Chap 1

② Limits at infinity (cont.)

Example:

$$\lim_{x\to\infty} \frac{1}{x^2 - x + 1} \overset{?}{=} \frac{1}{\infty^2 - \infty + 1} = \frac{1}{\underbrace{\infty - \infty}_{\text{indeterminate}} + 1}$$

$$\lim_{x\to\infty} \frac{1}{x\cdot(x-1)+1} = \frac{1}{\underbrace{\infty}\cdot(\infty - 1) + 1}$$

$$= \frac{1}{\infty\cdot\infty + 1} = \frac{1}{\infty + 1} = \frac{1}{\infty}$$

$$= 0$$

$$\lim_{x\to\infty} \sqrt{x+1} = \sqrt{\infty + 1} = \sqrt{\infty} = \infty$$

$$\lim_{x\to\infty} \frac{2x+1}{x+2} \overset{?}{=} \frac{2\cdot\infty + 1}{\infty + 2} = \frac{\infty + 1}{\infty + 2} = \frac{\infty}{\infty} \quad \text{(indeterminate)}$$

$$\lim_{x\to\infty} \frac{2x+1}{x+2} = \lim_{x\to\infty} \frac{\frac{2x+1}{x}}{\frac{x+2}{x}} = \frac{2 + \frac{1}{x}}{1 + \frac{2}{x}}$$

(plug in ∞)

$$= \frac{2 + \frac{1}{\infty}}{1 + \frac{2}{\infty}} = \frac{2+0}{1+0} = 2$$

Example:

$$\lim_{x \to \infty} \frac{x^2 + 2x + 2024}{2x^3 + x + 1}$$

$$= \lim_{x \to \infty} \frac{\frac{x^2}{x^2} + \frac{2x}{x^2} + \frac{2024}{x^2}}{\frac{2x^3}{x^2} + \frac{x}{x^2} + \frac{1}{x^2}}$$

$$= \lim_{x \to \infty} \frac{1 + \frac{2}{x} + \frac{2024}{x^2}}{2x + \frac{1}{x} + \frac{1}{x^2}} = \frac{1 + 0 + 0}{2 \cdot \infty + 0 + 0} = \frac{1}{\infty} = 0$$

Group work practice: (on Limits at infinity)

Find:

(1) $\lim_{x \to \infty} x^7 + x + 1$

(2) $\lim_{x \to \infty} x^3 - 2x^2 + 2$

(3) $\lim_{x \to \infty} \frac{1}{x^3 + x + 1}$

(4) $\lim_{x \to \infty} \frac{1}{x^3 - x + 1}$

(5) $\lim_{x \to \infty} \frac{2x^2 + x}{5x^2 + 20}$

(*) Limits at infinity of rational functions

A Rational function is a division / quotient of two polynomials

Let $f(x)$ be a rational function:

$$f(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \cdots + b_1 x + b_0}$$

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{a_n x^n}{b_m x^m}$$

Example:

$$\lim_{x \to \infty} \frac{2x^2 + x}{5x^2 + 20} = \lim_{x \to \infty} \frac{2x^2}{5x^2} = 2/5$$

$$\lim_{x \to \infty} \frac{2x^7 + x^5 + 2x^5 + x^4 + 20}{x^9 + x^8 + x + 1}$$

$$= \lim_{x \to \infty} \frac{2x^7}{x^9} = \lim_{x \to \infty} \frac{2}{x^2} = 0$$