Exam 1 : Tuesday Oct 8

On Thursday oct, 3 : Review Session

I will be availble on oct 7.

Practice : Find

① $\lim_{x \to 0} \frac{5x}{\sin 20x}$

② $\lim_{x \to 0} \frac{\sin x}{\sin 2x + x}$

③ $\lim_{x \to 0} \frac{2x^3 + \sin x}{x^3 - \sin 10x}$

Sol : Note when $x \to 0$ $(x \sim 0)$ $\sin x \sim x$

① $\lim_{x \to 0} \frac{5x}{\sin 20x} = \lim_{x \to 0} \frac{5x}{20x} = \lim_{x \to 0} \frac{5}{20} = 1/4$

② $\lim_{x \to 0} \frac{\sin x}{\sin 2x + x} = \lim_{x \to 0} \frac{x}{2x + x}$

$= \lim_{x \to 0} \frac{x}{3x} = 1/3$

③ $\lim_{x \to 0} \frac{2x^3 + \sin x}{x^3 - \sin 10x} = \lim_{x \to 0} \frac{2x^3 + x}{x^3 - 10x}$

plus 0 in

$$= \lim_{x \to 0} \frac{x(2x^2+1)}{x(x^2-10)} = \lim_{x \to 0} \frac{2x^2+1}{x^2-10} = -\frac{1}{10}$$

④ Is this correct?

$$\lim_{x \to 0} \frac{\sin(x+1)}{\sin(x+2)} = \lim_{x \to 0} \frac{x+1}{x+2}$$

No, because $x \to 0$, $x+2 \to 2$; $x+1 \to 1$. The arguments are not going to 0 so we cannot "drop" the sine function.

In fact, $\lim_{x \to 0} \frac{\sin(x+1)}{\sin(x+2)} = \frac{\sin(0+1)}{\sin(0+2)} = \frac{\sin 1}{\sin 2} \approx .925$

Chapter 2: The Derivatives

Section 2.2: The derivative function.

(*) Definition:

The derivative function of $f(x)$ is denoted by $f'(x)$

[$f'(x)$ reads f prime of x] and is defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Example : Find $f'(x)$

① $f(x) = 20x + 2024$

$$\frac{f(x+h) - f(x)}{h} = \frac{\overbrace{[20(x+h) + 2024]}^{f(x+h)} - \overbrace{[20x + 2024]}^{f(x)}}{h}$$

$$= \frac{\cancel{20x} + 20h + \cancel{2024} - \cancel{20x} - \cancel{2024}}{h}$$

$$= \frac{20h}{h} = 20$$

$$\Rightarrow \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} 20 = 20$$

② $f(x) = 2x^2 + 3x + 4$

$$\frac{f(x+h) - f(x)}{h} = \frac{\overbrace{2(x+h)^2 + 3(x+h) + 4}^{f(x+h)} \overbrace{- (2x^2 + 3x + 4)}^{f(x)}}{h}$$

$$= \frac{2(x+h)\cdot(x+h) + \cancel{3x} + 3h + \cancel{4} - 2x^2 - \cancel{3x} - \cancel{4}}{h}$$

$$= \frac{2(x^2 + xh + hx + h^2) + 3h - 2x^2}{h}$$

$$= \frac{\cancel{2x^2} + 2xh + 2hx + 2h^2 + 3h - \cancel{2x^2}}{h}$$

$$= \frac{4hx + 2h^2 + 3h}{h}$$

$$= \frac{\cancel{h} \cdot (4x + 2h + 3)}{\cancel{h}} = 4x + 2h + 3$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} 4x + 2h + 3$$

plug $h=0$ in

$$= 4x + 2 \cdot 0 + 3$$

$$= \boxed{4x + 3}$$

Practice: Find $f'(x)$

① $f(x) = 2024x + 7$

② $f(x) = x^2 + 3x + 2024$