Chapter 3 : Derivative in Graphing and Applications

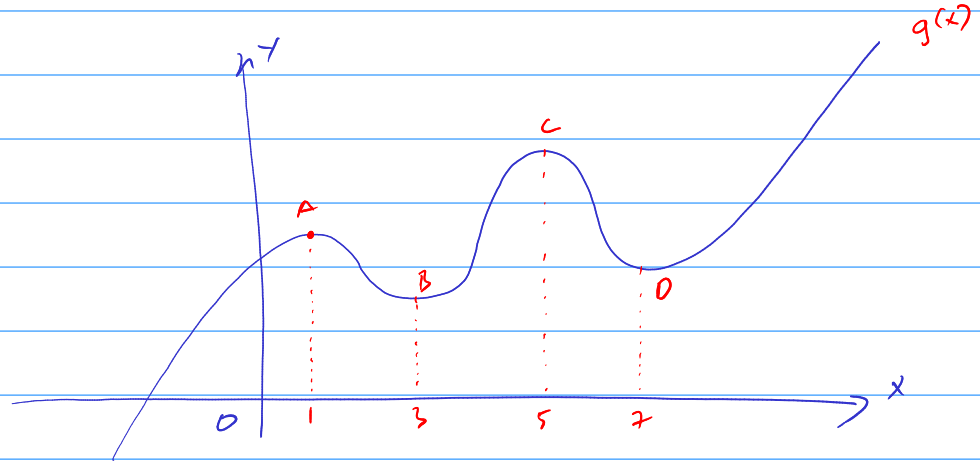
3.1 : Analysis of Function I: Increase, Decrease and concavity.

$$y = f(x) = x^7 + \ln x + e^x + \cos x$$

(*) To graph $f(x)$ we can graph points on $f(x)$ plug in x to find y values for many points of x

(*) We can add in the analysis of functions to understand it better.

(*)



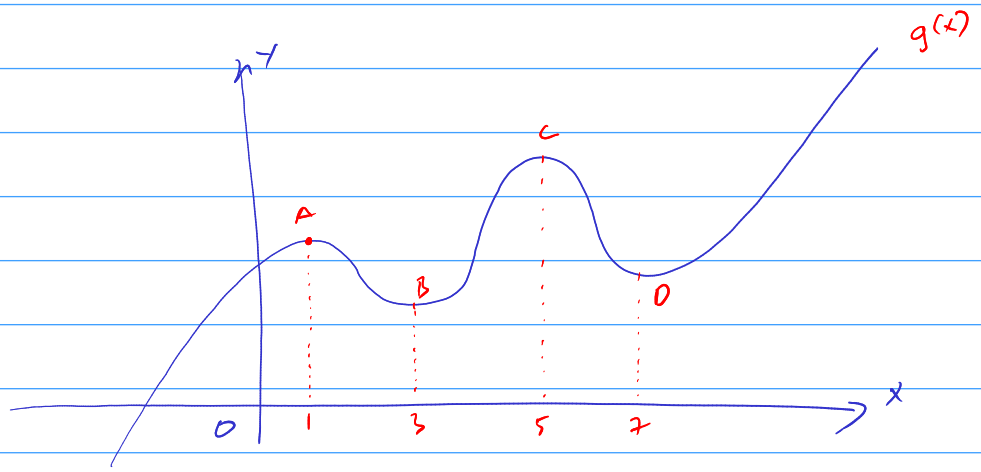
$g(x)$ is increasing on : $x < 1$ or $(-\infty, 1)$

$3 < x < 5$ or $(3, 5)$

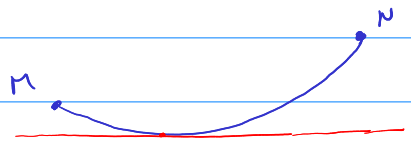
$x > 7$ or $(7, \infty)$

$g(x)$ is decreasing on $1 < x < 3$ or $(1, 3)$

$5 < x < 7$ or $(5, 7)$

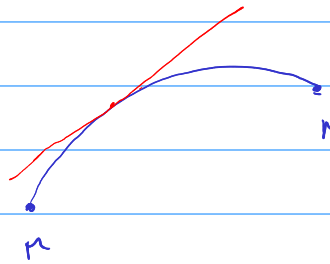


* $g(x)$ is concave upward:

