Time Series

Cross Sectional vs. Time Series Data

- Cross Sectional Data: Multiple objects observed at a particular point of time

Cross Sectional vs. Time Series Data

- Cross Sectional Data: Multiple objects observed at a particular point of time
- Examples: customers' behavioral data at today's update,companies' account balances at the end of the last year,patients' medical records at the end of the current month.

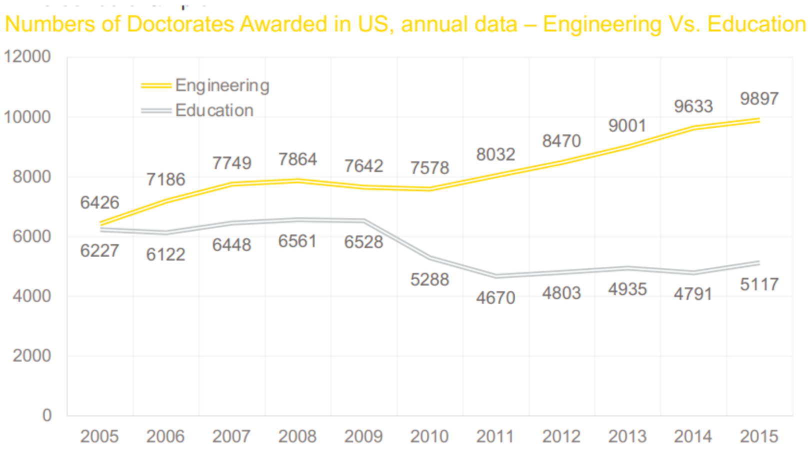
Cross Sectional vs. Time Series Data

- Time Series Data: One single object (product, country, sensor, ..) observed over multiple equally-spaced time periods

Cross Sectional vs. Time Series Data

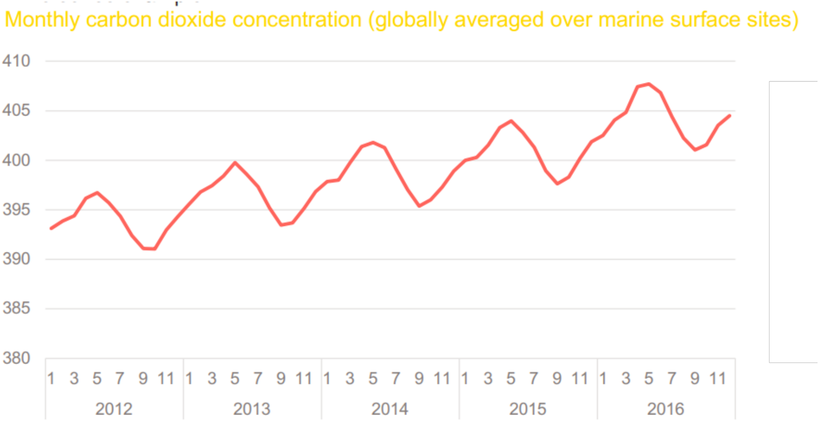
- Time Series Data: One single object (product, country, sensor, ..) observed over multiple equally-spaced time periods
- Examples: quarterly Italian GDP of the last 10 years, weekly supermarket sales of the previous year, yesterday's hourly temperature measurements.

Examples



Numbers of Doctorates Awarded in US, annual data – Engineering Vs. Education
Engineering
Education
12000
10000
8000
6000
4000
2000
0
6426
7186
7749
7864
7642
7578
8032
8470
9001
9633
9897
6227
6122
6448
6561
6528
5288
4670
4803
4935
4791
5117
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015

