6.1 : Exponential and Logarithmic Functions

(*) Exponential Functions

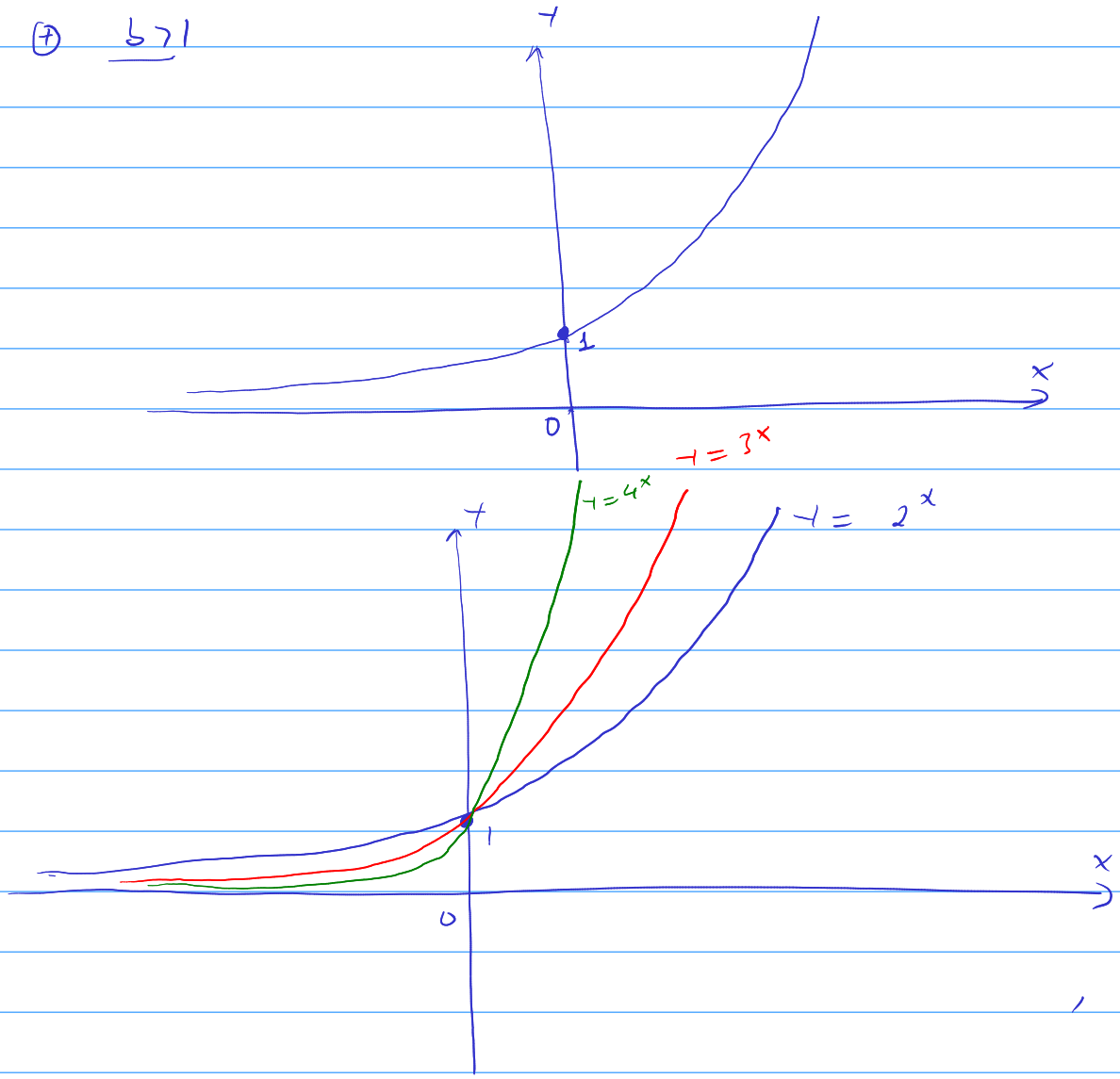
$$y = f(x) = b^x \qquad (b > 0,\ b \neq 1)$$

