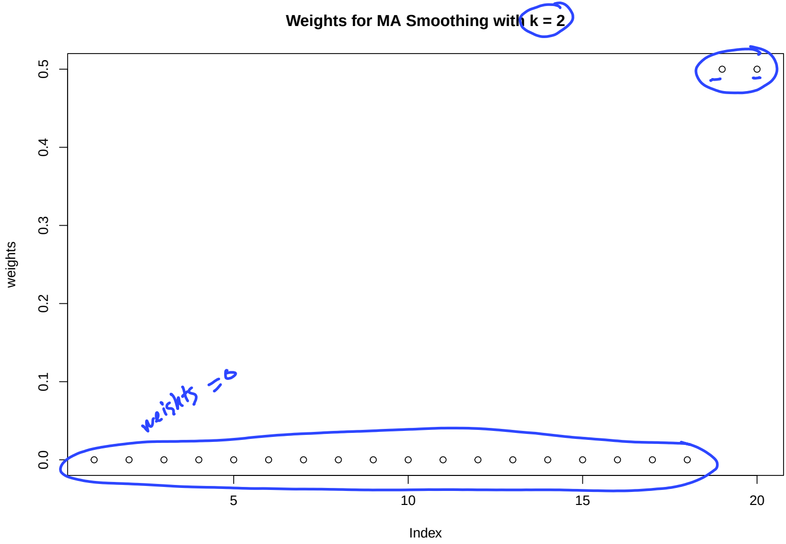
Time Series

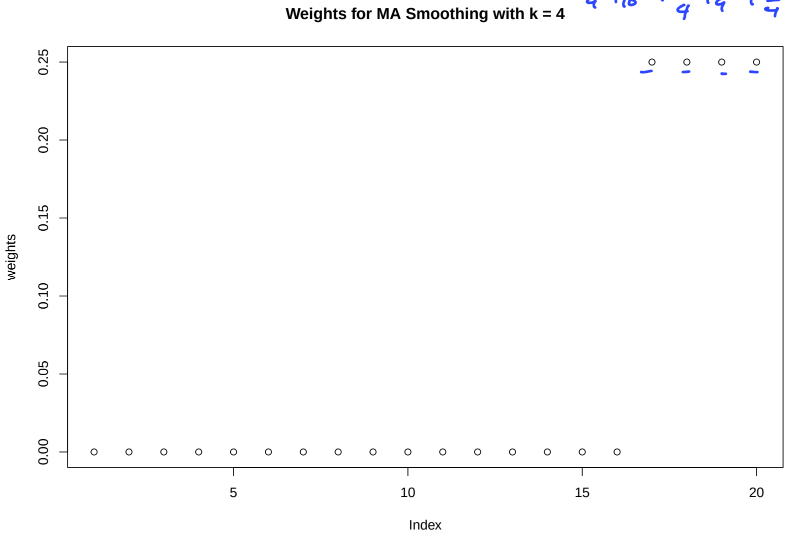
MA Weights Distribution

$S_{10} = \frac{Y_{10} + Y_9}{2}$

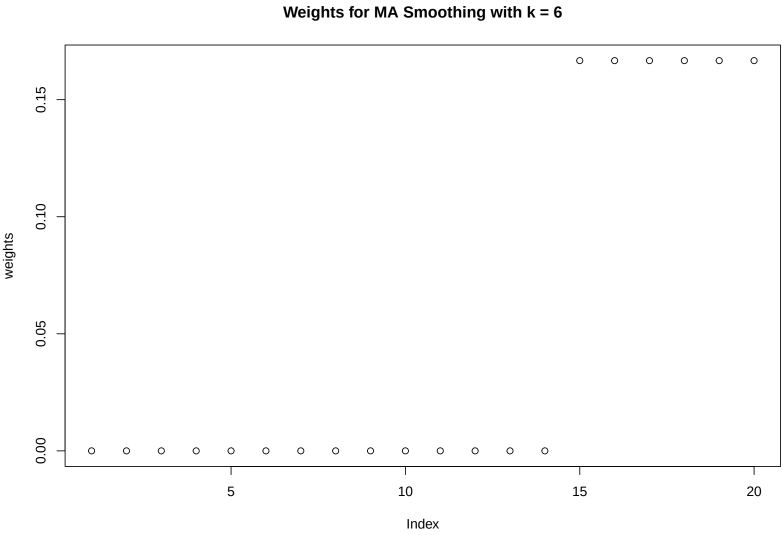


