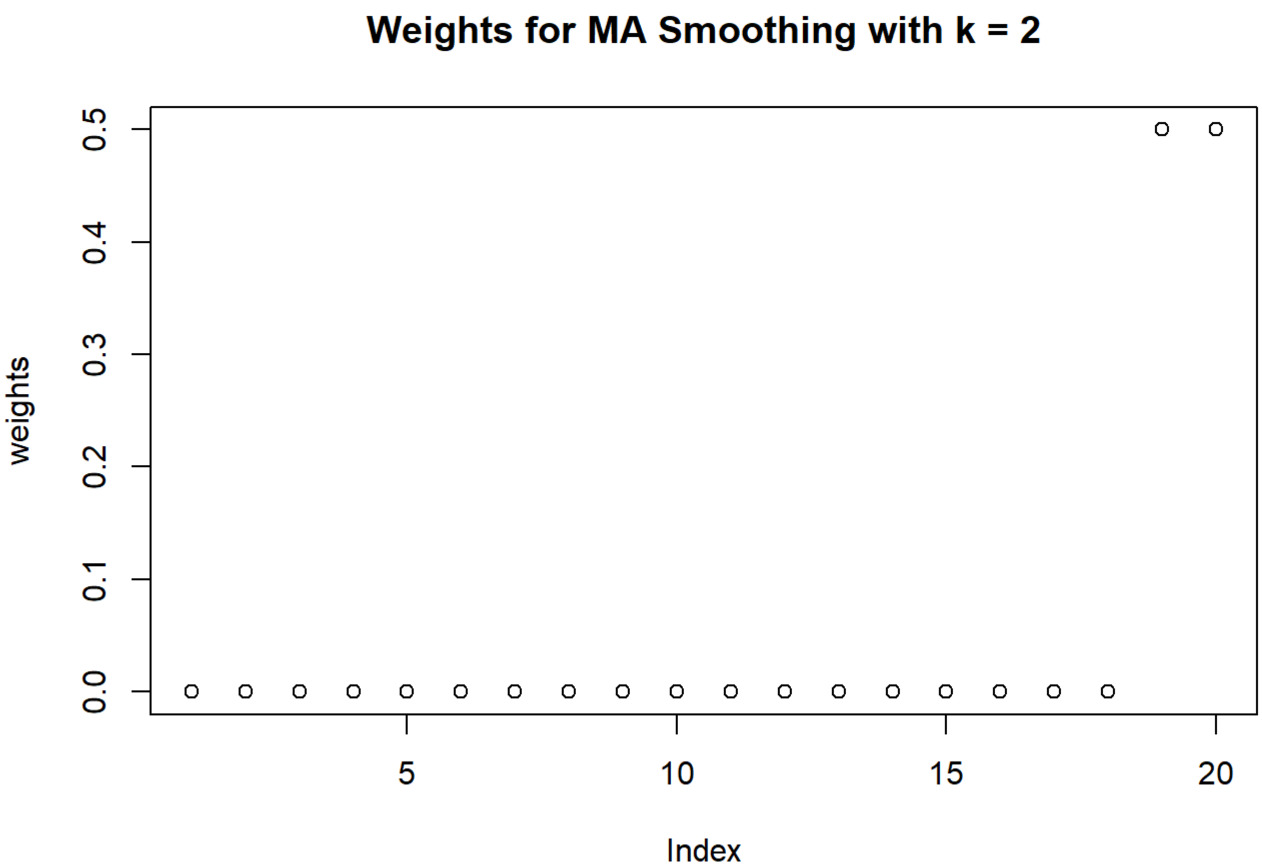
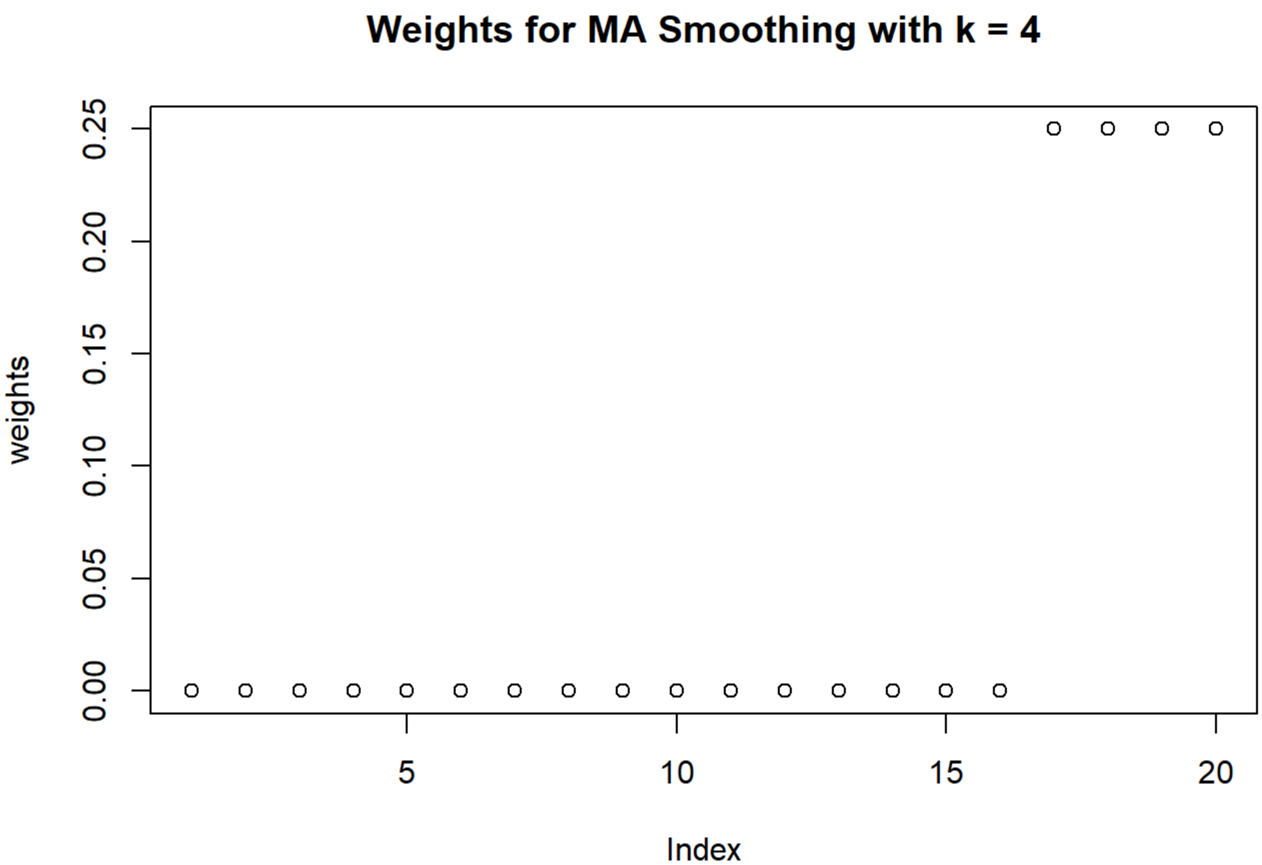
Time Series

