Exam 1 -Makeup

Name:

Notice: Calculators are not allowed.

Some formulas:

- The derivative of $f(x)$ is defined by the formula:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- An equation of the tangent line at $x = a$ is

$$y - f(a) = f'(a)(x - a)$$

Problem 1. (5 points each)

Find the following limits.

$$\lim_{x \to 2} \frac{x^2 - 4x + 4}{x^2 + 5x + 1}$$

$$\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 - 6x + 8}$$

$$\lim_{x \to \infty} \frac{2x^6 + 4x^4 + 3x^3}{7x^5 + 2024}$$

$$\lim_{x \to \infty} \frac{-2x^5 + 4x + 3}{3x^{13} + 5x + 6}$$

$$\lim_{x \to \infty} \frac{x + 1}{x^3 + 5x + 6}$$

$$\lim_{x \to 2} \frac{\sin 10x}{\sin 5x}$$

$$\lim_{x \to 0} \frac{\sin 8x}{\sin 4x}$$

$$\lim_{x \to 0} \frac{2x^3 + x^2 + 3\sin x}{3x^3 + 3\sin 5x}$$

Problem 2 (5 points each)

Find values of x, if any, at which the function is not continuous.

a. $f(x) = 3x^2 + \frac{1}{x-1} + 2024x + 1$

b. $f(x) = x^2 + \frac{3}{(x-2)(x-3)(x-4)} + 2024$

c. $f(x) = \frac{3}{x+1} + \frac{x-1}{x^2+7x+6}$

Problem 3. (5 points each)

Find a value of the constant k, if possible, that will make the function continuous everywhere.

a.

$$f(x) = \begin{cases} x^2, & x \leq 2 \\ x^2 - 3kx + 1, & x > 2 \end{cases}$$

b.

$$f(x) = \begin{cases} 3x^2 + 4x + 16, & x \leq 0 \\ -9x + k^2, & x > 0 \end{cases}$$

Problem 4. (17.5 points each)

a. Use the definition of derivatives to find $f'(x)$, and then find the tangent line to the graph of $y = f(x)$ at $x = 3$

$$f(x) = -5x^2 + 6x + 2$$

b. Use the definition of derivatives to find $f'(x)$, and then find the tangent line to the graph of $y = f(x)$ at $x = 4$

$$f(x) = \frac{4}{5x + 1}$$

c (Optional - 5 Points Extra Credits). Use the definition of derivatives to find $f'(x)$.

$$f(x) = x^5$$