MA Weights Distribution

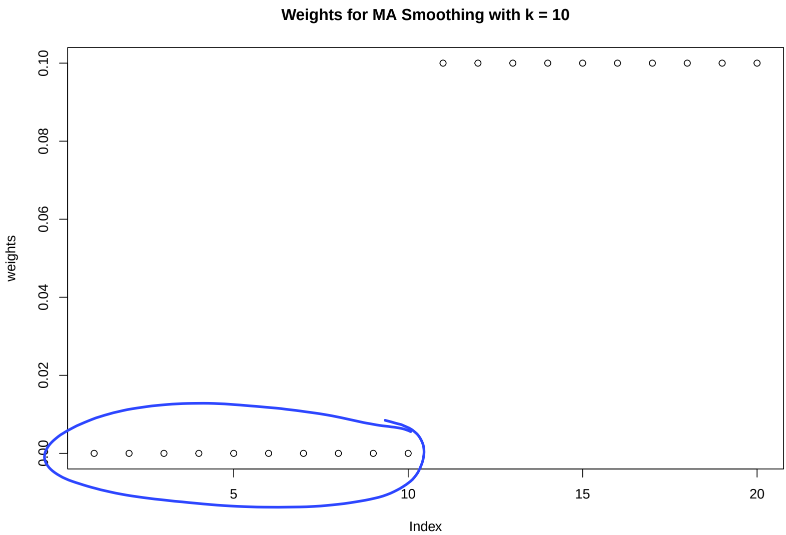
$$S_{10} = \frac{y_{10} + y_9 + y_8 + y_7}{4}$$
$$= \frac{1}{4} y_{10} + \frac{1}{4} y_9 + \frac{1}{4} y_8 + \frac{1}{4} y_7$$



