$$y = f(x) = x^3 - 9x^2 + 1$$

* **Increasing / decreasing**

$$f'(x) = 3x^2 - 18x = 0$$

$$\Rightarrow \quad 3x(x-6) = 0$$

$$\Rightarrow \quad x = 0, \quad x = 6$$

sign chart of $f'(x)$

$x = -1$ 0 6 x

$+ + + + +$ $- - - - - -$ $+ + + +$ $f'(x)$

$x < 0$:

$x = -1$

$\Rightarrow f'(-1) = 3(-1)(-1-6)$

$= (+)$

$0 < x < 6$

$x = 4$

$f'(4) = 3(4)(4-6)$

$(-)$

$f(x)$ increasing on $x < 0$ and $x > 6$

$f(x)$ decreasing on $0 < x < 6$

* **Concavity:**

$$f''(x) = 6x - 18 = 0$$

$$\Rightarrow \quad 6(x-3) = 0$$

$$\Rightarrow \quad x = 3$$

sign chart for $f''(x)$

3 x

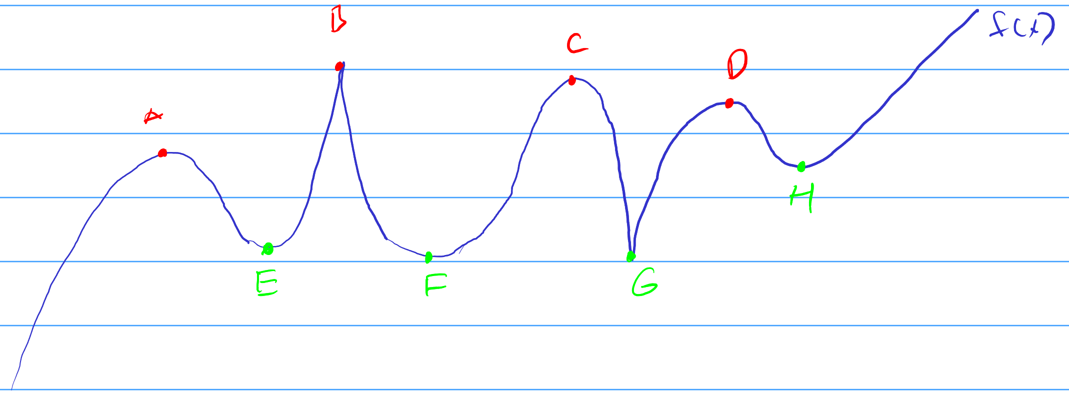
$- - - - -$ $+ + + + + +$ $f''(x)$

$f(x)$ concave down: $x < 3$

$f(x)$ concave up $x > 3$

Section 3.2: Analysis of functions II: Relative Extrema; Graphing polynomials

(*) Relative Extrema.



Relative maxima: A, B, C, D

Relative minima: E, F, G, H

All of these points are called relative extrema

(*) Critical points:

x_0 is a critical point of $f(x)$ if either

$f'(x_0) = 0$, or

$f'(x_0)$ DNE ($f(x)$ is not differentiable at x_0)

If $f'(x_0) = 0$, we call x_0 is a stationary point.

we have the following result:

A relative extrema occurs at critical points where $f'(x)$ changes the sign:

If $f'(x)$ changes the sign from $(-)$ to $(+)$ at $x = x_0$ then $(x_0, f(x_0))$ is a relative minimum.

If $f'(x)$ changes the sign from $(+)$ to $(-)$ at $x = x_0$ then $(x_0, f(x_0))$ is a relative maxima.

Example: Find all relative extrema of

$$f(x) = x^3 - 9x^2 + 1$$

Step 1: Find all critical points of $f(x)$

All points where $f'(x)$ DNE and

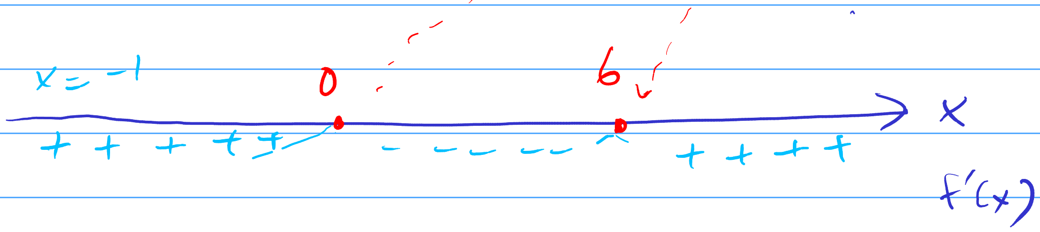
$f'(x) = 0$ (stationary points)

$$f'(x) = 3x^2 - 18x = 0$$

$x = 0$, $x = 6$ (from the previous example)

<u>Step 2</u>: Draw the sign chart and get the relative extrema.

Sign chart of $f'(x)$



relative maximum $(0, f(0)) = (0, 1)$

relative minimum $(6, f(6)) = (6, -107)$

$f(x) = x^3 - 9x^2 + 1 \Rightarrow f(0) = 1$

$$f(6) = 6^3 - 9 \cdot 6^2 + 1$$

$$= -107$$

Example : $f(x) = 3x^{5/3} - 15x^{2/3}$.

Find all critical points

Step 1 : Find all critical points

$$f'(x) = 3 \cdot \frac{5}{3} x^{5/3 - 1} - 15 \cdot 2/3 \cdot x^{2/3 - 1}$$

$$= 5x^{2/3} - 10x^{-1/3}$$

$$= 5\sqrt[3]{x^2} - 10 \cdot \frac{1}{\sqrt[3]{x}}$$

⊛ when $f'(x)$ DNE : $x = 0$

⊛ when $f'(x) = 0$ (stationary points)

$$5\sqrt[3]{x^2} - \frac{10}{\sqrt[3]{x}} = 0$$

$$\Rightarrow \frac{5\sqrt[3]{x^2}}{1} = \frac{10}{\sqrt[3]{x}}$$

$$\Rightarrow 5 \cdot \sqrt[3]{x^2} \cdot \sqrt[3]{x} = 10$$

$$\Rightarrow \sqrt[3]{x^3} = \frac{10}{5} \Rightarrow \boxed{x = 2}$$

7–14 Locate the critical points and identify which critical points are stationary points. ■

7. $f(x) = 4x^4 - 16x^2 + 17$ **8.** $f(x) = 3x^4 + 12x$