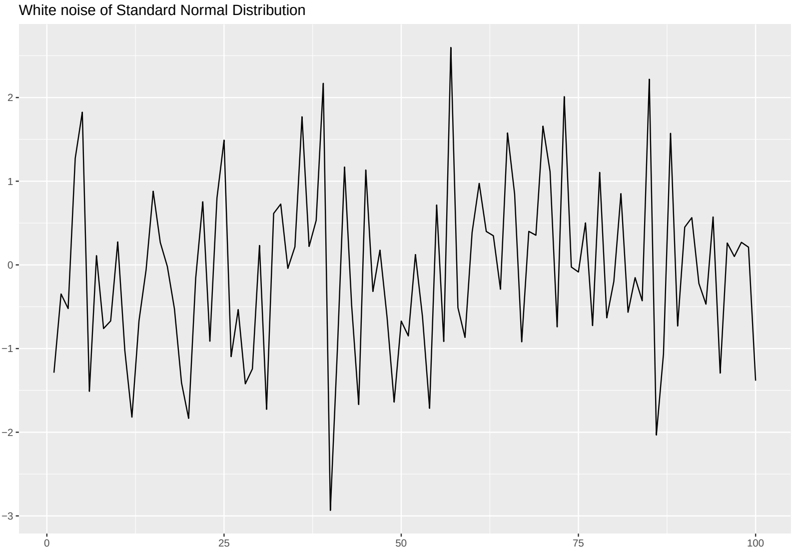
White Noise and Random Walks

White Noise

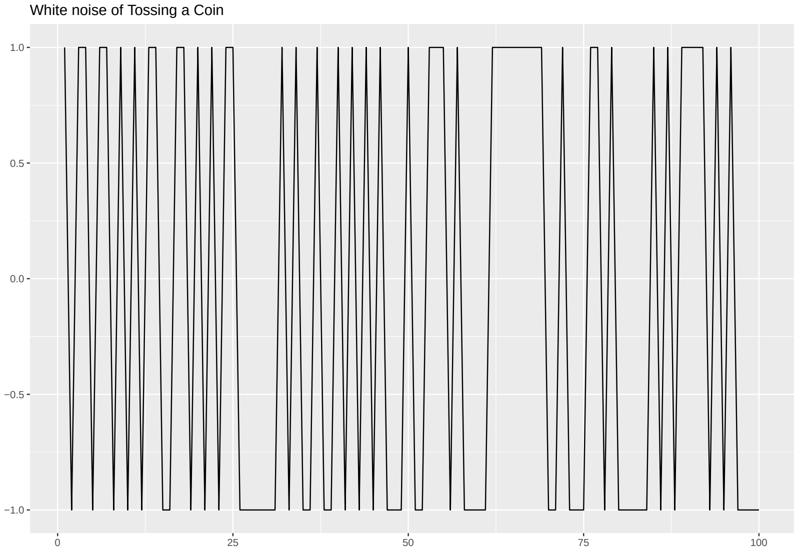
- y_t is a white-noise process (series) if y_1, y_2,...y_t... are i.i.d zero mean random variables from a certain distribution (usually normal)

Example

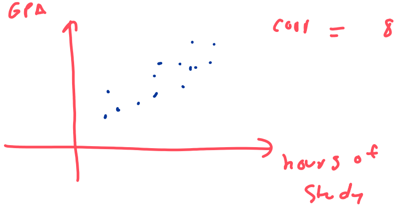


White noise of Standard Normal Distribution
2
1
0
−1
−2
−3
0
25
50
75
100

Example



Correlogram



- Autocorrelation lag with lag k is the the correlation between the time series y_t and y_{t-k}
- Autocorrelation lag with lag 0 is always 1
- The Correlogram is the plot of the autocorrelations for values of lag k $=$ 0, 1, 2,...

