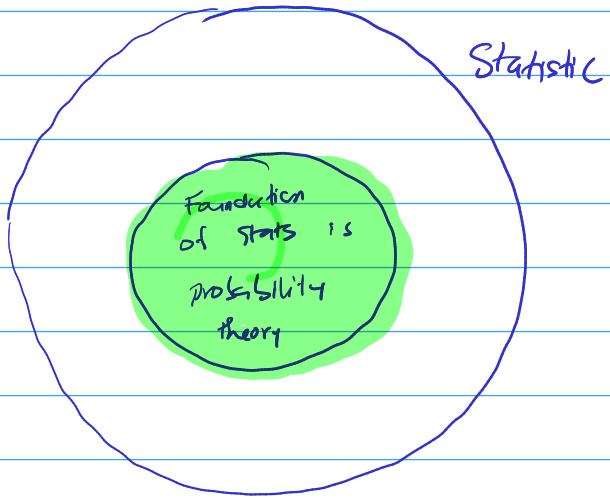
Probability Theory.



* Random variables and their probability distribution.

Example:

consider an experiment of tossing a fair coin one time.

let X be the number of tail observed.

what is the value for X ?

X could be 0 or X could be 1.

X is a random variable.

what is the pros. That $X = 0$?

$P(X = 0) = 1/2$

Similarly, the pob. $X = 1$ is

$P(X = 1) = 1/2$

$X = 0$ with prob 50%

$X = 1$ with prob 50%

we present this information as follows

X	0	1
$P(X)$	1/2	1/2

This is call the prob. distribution of X.

Example: Consider the experiment of tossing a fair coin twice.

Let X be the number of times we observe Tail.

X could be 0, 1, 2.

X is a random variable.

what is the prob. distribution of X?

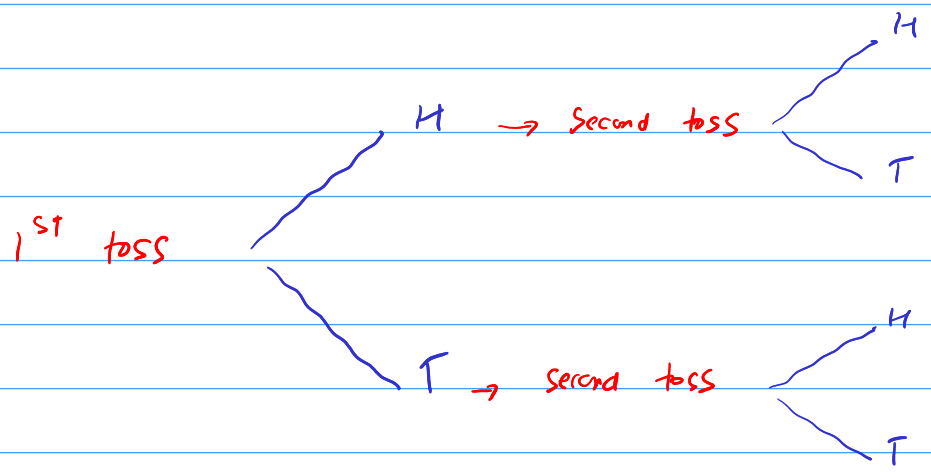
This means

$P(X = 0) = ?$

$P(X = 1) = ?$

$P(X = 2) = ?$

Let find the prob. distribution of X!



All the possibilities when tossing a coin twice:

HH, HT, TH, TT

$P(HH) = P(HT) = P(TH) = P(TT) = 1/4$

$P(X=0) = P(HH) = 1/4$

$P(X=1) = P(TH) + P(HT) = 1/4 + 1/4 = 1/2$

$P(X=2) = P(TT) = 1/4$

So the prob. distribution for X is

X	0	1	2	
P(X)	1/4	1/2	1/4	$\Sigma = 1$

notice that $P(X=0) + P(X=1) + P(X=2) = 1$

Example:

consider an experiment of rolling a die twice.

Let X be the sum of two numbers observed.

X is a random variable.

what are the values X can take?

1st rolling = {1, 2, 3, 4, 5, 6}

2nd rolling = {1, 2, 3, 4, 5, 6}

X = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

Find the prob. X = 5

$P(X = 5)$ = P(First time 1, Second time 4)

+ P(First time 4, Second time 1)

+ P(First time 2, second time 3)

+ P(First time 3, second time 2)

$P(\text{First time } 1, \text{ second time } 4) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

$P(\text{First time } 4, \text{ second time } 1) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

$P(\text{First time } 2, \text{ second time } 3) = 1/36$

$P(\text{First time } 3, \text{ second time } 2) = 1/36$

$\Rightarrow \quad P(X = 5) = \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \frac{4}{36} = \frac{1}{9}$

Assignment 8:

1. Consider an experiment of tossing a fair coin three times and let X be the number of times we observe Tail. Find the probability distribution of X.

2. Consider an experiment of rolling a die twice and let X be the summation of the two numbers observed. Find the probability X is 6.

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⑧ The Expected value of a random variable.

Consider a random variable with the prob. distribution below.