Example : $f(x) = 2024^x$

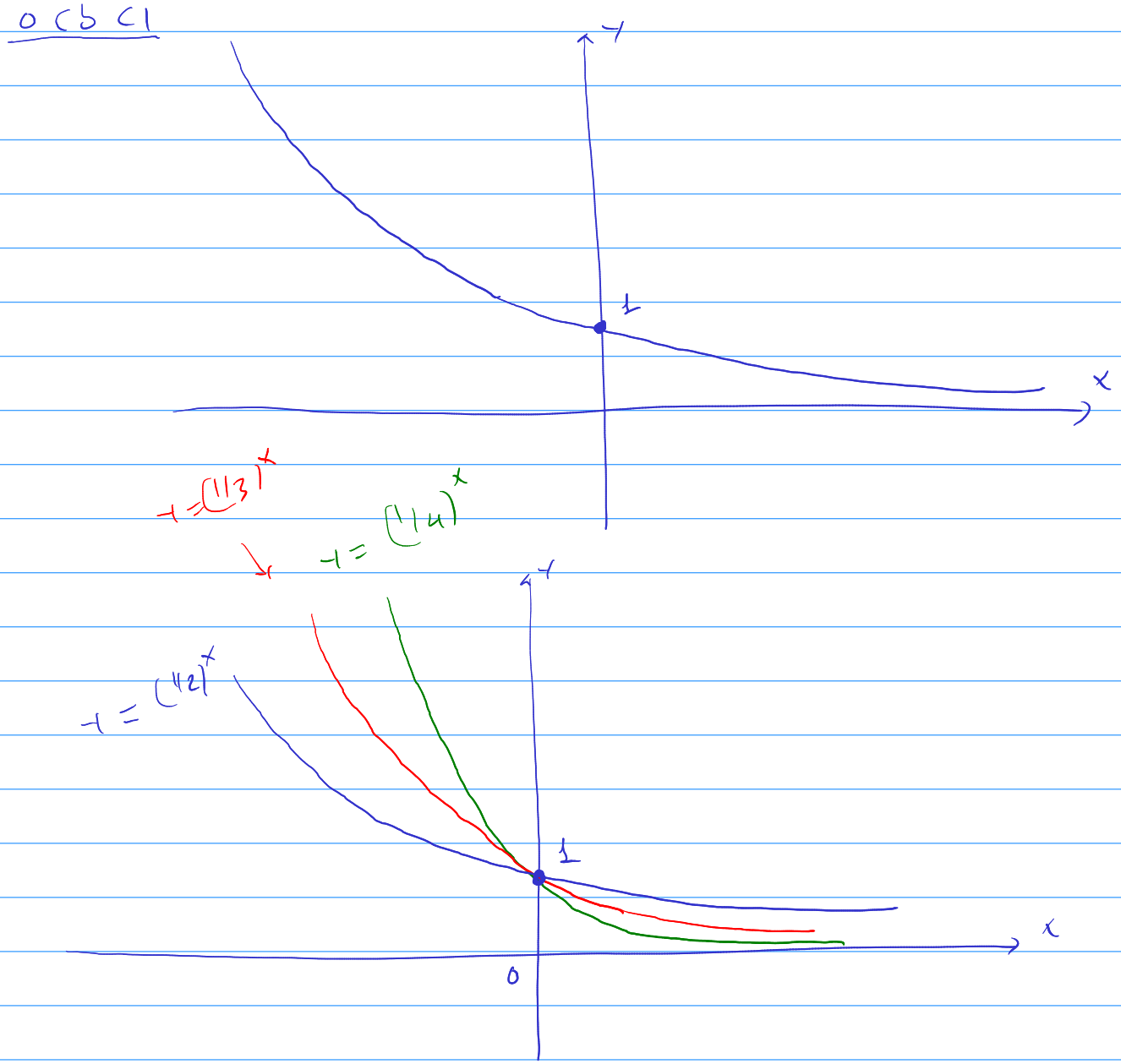
$h(x) = 3^x$

(*) Graphs of exp. functions

(1) $b > 1$



0 < b < 1



(*) Logarithmic Functions

⊕

$3^x = 9$ $\Leftrightarrow$ $x = 2$

