Chapter 4: Integration

Example:

(1) $(x^3)' = 3x^2$

$3x^2$ is the derivative of x^3

x^3 is an anti-derivative of $3x^2$

(2) $(x^3 + 2)' = 3x^2$

$3x^2$ is the derivative of $x^3 + 2$

$x^3 + 2$ is another anti-derivative of $3x^2$

we see that $3x^2$ has mutiple anti derivatives.

All anti-derivatives of $3x^2$ can be written

in this form: x^3 + a constant / number.

we write:

$$\int 3x^2 \, dx = x^3 + C$$

we can also say:

The integral of $3x^2$ is $x^3 + C$, OR

The antidrivatives of $3x^2$ are $x^3 + C$

Example : Find all the anti-derivatives of $\cos x$

$$(\sin x)' = \cos x$$

$$(\sin x + 10)' = \cos x$$

......

$$(\sin x + c)' = \cos x$$

All the anti-derivatives of $\cos x$ are $\sin x + C$

or c is a constant. OR we can just write

$$\int \cos x \, dx = \sin x + C$$

Example :

$$\int 4x^3 \, dx = x^4 + C$$

$$\int x^5 \, dx = \frac{x^6}{6} + C$$

[b/c : $\left(\frac{x^6}{6} + c\right)' = \frac{6x^5}{6} = x^5$

$$\int e^x \, dx = e^x + C$$

b/c : $(e^x + c)' = e^x$

Formulas :

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int x^{-1} \, dx = \int \frac{1}{x} \, dx = \ln|x| + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int b^x \, dx = \frac{b^x}{\ln b} + C$$

$$\int e^x \, dx = e^x + C$$

Example :

$$\int (x^3 + x^2 + 2x + 1) \, dx$$

$$= \int x^3 \, dx + \int x^2 \, dx + \int 2x \, dx + \int 1 \, dx$$

$$= \frac{x^4}{4} + C_1 + \frac{x^3}{3} + C_2 + 2\int x^1 dx + \int x^0 \, dx$$

$$= \frac{x^4}{4} + C_1 + \frac{x^3}{3} + C_2 + 2 \cdot \frac{x^2}{2} + C_3 + \frac{x^1}{1} + C_4$$

$$= \frac{x^4}{4} + \frac{x^3}{3} + x^2 + x + \underbrace{C_1 + C_2 + C_3 + C_4}_{C}$$

$$= \frac{x^4}{4} + \frac{x^3}{3} + x^2 + x + C$$

Formulas :

$$\int dx = x + C$$

$$\int k\, dx = kx + C$$

$$\int x\, dx = \frac{x^2}{2} + C$$

$$(x^3)' = 3x^2$$

$$x^3 \xrightarrow{\text{differentiation}} 3x^2$$

integration : $\int 3x^2\, dx = x^3 + C$

Differentiation : The process of finding the derivation of a function

Integration : The process of finding the anti-derivatives of a function.

Practice : Find :

1. $\int (x^7 - x^6 + 4x^3 + x^2 + 3)\, dx$

2. $\int \left(x + \frac{1}{x}\right) dx$

3. $\int (x+1)(x+2)\, dx$

4. $\int (x + \sin x + 2\cos x)\, dx$

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int x^{-1}\, dx = \int \frac{1}{x}\, dx = \ln|x| + C$$

$$\int \cos x\, dx = \sin x + C$$

$$\int \sin x\, dx = -\cos x + C$$

$$\int b^x\, dx = \frac{b^x}{\ln b} + C$$

$$\int e^x\, dx = e^x + C$$

(*) Definite Integral

$$\int_a^b f(x)\,dx \qquad\qquad \int f(x)\,dx$$

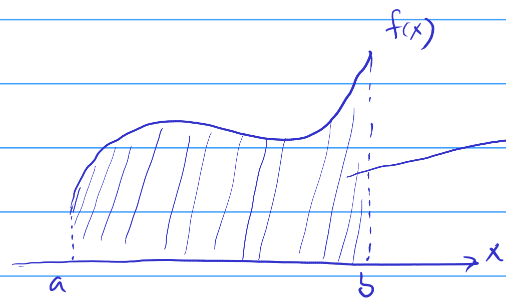
Definite integral (Indefinite) Integral

upper limit

$$\int_a^b f(x)\,dx$$

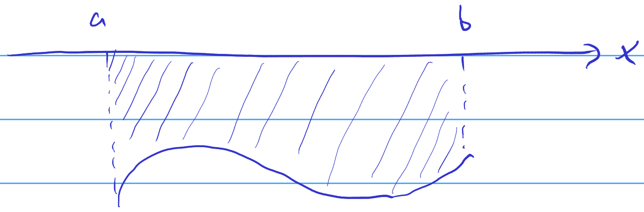
lower limit

① $f(x) > 0$



$\int_a^b f(x)\,dx$ is the area under the curve $f(x)$ from a to b

② $f(x) < 0$



$-\int_a^b f(x)\,dx$ is the area under the curve from a to b.