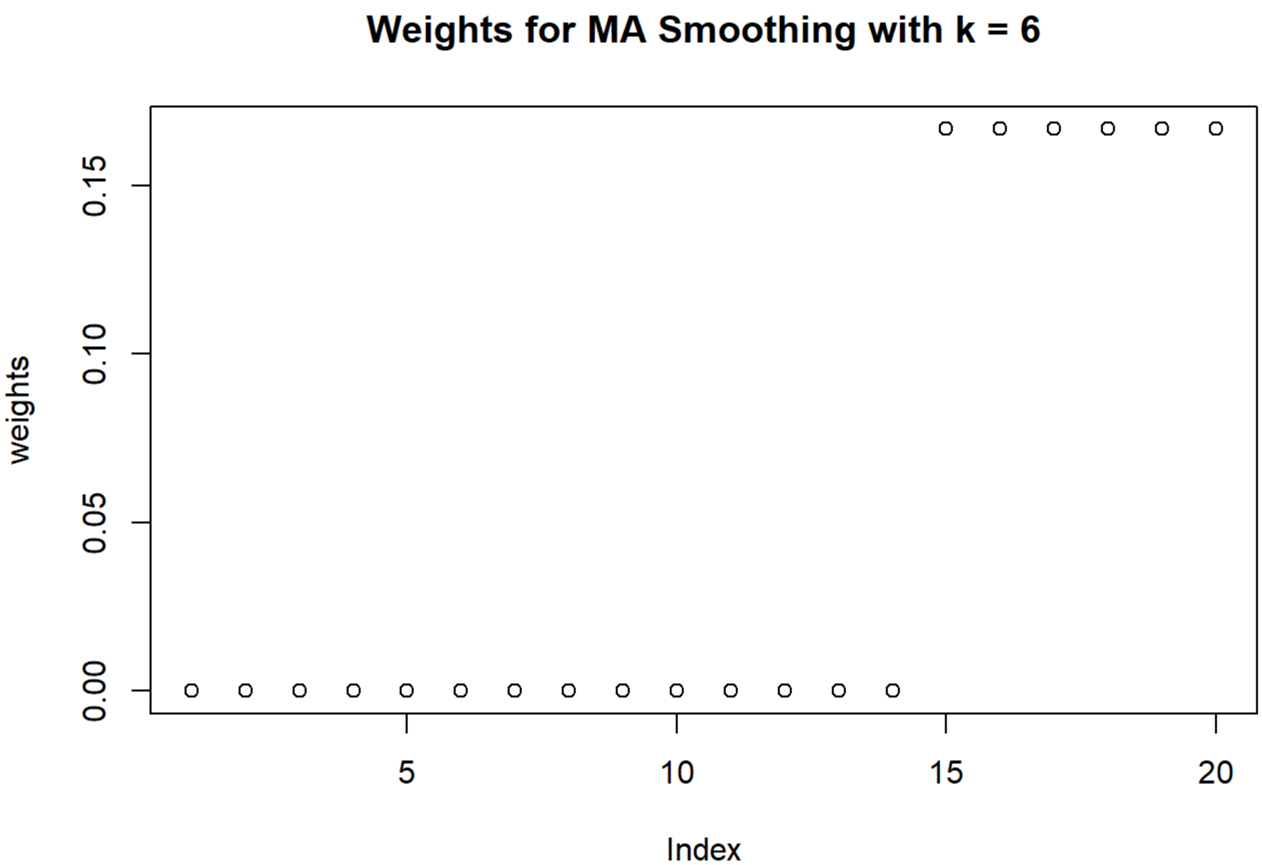
MA Weights Distribution



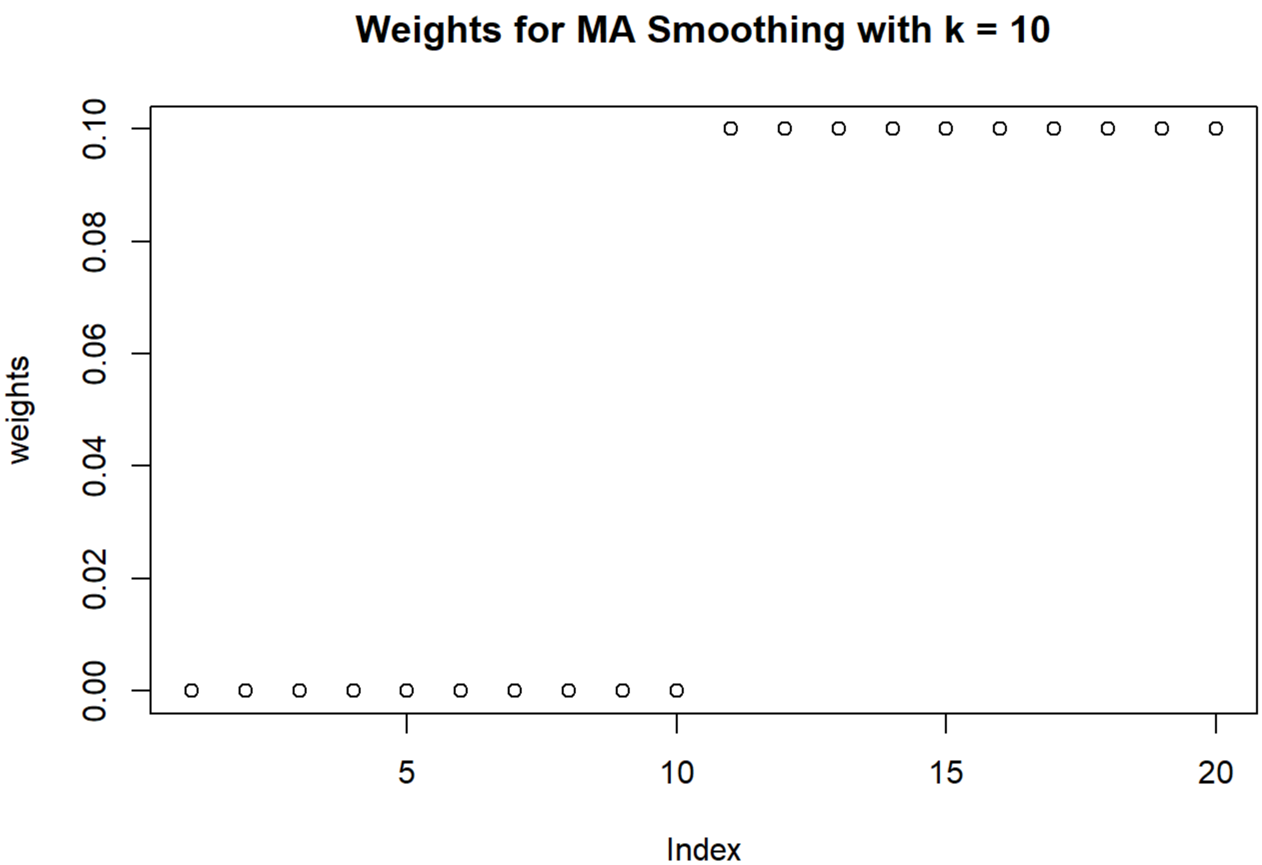
MA Weights Distribution



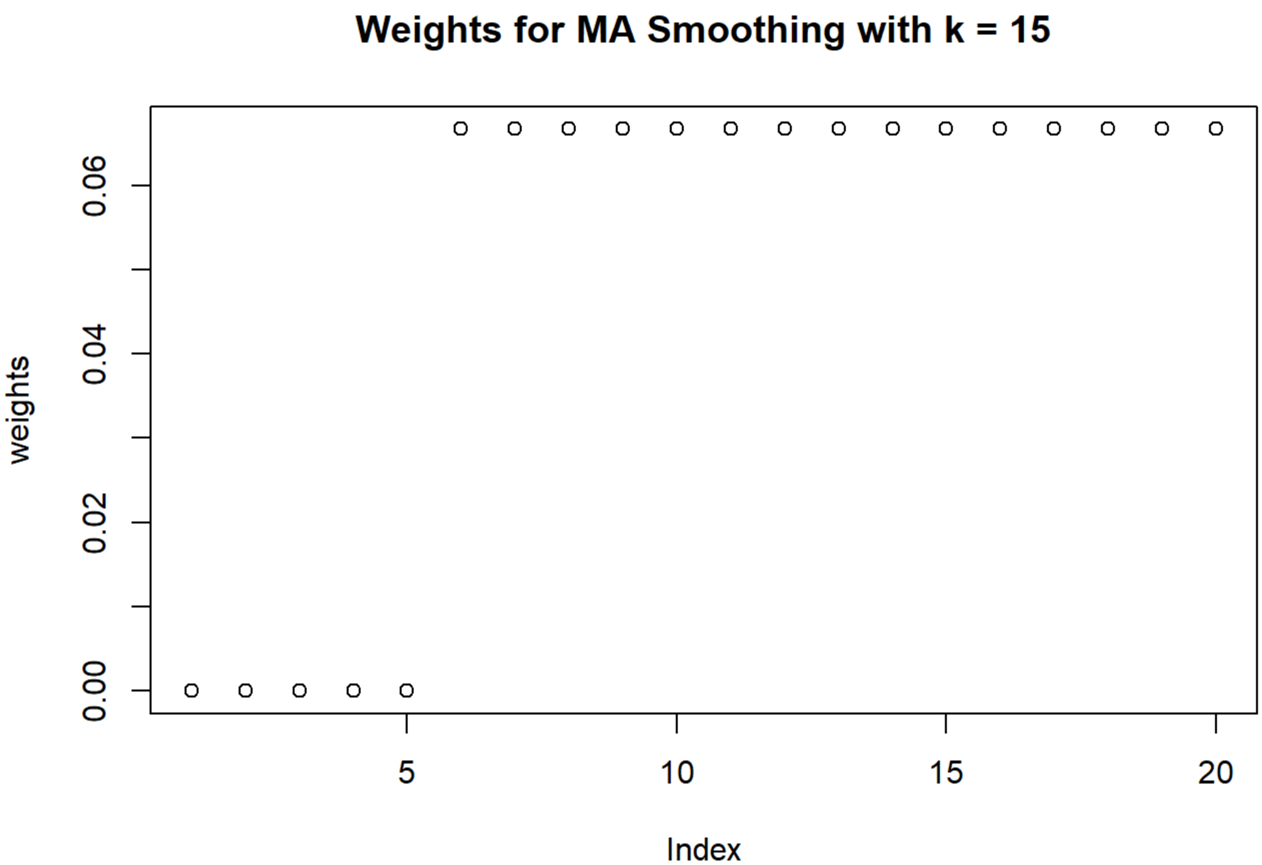
