cases} 7x-2, & x \le 1 \\ kx^2, & x > 1 \end{cases}$$

$\lim_{x\to 1} f(x)$ exists means $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$

$x\to 1^-$ means $x<1$ so we plug $x=1$ in $7x-2$

$x\to 1^+$ means $x>1$ so we plug $x=1$ in kx^2

$\Leftrightarrow \quad 7 \cdot 1 - 2 = k \cdot 1^2$

$5 = k$

The second cond. is satisfied when $k = 5$

we have: $\lim_{x \to 1} f(x) = 5 = f(1)$

This means the 3^{rd} cond. is also satisfied when $\boxed{k = 5}$

(b) $f(x) = \begin{cases} kx^2, & x \leq 2 \\ 2x + k, & x > 2 \end{cases}$

30. (a) $f(x) = \begin{cases} 9 - x^2, & x \geq -3 \\ k/x^2, & x < -3 \end{cases}$

$$f(x) = \begin{cases} 3k + x & \text{if } x < 5 \\ x^3 + k + x & \text{if } x \geq 5 \end{cases}$$