

# Time Series Decomposition

# Time Series Patterns

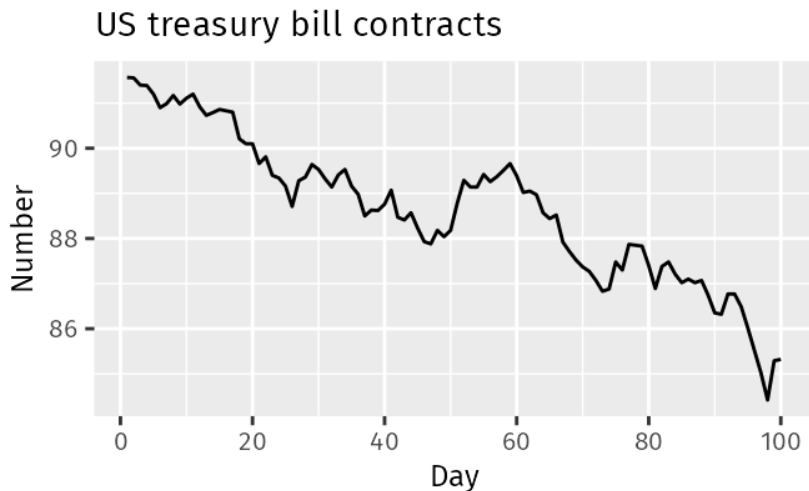
A time series may consist of

- ▶ trend,
- ▶ seasonality and
- ▶ cycles

# Trend

- ▶ A trend is a long-term increase or decrease in the data.
- ▶ Trend does not have to be linear.
- ▶ Sometimes we will refer to a trend as “changing direction”, when it might go from an increasing trend to a decreasing trend.

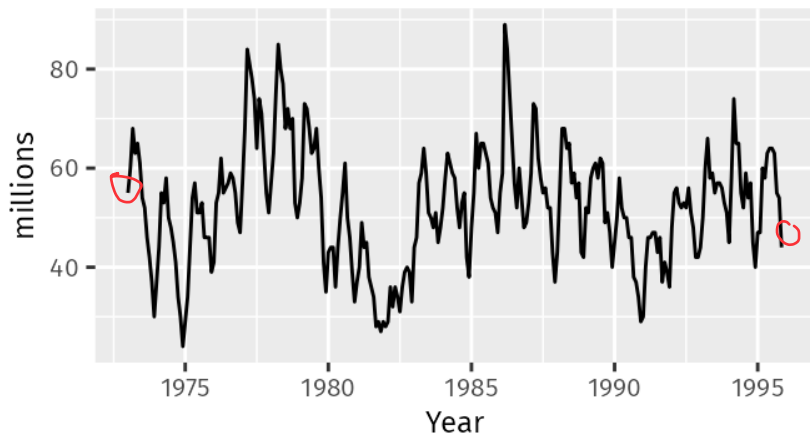
## Examples



- ▶ The US treasury bill contracts show results from the Chicago market for 100 consecutive trading days in 1981. There is a downward trend

## Examples

Sales of new one-family houses, USA



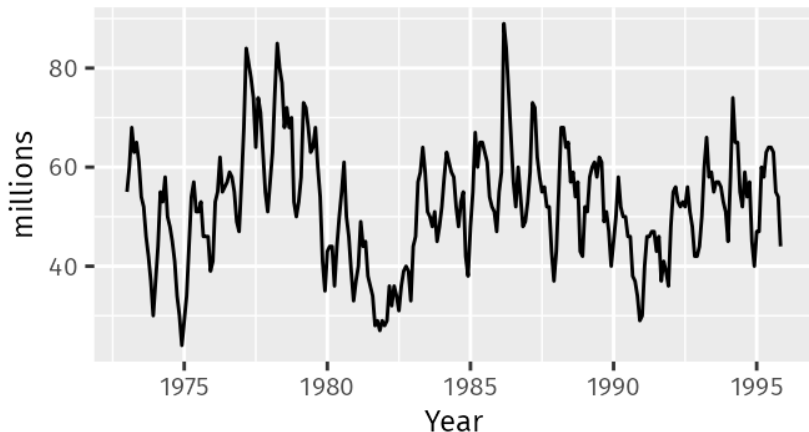
- ▶ There is no apparent trend in the data over this period

# Cycle and Seasonal

- ▶ cycle: repeated events over time that are not equally spaced
- ▶ seasonal: repeated events over time that are equally spaced

