Some properties of Expectation.

① $E(a \cdot X + b) = a \cdot E(X) + b$

X: Random variable

a and b are: constant numbers (non-random)

For example:

$$E(2X + 3) = 2 \cdot E(X) + 3$$

②

$$E(a) = a$$

where a is non-random / constant.

Ex: $E(100) = 100$

$E(-6) = -6$

③ $E(X + Y) = E(X) + E(Y)$

where X and Y are some random variables.

④ $E(X - Y) = E(X) - E(Y)$

where X and Y are some random variables.

Example: Rolling an unfair die, we obtain the following

X	1	2	3	4	5	6
P(X)	0.1	0.1	0.2	0.2	0.3	0.1

① $P(X \geq 3) = P(X=3) + P(X=4) + P(X=5) + P(X=6)$

$= 0.2 + 0.2 + 0.3 + 0.1 = 0.8$

another way:

$P(X \geq 3) = 1 - P(X \leq 2)$

$= 1 - (P(X=1) + P(X=2))$

$= 1 - (.1 + .1) = .8$

② Find prob. of X at least 4

$P(X \geq 4) = P(X=4) + P(X=5) + P(X=6)$

$= .2 + .3 + .1 = .6$

(3) Calculate $E(X)$

$$E(X) = 1 \times .1 + 2 \times .1 + 3 \times .2 + 4 \times .2 + 5 \times .3 + 6 \times .1$$

$$= .1 + .2 + .6 + .8 + 1.5 + .6$$

$$= 3.8$$

(4) $E(2X + 4)$

$$= E(2X) + E(4)$$

$$= 2E(X) + 4$$

$$= 2 \times 3.8 + 4 = 11.6$$

Another way:

X	1	2	3	4	5	6
$P(X)$	0.1	0.1	0.2	0.2	0.3	0.1

$2X + 4$	6	8	10	12	14	16
$P(X)$	0.1	0.1	0.2	0.2	0.3	0.1

$$E(2X+4) = 6 \times .1 + 8 \times .1 + 10 \times .2 + 12 \times .2 + 14 \times .3 + 16 \times .1$$

$$= 11.6$$

Assignment 16:

Suppose X has the following distribution.

X	1	2	3	4	5	6	7	8
$P(X)$	0.1	0.1	0.2	0.15	0.15	0.1	.15	.05

Calculate:

(1) $P(X > 6)$

(2) $P(X \geq 2)$

(3) Prob. of X being an even number.

(4) Prob of X being at least 3.

(5) $E(X)$

(6) $E(4X - 9)$