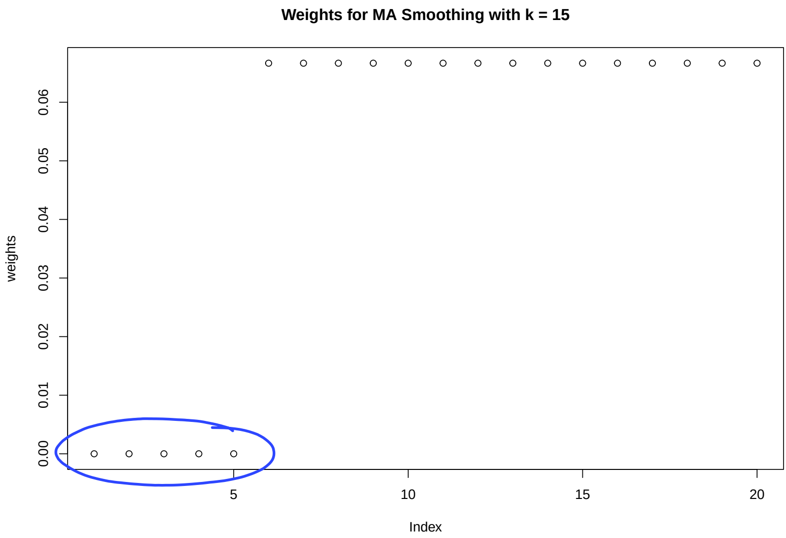
MA Weights Distribution



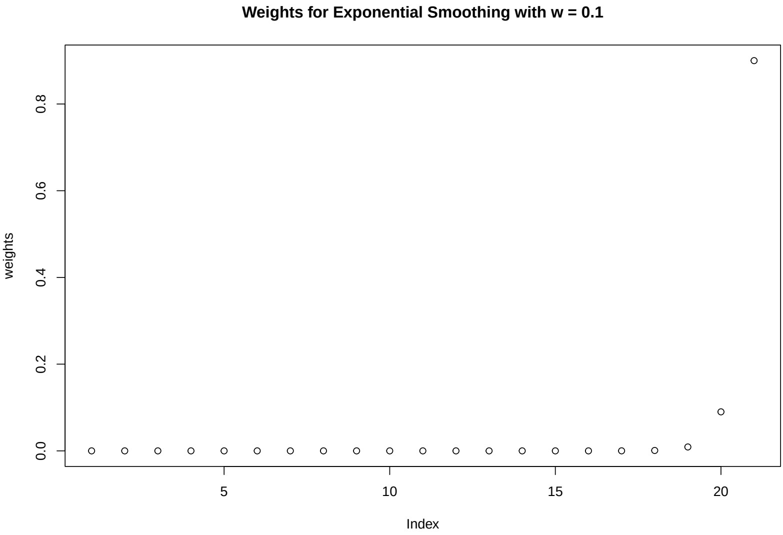
MA Weights Distribution



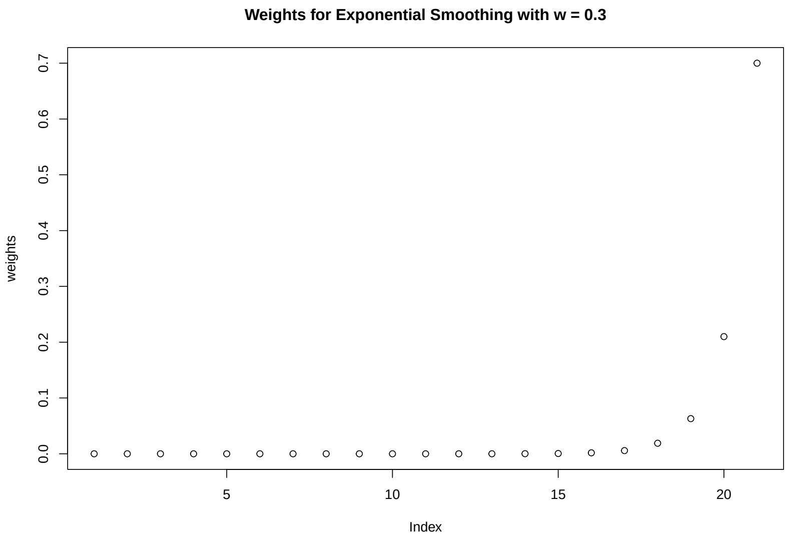
MA Weights Distribution



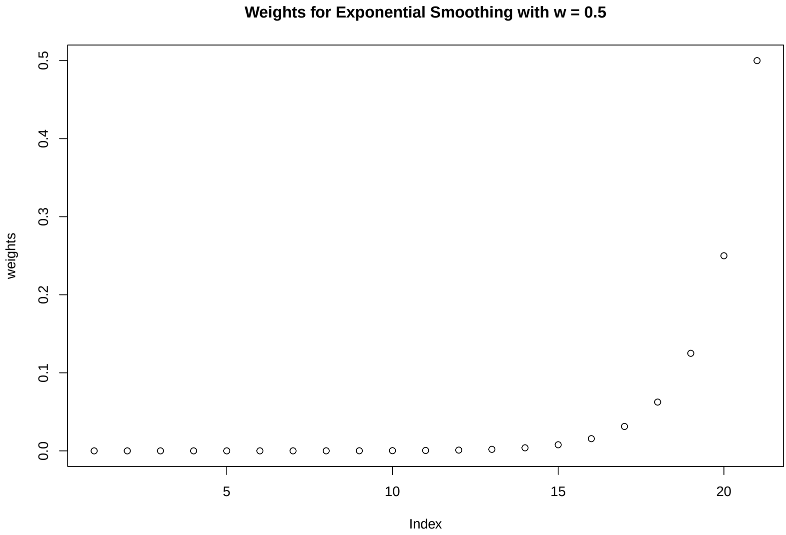
Exponential Weights Distribution



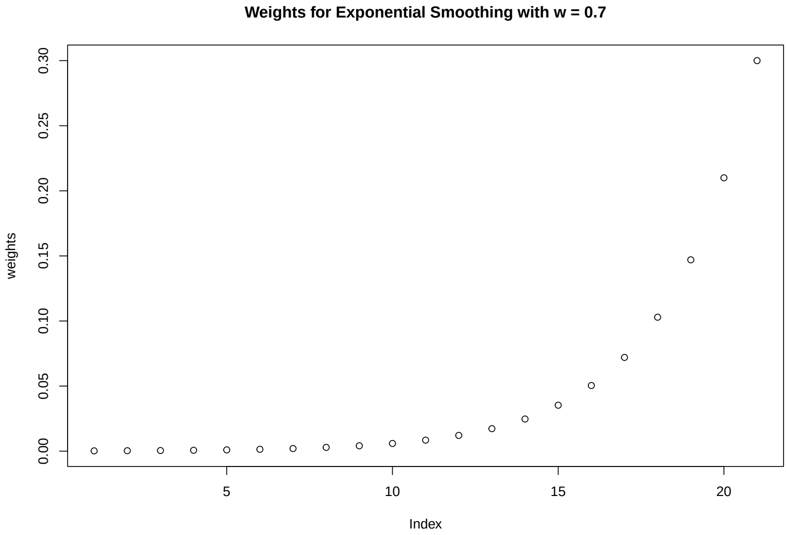