$y_t = 68, 70, 73, 67, 75$

$a_1 = \{70, 73, 67, 75\}$

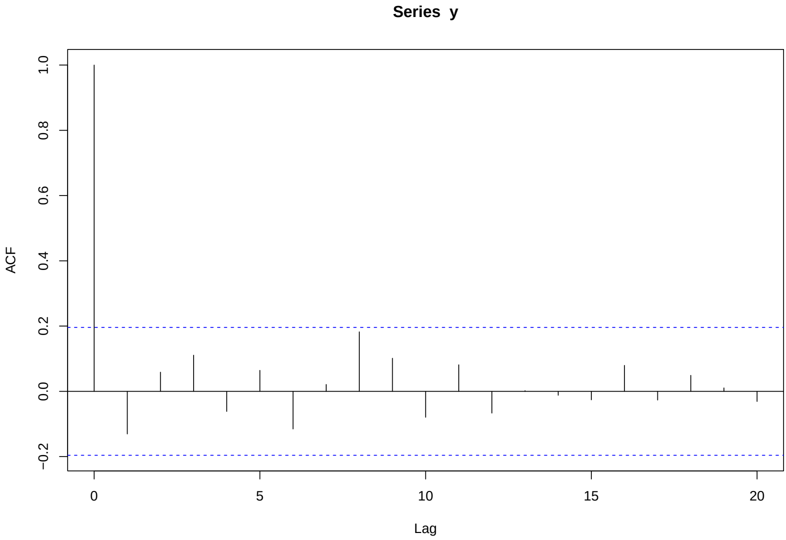
autocorr with lag 1 :

$a_2 = \{68, 70, 73, 67\}$

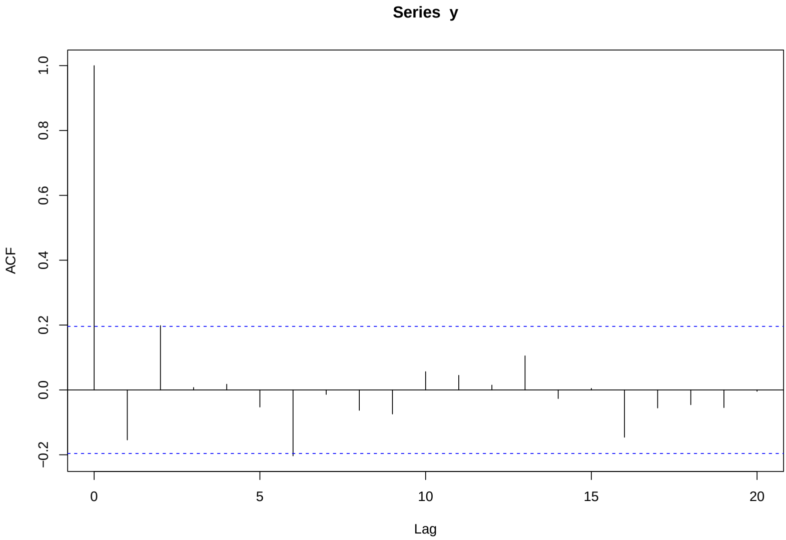
$\approx \text{corr}(a_1, a_2)$

Correlogram a white noise

- Correlogram of a white noise



Correlogram a white noise



Random Walk

Random Walk

- A time series y_t is called a random walk if

$$y_t = y_{t-1} + \epsilon_t,$$

  where ϵ_t is a white-noise
- A random walk can be written as

$$y_t = y_0 + \epsilon_1 + \epsilon_2 + ... + \epsilon_t$$

Random Walk with drift

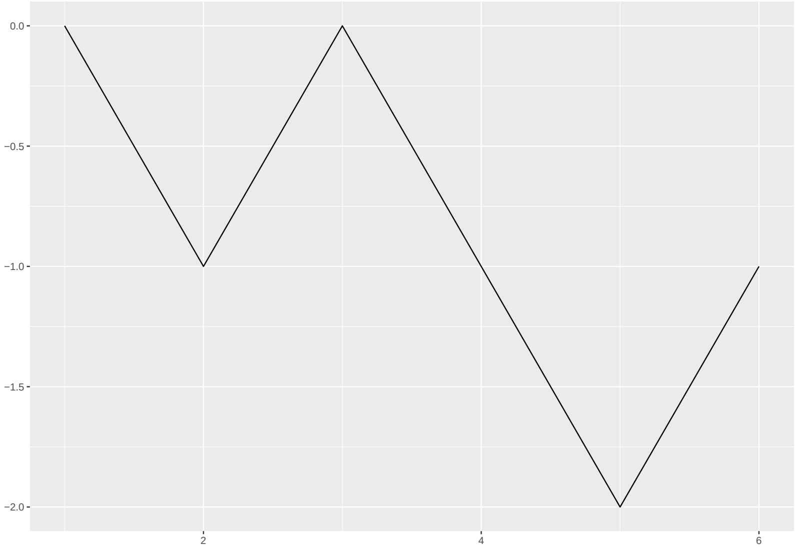
- A time series y_t is called a random walk if

