Question 1. *Calculation*

Given the following time series y_t.

t	y_t
1	1
2	3
3	5
4	8
5	12

a. Calculate the exponential smoothing series with $w = .2$

b. Calculate the double exponential smoothing series with $w = .2$

a. Calculate the exponential smoothing series with $w = .2$

y_t	
1	
3 ← y_2	
5 ← y_3	
8	
12	

$s_1 = y_1$, and

$s_t = s_{t-1} + (1-w)(y_t - s_{t-1})$

$= (1-w)y_t + ws_{t-1}$

$s_1 = y_1 = 1$

$s_2 = (1-w)y_2 + .2 \cdot s_1$

$= .8\, y_2 + .2\, s_1$

$= .8 * 3 + .2 * 1 = 2.6$

$s_3 = (1-.2) \cdot y_3 + .2 * s_2$

$= .8 * 5 + .2 * (2.6)$

$= 4 + .52 = 4.52$

$s_4 = .8 * y_4 + .2 * s_3$

$= .8 * 8 + .2 * 4.52$

$= 7.3$

$s_5 = .8 * y_5 + .2 * s_4$

$= .8 * 12 + .2 * 7.3$

$= 11.06.$

b. Calculate the double exponential smoothing series with $w = .2$

Y_t	$S_t^{(1)}$	$S_t^{(2)}$
1	1	1
3	2.6	2.28
5	4.52	4.072
8	7.3	6.6576
12	11.0608	10.16016

Similarly

$S_1^{(2)} = S_1^{(1)} = 1$

$S_t^{(2)} = (1-w) S_t^{(1)} + w S_{t-1}^{(2)}$

$S_2^{(2)} = (1-.2) \cdot S_2^{(1)} + .2 \cdot S_1^{(2)}$

$= .8 * 2.6 + .2 * .1$

$= 2.28$

Question 2. *Forecasting Linear Trend Time Series*

We assume the series in question 1 has linear trend. Use double exponential smoothing to estimate the linear trend (slope) of the time series. Use the estimated linear trend to predict the next value (y_8)

Estimate the trend: (slope)

$$b_1 = \frac{1-w}{w}(s_T^{(1)} - s_T^{(2)}) = \frac{1-.2}{.2}\left(S_5^{(1)} - S_5^{(2)}\right)$$

$= 3.52256$

$\hat{Y}_8 = \hat{Y}_{5+3} = S_5^{(1)} + b_1 \cdot 3$

$= 11.0608 + 3.52256 * 3$

$= 21.62848$