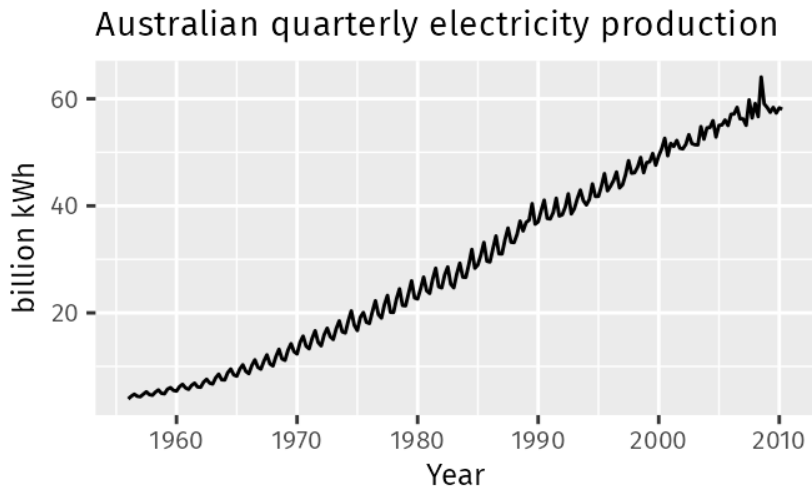
## Examples

Sales of new one-family houses, USA



- The monthly housing sales show strong seasonality within each year, as well as some strong cyclic behaviour with a period of about 6–10 years

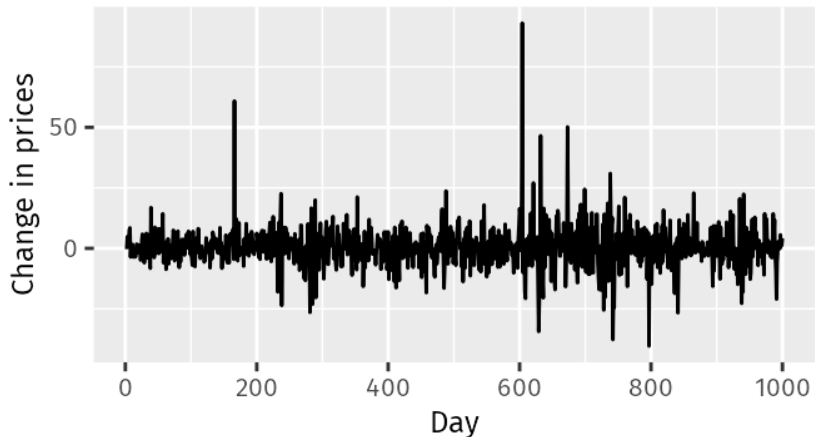
## Examples



The Australian quarterly electricity production shows a strong increasing trend, with strong seasonality. There is no evidence of any cyclic behavior here