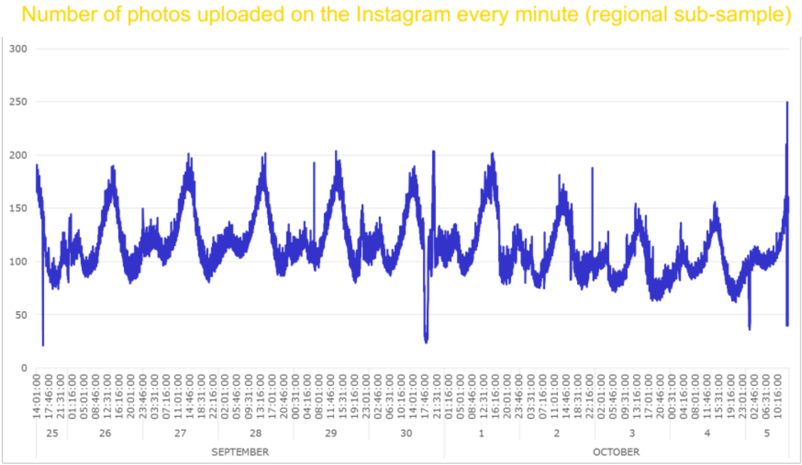
Examples


Monthly carbon dioxide concentration (globally averaged over marine surface sites)
410
405
400
395
390
385
380
1 3 5 7 9 11 1 3 5 7 9 11 1 3 5 7 9 11 1 3 5 7 9 11 1 3 5 7 9 11
2012
2013
2014
2015
2016

Examples


LinkedIn daily stock market closing price
lnkd
250
200
150
100
50
2012
2013
2014
2015
Index

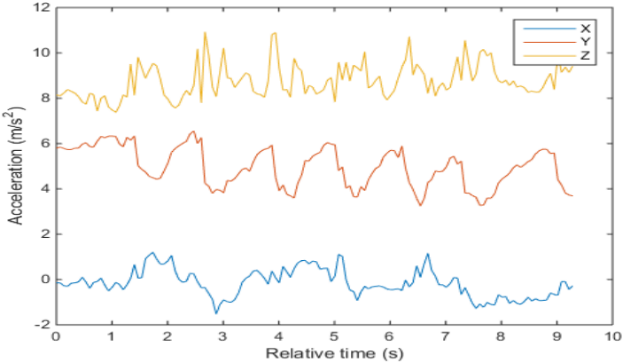
Examples



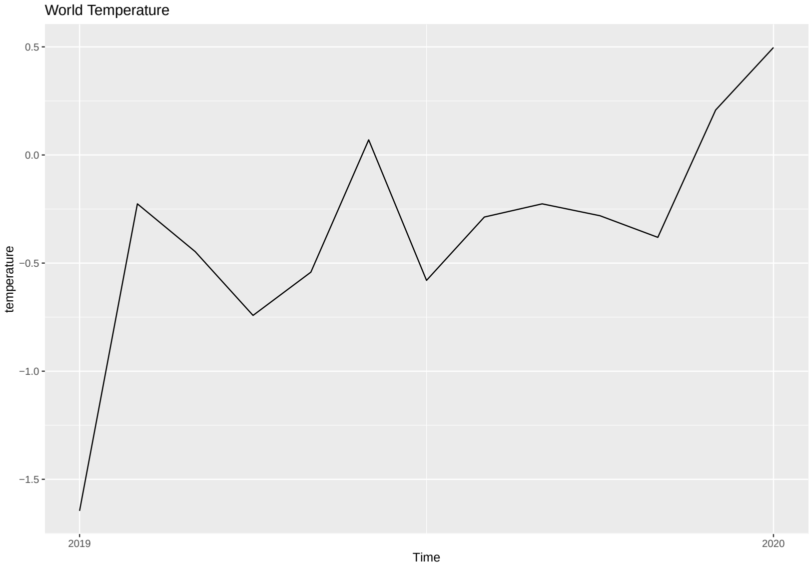
Number of photos uploaded on the Instagram every minute (regional sub-sample)
300
250
200
150
100
50
0
14:01:00
17:46:00
21:31:00
01:16:00
05:01:00
08:46:00
12:31:00
16:16:00
20:01:00
23:46:00
03:31:00
07:16:00
11:01:00
14:46:00
18:31:00
22:16:00
02:01:00
05:46:00
09:31:00
13:16:00
17:01:00
20:46:00
00:31:00
04:16:00
08:01:00
11:46:00
15:31:00
19:16:00
23:01:00
02:46:00
06:31:00
10:16:00
14:01:00
17:46:00
21:31:00
01:16:00
05:01:00
08:46:00
12:31:00
16:16:00
20:01:00
23:46:00
03:31:00
07:16:00
11:01:00
14:46:00
18:31:00
22:16:00
02:01:00
05:46:00
09:31:00
13:16:00
17:01:00
20:46:00
00:31:00
04:16:00
08:01:00
11:46:00
15:31:00
19:16:00
23:01:00
02:46:00
06:31:00
10:16:00
25
26
27
28
29
30
1
2
3
4
5
SEPTEMBER
OCTOBER

