

(k=2)

t	y _t	MA (k=2)	weights of y _t
1	y ₁		0
2	y ₂		0
3	y ₃		0
4	y ₄		1/2
5	y ₅	$\frac{y_4 + y_5}{2}$	1/2

① For MA

$$S_t = \frac{y_4 + y_5}{2} = \frac{1}{2} y_5 + \frac{1}{2} y_4 + 0 \cdot y_3 + 0 \cdot y_2 + 0 \cdot y_1$$

② For Exp. smoothing

$$S_t = .7 y_5 + .21 y_4 + .063 y_3 + .018 y_2 + .00567 y_1$$

$$\hat{s}_t = \frac{y_t + w y_{t-1} + w^2 y_{t-2} + \dots + w^t y_0}{1/(1-w)}$$

$$\hat{s}_t = (1-w) y_t + w(1-w) y_{t-1} + w^2(1-w) y_{t-2} + \dots + w^t(1-w) y_0$$

Another formula for exp. smoothing

$$\begin{aligned} \hat{s}_t &= \hat{s}_{t-1} + (1-w)(y_t - \hat{s}_{t-1}) \\ &= (1-w)y_t + w\hat{s}_{t-1} \end{aligned}$$

$$\hat{s}_0 = y_0$$

$$\hat{s}_1 = (1-w) y_1 + w \cdot \hat{s}_0$$

$$\hat{s}_2 = (1-w) y_2 + w \cdot \hat{s}_1$$

$$\vdots$$

$$\hat{s}_T = (1-w) y_T + w \hat{s}_{T-1}$$

Example

t	1	2	3	4	5
y_t	1	3	5	8	13
	τ_1	τ_2			

$$w = .8$$

② calculate the (single) exp. smoothing S_t

$$S_1 = \tau_1 = 1$$

$$S_2 = (1-w) \cdot \tau_2 + w \cdot S_1$$

$$= .2 \cdot \tau_2 + .8 \cdot 1 = 1.4$$

$$S_3 = .2 \cdot \tau_3 + .8 \cdot S_2 = .2 \cdot 5 + .8 \cdot 1.4$$

and so on...