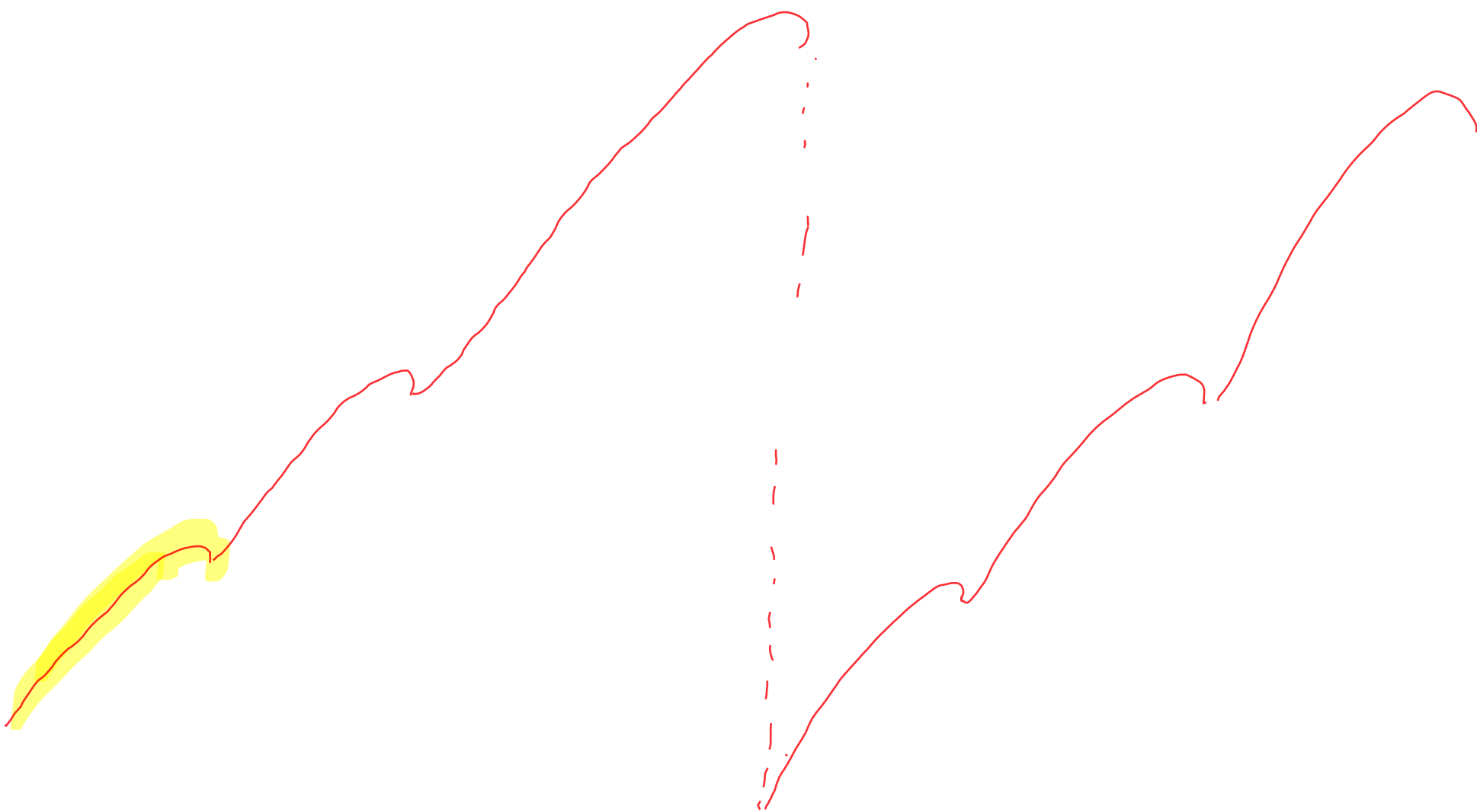
## Google daily changes in closing stock price



- ▶ No trend, seasonality or cyclic behaviour. There are random fluctuations which do not appear to be very predictable, and no strong patterns that would help with developing a forecasting model.

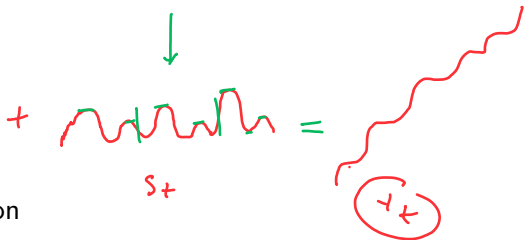
# Time series decomposition

- ▶ When we decompose a time series into components, we usually combine the trend and cycle into a single trend-cycle component (sometimes called the trend for simplicity).
- ▶ Three components: a trend-cycle component, a seasonal component, and a remainder component (containing anything else in the time series).



## Two decompositions

additive series:



► Additive Decomposition

$$y_t = S_t + T_t + R_t$$

► Multiplicative decomposition

⇒  $y_t = S_t \times T_t \times R_t$

$$Y_t = S_t \cdot T_t \cdot R_t$$

If we apply the additive model to  $Y_t$ , it would not be most appropriate.

we can convert  $Y_t$  to  $\log Y_t$  then apply the additive model.

$$\begin{aligned} \log Y_t &= \log (S_t \cdot T_t \cdot R_t) \\ &= \underline{\log S_t} + \underline{\log T_t} + \underline{\log R_t} \end{aligned}$$

## Relationship

- ▶ The log transformation will turn a multiplicative model to additive model.

## Central MA vs. MA

Link.