Examples

Acceleration detected by a smartphone sensors during a workout session (10 seconds)



Examples



What to do with time series?

- Understanding of specific series features or pattern
- Forecasting

Smoothing

Smoothing

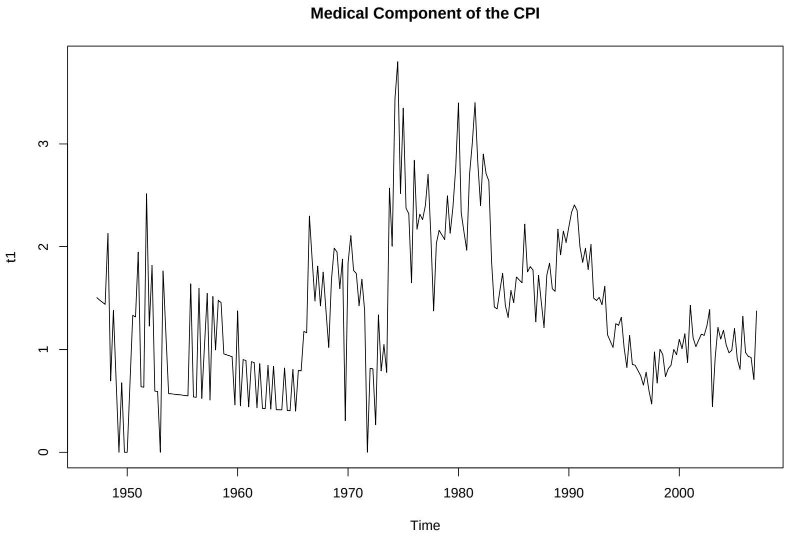
- Smoothing is usually done to reveal the series patterns and trends.

