

* Expectations and variances of Random Variables

Example :

Given the prob. distribution of X as follows.

x	1	2	3
$P(x)$	0.2	0.7	0.1

\nearrow \nearrow \nwarrow

X could be 1, 2 or 3.

Suppose that you are given 2 options

Option 1 : you will receive X thousands dollars

where X follows the above distribution.

You could receive 1, 2 or 3 thousands dollars.

we just don't know the amount.

Option 2 : you will receive for sure 2.2 thousands dollars.

which option would you choose ?

we will calculate the "expected" value of x for option 1 and compare that with 2.2 k.

The expected value of x or the expectation of x , denoted by: $E(x)$, is calculated as follows.

x	1	2	3
$P(x)$	0.2	0.7	0.1

$$\begin{aligned} E(x) &= 1 \cdot 0.2 + 2 \cdot 0.7 + 3 \cdot 0.1 \\ &= 0.2 + 1.4 + 0.3 \\ &= 1.9 \end{aligned}$$

This means, on average, we will receive about 1.9 (k) if we choose option 1.

Example : calculate $E(x)$

x	0	2	3	5
$P(x)$	0.1	0.3	0.3	0.3

$$\begin{aligned} E(x) &= 0 \times 0.1 + 2 \times 0.3 + 3 \times 0.3 + 5 \times 0.3 \\ &= 0 + 0.6 + 0.9 + 1.5 \end{aligned}$$

$$E(x) = 3.$$

(*) The variance of x , denoted by $V(x)$, measures how much x varies around $E(x)$.

x	0	1	2
$P(x)$	0.1	.8	0.1

$$\begin{aligned} E(x) &= 0 \times 0.1 + 1 \times 0.8 + 2 \times 0.1 \\ &= 1 \end{aligned}$$

$$V(X) = \underbrace{EX^2} - (EX)^2$$

x^2	0^2	1^2	2^2
$P(X)$	0.1	.8	0.1

$$EX^2 = 0^2 * 0.1 + 1^2 * .8 + 2^2 * .1$$

$$= 0 + .8 + .4$$

$$= \underline{\underline{1.2}}$$

$$V(X) = EX^2 - (EX)^2 = 1.2^2 - 1^2$$

$$= .44$$

Example :

x	1	2	3
$P(X)$.3	.5	.2

calculate EX and $V(X)$

$$\textcircled{1} \quad E(X) = 1 \cdot .3 + 2 \cdot .5 + 3 \cdot .2$$
$$= 1.9$$

$$\textcircled{2} \quad E X^2 = 1^2 \cdot .3 + 2^2 \cdot .5 + 3^2 \cdot .2$$
$$= .3 + 2 + 1.8 = 4.1$$

$$\Rightarrow V(X) = E X^2 - (E X)^2$$

$$= 4.1 - 1.9^2 = 0.49.$$

Assignment 15

①

x	1	2	6
$P(x)$	0.4	0.3	0.3

calculate $E(x)$ and $V(x)$

② which option is better?

Option 1: Receive x thousands \$

where x follows the distribution above.

Option 2: Receive for sure 3 thousands dollars.