MA Weights Distribution



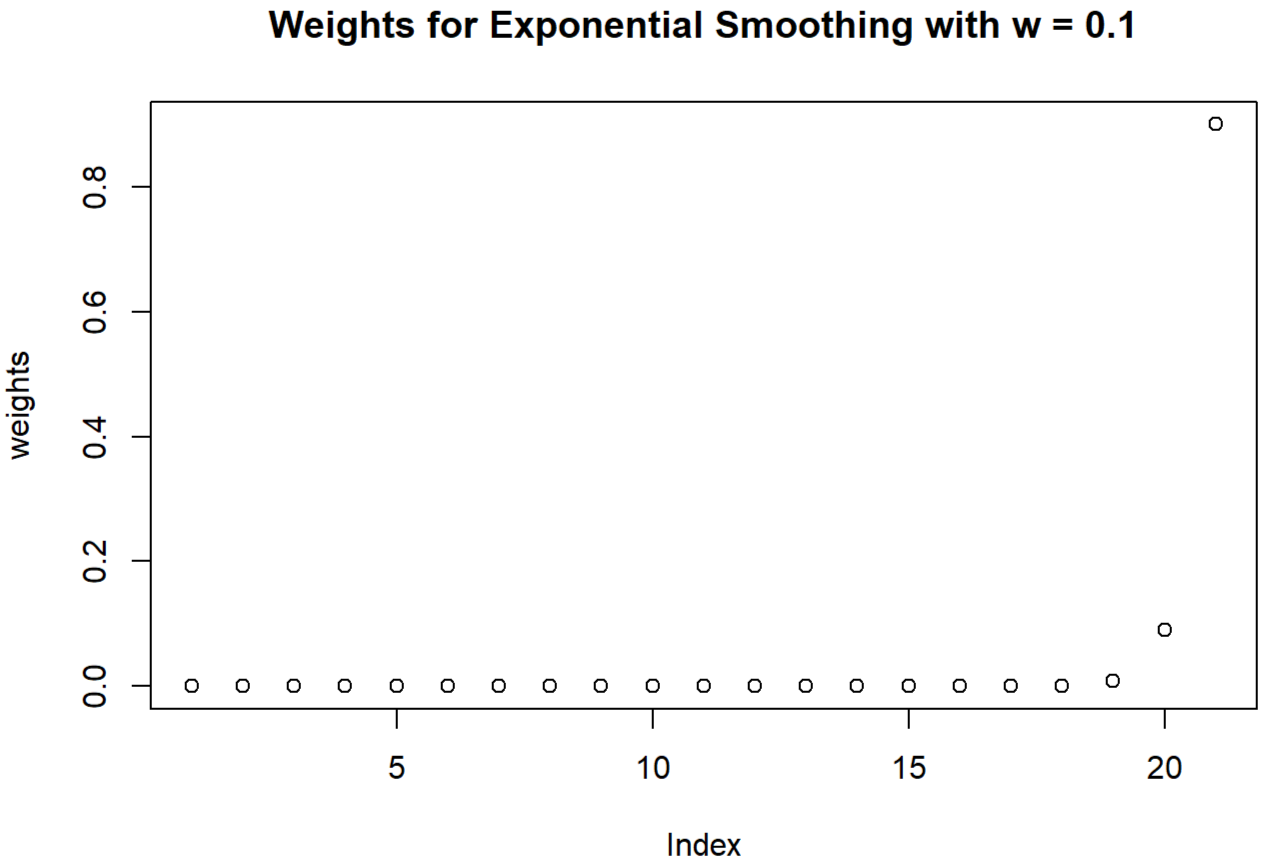
MA Weights Distribution



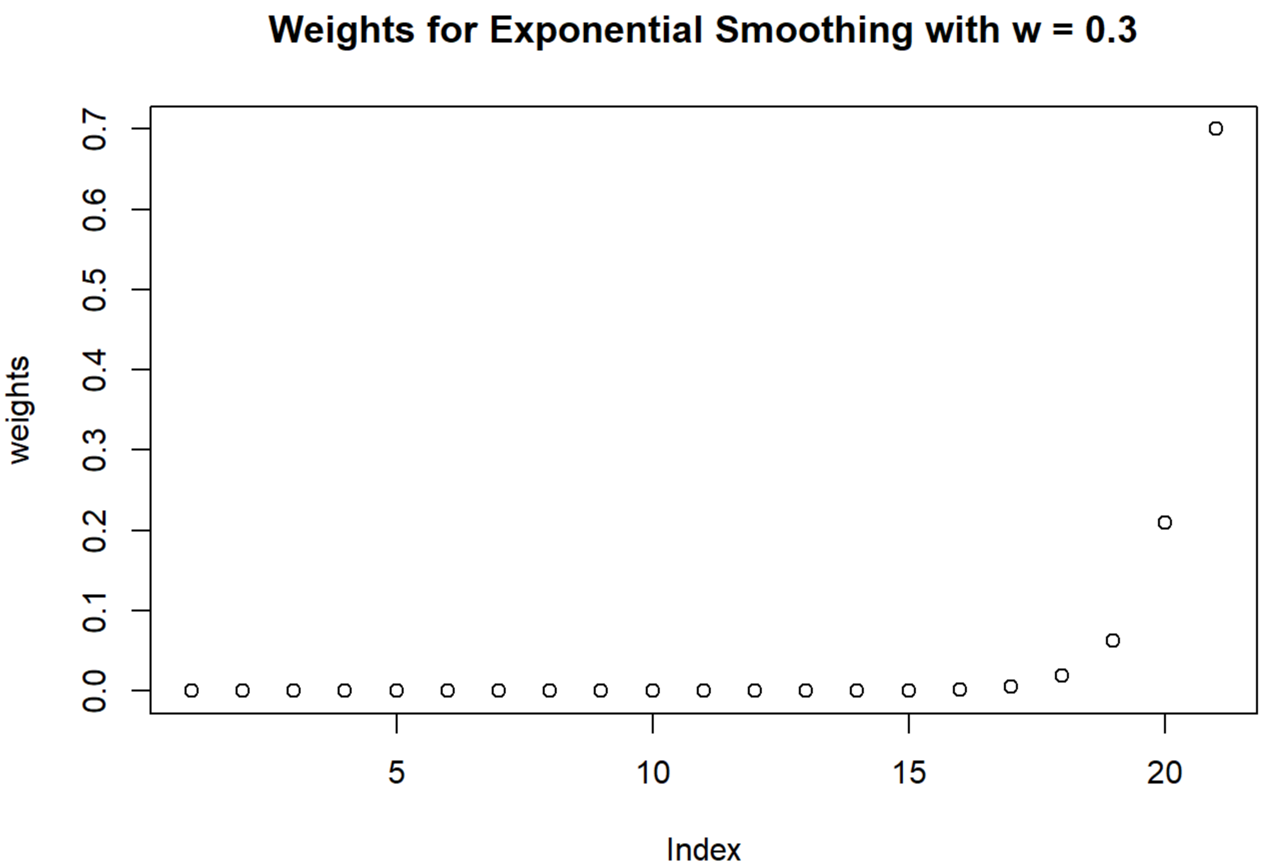
MA Weights Distribution



