(5) $f(x) = \frac{7}{x} + 20$

Step 1 : Rewrite f(x) in "power" form

$f(x) = 7 \cdot x^{-1} + 20$

Step 2 : $f'(x) = 7 \cdot (-1) \cdot x^{-2}$

$= -7x^{-2}$

(6) $f(x) = \frac{9}{x^2} + \frac{1}{x}$

Step 1 : power form :

$f(x) = 9x^{-2} + x^{-1}$

Step 2 :

$f'(x) = 9 \cdot (-2) \cdot x^{-3} - 1 \cdot x^{-2}$

$= -18x^{-3} - x^{-2}$

⑦ $f(x) = 20x + \dfrac{7}{x^3} + \dfrac{2}{\sqrt{x}} + 3$ $\rightarrow \dfrac{2}{x^{1/2}} \rightarrow 2x^{-1/2}$

$f(x) = 20x + 7x^{-3} + 2x^{-1/2} + 3$

$f'(x) = 20 + 7\cdot(-3)\cdot x^{-4} - 2\cdot(1/2)\cdot x^{-1/2-1}$

$f'(x) = 20 - 21x^{-4} - x^{-3/2}$

⑧ $f(x) = \dfrac{2x^3}{3} + \dfrac{x^5}{7} - \dfrac{2}{\sqrt[3]{x}} + \dfrac{5}{6\sqrt{x}} + 1$

$f(x) = \dfrac{2}{3}\cdot x^3 + \dfrac{1}{7}x^5 - 2x^{-1/3} + \dfrac{5}{6}\cdot x^{-1/2} + 1$

$f'(x) = 2x^2 + \dfrac{5}{7}x^4 + \dfrac{2}{3}x^{-4/3} - \dfrac{5}{12}\cdot x^{-3/2}$

⑨ $f(x) = \dfrac{1}{\sqrt{x^3}} + \dfrac{2}{x^3} + \dfrac{\sqrt[3]{x}}{4} + 1$

$f(x) = x^{-3/2} + 2x^{-3} + \dfrac{1}{4}\cdot x^{1/3} + 1$

$f'(x) = -3/2\cdot x^{-5/2} - 6x^{-4} + \dfrac{1}{12}x^{-2/3}$

(10) $f(x) = \sqrt[9]{x^2} + \dfrac{4}{\sqrt[6]{x^{31}}} + 2$

$f(x) = x^{2/9} + 4 \cdot x^{-31/6} + 2$

$f'(x) = \frac{2}{9} x^{-7/9} - \frac{62}{3} x^{-37/6}$

2.4. Product Rules and Quotient Rule

(*) <u>Product Rule</u>

$$[f(x) \cdot g(x)]' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

<u>Example</u>: Find $k'(x)$

(1) $k(x) = (x^2 + x) \cdot (x^3 + 2x^9)$

$\Rightarrow k'(x) = (x^2 + x)'(x^3 + 2x^9) + (x^2 + x) \cdot (x^3 + 2x^9)'$

$= (2x + 1)(x^3 + 2x^9) + (x^2 + x) \cdot (3x^2 + 18x^8)$

(*) <u>Quotient Rule</u>

$$\left[\frac{Top}{Bot}\right]' = \frac{(Top)' \cdot Bot - (Top) \cdot (Bot)'}{(Bot)^2}$$

Example: Find $f'(x)$

① $f(x) = \dfrac{x^2+1}{x^2-1}$

$$\left[\frac{x^2+1}{x^2-1}\right]' = \frac{(x^2+1)' \cdot (x^2-1) - (x^2+1)(x^2-1)'}{(x^2-1)^2}$$

quotient rule

$$= \frac{2x \cdot (x^2-1) - (x^2+1)(2x)}{(x^2-1)^2}$$

(Simplification) $$= \frac{\cancel{2x^3} - 2x - \cancel{2x^3} - 2x}{(x^2-1)^2}$$

$$= \frac{-4x}{(x^2-1)^2}$$

Practice: Find $f'(x)$

① $f(x) = (x^3 + 2x + 1)(x^4 - 2x^3 - x - 1)$

② $f(x) = (\sqrt{x} + 1) \cdot (x + 1)$

③ $f(x) = \dfrac{x}{x+1}$ (simplify the answer)

Solution:

① $f'(x) = (3x^2+2)(x^4-2x^3-x-1) + (x^3+2x+1)(4x^3-6x^2-1)$

② $f'(x) = \frac{1}{2}x^{-1/2} \cdot (x+1) + (\sqrt{x}+1)$

③ $f'(x) = \dfrac{(x)'(x+1) - x(x+1)'}{(x+1)^2}$

$= \dfrac{x+1-x}{(x+1)^2} = \dfrac{1}{(x+1)^2}$

(4) $f(x) = \dfrac{\sqrt{x}+1}{\sqrt{x}+2}$ (simplify the answer)

$$f'(x) = \frac{(\sqrt{x}+1)' \cdot (\sqrt{x}+2) - (\sqrt{x}+1) \cdot (\sqrt{x}+2)'}{(\sqrt{x}+2)^2}$$

$$\left[(\sqrt{x})' = (x^{1/2})' = \frac{1}{2} \cdot x^{1/2 - 1} = \frac{1}{2} x^{-1/2} \right]$$

$$= \frac{1/2 \cdot x^{-1/2} \cdot (\sqrt{x}+2) - (\sqrt{x}+1) \cdot 1/2 \cdot x^{-1/2}}{(\sqrt{x}+2)^2}$$

$$= \frac{1/2\ x^{-1/2} \left[(\sqrt{x}+2) - (\sqrt{x}+1) \right]}{(\sqrt{x}+2)^2}$$

$$= \frac{1/2\ x^{-1/2} \left[\cancel{\sqrt{x}} + 2 - \cancel{\sqrt{x}} - 1 \right]}{(\sqrt{x}+2)^2}$$

$$= \frac{1/2\ x^{-1/2}}{(\sqrt{x}+2)^2} = \frac{1}{2\sqrt{x}\,(\sqrt{x}+2)^2}$$

2.5: Derivatives of Trig. Functions

we have:

① $(\sin x)' = \cos x$

② $(\cos x)' = -\sin x$

Example: Find $f'(x)$

① $f(x) = x^2 \sin x$

$$[x^2 \cdot \sin x]' = (x^2)' \cdot \sin x + x^2 \cdot (\sin x)'$$

$$= 2x \sin x + x^2 \cdot \cos x$$

② $f(x) = \tan x$

quotient rule

$$(\tan x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{(\sin x)' \cdot \cos x - \sin x \cdot (\cos x)'}{(\cos x)^2}$$

$$= \frac{\cos x \cdot \cos x + \sin x \cdot \sin x}{(\cos x)^2}$$

$$= \frac{(\cos x)^2 + (\sin x)^2}{(\cos x)^2} = \frac{1}{(\cos x)^2}$$

Find

Practice : $f'(x)$

① $f(x) = \dfrac{x}{\sin x}$

② $f(x) = x^3 \cdot \cos x$