Regression ?

Measuring Performance in Classification Models

author: Son Nguyen

Reading Materials

- Max Kuhn. Chapter 11.

Two outcomes of classification models

- Predicted Probabilities
- Class Prediction

Examples

- Predicting if a passenger in the titanic is survived or not survived
- The outcome could look like this.

ID	Prob. of Survived	Prediction
1	0.55	Survived
2	0.2	Not Survived
3	0.94	Survived
4	0.63	Survived
5	0.9	Survived
6	0.35	Not Survived
7	0.84	Survived
8	0.38	Not Survived
9	0.01	Not Survived
10	0.68	Survived
11	0.71	Survived
12	0.45	Not Survived

Examples

- Notice that this model predicts “Survived” for passengers with the probabilities of being greater than 0.5
- 0.5 is called **cut-off value**.
- The cuff-off value is set by 0.5 by default.
- The cut-off value can be changed by the modeler.

Confusion Matrices

	Predicted Positive	Predicted Negative
Actual Positive	True Positive (TP)	False Negative (FN)
Actual Negative	False Positive (FP)	True Negative (TN)

Confusion Matrices

Actual Values

1 0

Predicted Values

1

0

Confusion Matrices - Example

- "Survived" = **"Positive"**
- "Not Survived" = **"Negative"**

ID	Prob. of Survived	Prediction	Truth	Evaluation
1	0.55	Survived	Survived	TP
2	0.2	Not Survived	Survived	FN
3	0.94	Survived	Survived	TP
4	0.63	Survived	Not Survived	FP
5	0.9	Survived	Survived	TP
6	0.35	Not Survived	Not Survived	TN
7	0.84	Survived	Not Survived	FP
8	0.38	Not Survived	Not Survived	TN
9	0.01	Not Survived	Not Survived	TN
10	0.68	Survived	Survived	TP
11	0.71	Survived	Survived	TP
12	0.45	Not Survived	Survived	FN

Confusion Matrices

	Predicted Positive	Predicted Negative
Actual Positive	5	2
Actual Negative	2	3

Model evaluation from Confusion Matrices

	Predicted Positive	Predicted Negative
Actual Positive	True Positive (TP)	False Negative (FN)
Actual Negative	False Positive (FP)	True Negative (TN)

$$\text{Misclassification Rate} = \frac{FN + FP}{\text{Total}} = \frac{FN + FP}{TN + TP + FN + FP}$$

$$\text{Accuracy} = \frac{TN + TP}{TN + TP + FN + FP}$$

$$\text{Sensitivity} = \frac{TP}{\text{Actual Positive}} = \frac{TP}{TP + FN}$$

$$\text{Specificity} = \frac{TN}{\text{Actual Negative}} = \frac{TN}{TN + FP}$$

Bryant's population: 1% have covid 19. Positive = has covid

Negative = does not have covid

Design a model to detect covid 19.

Consider a model to always has a negative outcome.

	Predicted Pos.	Predicted Neg.	
Actual pos.	0	10	10
Actual Neg.	10	990	990
	0	1000	

Accuracy: 99%

Sens. : 0

Precs. : 0

	Predicted Pos.	Predicted Neg.	
Actual pos.	10	0	10
Actual Neg.	990	0	990
	1000	0	1000

Sens. $= \frac{10}{10} = 100\%$

Prcs. $= \frac{10}{1000} = 1\%$

ACC $= \frac{10}{1000} = 1\%$

Model evaluation from Confusion Matrices

	Predicted Positive	Predicted Negative
Actual Positive	True Positive (TP)	False Negative (FN)
Actual Negative	False Positive (FP)	True Negative (TN)

$$\text{Precision} = \frac{TP}{TP + FP}$$

6	4	10
0	990	990
6	994	1000

$$\text{F1-Score} = 2 \cdot \frac{\text{Precision} \cdot \text{Sensitivity}}{\text{Precision} + \text{Sensitivity}} = \frac{2TP}{2TP + FN + FP}$$

Center measurement of 2 numbers . a and b

① The average / arithmetic mean of a, b

$$\frac{a+b}{2}$$

② Geometric mean : $\sqrt{a.b}$

③ Harmonic mean :

$$\frac{1}{\text{Harmonic mean}} = \frac{\frac{1}{a} + \frac{1}{b}}{2} = \frac{\frac{a+b}{ab}}{2}$$

$$\Leftrightarrow \quad \text{Harmonic mean} = \frac{2ab}{a+b}$$

F1-Score is the harmonic mean of <u>Sensitivity</u> and <u>Precision</u>.

