Section 1.1 (cont.)

Example : $y = \dfrac{x^2 - 9}{x - 3}$

Approach 3

$$\lim_{x \to 3} \frac{x^2-9}{x-3} = \lim_{x \to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} = \lim_{x \to 3} x+3 = 3+3$$
$$= 6$$

Notice: $x^2 - 9 = x^2 - 3^2 = (x-3)(x+3)$

in general:

$$x^2 - a^2 = (x-a)(x+a)$$

Example : $f(x) = \dfrac{x^2 - 5x + 6}{x - 2}$

Find $\displaystyle\lim_{x \to 2} \frac{x^2 - 5x + 6}{x - 2}$

Approach 1 :

x	y
1.9	-1.1
1.99	-1.01
1.999	-1.001
1.9999	-1.0001

$\downarrow$

-1

$$\lim_{x \to 2} y = -1$$

Approach 2: Fail because we cannot plug $x = 2$ in.

Approach 3

$$\lim_{x \to 2} \frac{x^2 - 5x + 6}{x - 2} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x-3)}{\cancel{(x-2)}} = \lim_{x \to 2} x - 3$$

$$= 2 - 3$$

$$= -1$$

Example: Find the follows

① $\displaystyle \lim_{x \to 1} \frac{x^2 - 1}{x - 1}$

② $\displaystyle \lim_{x \to 4} \frac{x^2 - 6x + 8}{x - 4}$

Solution:

①

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1} \frac{x^2 - 1^2}{x - 1} = \lim_{x \to 1} \frac{(x-1)(x+1)}{x - 1}$$

$$= \lim_{x \to 1} x + 1$$

$$= 1 + 1 = 2$$

(2) $\lim_{x \to 4} \frac{x^2 - 6x + 8}{x - 4} = \lim_{x \to 4} \frac{(x-2)(x-4)}{x-4}$

$= \lim_{x \to 4} x - 2 = 4 - 2$

$= 2$

* Infinite Limits and Limit at infinity.

Example

$\lim_{x \to 1^-} \frac{x+1}{x-1}$

Approach 2 and 3 both fail.

Approach 1

.9 .99 1

x	y
.999	-199
.9999	-1999
.99999	-19999
.999999	-199999
.999999999	-1999999

$\downarrow$

$-\infty$

$$\lim_{x \to 1^-} y = -\infty$$

$$\lim_{x \to 1^+} y = \infty$$

x	y
1.0001	20001
1.00001	200001
1.000001	2000001
	$\downarrow$
	∞

(*) Limit at infinity

$$\lim_{x \to \infty} (x^2 + 1) = \infty$$

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

<u>Section 1.2</u> : computing limits

$$\lim_{x \to a} f(x)$$

<u>Idea</u>: For the following function $f(x)$

1. Polynomials
2. Rational functions
3. Exponential / log functions
4. Trigonometric functions

we have:

$$\lim_{x \to a} f(x) = f(a)$$

if $f(a)$ exists.

<u>Example</u>:

$$\lim_{x \to 5} x^2 + 2x + 1 = 5^2 + 2.5 + 1$$

$$= 36$$

$$\lim_{x \to 2} \sqrt{x^2 + \sin x} = \sqrt{2^2 + \sin 2} = 2.2156\ldots$$

(*) $\lim_{x \to \infty} \frac{1}{x} = 0$

(*) if $f(x) \to \infty$ then $\frac{1}{f(x)} \to 0$

(*) $\infty \times \infty = \infty$

(*) $\infty + \infty = \infty$

(*) $\infty^2 = \infty$

(*) $\infty^k = \infty \qquad (k > 0)$

(*) $\frac{1}{\infty} = 0$

(*) $\infty - \infty$: indeterminate form

$\frac{\infty}{\infty}$:

Example :

$$\lim_{x \to \infty} \frac{1}{x^2 + x + 1} = \frac{1}{\infty^2 + \infty + 1} = \frac{1}{\infty} = 0$$

$$\lim_{x \to \infty} \frac{1}{x^2 - x + 1} = \frac{1}{\quad}$$