$3^x = 27$ $\Rightarrow$ $x = 3$

$3^x = 10$ $\Leftrightarrow$ $x = \log_3 10$

$4^x = 3 \iff x = \log_4 3$

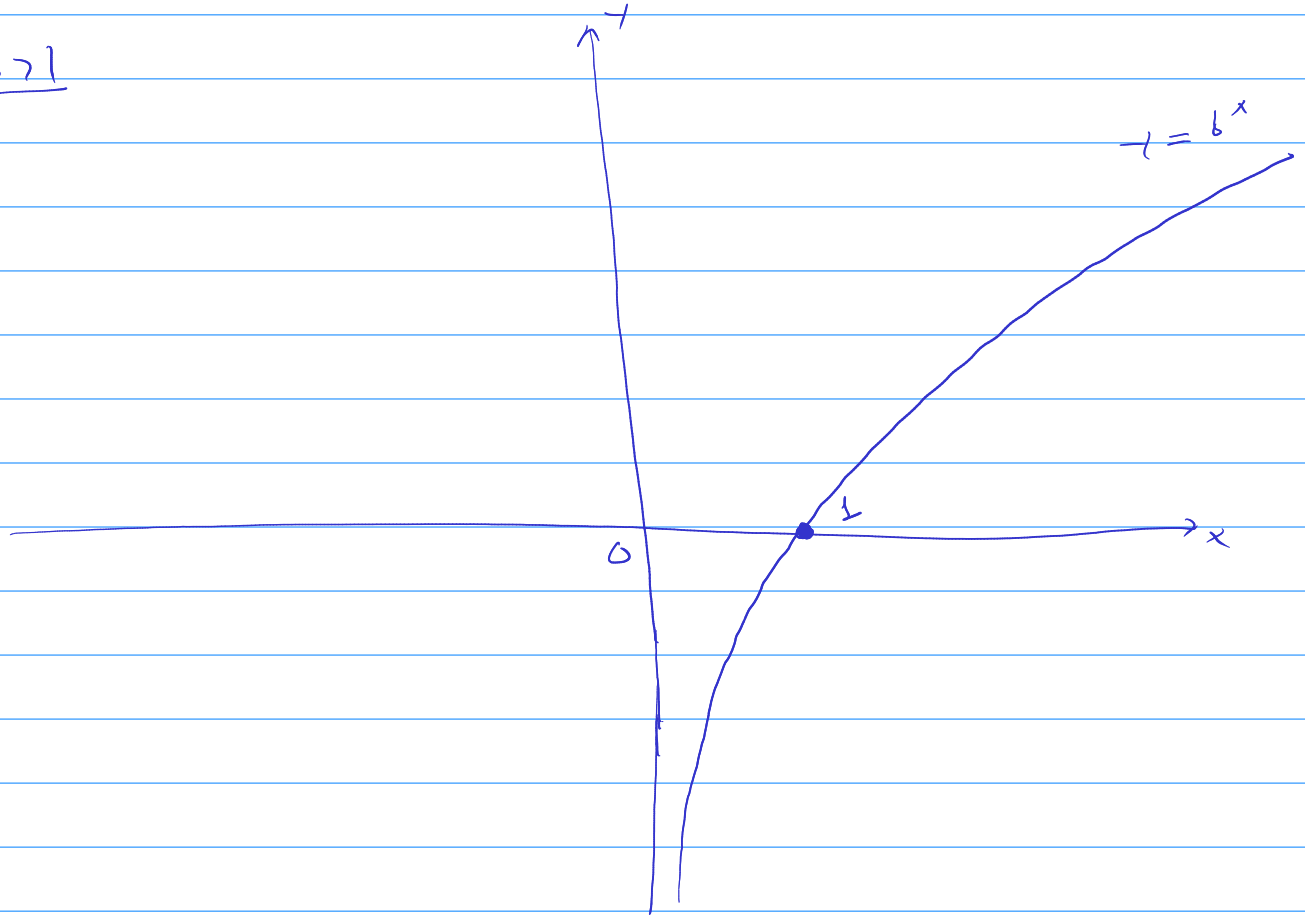
If $y = \log_b x$ then $x = b^y$ and vice versa.

