

Time Series

Cross Sectional vs. Time Series Data

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- ▶ Examples: customers' behavioral data at today's update, companies' account balances at the end of the last year, patients' medical records at the end of the current month.

Cross Sectional vs. Time Series Data

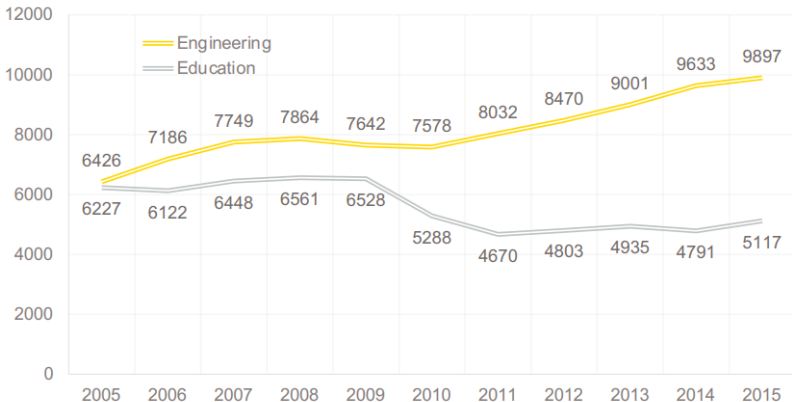
- ▶ Time Series Data: One single object (product, country, sensor, ..) observed over multiple equally-spaced time periods

Cross Sectional vs. Time Series Data

- ▶ Time Series Data: One single object (product, country, sensor, ..) observed over multiple equally-spaced time periods
- ▶ Examples: quarterly Italian GDP of the last 10 years, weekly supermarket sales of the previous year, yesterday's hourly temperature measurements.

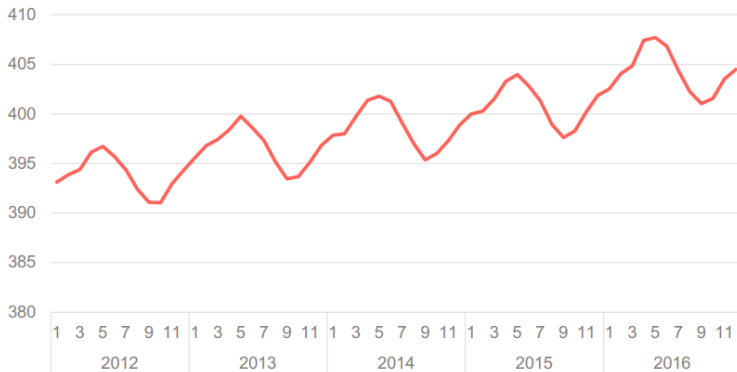
Examples

Numbers of Doctorates Awarded in US, annual data – Engineering Vs. Education



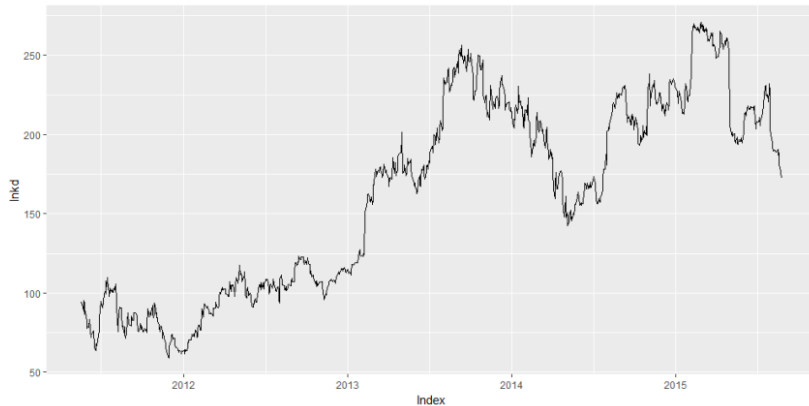
Examples

Monthly carbon dioxide concentration (globally averaged over marine surface sites)