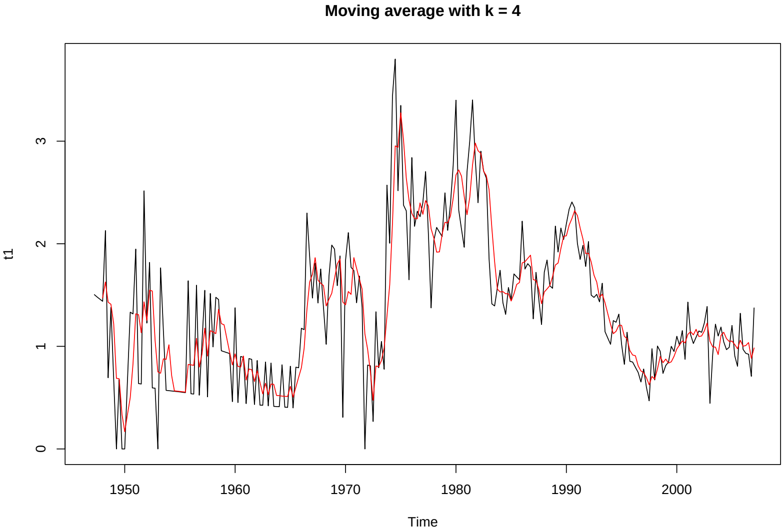
Simple Moving Average Smoothing

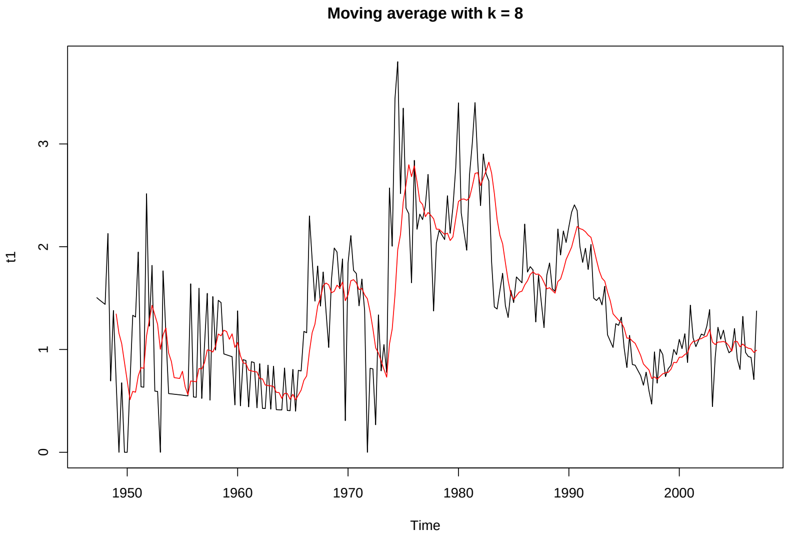
- Moving Average (MA) creates a new series by averaging the most recent observations from the original series.
- MA(k) creates s_t as follows.

$$s_t = \frac{y_t + y_{t-1} + ... + y_{t-k+1}}{k}$$

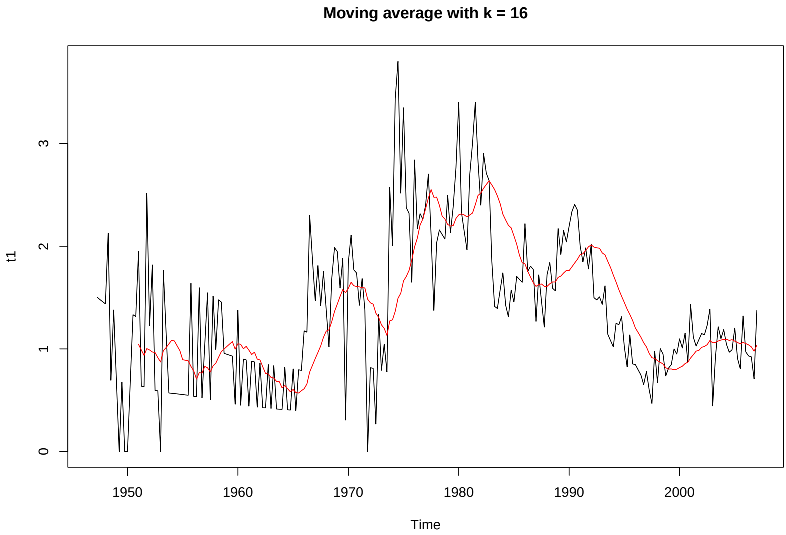
- Larger k will smooth the series more strongly