$$y_t = y_{t-1} + d + \epsilon_t,$$

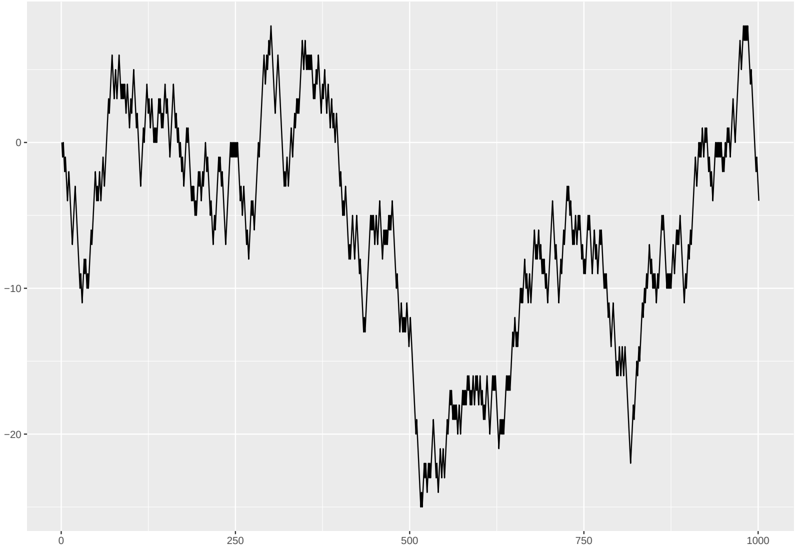
  where ϵ_t is a white-noise
- A random walk can be written as

$$y_t = y_0 + dt + \epsilon_1 + \epsilon_2 + ... + \epsilon_t$$

Example

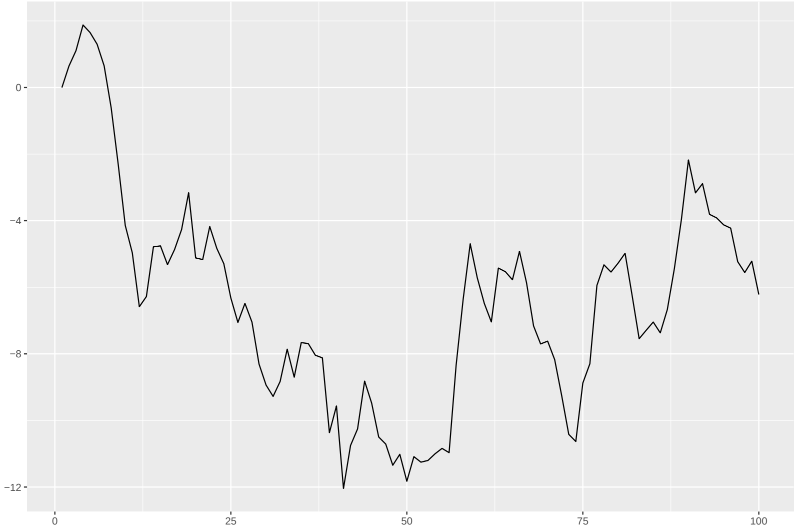


Example

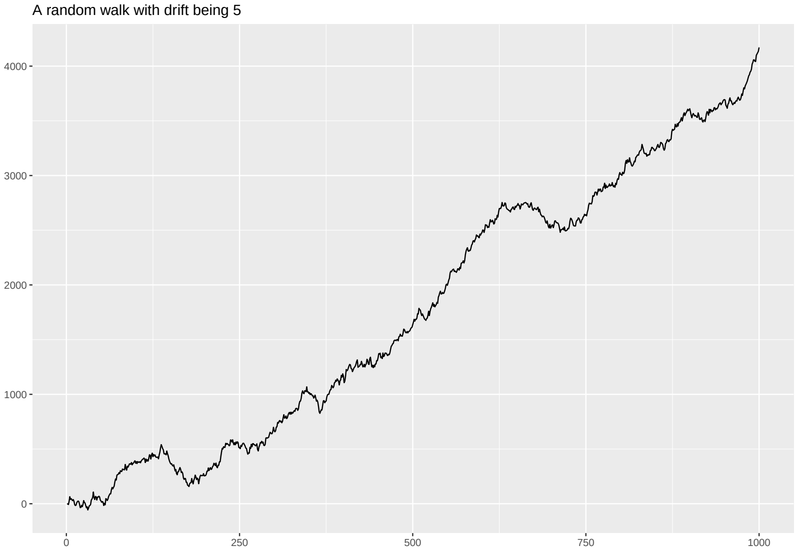


Example

A random walk:



Example


A random walk with drift being 5
4000
3000
2000
1000
0
0
250
500
750
1000

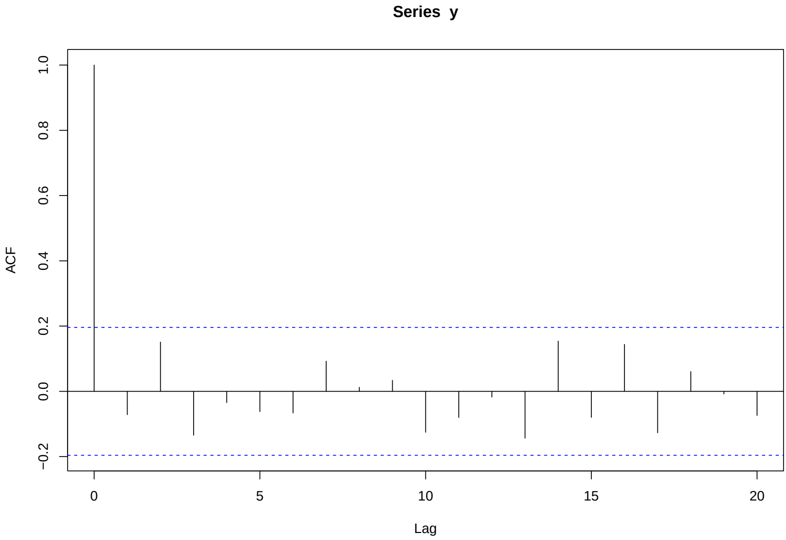
Forecasting with Random Walks

The Correlogram

- Autocorrelation lag with lag k is the the correlation between the time series y_t and y_{t-k}
- Autocorrelation lag with lag 0 is always 1
- The Correlogram is the plot of the autocorrelations for values of lag k $=$ 0, 1, 2,...