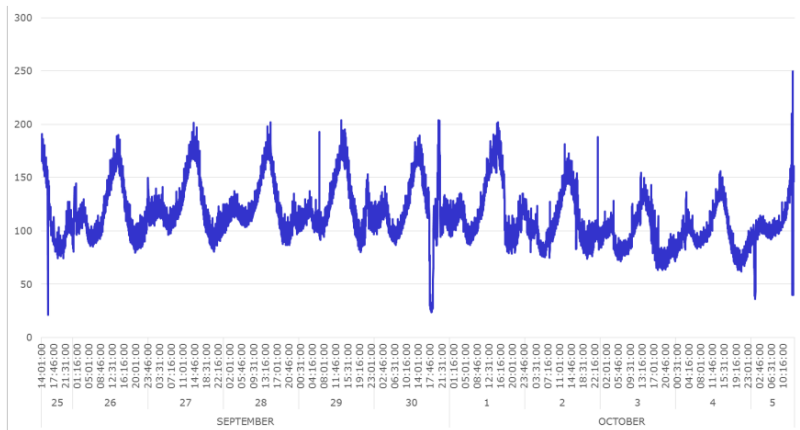
Examples

LinkedIn daily stock market closing price



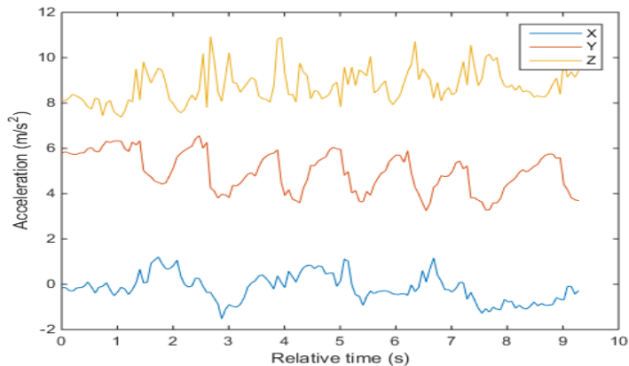
Examples

Number of photos uploaded on the Instagram every minute (regional sub-sample)

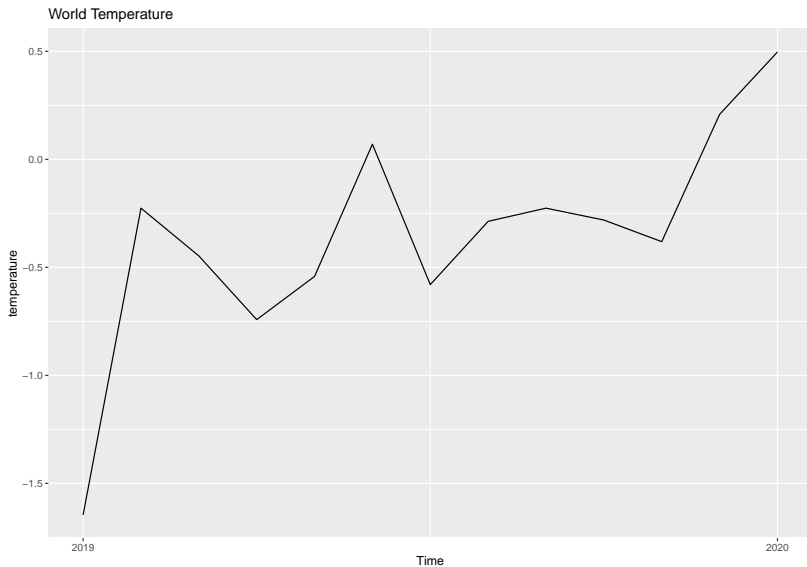


Examples

Acceleration detected by a smartphone sensors during a workout session (10 seconds)



Examples



What to do with time series?