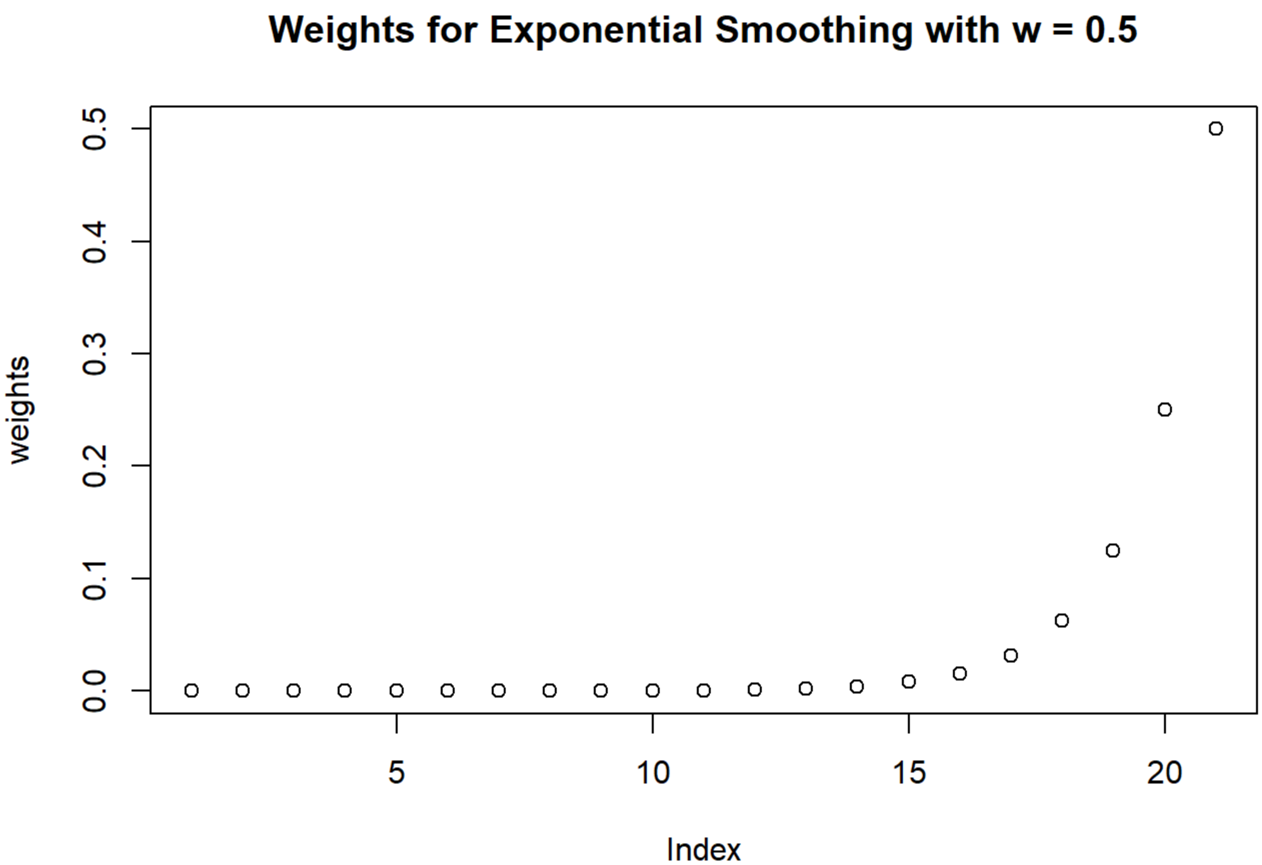
Exponential Weights Distribution



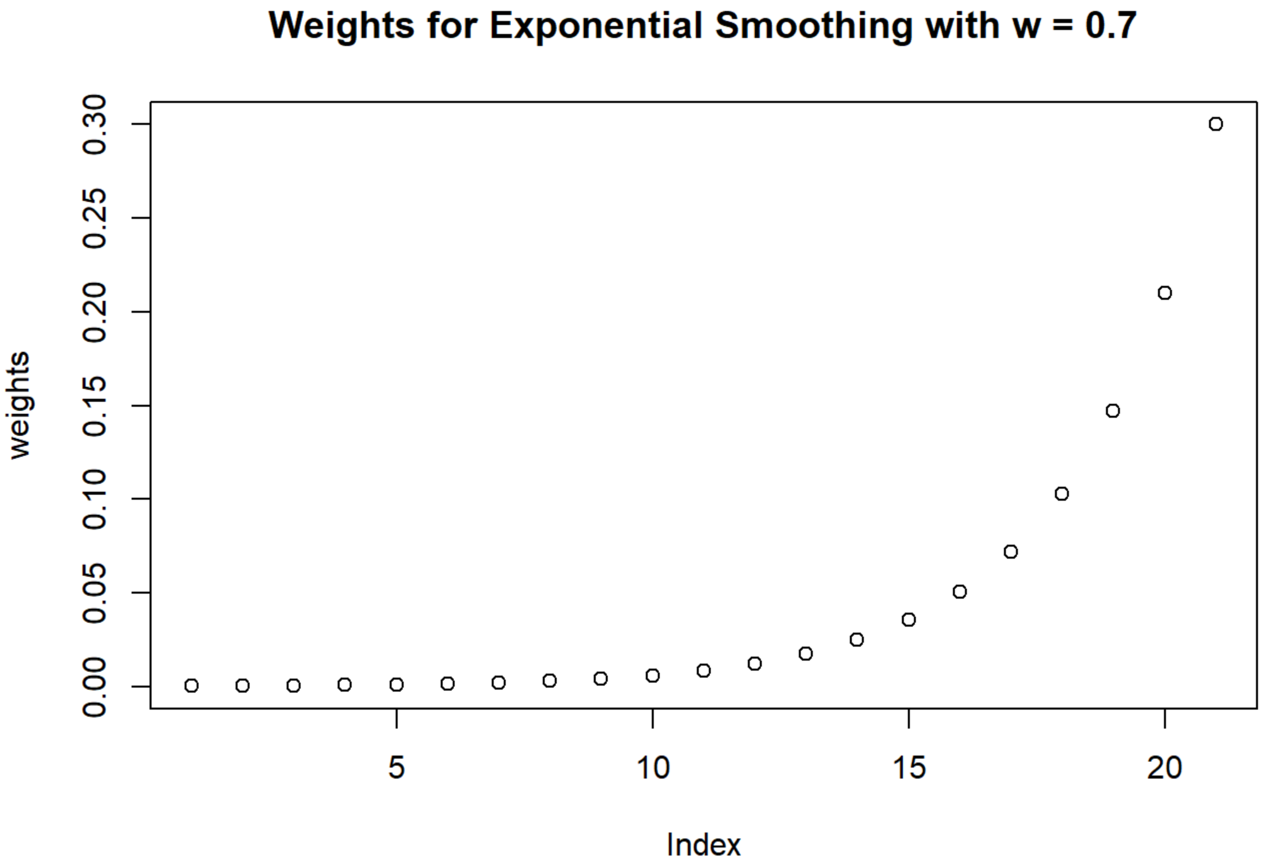
Exponential Weights Distribution



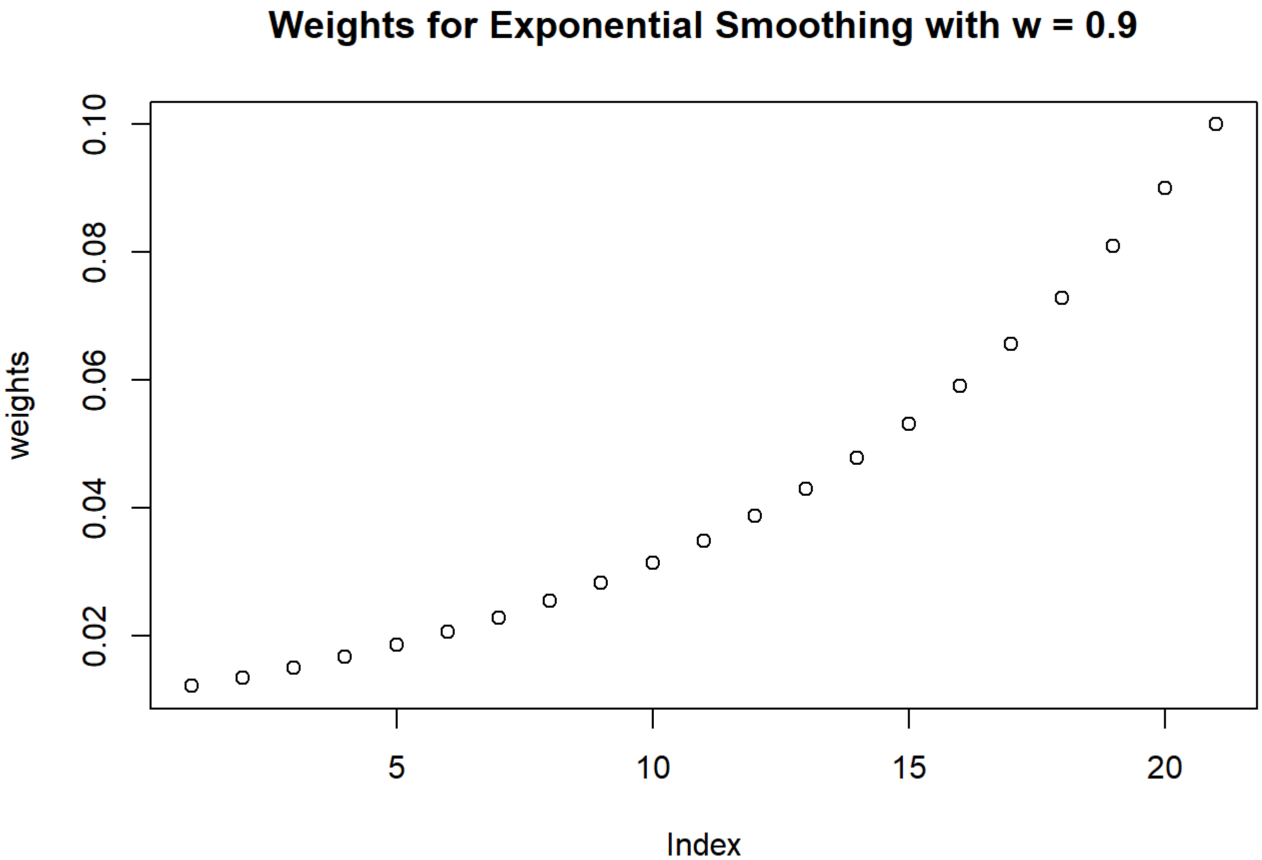