Medical Component of the CPI
t1
0
1
2
3
1950
1960
1970
1980
1990
2000
Time


Moving average with k = 4
t1
Time
0
1
2
3
1950
1960
1970
1980
1990
2000



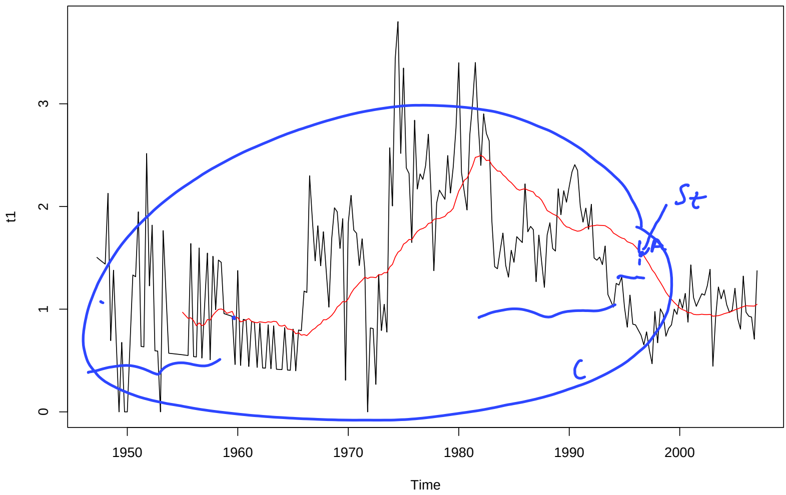
Moving average with k = 8
t1
0
1
2
3
1950
1960
1970
1980
1990
2000
Time

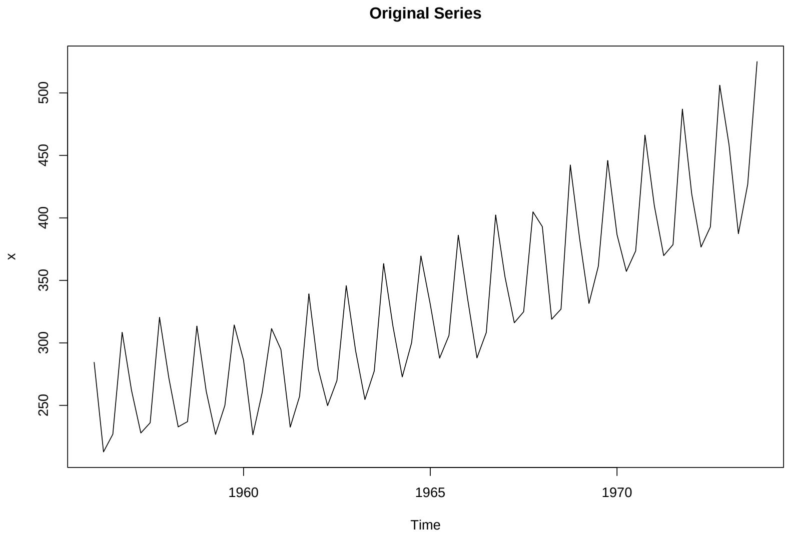


Moving average with k = 16
t1
0
1
2
3
Time
1950
1960
1970
1980
1990
2000

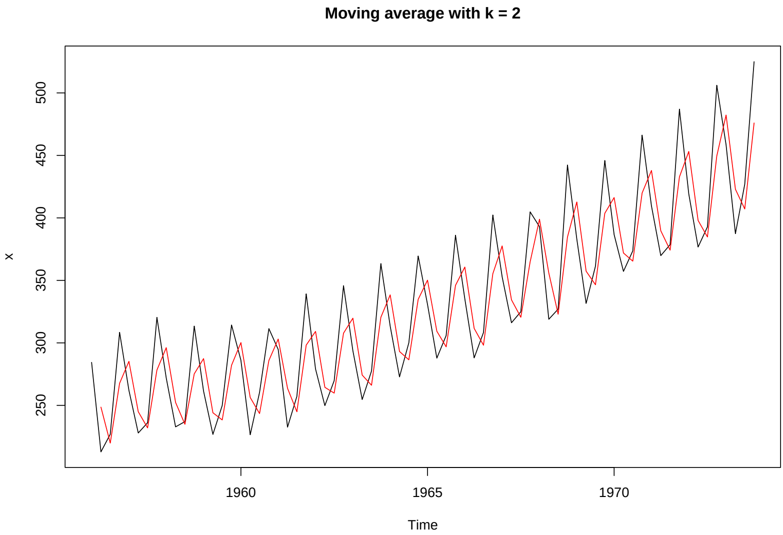
$$S_{10} = \frac{y_{12} + y_{11} + y_{10} + y_9 + y_8}{5}$$