- ▶ Understanding of specific series features or pattern
- ▶ Forecasting

Smoothing

Smoothing

- ▶ Smoothing is usually done to reveal the series patterns and trends.

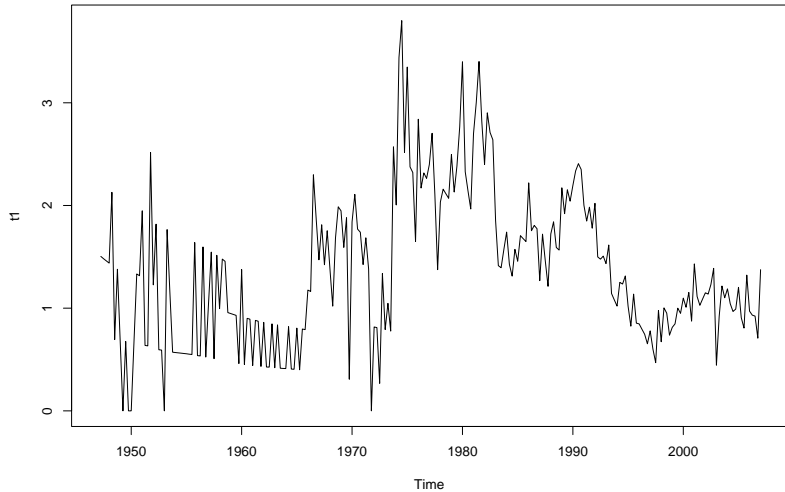
Simple Moving Average Smoothing

- ▶ Moving Average (MA) creates a new series by averaging the most recent observations from the original series.
- ▶ MA(k) creates s_t as follows.

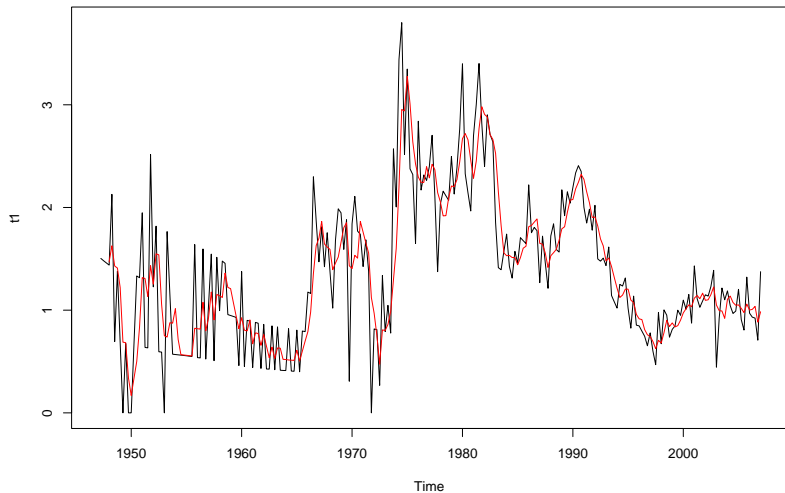
$$s_t = \frac{y_t + y_{t-1} + \dots + y_{t-k+1}}{k}$$