Exponential Weights Distribution



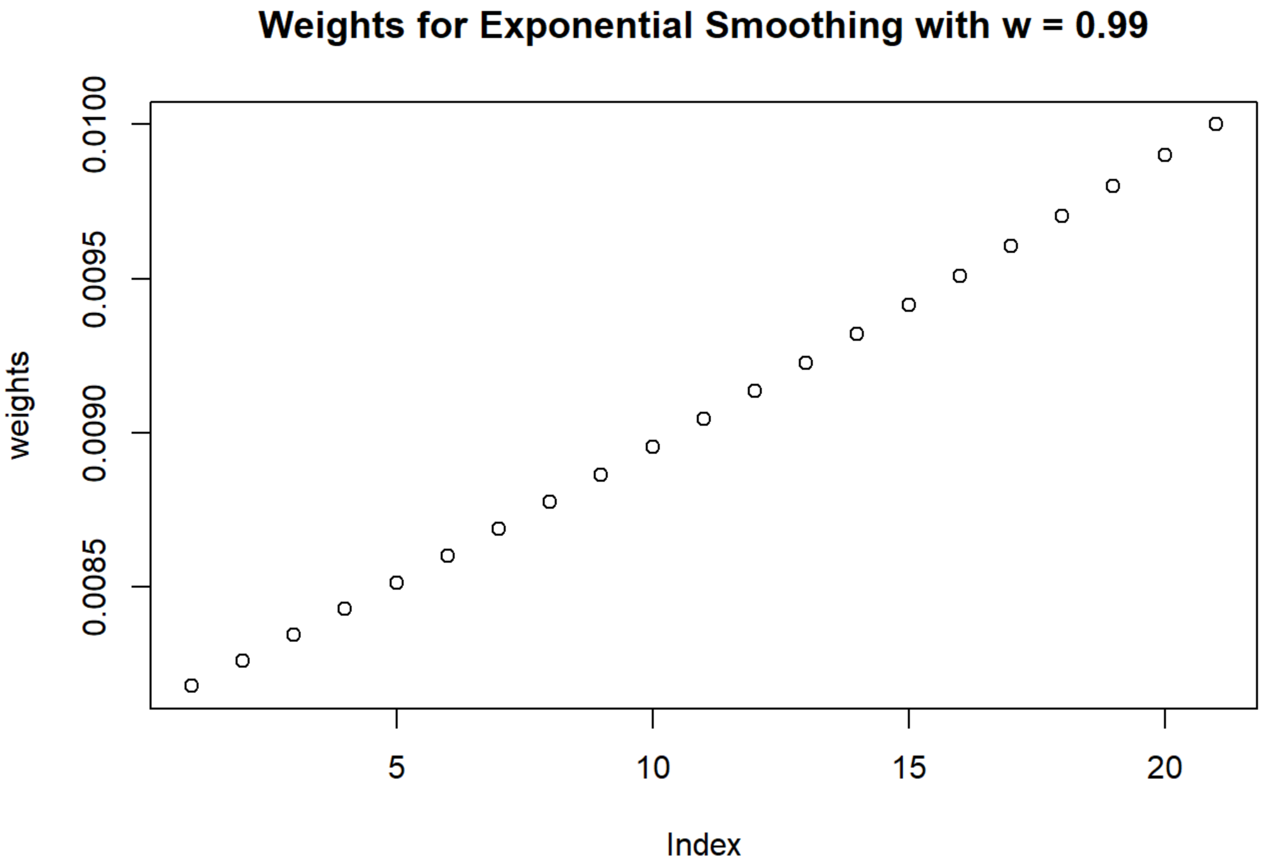
Exponential Weights Distribution



Exponential Weights Distribution



Exponential Weights Distribution



Exponential Smoothing

- MA distributes the weight equally to the recent observations
- Exponential Smoothing controls the weights of the recent observations by w