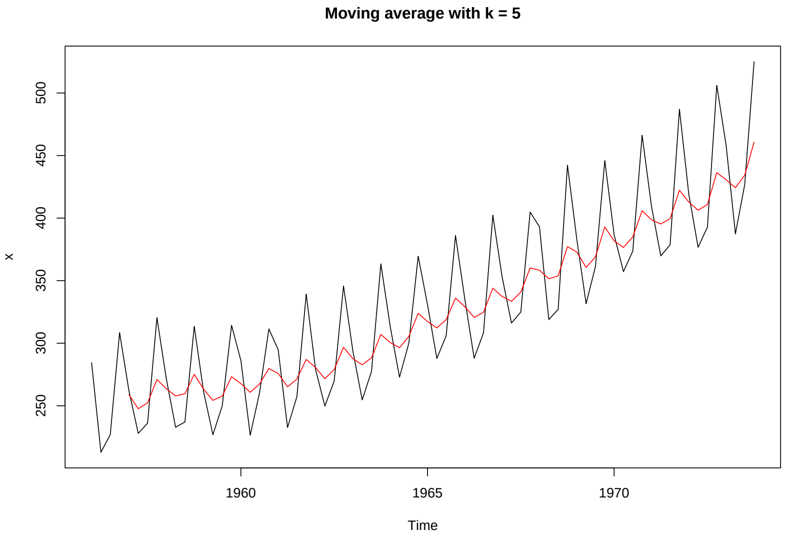
Moving average with k = 32

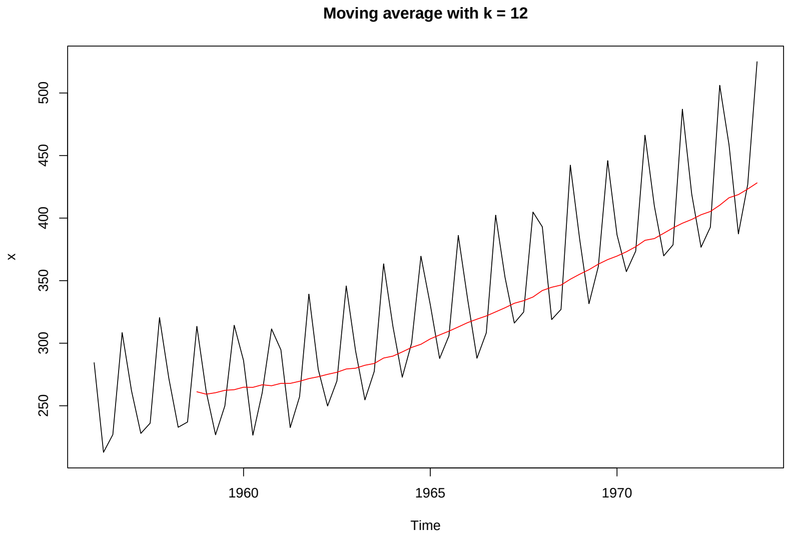




Original Series
x
500
450
400
350
300
250
1960
1965
1970
Time

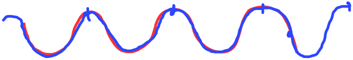

Moving average with k = 2
x
500
450
400
350
300
250
1960
1965
1970
Time


Moving average with k = 5
x
250
300
350
400
450
500
1960
1965
1970
Time



Moving average with k = 12
x
250
300
350
400
450
500
1960
1965
1970
Time

Forecasting



- We can use MA smoothing for forecasting
- We have

definition of smoothing

$$
\begin{aligned}
s_t &= \frac{y_t + y_{t-1} + \ldots + y_{t-k+1}}{k} \\
&= \frac{y_t + y_{t-1} + \ldots + y_{t-k+1} + y_{t-k} - y_{t-k}}{k} \\
&= \frac{y_t + \left(y_{t-1} + \ldots + y_{t-k+1} + y_{t-k}\right) - y_{t-k}}{k} \\
&= \frac{y_t + k s_{t-1} - y_{t-k}}{k} \\
s_t &= s_{t-1} + \underbrace{\frac{y_t - y_{t-k}}{k}}_{\text{can we drop this term?}}
\end{aligned}
$$

