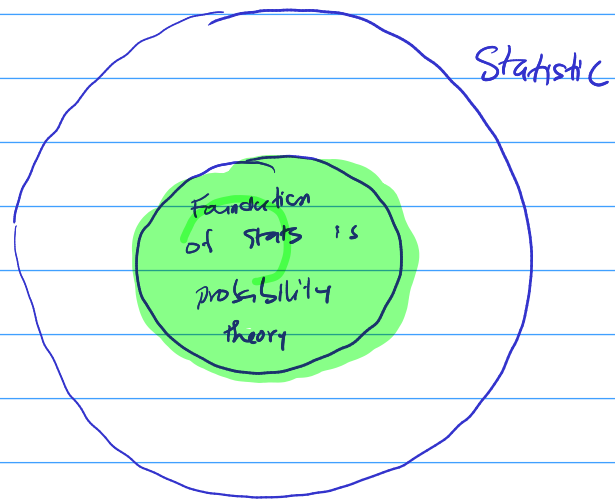


Probability Theory.



⊛ Random Variables and their probability distribution.

Example :

consider an experiment of tossing a fair coin one time.

let X be the number of tail observed.

what is the value for X ?

X could be 0 or X could be 1.

X is a random variable.

what is the prob. that $X = 0$?

$$P(X=0) = 1/2$$

Similarly, the prob. $X = 1$ is

$$P(X=1) = 1/2$$

$X = 0$ with prob 50%

$X = 1$ with prob 50%

we present this information as follows

X	0	1
$P(X)$	$1/2$	$1/2$

This is call the prob. distribution of X .

Example : Consider the experiment of tossing a fair coin twice.

Let X be the number of times we observe Tail.

X could be 0, 1, 2.

X is a random variable.

What is the prob. distribution of X ?

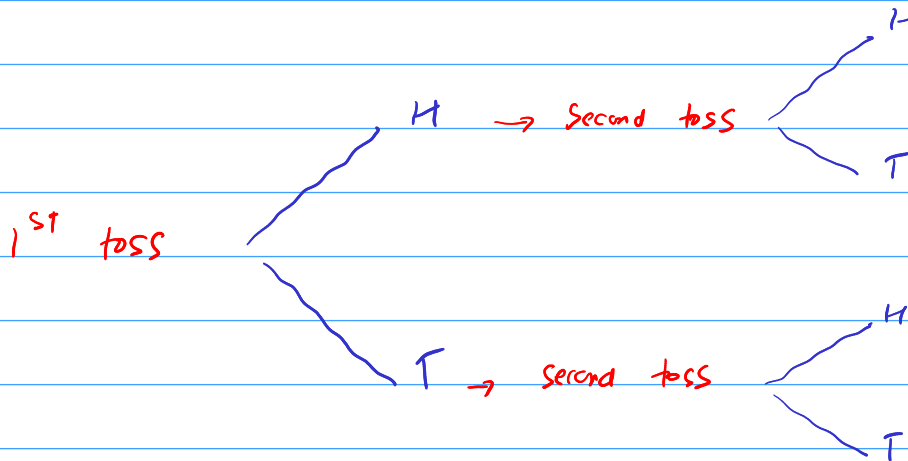
This means

$$P(X = 0) = ?$$

$$P(X = 1) = ?$$

$$P(X = 2) = ?$$

let find the prob. distribution of X !



All the possibilities when tossing a coin twice:

HH, HT, TH, TT

$$P(HH) = P(HT) = P(TH) = P(TT) = 1/4$$

$$P(X=0) = P(HH) = 1/4$$

$$P(X=1) = P(TH) + P(HT) = 1/4 + 1/4 = 1/2$$

$$P(X=2) = P(TT) = 1/4$$

So the prob. distribution for X is

x	0	1	2	
$P(x)$	$1/4$	$1/2$	$1/4$	$\Sigma = 1$

notice that $P(X=0) + P(X=1) + P(X=2) = 1$

Example:

consider an experiment of rolling a die twice.

let X be the sum of two numbers observed.

X is a random variable.

what are the values X can take?

1st rolling = $\{1, 2, 3, 4, 5, 6\}$

2nd rolling = $\{1, 2, 3, 4, 5, 6\}$

$X = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

Find the prob. $X = 5$

$$P(X = 5) = P(\text{First time } 1, \text{ Second time } 4)$$

$$+ P(\text{First time } 4, \text{ Second time } 1)$$

$$+ P(\text{First time } 2, \text{ Second time } 3)$$

$$+ P(\text{First time } 3, \text{ Second time } 2)$$

$$P(\text{First time } 1, \text{ Second time } 4) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

$$P(\text{First time } 4, \text{ Second time } 1) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

$$P(\text{First time } 2, \text{ second time } 3) = 1/36$$

$$P(\text{First time } 3, \text{ second time } 2) = 1/36$$

$$\Rightarrow P(X = 5) = \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \frac{4}{36} = \frac{1}{9}$$

Assignment 8:

1. Consider an experiment of tossing a fair coin three times and let X be the number of times we observe Tail. Find the probability distribution of X .
2. Consider an experiment of rolling a die twice and let X be the summation of the two numbers observed. Find the probability X is 6.