Exam 2 is scheduled for Tuesday, Nov 12.

Review is on Thursday, Nov 7

(*) Chain Rule on Exp./Log functions

(1) $$\left[b^{g(x)} \right]' = b^{g(x)} \cdot \ln b \cdot g'(x)$$

[Note: outside function is exp. and the inside is $g(x)$]

Example:

(1) $f(x) = 2024^{\cos x} \Rightarrow f'(x) = 2024^{\cos x} \cdot \ln 2024 \cdot (\cos x)'$

$$= 2024^{\cos x} \cdot \ln 2024 \; (-\sin x)$$

(2) $f(x) = e^{x^2+x+1}$

$$\Rightarrow f'(x) = e^{x^2+x+1} \cdot \ln e \cdot (x^2+x+1)'$$

$$= e^{x^2+x+1} \cdot (2x+1)$$

[Note: $\ln e = 1$]

② $$\left(\log_6 [\, g(x) \,]\right)' = \frac{g'(x)}{g(x) \cdot \ln 6}$$

special case:

$$\left(\ln [g(x)]\right)' = \frac{g'(x)}{g(x)}$$

Example Find $f'(x)$

① $$f(x) = \log_7 (\underbrace{\sqrt{x} + x^3}_{g(x)})$$

$$\Rightarrow \quad f'(x) = \frac{(\sqrt{x} + x^3)'}{(\sqrt{x} + x^3) \ln 7}$$

$$= \frac{1/2 \cdot x^{-1/2} + 3x^2}{(\sqrt{x} + x^3) \cdot \ln 7}$$

Practice : Find $f'(x)$

① $$f(x) = e^{\sin x + \cos x + x^7}$$

$$\Rightarrow \quad f'(x) = e^{\sin x + \cos x + x^7} \cdot (\cos x - \sin x + 7x^6)$$

② $$f(x) = \log_9 (x^3 + x^2 + \sin x)$$

$$f'(x) = \frac{(x^3 + x^2 + \sin x)'}{(x^3 + x^2 + \sin x) \cdot \ln 9} = \frac{3x^2 + 2x + \cos x}{(x^3 + x^2 + \sin x) \cdot \ln 9}$$

③ $f(x) = \ln(\sin x \cdot \cos x)$

$$f'(x) = \frac{(\sin x \cdot \cos x)'}{\sin x \cdot \cos x} = \frac{(\sin x)' \cdot \cos x + \sin x \cdot (\cos x)'}{\sin x \cdot \cos x}$$

$$= \frac{\cos^2 x - \sin^2 x}{\sin x \cdot \cos x}$$

Section 2.7 : Implicit Differentiation

$y = f(x) = x^x \qquad f'(x) = ?$

usually the equation of a function is in the form

$$y = f(x)$$

then y' or $\frac{dy}{dx}$ can be called explicit differentiation

But sometime a function can be in the form

$$f(x, y) = 0$$

For example:

$$x^3 + y^3 + 1 = 0$$

In this case the function y is given implicitly.

If we want to find y' in this case then we can either

① Solve y by itself explicitly then find y' explicitly.

$$x^3 + y^3 + 1 = 0$$

$$\Rightarrow \quad y^3 = -x^3 - 1$$

$$\Rightarrow \quad y = \sqrt[3]{-x^3 - 1} = (-x^3 - 1)^{1/3}$$

$$\Rightarrow \quad \boxed{y' = \frac{1}{3}(-x^3 - 1)^{-2/3} \cdot (-3x^2)}$$

② OR we can find y' implicitly also called implicit differentiation. We do not have to get y by itself

$$x^3 + y^3 + 1 = 0$$

$$(x^3 + y^3 + 1)' = (0)'$$

$$\Rightarrow \quad (x^3)' + (y^3)' + (1)' = 0$$

$$\Rightarrow \quad 3x^2 + 3y^2 \cdot y' = 0$$

$$\Rightarrow \quad 3y^2 \cdot y' = -3x^2$$

$$\Rightarrow \quad y' = \frac{-3x^2}{3y^2}$$

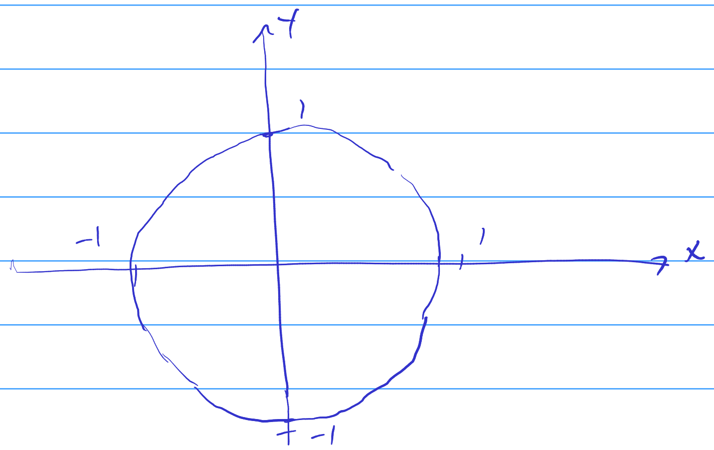
$$\Rightarrow \quad \boxed{y' = \frac{-x^2}{y^2}}$$

(*) Remark: Implicit differentiation can be used to find y' even when y is not a function of x.
Explicit differentiation may not be practical to find y' when y is not a function of x.

Example: $x^2 + y^2 - 1 = 0$

y is not a function of x b/c one input x can produce more than 1 output y.

Say $x = 0$, $y = 1$ or $y = -1$



Let find y' implicitly :

$$x^2 + y^2 - 1 = 0$$

$$\Rightarrow (x^2 + y^2 - 1)' = 0$$

$$\Rightarrow 2x + 2y \cdot y' = 0$$

$$\Rightarrow 2y \cdot y' = -2x$$

$$\Rightarrow y' = \frac{-2x}{2y}$$

$$\Rightarrow y' = -\frac{x}{y}$$