(*) Limits at infinity (cont.)

Result from last session

(1)
$$\lim_{x\to\infty} \frac{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \cdots + b_1 x + b_0} = \lim_{x\to\infty} \frac{a_n x^n}{b_m x^m}$$

(2)
$$\lim_{x\to-\infty} \frac{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \cdots + b_1 x + b_0} = \lim_{x\to-\infty} \frac{a_n x^n}{b_m x^m}$$

Example

$$\lim_{x\to+\infty} \frac{5x^2 - 4x}{2x^2 + 3} = \lim_{x\to\infty} \frac{5x^2}{2x^2} = 5/2$$

(3)
$$\lim_{x\to\infty} a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 = \lim_{x\to\infty} a_n x^n$$

(4)
$$\lim_{x\to-\infty} a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 = \lim_{x\to-\infty} a_n x^n$$

Example:

$$\lim_{x\to\infty} (1 + 2x - 3x^5) = \lim_{x\to\infty} -3x^5 = -\infty$$

$$\lim_{x\to\infty} \sqrt{x^2 - 2x + 20} = \lim_{x\to\infty} \sqrt{x^2} = \infty$$

29. $\lim_{x\to-\infty} \dfrac{\sqrt{3x^4 + x}}{x^2 - 8} = \lim_{x\to-\infty} \dfrac{\sqrt{3x^4}}{x^2} = \lim_{x\to\infty} \dfrac{\sqrt{3}\cdot\sqrt{x^4}}{x^2} = \sqrt{3}$

[Notice: $\sqrt{x^4} = (x^4)^{1/2} = x^{4\cdot 1/2} = x^2$]

Practice: Find

(1) $\lim_{x\to\infty} \dfrac{10}{2x^3 - 10x^2 - 100x + 2}$

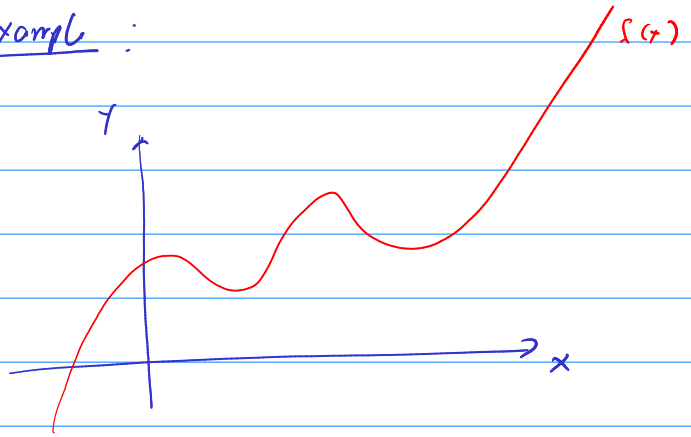
(2) $\lim_{x\to\infty} \dfrac{3x^6 - 6x^3 + x}{1 - x - x^{10}}$

(3) $\lim_{x\to\infty} \dfrac{6x^3 + x + 1}{7x^3 - 2x - 1}$

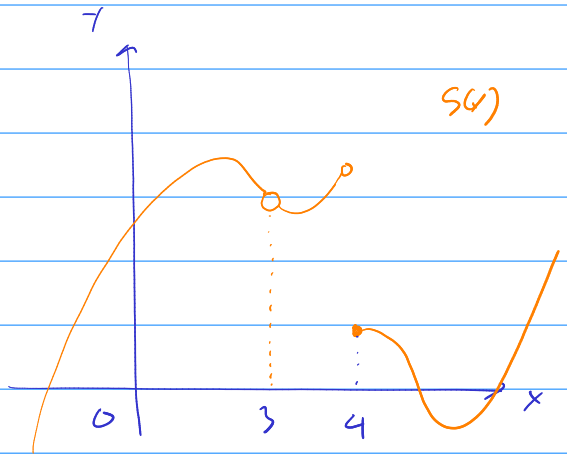
(4) $\lim_{x\to\infty} \dfrac{\sqrt{3x+1}}{\sqrt{4x+2}}$

1.5 Continuity

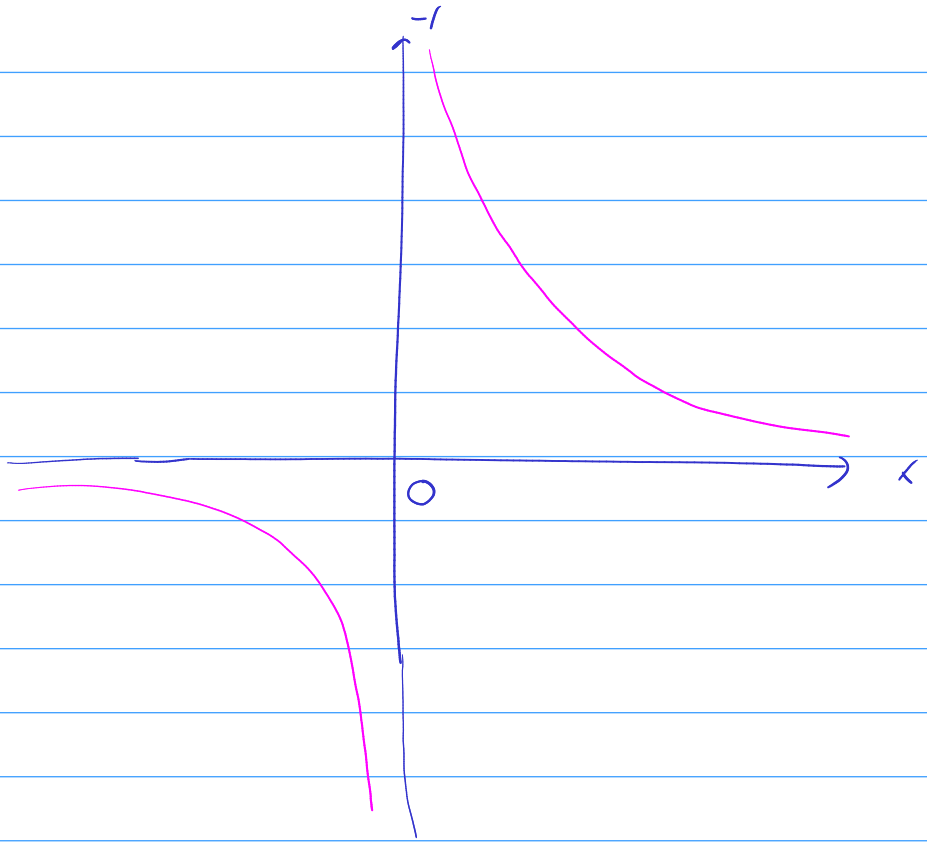
Example:



f(x) is a continuous function



g(x) is not a continuous function
g(x) is discont. at $x = 3, 4$



This function is not continuous at $x = 0$

(*) Def.

The function $y = f(x)$ is continuous at $x = a$ if and only if (iff) the following 3 conditions are satisfied

(1) $f(a)$ exists

(2) $\lim_{x \to a} f(x)$ exists [means: $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$]

(3) $\lim_{x \to a} f(x) = f(a)$

Example: Find some example of discontinuity when the first condition is NOT satisfied.

(1) Example of functions that are not continuous at $x = 7$

$f(x) = \dfrac{1}{x-7}$ is not cont. at $x = 7$ b/c

$f(7)$ does not exist (DNE)

$g(x) = \dfrac{10}{(x-7)^3}$, $h(x) = \sqrt{-x}$

$k(x) = \sqrt{1-x}$

are some example where the function is not cont. at $x = 7$

b/c the func. DNE at $x = 7$