$(k=2)$

t	y_t	MA $(k=2)$	weights of y_t
1	y_1		0
2	y_2		0
3	y_3		0
4	y_4		1/2
5	y_5	$\frac{y_4+y_5}{2}$	1/2

① For MA

$$S_t = \frac{y_4+y_5}{2} = \frac{1}{2}y_5 + \frac{1}{2}y_4 + 0\cdot y_3 + 0\cdot y_2 + 0\cdot y_1$$

② For Exp. smoothing

$$S_t = .7\ y_5 + .21\cdot y_4 + .063\ y_3 + .018\cdot y_2 + .00567\ y_1$$

$$\hat{s}_t = \frac{y_t + wy_{t-1} + w^2 y_{t-2} + \ldots + w^t y_0}{1/(1-w)}$$

$$\hat{s}_t = (1-w)\ y_t + w(1-w)\ y_{t-1} + w^2(1-w)\ y_{t-2} + \cdots + w^t(1-w)\ y_0$$

$$\begin{aligned}\hat{s}_t &= \hat{s}_{t-1} + (1-w)(y_t - \hat{s}_{t-1}) \\ &= (1-w)y_t + w\hat{s}_{t-1}\end{aligned}$$

$\hat{S}_0 = y_0$

$\hat{S}_1 = (1-w)\cdot y_1 + w\cdot \hat{S}_0$

$\hat{S}_2 = (1-w)\ y_2 + w\cdot \hat{S}_1$

$\vdots$

$\hat{S}_T = (1-w)\cdot y_T + w\ \hat{S}_{T-1}$

t	1	2	3	4	5
y_t	1	3	5	8	13
	y_1	y_2			

$w = .8$

⊛ calculate the (single) exp. smoothing S_t

$S_1 = y_1 = 1$

$S_2 = (1-w) \cdot y_2 + w \cdot \underline{S_1}$

$= .2 \cdot y_2 + .8 \cdot = 1.4$

$S_3 = .2 \cdot y_3 + .8 * S_2 = .2 * 5 + .8 * 1.4$