- ▶ Larger k will smooth the series more strongly

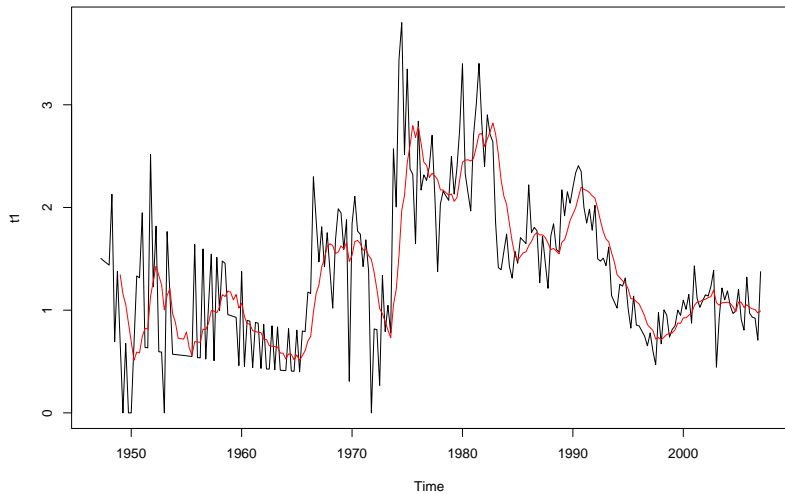
Medical Component of the CPI



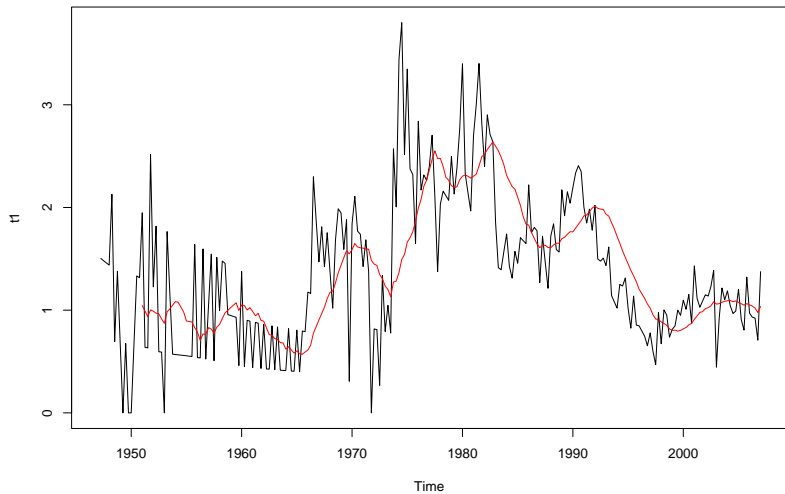
Moving average with $k = 4$



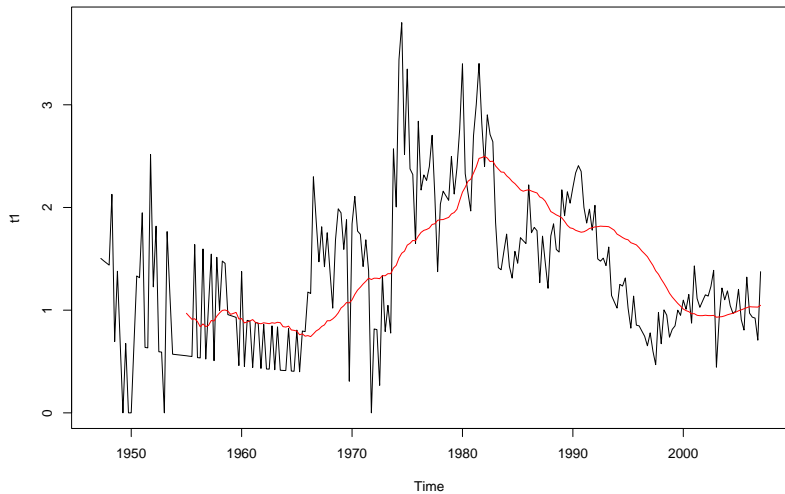
Moving average with $k = 8$



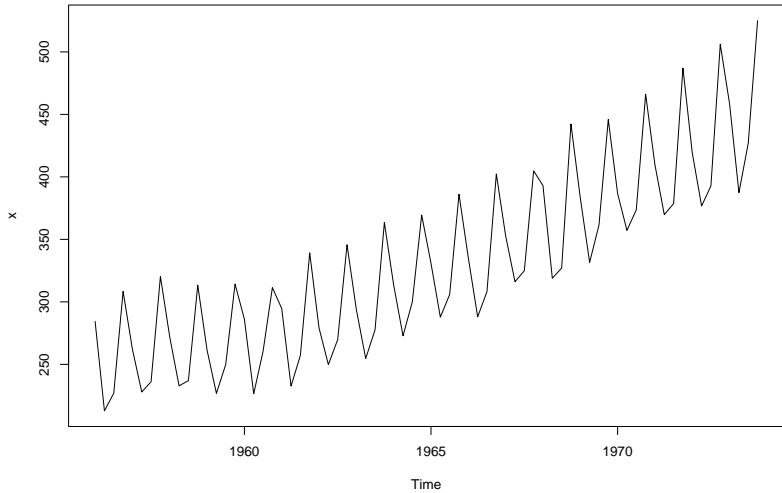
Moving average with $k = 16$



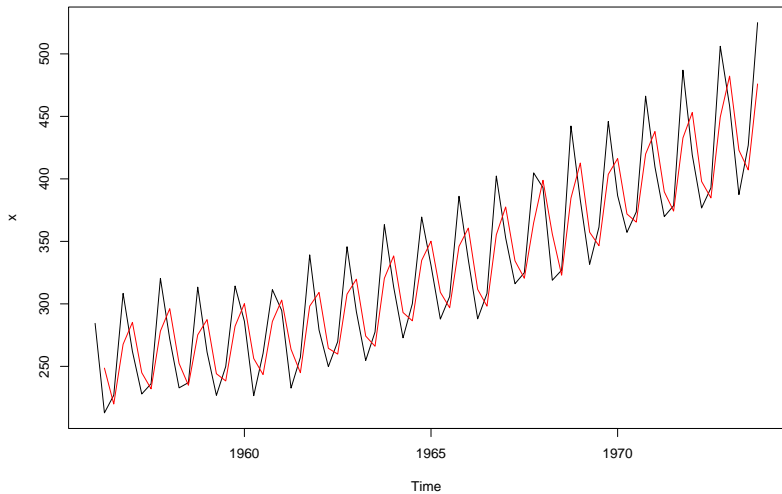
Moving average with $k = 32$



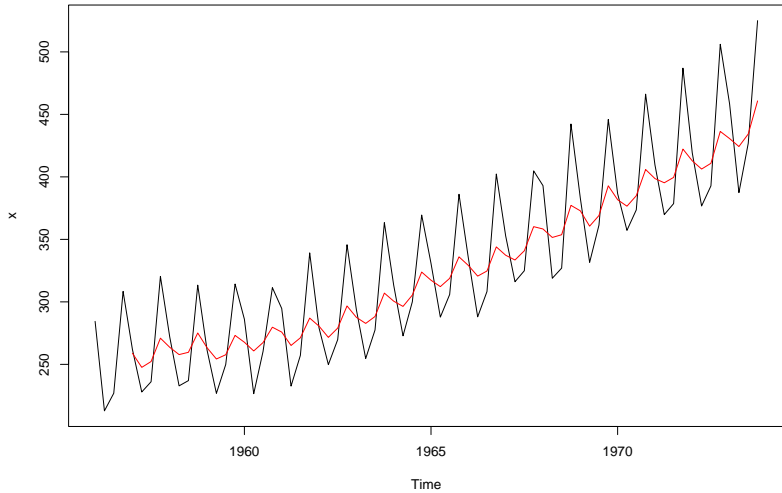
Original Series



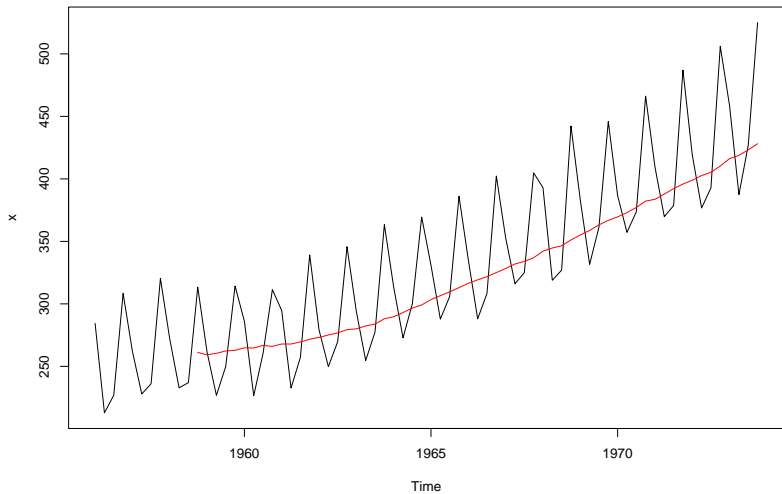
Moving average with $k = 2$



Moving average with $k = 5$



Moving average with $k = 12$



Forecasting

- ▶ We can use MA smoothing for forecasting
- ▶ We have