arith mean $\qquad$ geometric mean $\qquad$ Harmonic

$$\frac{a+b}{2} \geq \sqrt{ab} \geq \frac{2ab}{a+b}$$

$a = .7$

$b = .8$

$\Rightarrow$

$$\frac{a+b}{2} = .75$$

$$\sqrt{ab} = .748$$

$$\frac{2ab}{a+b} = .7467$$

$$\frac{1}{F_1\text{-score}} = \frac{\frac{1}{\text{Precision}} + \frac{1}{\text{Sensitivity}}}{2}$$

F_1-score is called the harmonic-mean of Precision and Sensitivity

Confusion Matrices

	Predicted Positive	Predicted Negative
Actual Positive	TP $= 5$	FN $= 2$
Actual Negative	FP $= 2$	TN $= 3$

$$\text{Misclassification Rate} = 4/12$$

$$\text{Accuracy} = 8/12$$

$$\text{Sensitivity} = 5/7$$

$$\text{Specificity} = 3/5$$

$$\text{Precision} = 5/7; \text{F1-Score} = 5/7$$

ROC Curves

- Notice that all of the measures calculated in the last slide are based on the **cut-off 0.5**
- What if we change the cut-off value, **c**?

ROC Curves

- What is the best cut-off value?

predict all pos

Cut-off Values	Sensitivity	Specificity
c = 0	1.0000000	0.0
c = 0.1	1.0000000	0.2
c = 0.2	0.8571429	0.2
c = 0.3	0.8571429	0.2
c = 0.4	0.8571429	0.6
c = 0.5	0.7142857	0.6
c = 0.6	0.5714286	0.6
c = 0.7	0.4285714	0.8
c = 0.8	0.2857143	0.8
c = 0.9	0.1428571	1.0
c = 1	0.0000000	1.0

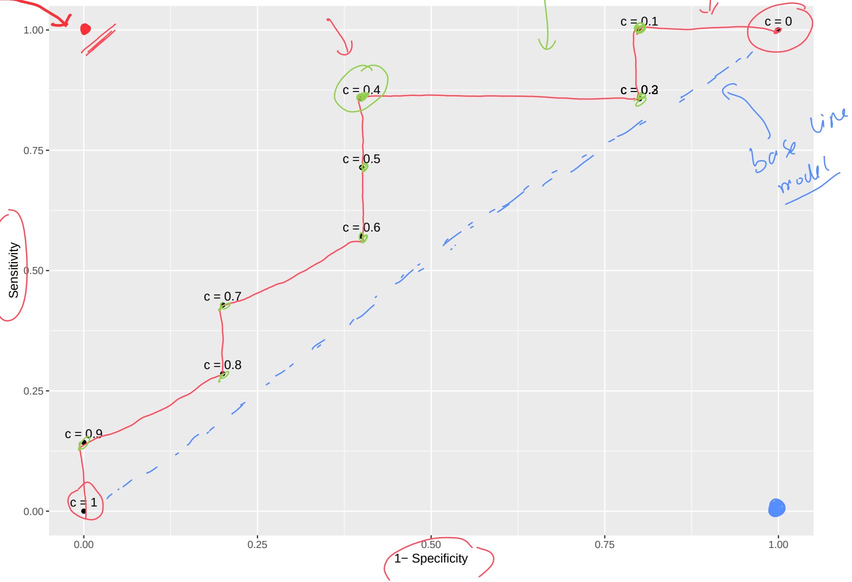
predict all neg.

ROC

perfect model

model 1

- **Question**: What is the best cut-off value?



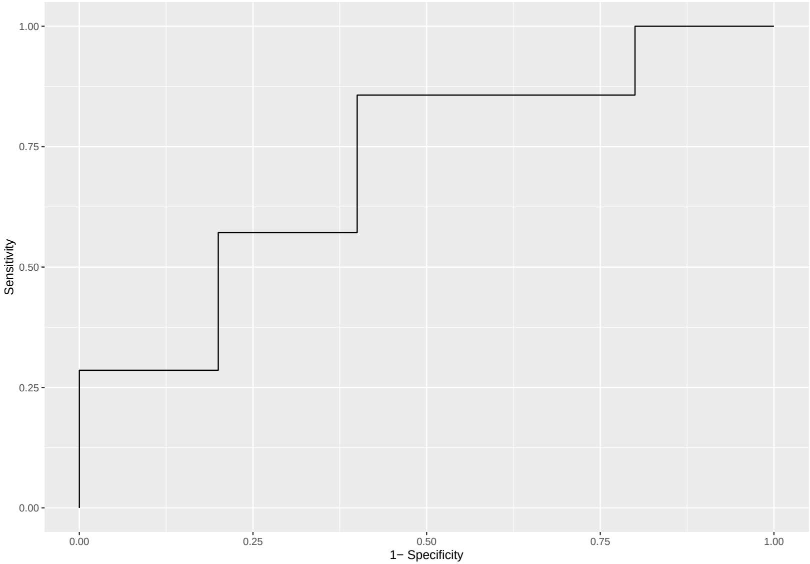
ROC Curve

- **Question**: What is the best cut-off value?
- **Answer**: $c = 0.4$ is the best cut-off value

