Final Exam - Math 121.

Name:

Notice: Calculators are not allowed.

Problem 1. (5 points each)

Find the following limits.

$$\lim_{x\to 2} \frac{x^2+4x+7}{x^2+5x+1}$$

$$\lim_{x\to 2} \frac{x^2-5x+6}{x^2-7x+10}$$

$$\lim_{x\to \infty} \frac{2x^5+4x^4+3x^3}{7x^5+2024}$$

$$\lim_{x \to \infty} \frac{-2x^8 + 4x + 3}{3x^3 + 5x + 6}$$

$$\lim_{x \to \infty} \frac{7x + 3}{x^3 + 5x + 6}$$

$$\lim_{x \to 2} \frac{\sin x}{\sin 7x}$$

$$\lim_{x \to 0} \frac{\sin 10x}{\sin 5x}$$

$$\lim_{x \to 0} \frac{2x^2 + x + \sin 3x}{3x^2 + 3\sin 5x}$$

Problem 2 (5 points each)

Find values of x, if any, at which the function is not continuous.

a. $f(x) = 3x^2 + \frac{x}{x-3} + 2024x + 1$

b. $f(x) = x^2 + \frac{3}{(x-1)(x-2)(x-3)} + 2024$

c. $f(x) = \frac{3}{x+1} + \frac{x-1}{x^2-7x+6}$

Problem 3. (5 points each)

Find a value of the constant k, if possible, that will make the function continuous everywhere.

a.

$$f(x) = \begin{cases} x^2, & x \leq 1 \\ x^2 - 3kx + 1, & x > 1 \end{cases}$$

b.

$$f(x) = \begin{cases} 3x^2 + 4x + 1, & x \leq 0 \\ -9x + k^2, & x > 0 \end{cases}$$

Problem 4. (17.5 points each)

a. Use the definition of derivatives to find $f'(x)$, and then find the tangent line to the graph of $y = f(x)$ at $x = 3$

$$f(x) = 5x^2 - 6x + 1$$

b. Use the definition of derivatives to find $f'(x)$, and then find the tangent line to the graph of $y = f(x)$ at $x = 4$

$$f(x) = \frac{5}{3x + 1}$$

c (Optional - 5 Points Extra Credits). Use the definition of derivatives to find $f'(x)$.

$$f(x) = x^4$$

Problem 1.

(5 points each) Find $f'(x)$.

$$f(x) = -\frac{2x}{3} + \frac{5x^2}{3} - \frac{1}{\sqrt[6]{x^3}} + \frac{1}{\sqrt{x}} + 2024x^2 + x + 2024$$

$$f(x) = (\sqrt{x} + 1)(x^2 + 2x + 1)$$

$$f(x) = \frac{x^3 + 1}{x^3 - 1} \text{ (Simplify your answers.)}$$

$$f(x) = 3x^2 \cos x$$

$$f(x) = \frac{\sin x}{x}$$

$$f(x) = \sin\left(x + \sin x\right)$$

$$f(x) = \cos^2 x$$

$$f(x) = \sin\left(x \cos x\right)$$

$$f(x) = \left(3 \sin x - 2 \cos x\right)^2$$

$$f(x) = e^x + 17^x - 2 \log_3 x + 8 \ln x - \frac{3 \log_2 x}{2} + \frac{\log_9 x}{3} + 2024x + 1$$

$$f(x) = \log_3\left(\sqrt{x} + x^2 + 1\right)$$

$$f(x) = 100^{\cos x - \sin x + 3x^2}$$

$$f(x) = e^{x\cos x}$$

Problem 2

(8 points each)

$$y + x^2 y - x = 1$$

(a) Find dy/dx or y' by differentiating implicitly.

(b) Solve the equation for y as a function of x, and find dy/dx from that equation.

(c) Find an equation for the tangent line at the point (1, 1)

Problem 3

(5 points each)

(a) Find the local linear approximation of $f(x) = \sqrt[4]{x}$ at $x_0 = 1$

(b) Use the local linear approximation obtained in part (a) to approximate $\sqrt[4]{1.1}$

Problem 1.

Given that

$$f(x) = x^3 - 6x^2 + 9x + 1$$

Find all the intervals where

a. $f(x)$ is increasing
b. $f(x)$ is decreasing
c. $f(x)$ is concave upward
d. $f(x)$ is concave downward

Problem 2

Find all the relative extrema of

$$f(x) = 2x^3 - 9x^2 + 12x + 1$$

Problem 3

Find an relative extrema of $f(x) = x^4 - x^2 - x + 1$ using gradient descent.

Problem 4

Find the absolute maximum and absolute minimum of $f(x) = 2x^3 - 15x^2 + 36x + 1$ on the interval [0, 1].

Problem 5

The given equation has one (real) solution. Approximate the solution by Newton's method.

$$x^3 + x - 1 = 0$$