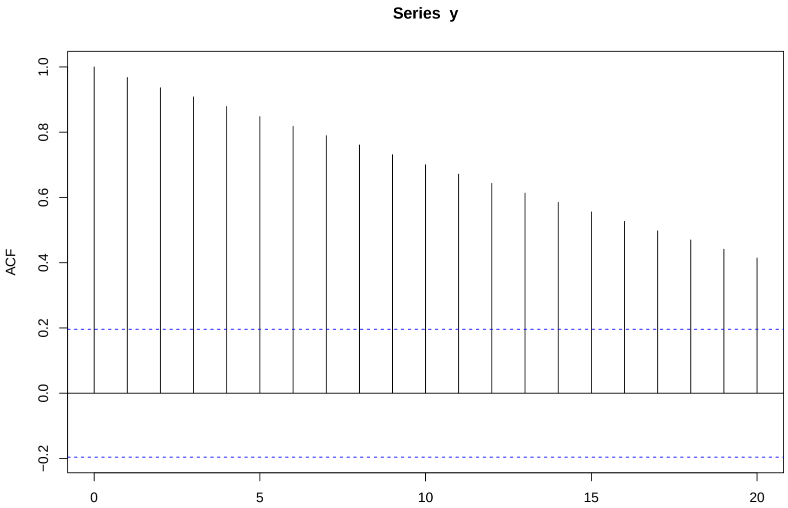
The Correlogram - Example

- Correlogram of a white noise

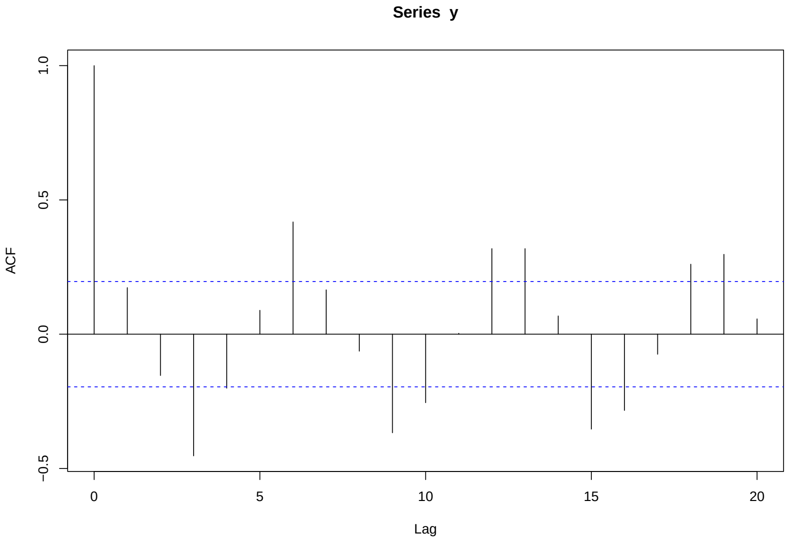


The Correlogram - Example

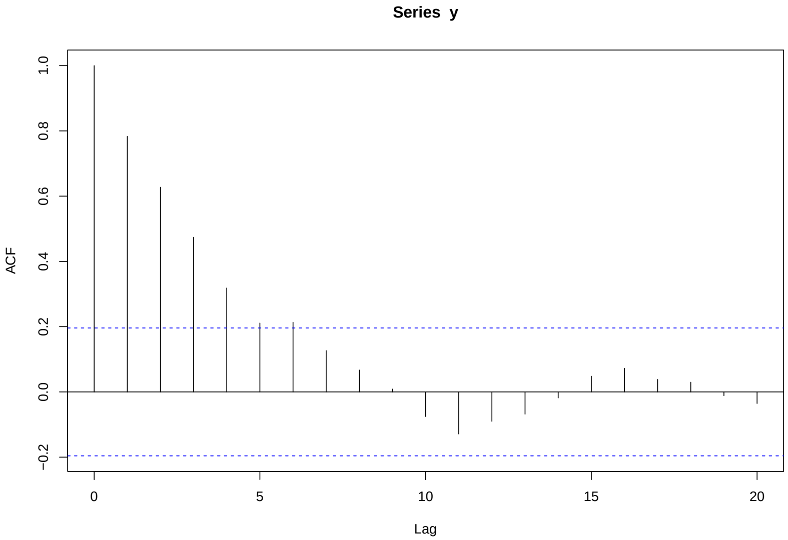
- Correlogram of a time series with trend
- Usually a trend in the data will show in the correlogram as a slow decay in the autocorrelation