ROC Curve

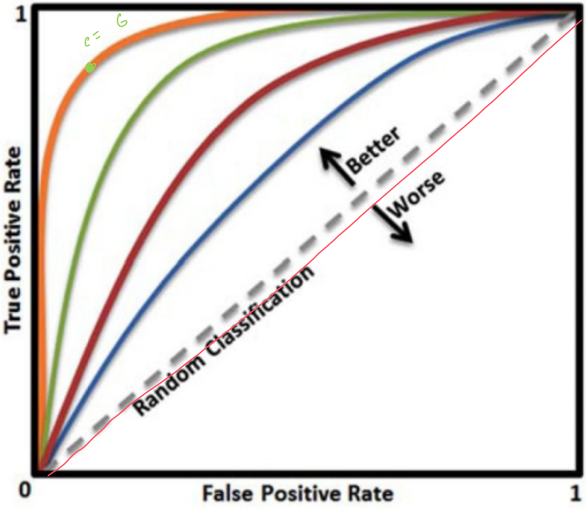
- Each cut-off value **c** results a pair of (1-Specificity, Sensitivity) or (TP Rate, FP Rate)
- The collections of all these pairs/points for all the cut-off values is the Receiver operating characteristic Curve (ROC Curve)

ROC Curve of the example model

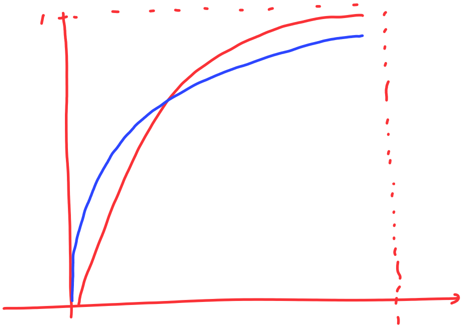


- The curve is not very smooth because the data is very small
- With bigger data, the ROC curve will be very “smooth”

ROC Curve



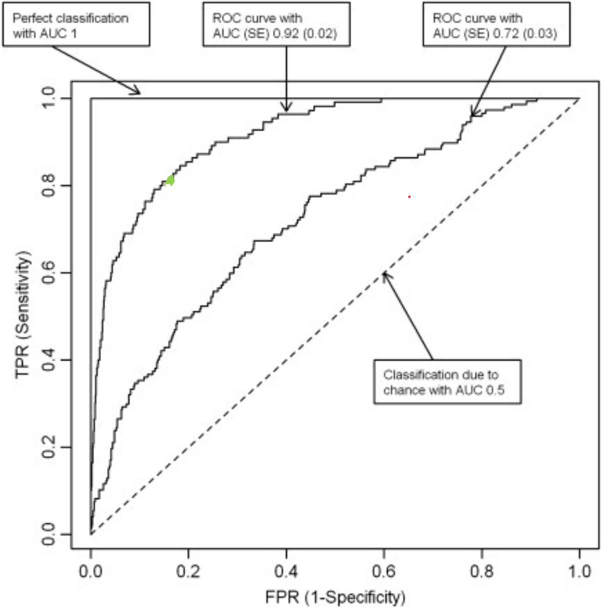
1
0
1
True Positive Rate
False Positive Rate
c = 6
Better
Worse
Random Classification