$$\hat{s}_t = \frac{y_t + wy_{t-1} + w^2 y_{t-2} + \ldots + w^t y_0}{1/(1-w)}$$

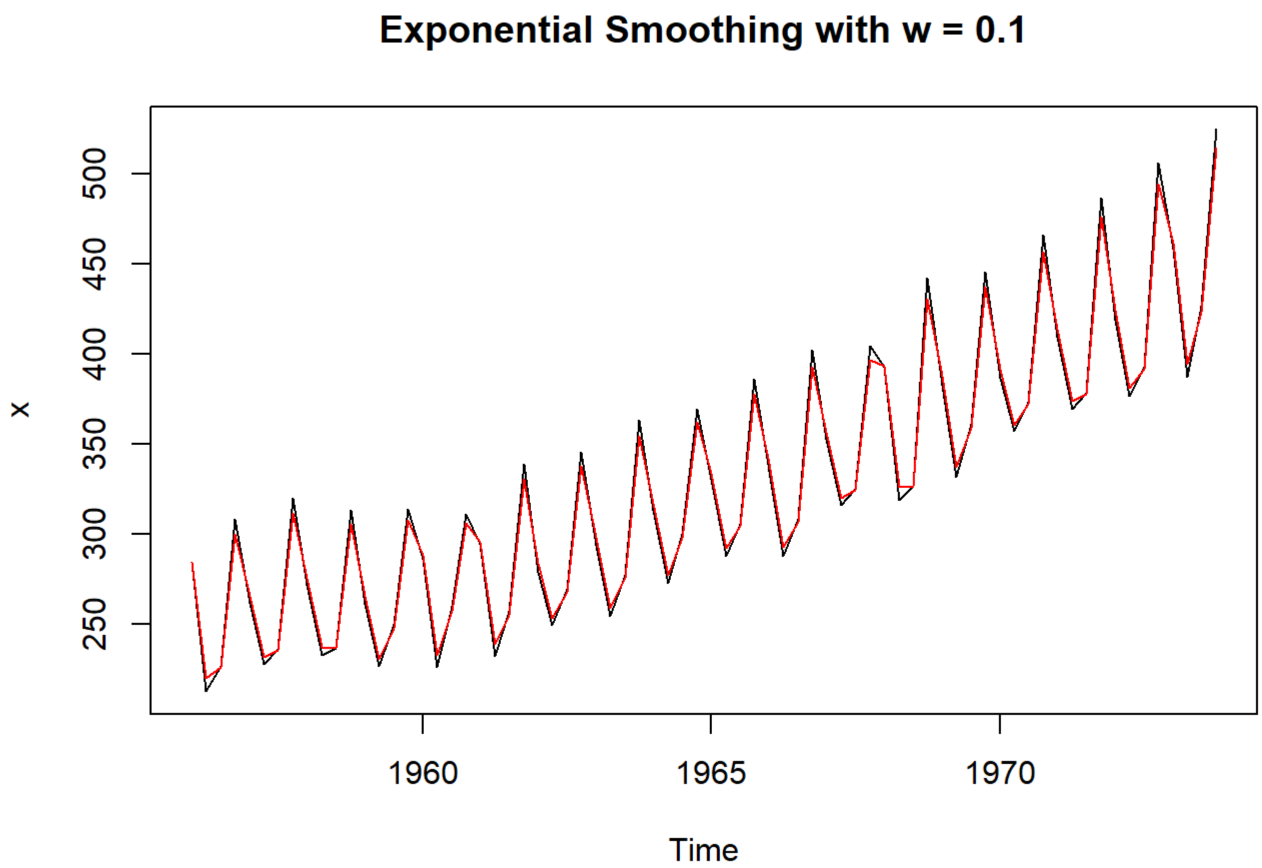
- Smaller w smooths the series more lightly.
- Greater w smooths the series more strongly.