## Classical Decomposition

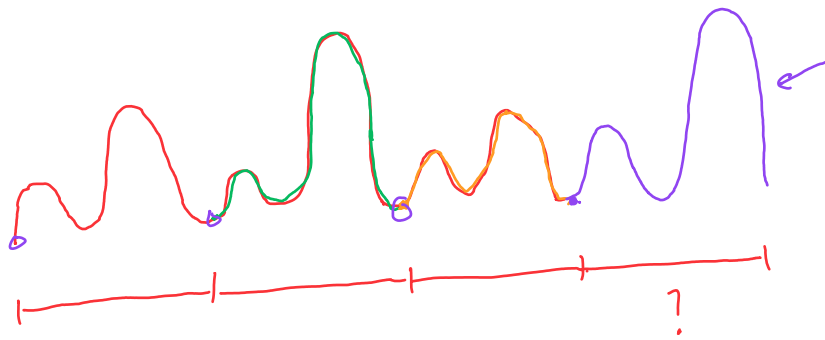
- ▶ In classical decomposition, we assume that the seasonal component is constant from year to year

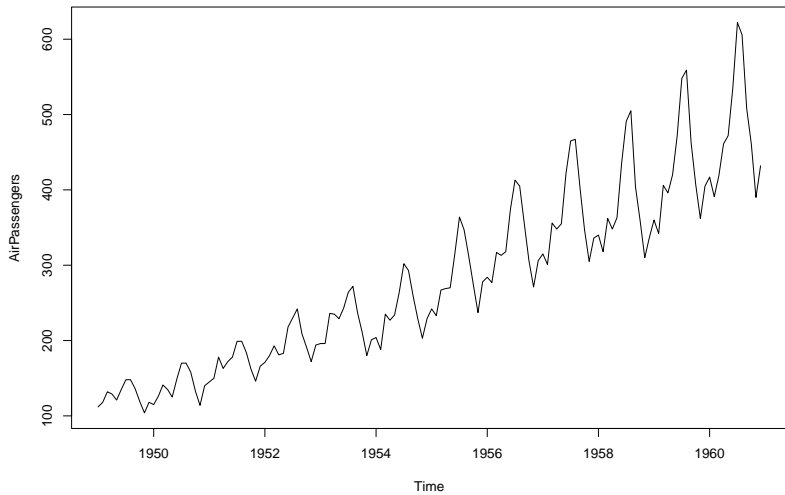


# Classical Decomposition

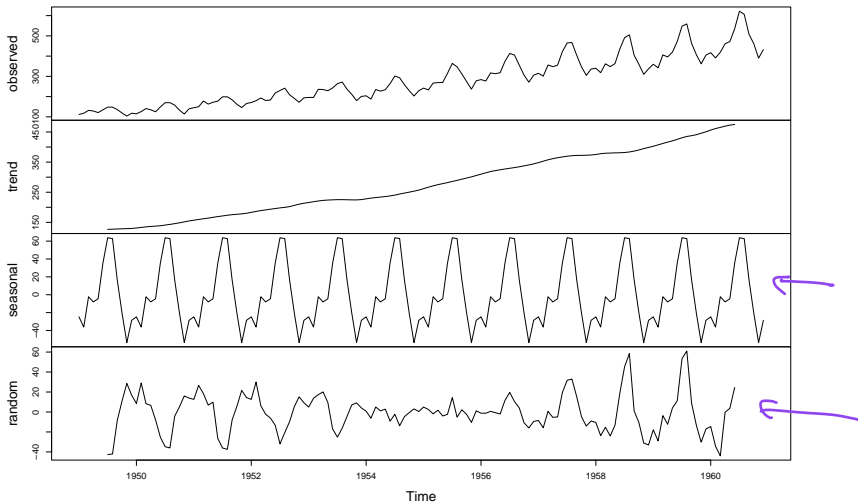
- ▶ Trend Estimate: Smooth the data using centred moving average (CMA) of the order equal to the periodicity of the data
- ▶ Detrend the data: calculate the detrended series by subtracting/dividing the trend estimate
- ▶ Seasonal Estimate: Average value for each period is calculated based on the de-trended series.
- ▶ Random/Remainder: subtract from the series the trend and seasonal component estimations

Seasonal component:

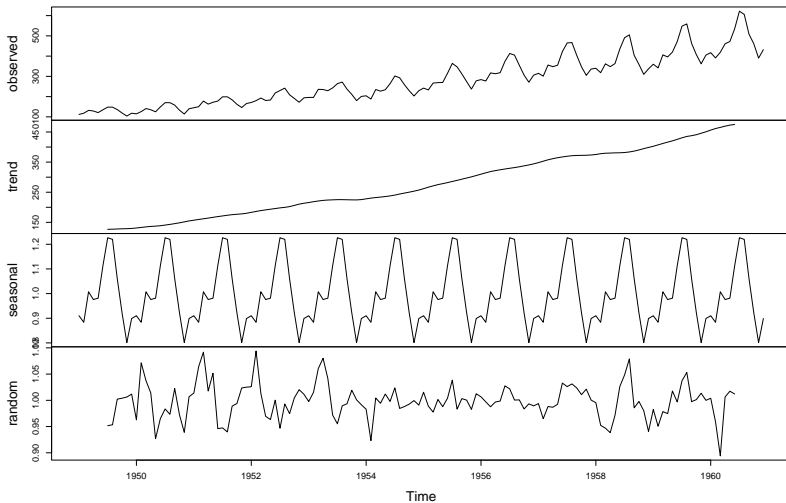




Decomposition of additive time series



**Decomposition of multiplicative time series**



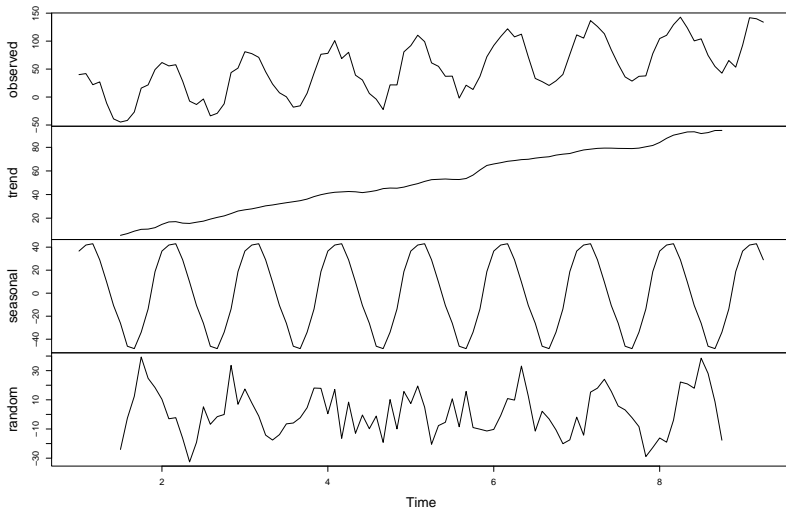
sequence: 1, 2, 2, ... 100 (trend)

seasonal

```
y <- ts(c(1:100) +  
        50*cos(seq(0,16*pi,length.out=100)) +  
        rnorm(100,0,10),frequency=12)
```

random normal  
with mean 0  
sd = 10

**Decomposition of additive time series**



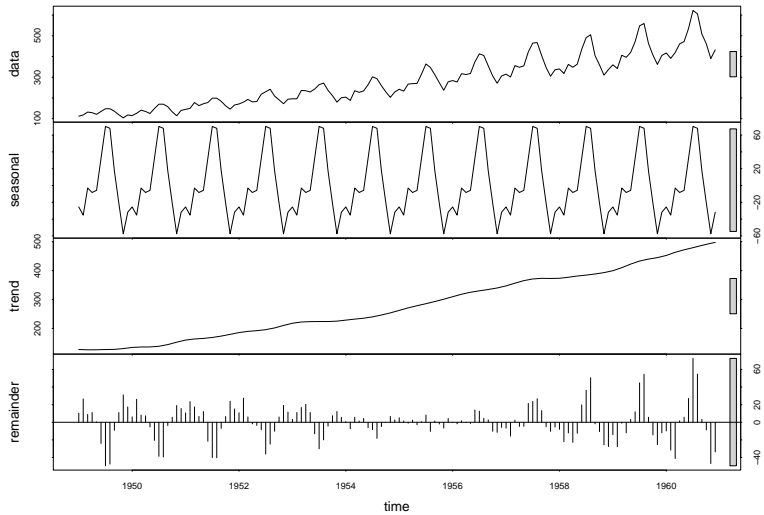
# STL Decomposition

- ▶ Classical Decomposition uses Central Moving Average to estimate the trend
- ▶ STL uses LOESS (locally estimated scatterplot smoothing) to estimate the trend
- ▶ Link
- ▶ The seasonal component is estimated using moving averages of the de-trend series



- ▶ STL will handle any type of seasonality
- ▶ STL is a versatile and robust method for decomposing time series
- ▶ STL can be implemented using `stl` function

```
data("AirPassengers")  
stl_Decomposition <- stl(AirPassengers,  
                          s.window = "periodic")  
plot(stl_Decomposition)
```



## Other Decomposition Methods

- ▶ X-11 decomposition: originated in the US Census Bureau and Statistics Canada
- ▶ SEATS decomposition: developed at the Bank of Spain, and is now widely used by government agencies around the world