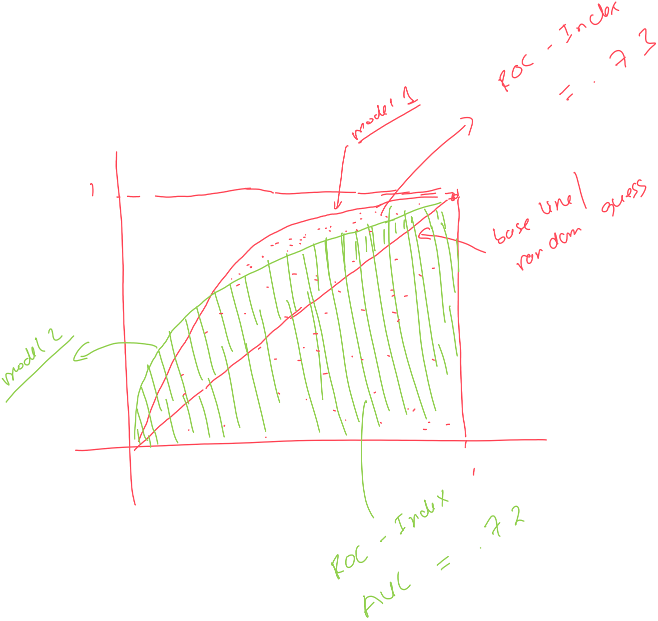
ROC Curve

- The closer the curve to the point (0,1) the better the model
- The best cut-off value is at the point closest to (0,1)
- (0,1) is the **perfect point**, resulting 0 misclassification model.
- At (0,0) the model predicts everything positive
- At (1,1) the model predicts everything negative
- The ROC of the random guess model is the diagonal

ROC Curve

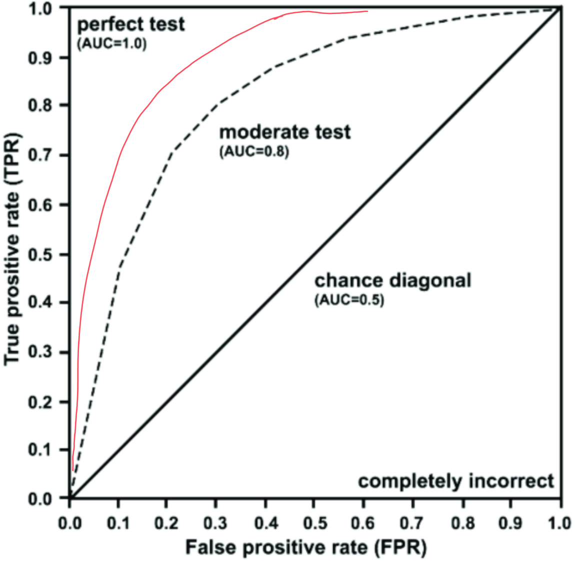


- AUC = Area Under the (ROC) Curve



Model 1 is better in terms of ROC - Index

ROC Curve


perfect test
(AUC=1.0)
moderate test
(AUC=0.8)
chance diagonal
(AUC=0.5)
completely incorrect
True prositive rate (TPR)
False prositive rate (FPR)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

ROC Index

- ROC Index is the area under the ROC Curve

ROC Index - Area Under the Curve (AUC)

- The closer the AUC to 1 the better the model
- The closer the AUC to $1/2$ the worse the model
- Model with AUC $= 1/2$ is as good as a random guess or guessing by tossing a coin
- **Question**: What if the AUC less than $1/2$? Are models with AUC less than $1/2$ **useless**?

Another Question

- **Question**: Is the model with the misclassification rate of 100% the most **useless** model?

Answer

- **Question**: Is the model with the misclassification rate of 100% an useless model?
- *Answer*: No, by flipping the predictions of the models, one gets the **perfect model** with 0 misclassification rate.

Back to the Question

- **Question**: What if the AUC less than 1/2? Are models with AUC less than 1/2 **useless**?
- **Answer**: Model with AUC less than 1/2 could be made to be better by flipping the predictions (if the model predicts positve, flip it to predict negative)

Cumulative Lift

- In the dataset, the ratio of "Survived" is $7/12 = 58.33\%$
- This mean that if we pick **randomly** a passenger in the this group, the chance of picking a "Survived" passenger is 58.33%
- **Question**: If we want to pick a "Survived" passenger, is there a better way than pick randomly?

Cumulative Lift

- **Question**: If we want to pick a "Survived" passenger, is there a better way than pick randomly?
- **Answer**: Yes, we should pick the one with the highest predictied probability.

Cumulative Lift

Order	Predicted Probabilities	True Values
1	0.94	1
2	0.90	1
3	0.84	0
4	0.71	1
5	0.68	1
6	0.63	0
7	0.55	1
8	0.45	1
9	0.38	0
10	0.35	0
11	0.20	1
12	0.01	0

- Pick randomly, "success rate" is 58.33%
- Pick the top 1, success rate is $1/1 = 100\%$
- We say, at $1/12 = 8.33\%$, the model lift is $100/58.33 = 1.71$

Cumulative Lift

Order	Predicted Probabilities	True Values
1	0.94	1
2	0.90	1
3	0.84	0
4	0.71	1
5	0.68	1
6	0.63	0
7	0.55	1
8	0.45	1
9	0.38	0
10	0.35	0
11	0.20	1
12	0.01	0

