Home work problems

9. $\lim\limits_{x \to -1} \dfrac{x^2 + 6x + 5}{x^2 - 3x - 4}$

plug -1 in : $\dfrac{(-1)^2 + 6(-1) + 5}{(-1)^2 + 3 - 4} = \dfrac{0}{0}$ (indeterminate)

we will simplify it before pluging $x = -1$ in

$$\lim_{x \to -1} \frac{x^2 + 6x + 5}{x^2 - 3x - 4} = \lim_{x \to -1} \frac{(x+1)(x+5)}{(x+1)(x-4)} = \lim_{x \to -1} \frac{x+5}{x-4} = \frac{-1+5}{-1-4}$$

$$= \boxed{-\frac{4}{5}}$$

1.5. Continuity.

<u>Example</u> : Given $\boxed{f(x) = \dfrac{1}{x-1}}$

ⓐ Is $f(x)$ continuous at $\boxed{x = 2}$?

Let check the 3 conditions

① Does $f(2)$ exist ? Yes : $f(2) = \dfrac{1}{2-1} = 1$ ✓

② Does $\lim\limits_{x \to 2} f(x)$ exist ? Yes $\lim\limits_{x \to 2} f(x) = \dfrac{1}{2-1} = 1$ ✓

③ Is $\lim\limits_{x \to 2} f(x) = f(2)$? Yes, they both equal 1

✓

Because all 3 conditions are satisfied, $f(x)$ is cont.

at $x = 2$.

5. Find values of x, if any, at which f is not continuous.

The function is continuous at any point except $x = 1$ b/c

the first condition is not satisfied at $x = 1$ or $f(1)$

DNE.

Example : (HW problems)

11–22 Find values of x, if any, at which f is not continuous.

11. $f(x) = 5x^4 - 3x + 7$ **12.** $f(x) = \sqrt[3]{x - 8}$

There is no values of x at which $f(x)$ is not cont.

13. $f(x) = \dfrac{x + 2}{x^2 + 4}$ **14.** $f(x) = \dfrac{x + 2}{x^2 - 4}$

↓

not cont. at $x = 2$, $x = -2$

cont. every where

Result: 1.   If $f(x)$ is a polynomial, it is cont. everywhere.

2.   If $f(x)$ is a rational function (ratio of 2 polynomials) then $f(x)$ is cont. at values of x in ==the domain== of $f(x)$.

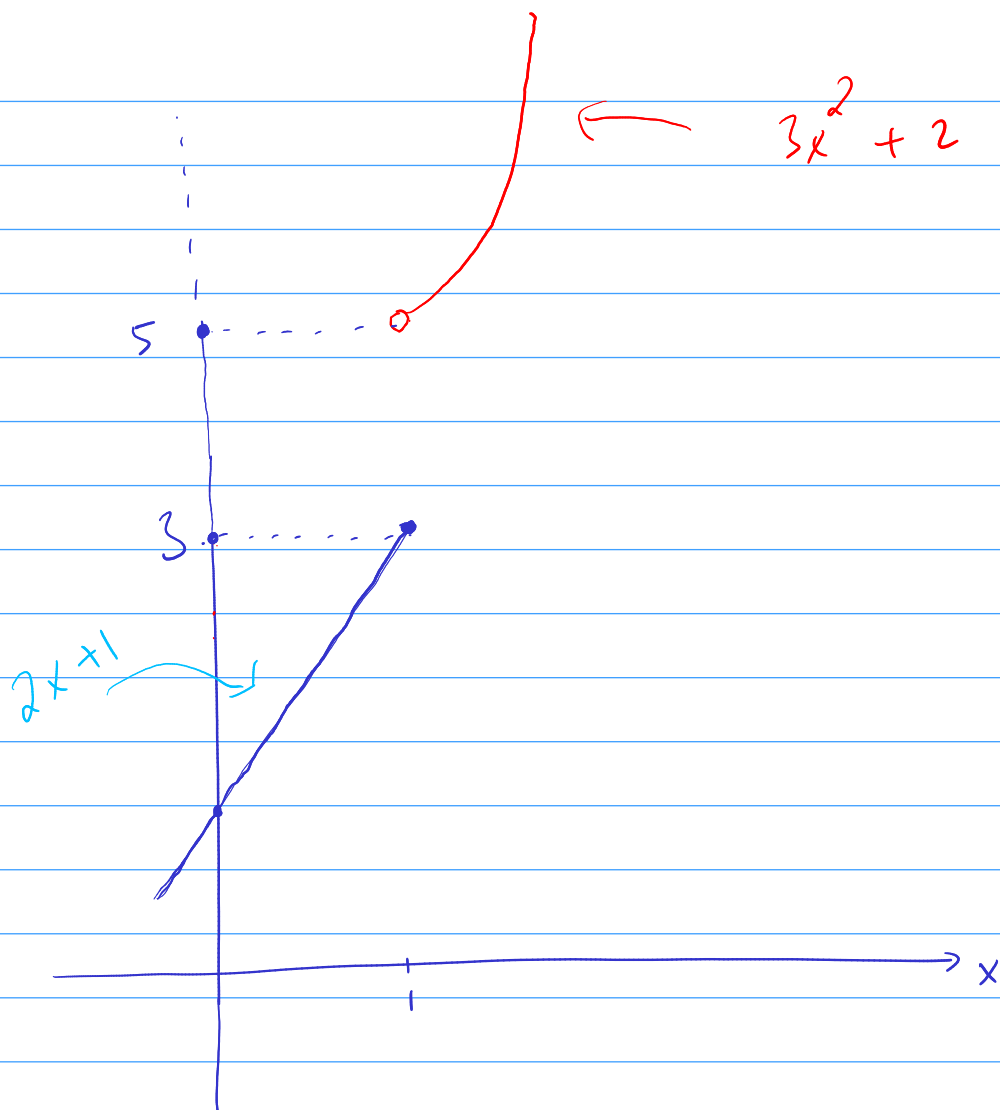OR :   If $f(x) = \dfrac{P(x)}{q(x)}$ then $f(x)$ is not cont. at x where $q(x) = 0$ and cont. everywhere else.

Example :   At which points, $f(x)$ is not cont. ?

$$f(x) = \begin{cases} 2x + 1, & x \leq 1 \\ \\ 3x^2 + 2, & x > 1 \end{cases}$$

notice that $f(x)$ exists everywhere so the 1st condition is always satisfied for any value of x.

Is there anywhere the limit DNE ? (2nd condition)

$3x^2 + 2$

$2x + 1$

$\lim_{x \to 1^-} f(x) = 2 \cdot 1 + 1 = 3$

$\lim_{x \to 1^+} f(x) = 3(1)^2 + 2 = 5$

$\Rightarrow \quad \lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$

$\Rightarrow \quad \lim_{x \to 1} f(x) \quad DNE. \quad \Rightarrow \quad f(x) \text{ is not cont.}$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad at \quad x = 1.$

Find value of x where $f(x)$ is not cont.

①

$$f(x) = \frac{x^2 + 1}{x^2 - 9}$$

② $f(x) = \sqrt[3]{x + 1}$