

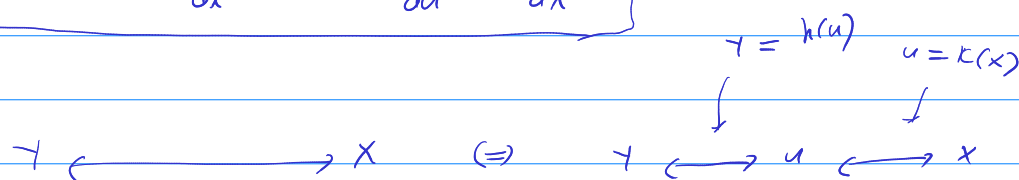
① Chain Rule

② Let  $y = g(x) = h(k(x))$ . we can find  $g'(x)$

as follows.

Let  $u = k(x)$ .

$$g'(x) = \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$



$$y \longleftrightarrow v \longleftrightarrow u \longleftrightarrow x$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$$

Example: Find  $f'(x)$  or  $dy/dx$

$$y = f(x) = (\sin x)^{2024}$$

Let  $u = \sin x$ . Then  $y = u^{2024}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \left( 2024 \cdot u^{2023} \right) \cdot \cos x$$

$$= 2024 \cdot (\sin x)^{2023} \cdot \cos x$$

$$\textcircled{2} \quad y = \left( \underbrace{x^2 + x + 1}_u \right)^{100} \quad \leftarrow$$

$$u = x^2 + x + 1 \Rightarrow \frac{du}{dx} = 2x + 1$$

$$y = u^{100} \Rightarrow \frac{dy}{du} = 100 \cdot u^{99}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 100 \cdot u^{99} \cdot (2x + 1)$$

$$= 100 (x^2 + x + 1)^{99} \cdot (2x + 1)$$

$$\textcircled{3} \quad y = \cos \left( \underbrace{x^2 + 3x + 2}_u \right)$$

$$u = x^2 + 3x + 2 \Rightarrow \frac{du}{dx} = 2x + 3$$

$$y = \cos u \Rightarrow \frac{dy}{du} = -\sin u$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = (-\sin u) \cdot (2x + 3)$$

$$= \left[ -\sin(x^2 + 3x + 2) \right] (2x + 3)$$

Practica : Find  $dy/dx$

$$\textcircled{1} \quad y = (\tan x)^{2025}$$

$$\textcircled{2} \quad y = \tan(x^2 + 9x)$$

⊛ Apply chain rule to power functions

$$y = [g(x)]^n$$

$$\Rightarrow \frac{dy}{dx} = n [g(x)]^{n-1} \cdot g'(x)$$

Example :  $y = (\tan x)^{2025}$

$$\Rightarrow \frac{dy}{dx} = 2025 \cdot (\tan x)^{2024} \cdot (\tan x)'$$

$$= \boxed{2025 \cdot (\tan x)^{2024} \cdot \sec^2 x}$$

⊛ Apply chain rules to trig. function

①  $y = \sin(g(x))$

$$\Rightarrow \frac{dy}{dx} = [\cos g(x)] \cdot g'(x)$$

②  $y = \cos g(x) \Rightarrow \frac{dy}{dx} = -[\sin g(x)] \cdot g'(x)$

③  $y = \tan g(x) \Rightarrow \frac{dy}{dx} = [\sec^2 g(x)] \cdot g'(x)$

④  $y = \cot g(x) \Rightarrow \frac{dy}{dx} = -[\csc^2 g(x)] \cdot g'(x)$

⑤  $y = \sec g(x)$

$$\Rightarrow \frac{dy}{dx} = [\sec g(x)] \cdot [\tan g(x)] \cdot g'(x)$$

$$(5) \quad y = \csc g(x)$$

$$\Rightarrow \frac{dy}{dx} = -[\csc g(x)] \cdot [\cot g(x)] \cdot g'(x)$$

Example :  $y = \tan (x^2 + 9x)$

$$\text{By (3), } \frac{dy}{dx} = [\sec^2 (x^2 + 9x)] \cdot (x^2 + 9x)'$$

$$= [\sec^2 (x^2 + 9x)] \cdot (2x + 9)$$

Example :

$$(1) \quad y = \sin (\cos x)$$

$$\Rightarrow \frac{dy}{dx} = [\cos (\cos x)] \cdot (\cos x)'$$

$$= \cos (\cos x) \cdot -\sin x$$

$$= -\cos (\cos x) \cdot \sin x$$

$$(2) \quad y = \cos (\sqrt{x} + 1)$$

$$\frac{dy}{dx} = -[\sin (\sqrt{x} + 1)] \cdot (\sqrt{x} + 1)'$$

$$= -[\sin (\sqrt{x} + 1)] \cdot \left( \frac{1}{2} \cdot x^{-1/2} \right)$$

Practise :

(1)  $y = (\sin x + \cos x)^{100}$

(2)  $y = \sin(\sqrt{x} + x)$

(3)  $y = \tan(x^3 + x + 1)$

(4)  $y = \tan(\sin x + x^2)$