- Pick randomly, "success rate" is 58.33%
- Pick the top 2, success rate is $2/2 = 100\%$
- We say, at $2/12 = 16.67\%$, the model lift is $100/58.33 = 1.71$

Cumulative Lift

Order	Predicted Probabilities	True Values
1	0.94	1
2	0.90	1
3	0.84	0
4	0.71	1
5	0.68	1
6	0.63	0
7	0.55	1
8	0.45	1
9	0.38	0
10	0.35	0
11	0.20	1
12	0.01	0

- Pick randomly, "success rate" is 58.33%
- Pick the top 2, success rate is $2/2 = 100\%$
- We say, at $2/12 = 16.67\%$, the model lift is $100/58.33 = 1.71$

Cumulative Lift

Order	Predicted Probabilities	True Values
1	0.94	1
2	0.90	1
3	0.84	0
4	0.71	1
5	0.68	1
6	0.63	0
7	0.55	1
8	0.45	1
9	0.38	0
10	0.35	0
11	0.20	1
12	0.01	0

- Pick randomly, "success rate" is 58.33%
- Pick the top 3, success rate is $2/3 = 66.66\%$
- We say, at $3/12 = 25\%$, the model lift is $66.66/58.33 = 1.14$

Cumulative Lift

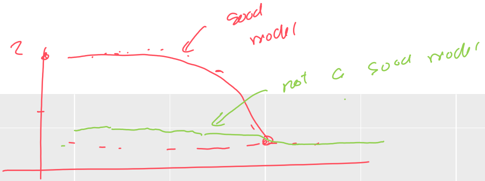
Order	Predicted Probabilities	True Values
1	0.94	1
2	0.90	1
3	0.84	0
4	0.71	1
5	0.68	1
6	0.63	0
7	0.55	1
8	0.45	1
9	0.38	0
10	0.35	0
11	0.20	1
12	0.01	0

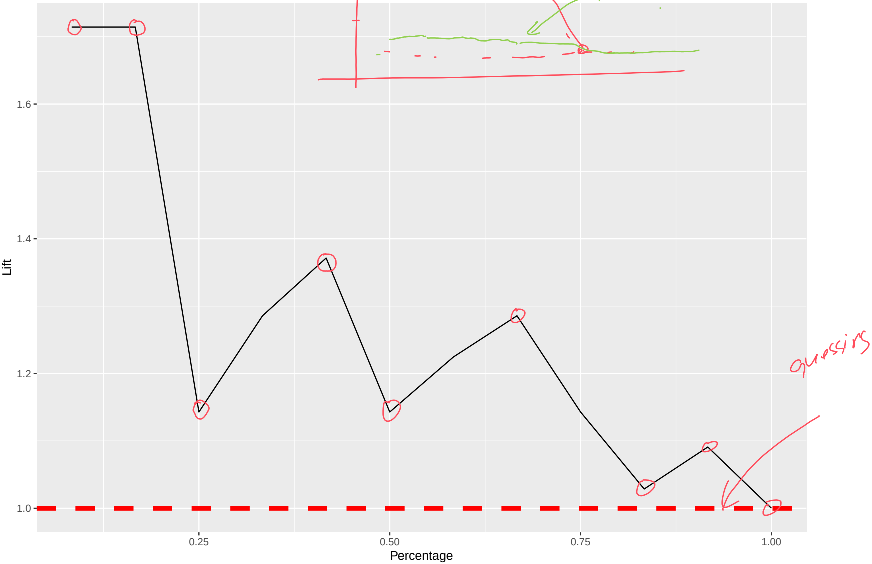
- Pick randomly, "success rate" is 58.33%
- Pick the top 4, success rate is $3/4 = 75\%$
- We say, at $4/12 = 25\%$, the model lift is $75/58.33 = 1.28$

Cumulative Lift

Percentage	Lift
0.0833333	1.714286
0.1666667	1.714286
0.2500000	1.142857
0.3333333	1.285714
0.4166667	1.371429
0.5000000	1.142857
0.5833333	1.224490
0.6666667	1.285714
0.7500000	1.142857
0.8333333	1.028571
0.9166667	1.090909
1.0000000	1.000000

Cumulative Lift





Cumulative % Response

Percentage	Percent_Response
0.0833333	1.0000000
0.1666667	1.0000000
0.2500000	0.6666667
0.3333333	0.7500000
0.4166667	0.8000000
0.5000000	0.6666667
0.5833333	0.7142857
0.6666667	0.7500000
0.7500000	0.6666667
0.8333333	0.6000000
0.9166667	0.6363636
1.0000000	0.5833333

Cumulative % Response

(positive response)