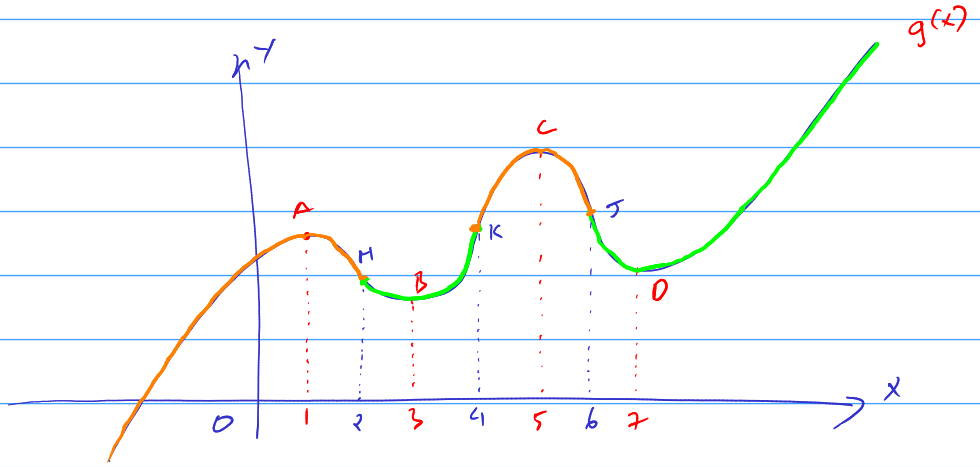


the graph is above the tangent line

* concave downward



the graph is below the tangent line



(*) concave up on : $2 < x < 4$ and $x > 6$

OR

$(2, 4)$ and $(6, \infty)$

(*) Concave down: $x < 2$ and $4 < x < 6$

OR $(-\infty, 2)$ and $(4, 6)$

(*) Using the derivative to find the info. about increase / decrease and concavity.

(1) If $f'(x) > 0$ on (a, b) then $f(x)$ is increasing on (a, b)

(2) If $f'(x) < 0$ on (a, b) then $f(x)$ is decreasing on (a, b)

(3) If $f''(x) > 0$ on (a, b) then $f(x)$

is concave upward on (a,b)

④ $f''(x) < 0$ on (a,b) then $f(x)$ is concave downward on (a,b)

Example: $f(x) = x^4$

(The 1st derivative) $f'(x) = 4x^3$

(The 2nd derivative) $f''(x) = [f'(x)]' = (4x^3)' = 12x^2$

The 3rd derivative $f'''(x) = (12x^2)' = 24x$

Example: $y = f(x) = x^3 - 3x^2 + 1$

Find all intervals where ⊕ $f(x)$ is increasing

⊕ $f(x)$ is decreasing

⊕ $f(x)$ concave up

⊕ $f(x)$ concave down

$$f(x) = x^3 - 3x^2 + 1$$

Step 1 : Find $f'(x)$ and factor it

$$f'(x) = 3x^2 - 6x$$

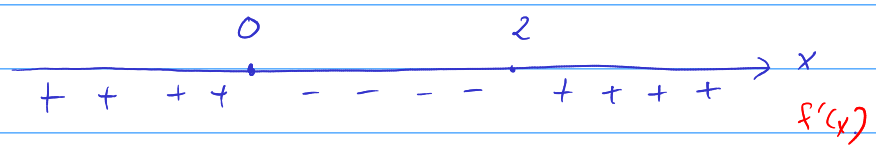
$$= 3x(x-2)$$

Step 2 : Solve $f'(x) = 0$

$$3x(x-2) = 0$$

$$x = 0, \quad x = 2$$

Step 3 : Get the sign chart of $f'(x)$



$x < 0$: plug in a number $x < 0$, say $x = -10$

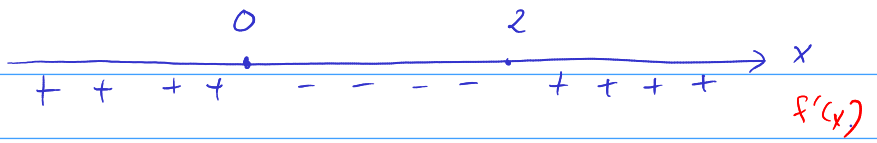
$$f'(-10) = 3(-10)(-10-2) > 0 \quad (+)$$

$0 < x < 2$: plug in $x = 1$

$$f'(1) = 3 \cdot (1)(1-2) < 0 \quad (-)$$

$x > 2$: plug in $x = 3$

$$f'(3) = 3(3)(3-2) > 0 \quad (+)$$



$f(x)$ is increasing on $(-\infty, 0)$ and $(2, \infty)$

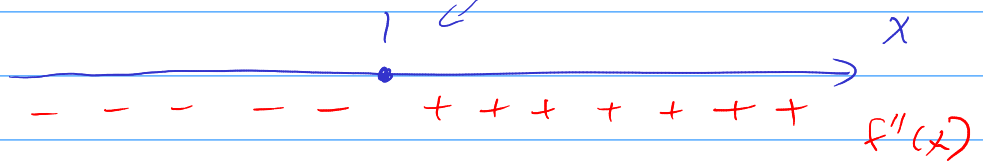
$f(x)$ is decreasing on $(0, 2)$

For concavity:

$$f''(x) = (3x^2 - 6x)' = 6x - 6$$

$$= 6(x-1)$$

$$f''(x) = 0 \iff \boxed{x = 1}$$



$f(x)$ is concave upward on $(1, \infty)$

$f(x)$ is concave down on $(-\infty, 1)$

Example: $y = f(x) = x^3 - 9x^2 + 1$

Find all intervals where
- ⊕ $f(x)$ is increasing
- ⊖ $f(x)$ is decreasing
- ⊕ $f(x)$ concave up
- ⊖ $f(x)$ concave down