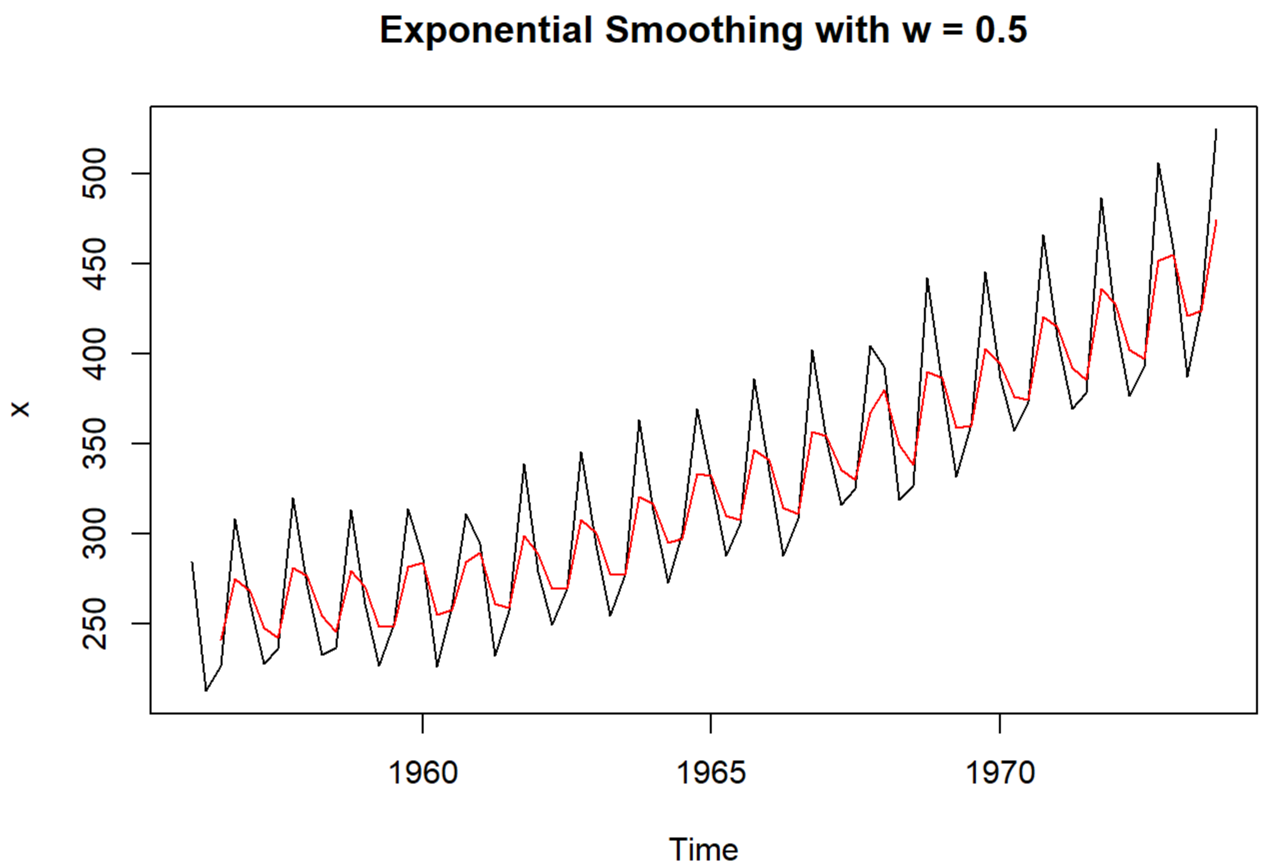
Exponential Smoothing

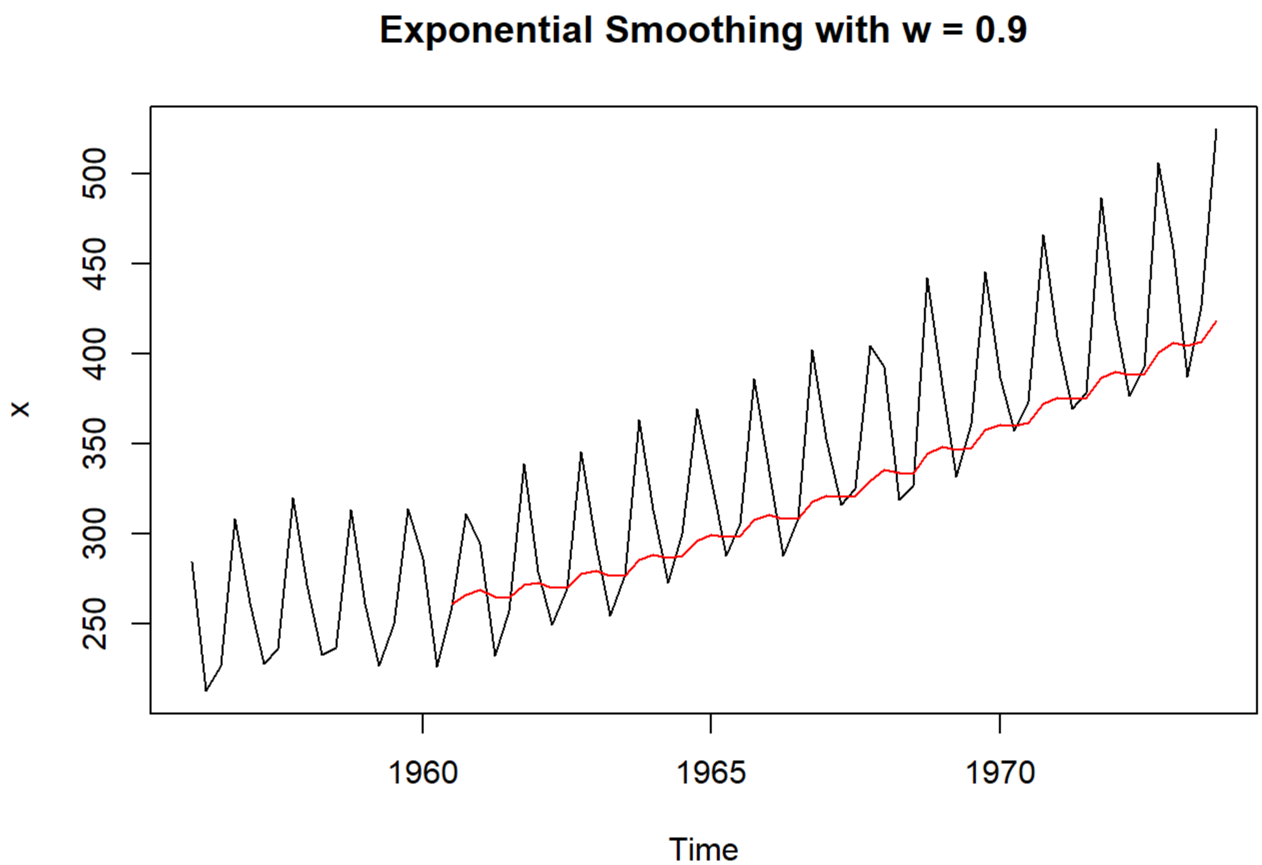
- We have

$$\begin{aligned}\hat{s}_t &= \hat{s}_{t-1} + (1-w)(y_t - \hat{s}_{t-1}) \\ &= (1-w)y_t + w\hat{s}_{t-1}\end{aligned}$$

- When $w \to 0$, $\hat{s}_t \to y_t$, or little smoothing has taken


Exponential Smoothing with w = 0.1
x
250
300
350
400
450
500
1960
1965
1970
Time


Exponential Smoothing with w = 0.5
x
250
300
350
400
450
500
1960
1965
1970
Time



Double Exponential Smoothing

We can use double smoothing to identify the trend and forecast linear trend time series as follows.

- Step 1: Create a smoothed series: $\hat{s}_t^{(1)} = (1-w)y_t + w\hat{s}_{t-1}^{(1)}$
- Step 2: Create a double smoothed series: $\hat{s}_t^{(2)} = (1-w)\hat{s}_t^{(1)} + w\hat{s}_{t-1}^{(2)}$
- Step 3: Estimate the trend:

$$b_1 = \frac{1-w}{w}(\hat{s}_T^{(1)} - \hat{s}_T^{(2)})$$

- Step 4: Forecast

$$\hat{y}_{T+l} = \hat{s}_T^{(1)} + b_1 \cdot l$$

Example

You are given the following time series

t	**1**	**2**	**3**	**4**	**5**
y_t	1	3	5	8	13

- Assume that this is a linear trend time series. Using double exponential smoothing with $w = .8$ to estimate the trend (slope) and forecast y_6.