if $x = b^y$ then $y = \log_b x$

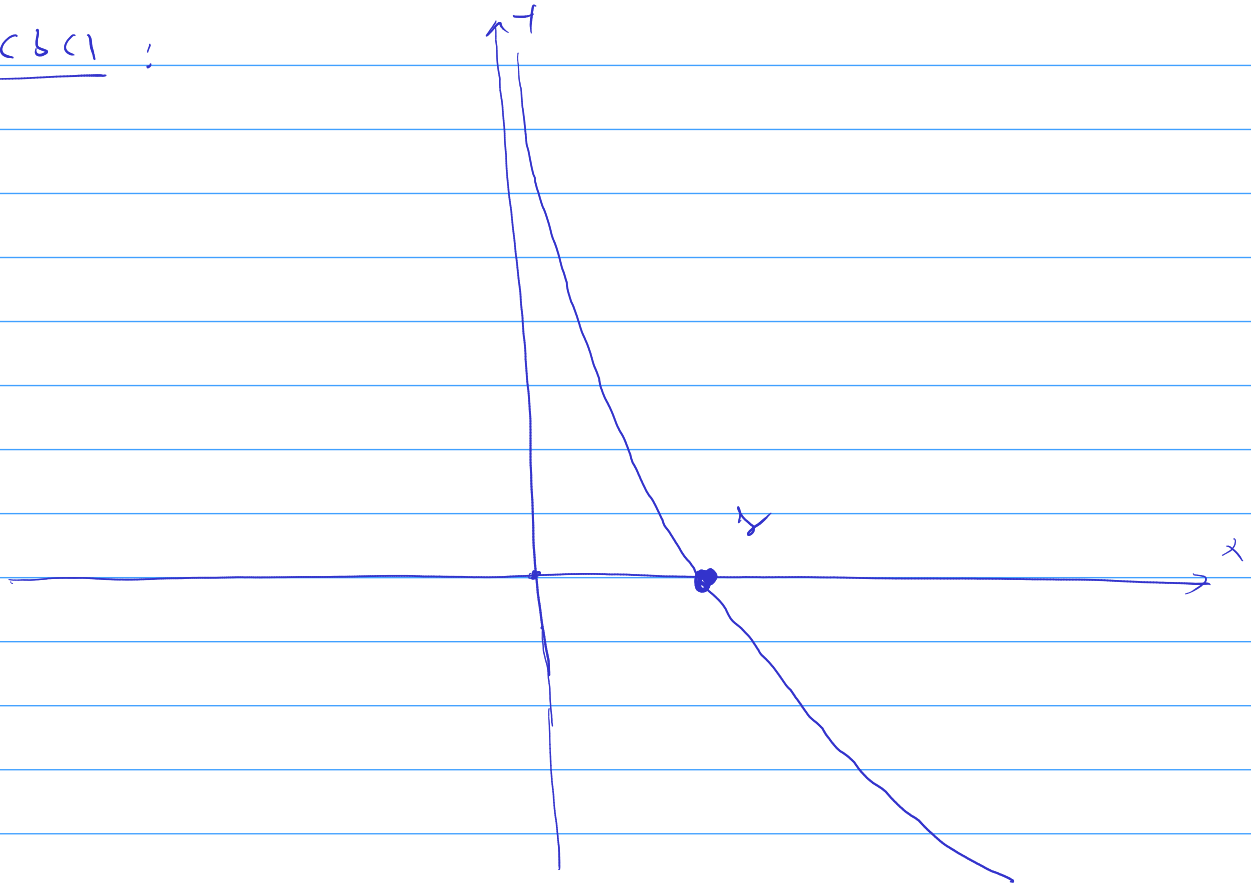
$b > 0, \; b \neq 1$.

(*) Graphs of log functions

$b > 1$



0 < b < 1 :



(*) Number e :

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

$e = 2.71828\ldots$

we write $\log_e x$ as $\ln x$

(*) Derivatives of Exponential and log. functions

$$\left(b^x\right)' = b^x \cdot \ln b$$

$$(e^x)' = e^x$$

[bic: $(e^x)' = e^x \cdot \ln e = e^x$]

$$(\log_b x)' = \frac{1}{x \cdot \ln b}$$

$$(\ln x)' = \frac{1}{x}$$

Example: Find $f'(x)$

① $f(x) = 2^x + 3^x$

$\Rightarrow f'(x) = (2^x + 3^x)'$

$= 2^x \cdot \ln 2 + 3^x \cdot \ln 3$

② $f(x) = \log_3 x + 2 \cdot \log_7 x$

$\Rightarrow f'(x) = \frac{1}{x \cdot \ln 3} + \frac{2}{x \cdot \ln 7}$

(3) $f(x) = x \cdot \ln x$

$f'(x) = (x \cdot \ln x)' = (x)' \cdot \ln x + x \cdot (\ln x)'$

$= \ln x + x \cdot \frac{1}{x}$

$= \ln x + 1$

Practice: Find $f'(x)$

(1) $f(x) = 2024^x + 7^x - 9^x$

(2) $f(x) = 3^x \cdot \sin x$

(3) $f(x) = \log_9 x - 6 \ln x + 3 \log_4 x$

(4) $f(x) = \frac{x^2}{\log_2 x}$

(5) $f(x) = \frac{\tan x}{\log_6 x}$