

# Exam 1 - Practice 1

*Name:*

*Notice: Calculators are not allowed.*

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## Some formulas:

- The derivative of  $f(x)$  is defined by the formula:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- An equation of the tangent line at  $x = a$  is

$$y - f(a) = f'(a)(x - a)$$

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## Problem 1.

Find the following limits.

a.

$$\lim_{x \rightarrow 1} \frac{x^2 + 4x + 3}{x^2 + 5x + 4}$$

b.

$$\lim_{x \rightarrow -1} \frac{x^2 + 4x + 3}{x^2 + 5x + 6}$$

c.

$$\lim_{x \rightarrow \infty} \frac{2x^2 + 4x + 3}{3x^2 + 5x + 6}$$

d.

$$\lim_{x \rightarrow \infty} \frac{2x^2 + 4x + 3}{3x^3 + 5x + 6}$$

e.

$$\lim_{x \rightarrow \infty} \frac{2x^5 + 4x + 3}{3x^3 + 5x + 6}$$

f.

$$\lim_{x \rightarrow 1} \frac{\sin 3x}{\sin 5x}$$

g.

$$\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 5x}$$

h.

$$\lim_{x \rightarrow 0} \frac{x^2 + \sin 3x}{x + \sin 5x}$$

**Problem 2**

Find values of  $x$ , if any, at which the function is not continuous.

a.  $f(x) = x^2 + \frac{x}{3} + 2024$

b.  $f(x) = x^2 + \frac{3}{x-1} + 2024$

c.  $f(x) = \frac{3}{2x+5} + \frac{x-1}{x^2-5x+6}$

**Problem 3.**

Find a value of the constant  $k$ , if possible, that will make the function continuous everywhere.

a.

$$f(x) = \begin{cases} x - 2, & x \leq 2 \\ kx^2 + k, & x > 2 \end{cases}$$

b.

$$f(x) = \begin{cases} x^2 + x + 4, & x \leq 0 \\ -9x + k^2, & x > 0 \end{cases}$$

**Problem 4.**

- a. Use the definition of derivatives to find  $f'(x)$ , and then find the tangent line to the graph of  $y = f(x)$  at  $x = 1$

$$f(x) = 2x^2 - 3x + 4$$

- b. Use the definition of derivatives to find  $f'(x)$ , and then find the tangent line to the graph of  $y = f(x)$  at  $x = 0$

$$f(x) = \frac{1}{x+1}$$