# Poisson Distribution

Example:

(1)

Let X be the number of text messages you receive a day. Suppose that you receive on average 5 texts a day.

X here is an example of a possion distribution with lambda = 5 ($\lambda = 5$).

(2) Let X be the number of goals in a soccer match at the World Cup. Suppose a match has on average about 3 goals.

X follows a Poisson distribution with $\lambda = 3$.

Definition:

Poisson Distribution is the distribution of the number of events occuring in a given time period, given the average number of times the events occurs during that period.

The average number of times the event occurs is called lambda ($\lambda$).

$$X \sim \text{Poisson}(\lambda)$$

(*) Some properties of Poisson Distribution.

(1) $EX = \lambda$

(2) $V(X) = \lambda$

(3) $P(X = k) = \frac{\lambda^k \cdot e^{-\lambda}}{k!} = p(k)$

(This is the prob. distribution function of Poisson ($\lambda$))

Example:

(1) Suppose you receive about 5 texts a day on average. Find the prob. that you receive no texts today.

$$X \sim \text{Poisson}(5)$$

$$P(X = 0) = \frac{5^0 \cdot e^{-5}}{0!} \approx .006738$$

Find the prob. of you receiving exactly 5 texts today.

$$P(X = 5) = \frac{5^5 \cdot e^{-5}}{5!} = .1755$$

② Let $X$ be the number of goals in a soccer match at the World Cup. Suppose a match has on average about 3 goals.

Find the prob. that a match has almost 3 goals.

$$P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$$

we have $P(X = k) = \frac{3^k \cdot e^{-3}}{k!}$

$$P(X=0) = \frac{3^0 \cdot e^{-3}}{0!} = .0498$$

$$P(X=1) = \frac{3^1 \cdot e^{-3}}{1!} = .1494$$

$$P(X=2) = \frac{3^2 \cdot e^{-3}}{2!} = .2240$$

$$P(X=3) = \frac{3^3 \cdot e^{-3}}{3!} = .2240$$

$$\Rightarrow P(X<3) = .0498 + .1494 + .224 + .224$$

$$= .6472.$$

Find the prob. that the rock will have at least 3 goals.

$$P(X \geq 3) = P(X=3) + P(X=4)$$

$$+ P(X=5) + P(X=6)$$

. . . . . .

$$P(x \geq 3) = 1 - P(x \leq 2)$$

$$= 1 - P(x=0) - P(x=1) - P(x=2)$$

$$= 1 - .0498 - .1494 - .224$$

$$= .5768.$$