$$\begin{aligned}s_t &= \frac{y_t + y_{t-1} + \dots + y_{t-k+1}}{k} \\&= \frac{y_t + y_{t-1} + \dots + y_{t-k+1} + y_{t-k} - y_{t-k}}{k} \\&= \frac{y_t + \left(y_{t-1} + \dots + y_{t-k+1} + y_{t-k} \right) - y_{t-k}}{k} \\&= \frac{y_t + k s_{t-1} - y_{t-k}}{k} \\&= s_{t-1} + \frac{y_t - y_{t-k}}{k}\end{aligned}$$

Forecasting

- ▶ If there is no trend in y_t the second term $(y_t - y_{t-k})/k$ can be ignored
- ▶ Forecasting l lead time into future by (\hat{y}_{T+l}) is a forecast of y_{T+l}

$$\hat{y}_{T+l} = s_T$$

- ▶ If there is a linear trend in a series, we can use the double moving average to estimate the trend (slope)

Double MA

Double MA

- ▶ Linear trend time series:

$$y_t = \beta_0 + \beta_1 t + \epsilon_t$$

- ▶ Step 1: MA Smooth the series

$$s_t^{(1)} = \frac{y_t + y_{t-1} + \dots + y_{t-k+1}}{k}$$

- ▶ Step 2: MA Smooth the smoothed series

$$s_t^{(2)} = \frac{s_t^{(1)} + s_{t-1}^{(1)} + \dots + s_{t-k+1}^{(1)}}{k}$$

- ▶ Step 3: Calculate the linear trend/slope

$$b_1 = \hat{\beta}_1 = \frac{2}{k-1} \left(s_T^{(1)} - s_T^{(2)} \right)$$

Forecasting

- Forecasting l lead time into future by

$$\hat{y}_{T+l} = s_T^{(1)} + b_1 \cdot l$$

You are given the following time series

t	1	2	3	4	5
y_t	1	3	5	8	13

- ▶ Forecasting y_6 using simple moving average with $k = 2$
- ▶ Forecasting y_6 using double moving average with $k = 2$