X	0	1
$P(X)$	1/2	1/2

Let's play a game. You have 2 options

① Take $300 dollars.

② You have $\$ X$ thousands dollars.

which option would you take? would you change your mind if option (1) offers $600?

we will calculate "The expected value" of X to make the decision.

$$EX = \sum x \cdot p(x)$$

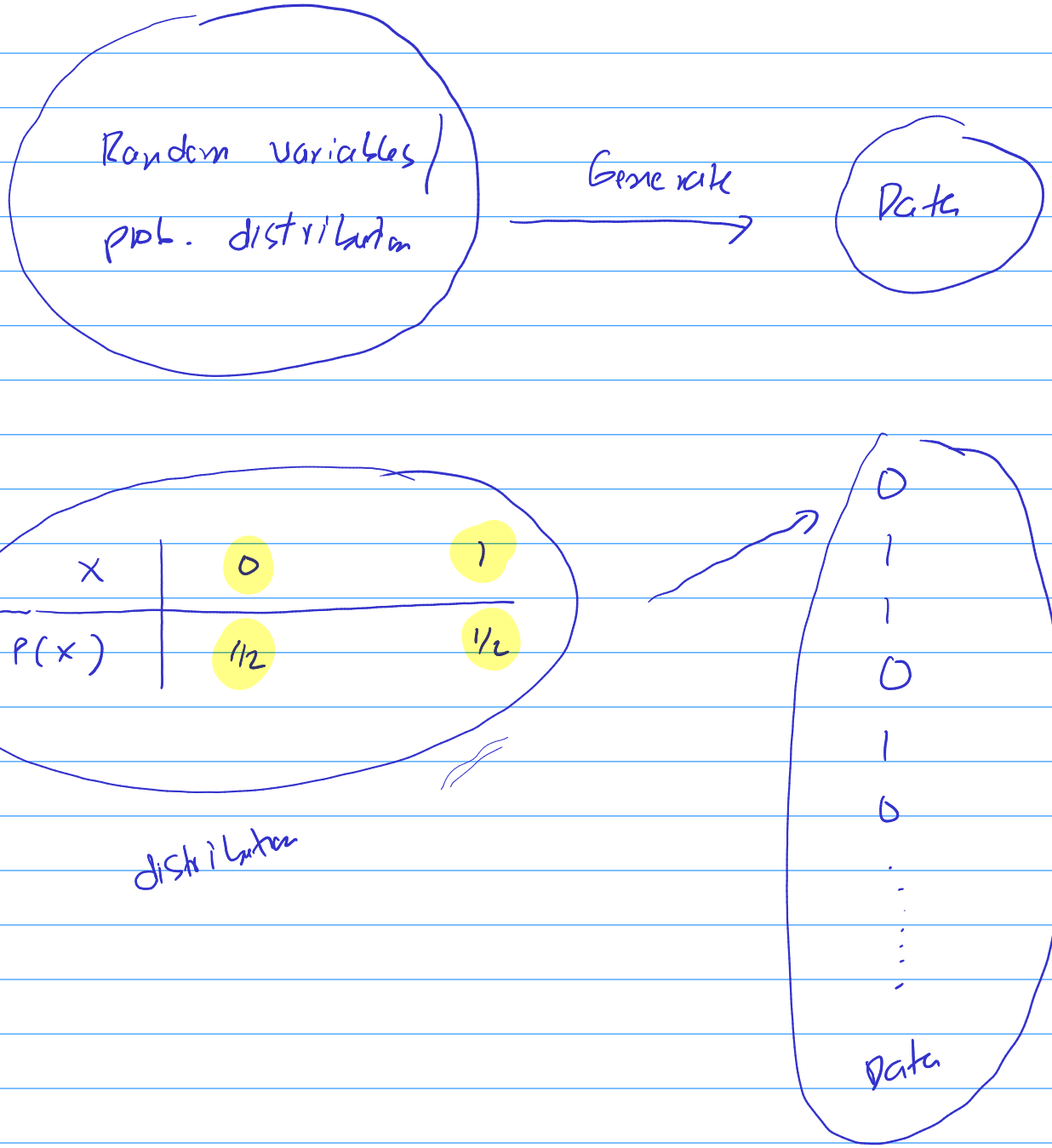
Expected value of X

x	0	1
p(x)	1/2	1/2

$$\Rightarrow EX = 0 \cdot 1/2 + 1 \cdot 1/2 = .5$$

$$\Rightarrow \boxed{EX = .5}$$

⊛ Data simulation / Generation by random variables.



Consider a random variable X with the distribution below.

x	$P(x)$
1	.5
2	.3
3	.1
4	.1

(1)

Find EX :

$$EX = 1 * .5 + 2 * .3 + 3 * .1 + 4 * .1$$

$$= .5 + .6 + .3 + .4 = 1.8$$

$\Rightarrow$ $\boxed{EX = 1.8}$

(2) Simulate 10000 of X. Calculate the frequency of each values of X and compare the frequencies with the distribution.

(3) Calculate the average of X and E(x)