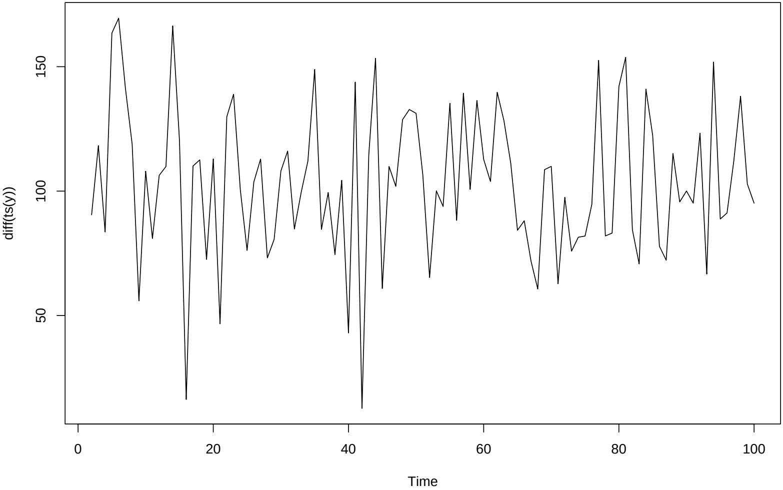
The Correlogram - Example



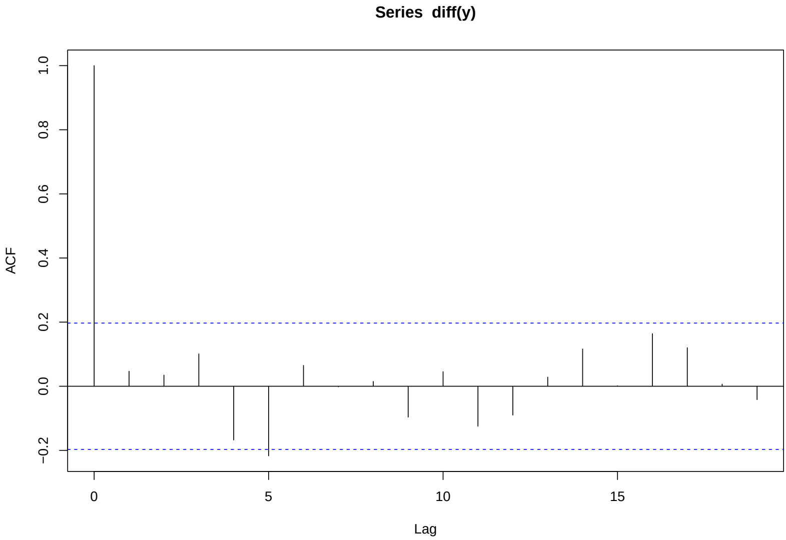
The Correlogram of Random Walks



Differencing Time Series



The Correlogram of Random Walks



Random Walks and Stocks

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[1] "MSFT"
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Estimate the random walk model

For a given time series y we can fit the random walk model with a drift by

- first differencing the data,
- then fitting the white noise (WN) model to the differenced data using the arima() command with the order = c(0, 0, 0)) argument.
- The arima() command displays information or output about the fitted model. Under the Coefficients: heading is the estimated drift variable, named the intercept. Its approximate standard error (or s.e.) is provided directly below it. The variance of the WN part of the model is also estimated under the label sigma^2.

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[1] "MSFT"


Call:
arima(x = dy, order = c(0, 0, 0))

Coefficients:
      intercept
         0.5923
s.e.     0.2815

sigma^2 estimated as 22.34:  log likelihood = -838.15,  aic
```

Forecasting with Random Walks

Suppose that we know $y_0, y_1, ..., y_T$ and we want to forecast y_{T+l} for some fixed $l > 0$

- Point forecast: the estimated l step-ahead is

$$\hat{y}_{T+l} = y_T + l\hat{\mu}_c,$$

where $\hat{\mu}_c$ is the estimated mean of the white-noise. $\hat{\mu}_c$ can be $\bar{c}$

$$\bar{c} = \frac{\epsilon_1 + \epsilon_2 + ... + \epsilon_T}{T}$$

- The standard error of the forecast is $s_c\sqrt{l}$, where s_c is the estimated standard deviation of σ_c,

$$s_c^2 = \frac{1}{n-1}\sum_{i=1}^{T}\left(\epsilon_i - \bar{c}\right)^2$$

Example

You are given:

i) The random walk model

$$y_t = y_0 + \epsilon_1 + \epsilon_2 + \epsilon_3 + ... + \epsilon_t,$$

where $\epsilon_i, (i = 1, 2, ..., t)$ denote observations from a white noise process.

ii) The following ten observed values of ϵ_t:

t	1	2	3	4	5	6	7	8	9	10
y_t	2	5	10	13	18	20	24	25	27	30

iii) $y_0 = 0$

Calculate the 9 step-ahead forecast, $\hat{y}_{19}$.

Example

You are given:

i) The random walk model

$$y_t = y_0 + \epsilon_1 + \epsilon_2 + \epsilon_3 + ... + \epsilon_t,$$

where $\epsilon_i, (i = 1, 2, ..., t)$ denote observations from a white noise process.

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t	1	2	3	4	5	6	7	8	9	10
y_t	2	5	10	13	18	20	24	25	27	30

iii) $y_0 = 0$

Calculate the standard error of the 9 step-ahead forecast, $\hat{y}_{19}$.

We have

$\epsilon_t = y_t - y_{t-1} \implies \epsilon_1, \epsilon_2, ..., \epsilon_{10} = 2, 3, 5, 3, 5, 2, 4, 1, 2, 3$

$$\implies \bar{c} = \frac{\epsilon_1 + \epsilon_2 + ... + \epsilon_{10}}{10} = 3$$

$$\implies s_c^2 = \frac{1}{9} \sum_{i=1}^{10} \left(\epsilon_i - 3\right)^2 = 16/9$$

Hence, the standard error is $s_c\sqrt{l} = \frac{4}{3}\sqrt{9} = 4$