$\iff y_t - y_{t-k} = 0$

If Y_t does not have trend (not going up or down) then

we can assume that $Y_t \approx Y_{t-k}$ and $S_t \approx S_{t-1}$.

Then we can use S_{t+1} as a forecast for Y_{t+1}

Forecasting

- If there is no trend in y_t the second term $(y_t - y_{t-k})/k$ can be ignored
- Forecasting l lead time into future by ($\hat{y}_{T+l}$ is a forecast of y_{T+l})

$$\hat{y}_{T+l} = s_T$$

- If there is a linear trend in a series, we can use the double moving average to estimate the trend (slope)

Double MA

Double MA

- Linear trend time series:

$$y_t = \beta_0 + \beta_1 t + \epsilon_t$$

- Step 1: MA Smooth the series

$$s_t^{(1)} = \frac{y_t + y_{t-1} + ... + y_{t-k+1}}{k}$$

- Step 2: MA Smooth the smoothed series

$$s_t^{(2)} = \frac{s_t^{(1)} + s_{t-1}^{(1)} + ... + s_{t-k+1}^{(1)}}{k}$$

- Step 3: Calculate the linear trend/slope

$$b_1 = \hat{\beta}_1 = \frac{2}{k-1}\left(s_T^{(1)} - s_T^{(2)}\right)$$

Forecasting

- Forecasting l lead time into future by

$$\boxed{\hat{y}_{T+l} = s_T^{(1)} + b_1 \cdot l}$$

notice: ① $y_1, y_2, \ldots y_T$ are known

$y_{T+1}, y_{T+2}, \ldots$ are unknown and needes

to be forecast

② $\hat{y}_{T+l}$ is a forecast for y_{T+l}

You are given the following time series

t	1	2	3	4	5
y_t	1	3	5	8	13

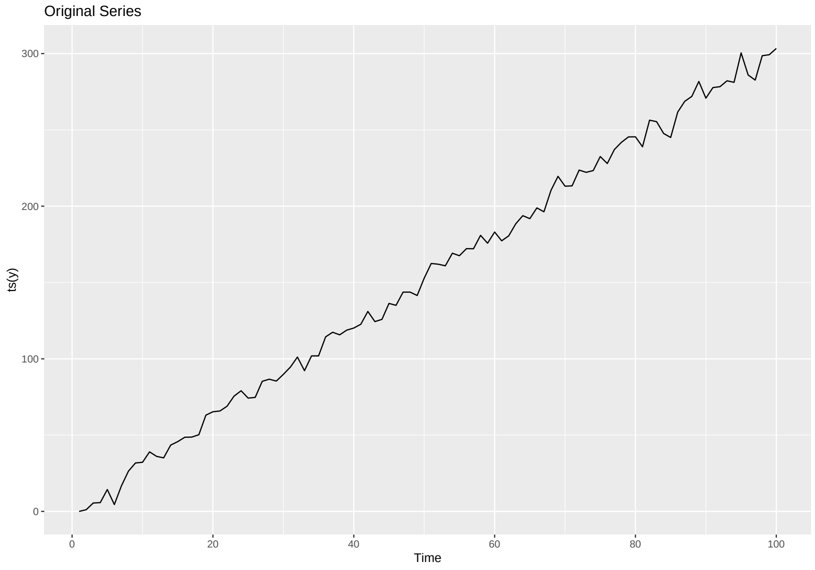
- Forecasting y_6 using simple moving average with $k = 2$
- Forecasting y_6 using double moving average with $k = 2$

