Exam 2 - Practice 1

Notice:

- *Calculators are not allowed.*
- *Exam 2 is scheduled for Tuesday, Nov 12.*
- *A page of formula is allowed. Only formulas are allowed on the page. The page will be checked during the exam.*

Problem 1.

Find $f'(x)$.

$$f(x) = \frac{2x^3}{3} + \frac{x^5}{7} - \frac{2}{\sqrt[3]{x}} + \frac{5}{6\sqrt{x}} + 2024$$

$f(x) = \frac{2}{3}x^3 + \frac{1}{7}x^5 - 2x^{-1/3} + \frac{5}{6}x^{-1/2} + 2024$

$\Rightarrow f'(x) = \frac{2}{3}\cdot 3x^2 + \frac{1}{7}\cdot 5x^4 - 2\left(-\frac{1}{3}\right)x^{-4/3} + \frac{5}{6}\cdot\left(-\frac{1}{2}\right)\cdot x^{-3/2}$

$= 2x^2 + \frac{5}{7}x^4 + \frac{2}{3}x^{-4/3} - \frac{5}{12}x^{-3/2}$

Formulas

$(x^n)' = n\cdot x^{n-1}$

$\sqrt[k]{x} = x^{1/k}$

$\sqrt[k]{x^n} = x^{n/k}$

$\frac{1}{x^k} = x^{-k}$

$$f(x) = (\sqrt{x} + 1)(x + 1)$$

$f(x) = (x^{1/2} + 1)(x+1)$

product rule

$(f\cdot g)' = f'\cdot g + f\cdot g'$

product rule

$\Rightarrow f'(x) = (x^{1/2}+1)'\cdot(x+1) + (x^{1/2}+1)(x+1)'$

$= \frac{1}{2}x^{-1/2}\cdot(x+1) + (x^{1/2}+1)\cdot 1$

$$f(x) = \frac{x-1}{x+1} \text{ (Simplify your answers.)}$$

$f'(x) = \dfrac{(x-1)'\cdot(x+1) - (x-1)(x+1)'}{(x+1)^2}$

Quotient rule

$= \dfrac{x+1-(x-1)}{(x+1)^2} = \dfrac{2}{(x+1)^2}$

$$f(x) = x \sin x$$

$\Rightarrow$ $f'(x) = x' \cdot \sin x + x \cdot (\sin x)'$

$= \sin x + x \cdot \cos x$

$(\sin x)' = \cos x$

$$f(x) = \frac{x}{\tan x}$$

$f'(x) = \dfrac{(x)' \cdot \tan x - x \cdot (\tan x)'}{(\tan x)^2}$

$= \dfrac{\tan x - x \cdot \sec^2 x}{(\tan x)^2}$

$(\tan x)' = \sec^2 x$

$$f(x) = \cos^{2024} x$$

$f'(x) = 2024 \cos^{2023} x \cdot (\cos x)'$

$= 2024 \cos^{2023} x \cdot (-\sin x)$

$(\cos x)' = -\sin x$

$\left([g(x)]^n\right)' = n \cdot (g(x))^{n-1} \cdot g'(x)$

$$f(x) = \cos(3x^2 + x + 1)$$

$[\cos(g(x))]' = [-\sin g(x)] \cdot g'(x)$

$f'(x) = [-\sin(3x^2 + x + 1)] \cdot (3x^2 + x + 1)'$

$= [-\sin(3x^2 + x + 1)] \cdot (6x + 1)$

$$f(x) = \tan\left(\cos x + \sqrt{x}\right)$$

$\left[\tan(g(x))\right]' = \left[\sec^2 g(x)\right] \cdot g'(x)$

$f'(x) = \left[\sec^2(\cos x + \sqrt{x})\right] \cdot (\cos x + \sqrt{x})'$

$= \left[\sec^2(\cos x + \sqrt{x})\right] \cdot \left(-\sin x + \frac{1}{2}x^{-1/2}\right)$

$$f(x) = \left(\cos x + \sin x\right)^{2024}$$

$f'(x) = 2024\,(\cos x + \sin x)^{2023} \cdot (\cos x + \sin x)'$

$= 2024\,(\cos x + \sin x)^{2023}\,(-\sin x + \cos x)$

$$f(x) = 2024^x + 7^x - 2\log_9 x + 3\ln x - \frac{3\log_2 x}{5} + \frac{\log_7 x}{3} + 2024$$

$$f(x) = \log_7\left(\sqrt{x} + x^2 + x + 1\right)$$

$$f(x) = e^{\sin x + \tan x + 2x^3}$$

$$f(x) = e^{x \sin x}$$

Problem 2

$$y + xy - 2x^3 = 1$$

(a) Find dy/dx or y' by differentiating implicitly.

(b) Solve the equation for y as a function of x, and find dy/dx from that equation.

Problem 3

Given the equation

$$x^3 + y^3 = 3xy$$

1. Use implicit differentiation to find dy/dx or y'

2. Find an equation for the tangent line at the point (3/2, 3/2)

Problem 4

(a) Find the local linear approximation of $f(x) = \sqrt{x}$ at $x_0 = 1$

(b) Use the local linear approximation obtained in part (a) to approximate $\sqrt{1.1}$