Exponential Weights Distribution



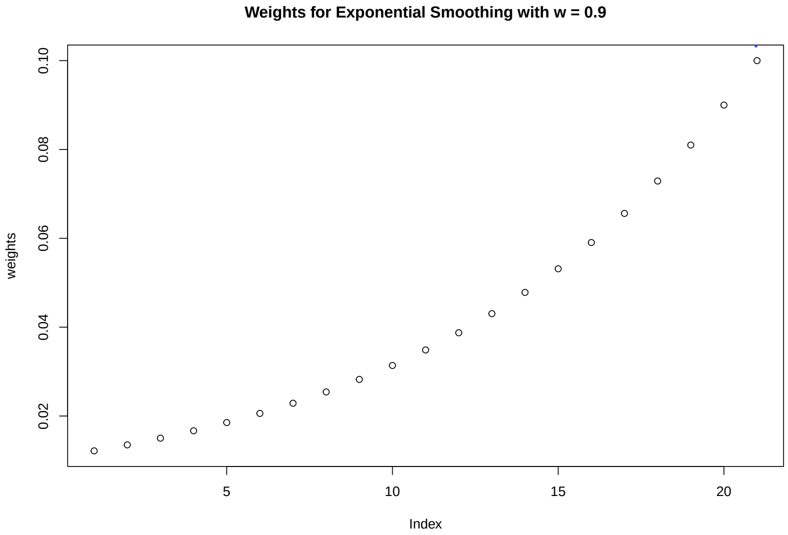
Exponential Weights Distribution



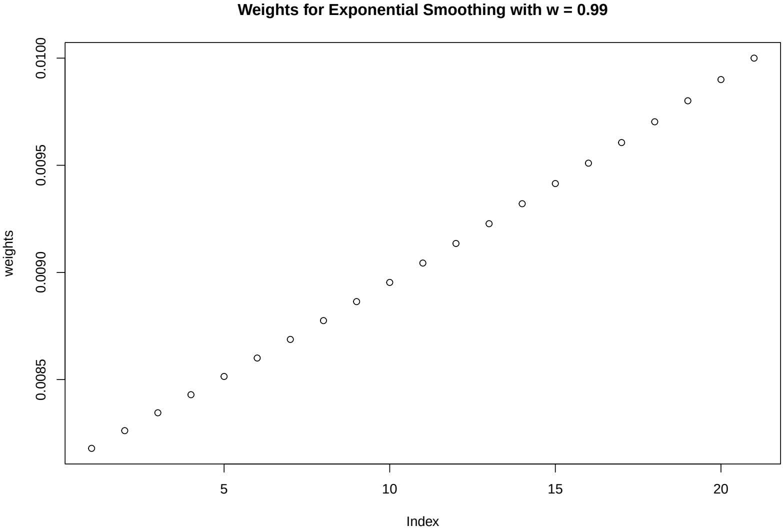
Exponential Weights Distribution



Exponential Weights Distribution



Exponential Weights Distribution



Exponential Smoothing

- MA distributes the weight equally to the recent observations