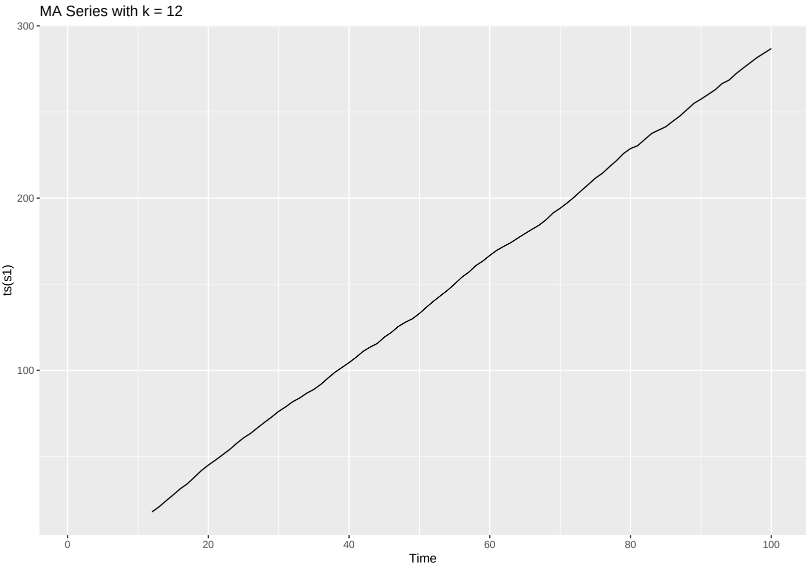
Example

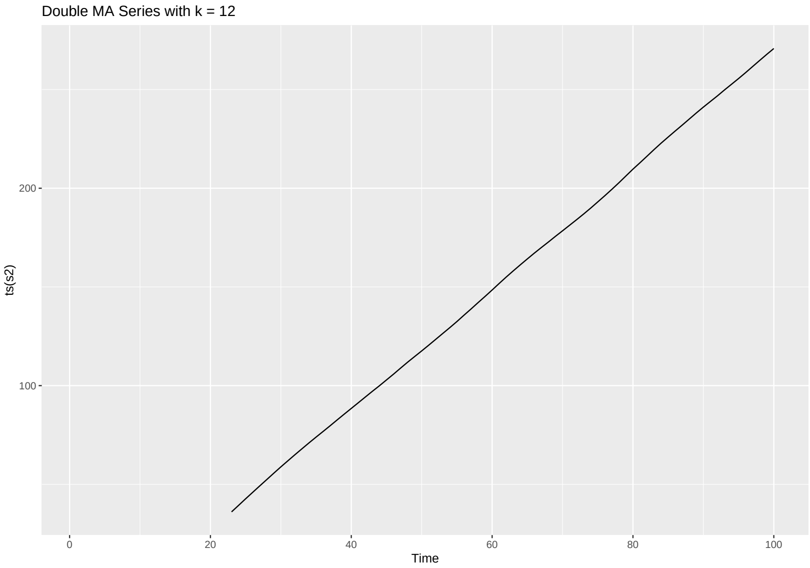
- We simulate 100 data points ($T = 100$) of

$$y_t = 1 + 3t + \epsilon,$$

where, $\epsilon \sim N(0, 5^2)$.


Original Series
ts(y)
300
200
100
0
Time
0
20
40
60
80
100


MA Series with k = 12
300
200
100
ts(s1)
0
20
40
60
80
100
Time


Double MA Series with k = 12
200
100
ts(s2)
0
20
40
60
80
100
Time

- Using the above steps, the estimated trend is $b_1 = 2.92$
- The forecast for the next points from y_{100} is

$$\hat{y}_{100+l} = s_{100} + b_1 \cdot l = s_{100} + 2.92 \cdot l$$