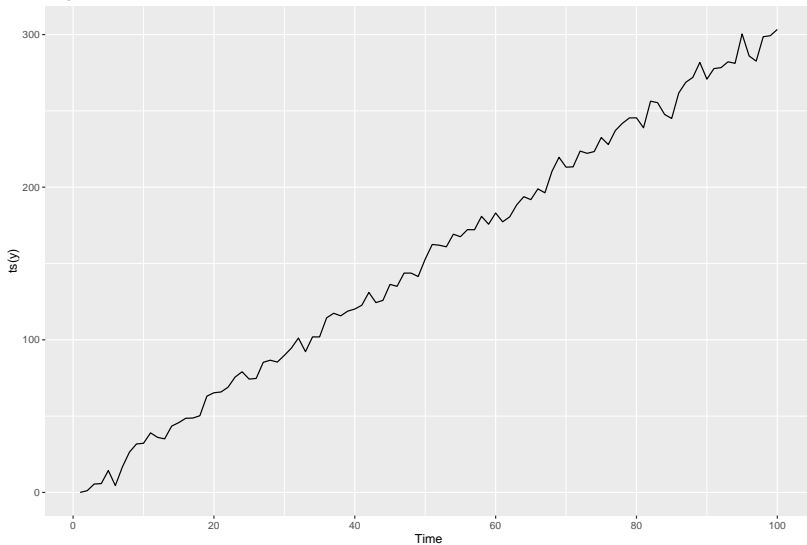
Example

- ▶ We simulate 100 data points ($T = 100$) of

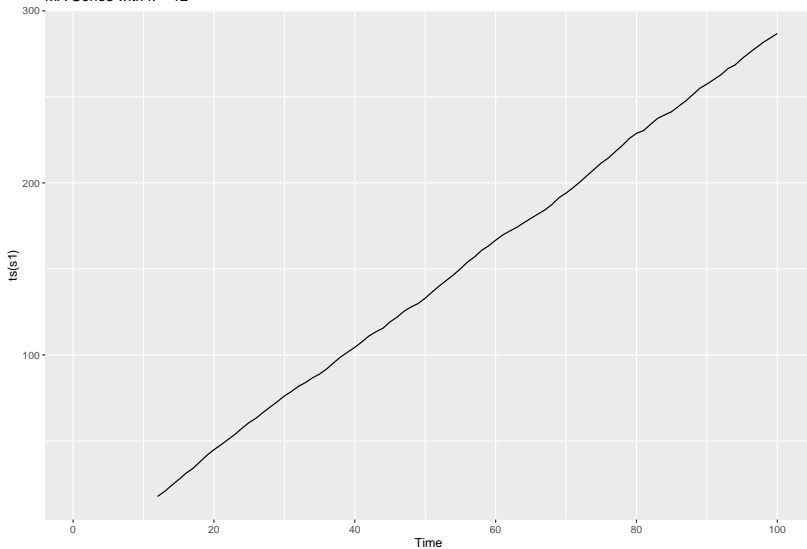
$$y_t = 1 + 3t + \epsilon,$$

where, $\epsilon \sim N(0, 5^2)$.

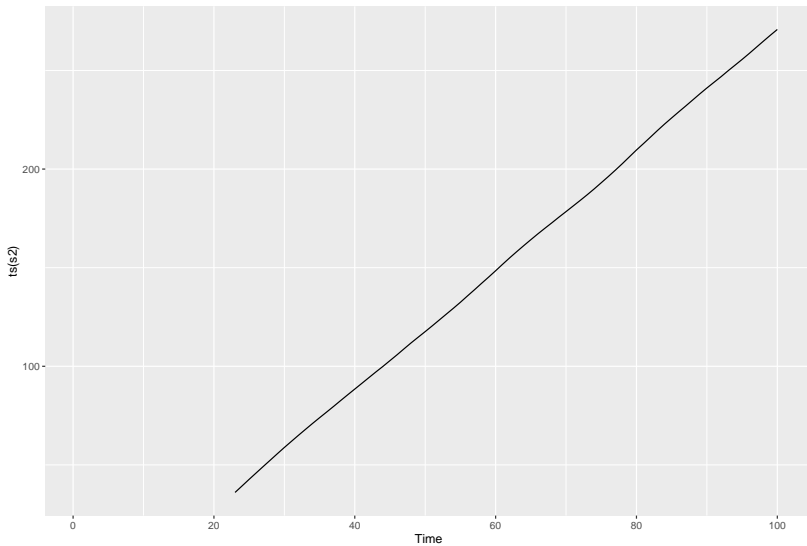
Original Series



MA Series with $k = 12$



Double MA Series with $k = 12$



- ▶ Using the above steps, the estimated trend is $b_1 = 2.92$
- ▶ The forecast for the next points from y_{100} is

$$\hat{y}_{100+l} = s_{100} + b_1 \cdot l = s_{100} + 2.92 \cdot l$$