- Exponential Smoothing controls the weights of the recent observations by w

$$s_t = \frac{y_t + wy_{t-1} + w^2 y_{t-2} + \ldots + w^t y_0}{1/(1-w)}$$

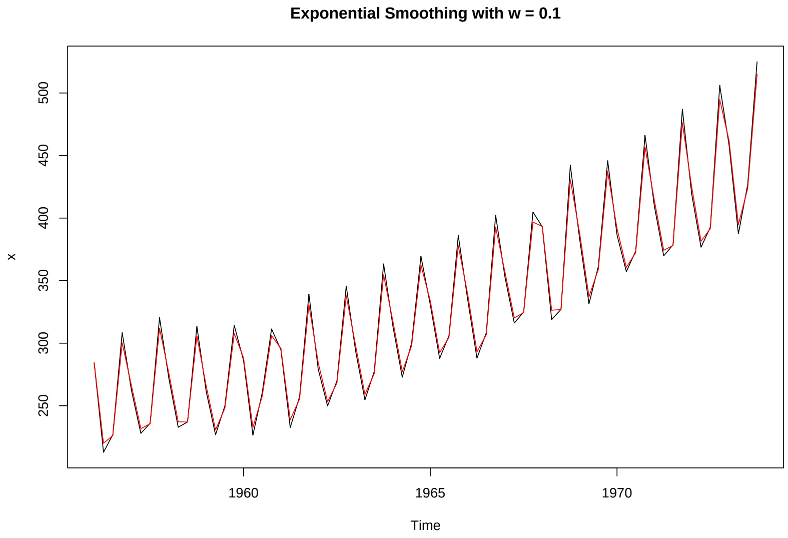
- Smaller w smooths the series more lightly.
- Greater w smooths the series more strongly.

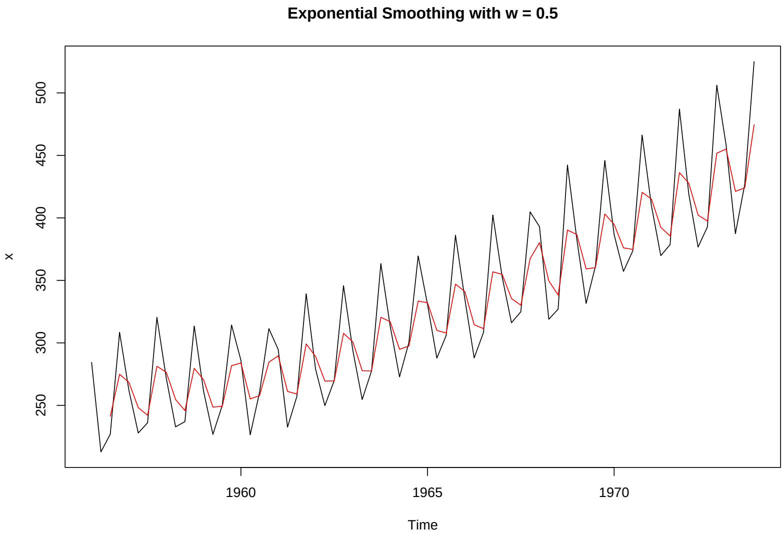
Another Formula:

- Exponential Smoothing can be calculated by

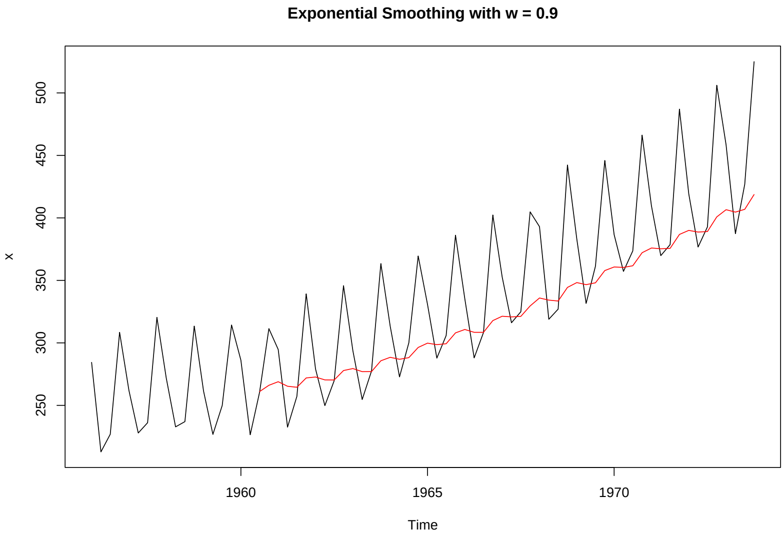
$$
\begin{aligned}
s_1 &= y_1, \text{and} \\
s_t &= s_{t-1} + (1-w)(y_t - s_{t-1}) \\
&= (1-w)y_t + w s_{t-1}
\end{aligned}
$$

- Notice that: when $w \to 0$, $s_t \to y_t$, or little smoothing has taken.


Exponential Smoothing with w = 0.1
x
250
300
350
400
450
500
1960
1965
1970
Time



Exponential Smoothing with w = 0.5
x
250
300
350
400
450
500
1960
1965
1970
Time


Exponential Smoothing with w = 0.9
x
250
300
350
400
450
500
1960
1965
1970
Time

Double Exponential Smoothing

Double Exponential Smoothing

(linear trend)

We can use double smoothing to identify the trend and forecast linear trend time series as follows.

- Step 1: Create a smoothed series: $s_t^{(1)} = (1-w)y_t + ws_{t-1}^{(1)}$
- Step 2: Create a double smoothed series: $s_t^{(2)} = (1-w)s_t^{(1)} + ws_{t-1}^{(2)}$
- Step 3: Estimate the trend:

$$b_1 = \frac{1-w}{w}(s_T^{(1)} - s_T^{(2)})$$

- Step 4: Forecast

$$\hat{y}_{T+l} = s_T^{(1)} + b_1 \cdot l$$

t	y_t
1	1
2	3 $\leftarrow y_2$
3	5 $\leftarrow y_3$
4	8
5	12

$$s_1 = y_1, \text{ and}$$

$$s_t = s_{t-1} + (1-w)(y_t - s_{t-1})$$

$$= \boxed{(1-w)y_t + w\underline{s_{t-1}}}$$

$$s_1 = y_1 = 1$$

$$s_2 = (1-w)\,y_2 + .2 \cdot s_1$$

$$= .8\,y_2 + .2\,s_1$$

$$= .8 * 3 + .2 * 1 = 2.6$$

$$s_3 = (1-.2)\cdot y_3 + .2 * s_2$$

$$= .8 * 5 + .2 * (2.6)$$

$$= 4 + .52 = \boxed{4.52}$$

$$s_4 = .8 * y_4 + .2 * s_3$$

$$= .8 * 8 + .2 * 4.52$$

$$= 7.3$$

$$s_5 = .8 * y_5 + .2 * s_4$$

$$= .8 * 12 + .2 * 7.3$$

$$= 11.06.$$

Example

You are given the following time series

t	1	2	3	4	5
y_t	1	3	5	8	13

- Assume that this is a linear trend time series. Using double exponential smoothing with $w = .8$ to estimate the trend (slope) and forecast y_6.