Use the definition of derivatives to find $f'(x)$

$$f'(x) = \frac{1}{x+1}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h}$$

$$= \frac{\frac{x+1 - (x+h+1)}{(x+h+1)\cdot(x+1)}}{h}$$

[note: $\frac{1}{3} - \frac{1}{4} = \frac{4-3}{3\cdot 4}$]

$$= \frac{\frac{\cancel{x}+\cancel{1} - \cancel{x} - h - \cancel{1}}{(x+h+1)\cdot(x+1)}}{h}$$

$$= \frac{-\cancel{h}}{(x+h+1)(x+1)} \cdot \frac{1}{\cancel{h}}$$

$$= \frac{-1}{(x+h+1)(x+1)}$$

plus $h=0$ in, we have: $f'(x) = \dfrac{-1}{(x+1)(x+1)}$

$$= -\frac{1}{(x+1)^2}$$

Practice: Find $f'(x)$ using the def. of der.

$$f(x) = \frac{1}{3x+4}$$

$f(x)$

$$\frac{f(x+h) - f(x)}{h} = \frac{\dfrac{1}{3(x+h)+4} - \dfrac{1}{3x+4}}{h}$$

$$= \frac{3x+4 - [3(x+h)+4]}{[3(x+h)+4]\cdot(3x+4)} \cdot \frac{1}{h}$$

$$\left[\frac{1}{3} - \frac{1}{4} = \frac{4}{3\cdot 4} - \frac{3}{4\cdot 3} = \frac{4-3}{4\cdot 3}\right]$$

$$= \frac{\cancel{3x} + \cancel{4} - \cancel{3x} - 3h - \cancel{4}}{(3x+3h+4)(3x+4)} \cdot \frac{1}{h}$$

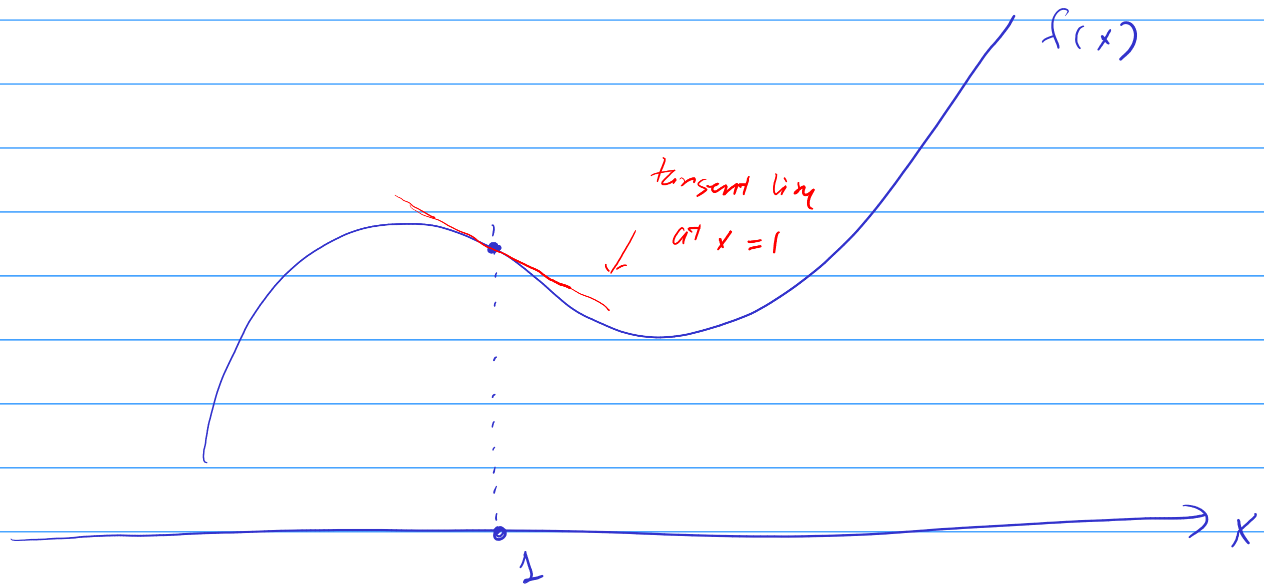
$$= \frac{-3\cancel{h}}{(3x+3h+4)(3x+4)} \cdot \frac{1}{\cancel{h}}$$

$$= \frac{-3}{(3x+3h+4)(3x+4)}$$

Plug $h=0$ in: $f'(x) = \dfrac{-3}{(3x+4)^2}$

(*) A meaning of derivatives.

$f'(a)$ is the slope of the tangent line at $x=a$.



$f'(1)$ is the slope of the red line (tangent line at $x=1$)

Practice : write the equation of the tangent line at $x = 1$: $f(x) = \frac{1}{x+1}$

$$f'(x) = -\frac{1}{(x+1)^2}$$

the equation of the tangent line at $x = 1$ is

$$y - f(1) = f'(1)(x - 1)$$

$$f(1) = \frac{1}{1+1} = \frac{1}{2}, \quad f'(1) = -\frac{1}{(1+1)^2} = -\frac{1}{4}$$

$$\Rightarrow y - \frac{1}{2} = -\frac{1}{4}(x-1)$$

$$y = -\frac{1}{4}x + \frac{1}{4} + \frac{1}{2}$$

$$\Rightarrow \boxed{y = -\frac{1}{4}x + \frac{3}{4}}$$

Practice : Use the definition of derivatives to find $f'(x)$, and then find the tangent line to the graph of $y = f(x)$ at $x = 0$

① $f(x) = \dfrac{2}{3x+1}$

② $f(x) = 3x^2 + x + 1$

③ $f(x) = \dfrac{1}{x^2+1}$