

Time Series Decomposition

Time Series Patterns

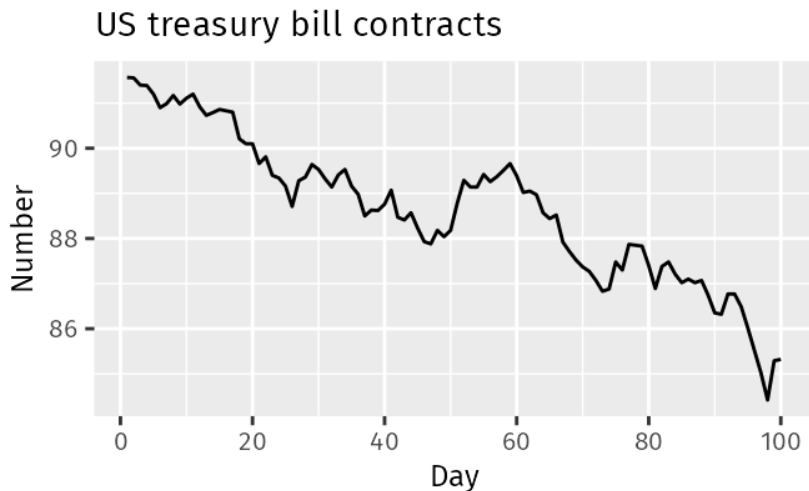
A time series may consist of

- ▶ trend,
- ▶ seasonality and
- ▶ cycles

Trend

- ▶ A trend is a long-term increase or decrease in the data.
- ▶ Trend does not have to be linear.
- ▶ Sometimes we will refer to a trend as “changing direction”, when it might go from an increasing trend to a decreasing trend.

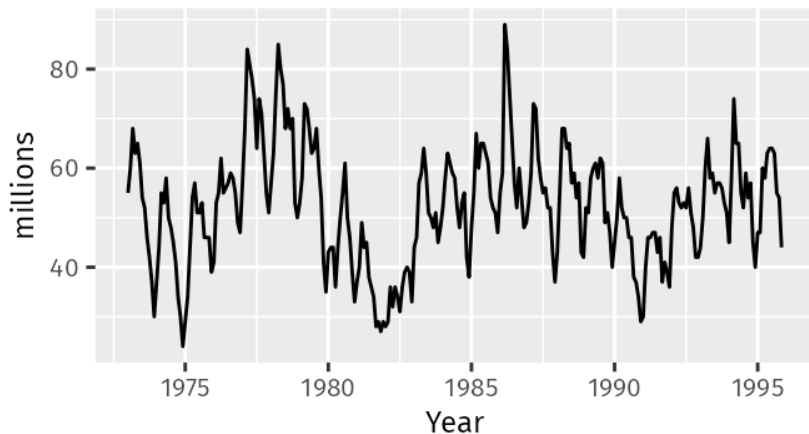
Examples



- ▶ The US treasury bill contracts show results from the Chicago market for 100 consecutive trading days in 1981. There is a downward trend

Examples

Sales of new one-family houses, USA



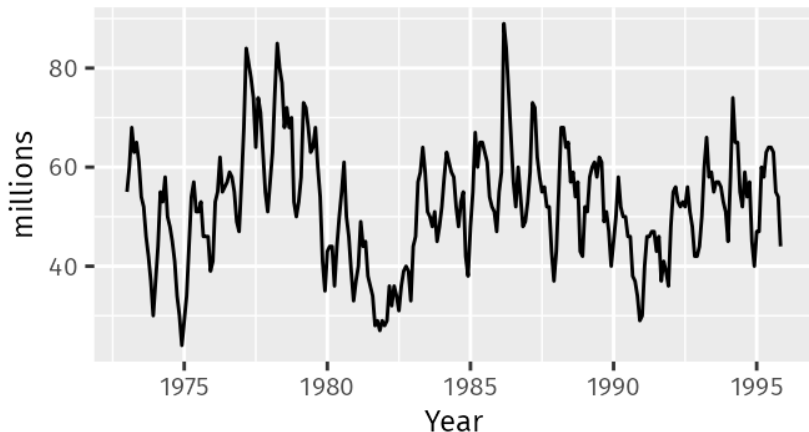
- There is no apparent trend in the data over this period

Cycle and Seasonal

- ▶ cycle: repeated events over time that are not equally spaced
- ▶ seasonal: repeated events over time that are equally spaced

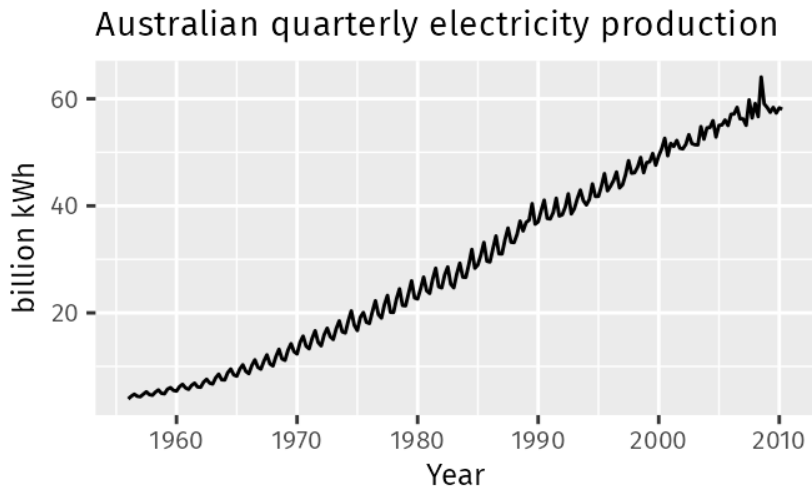
Examples

Sales of new one-family houses, USA



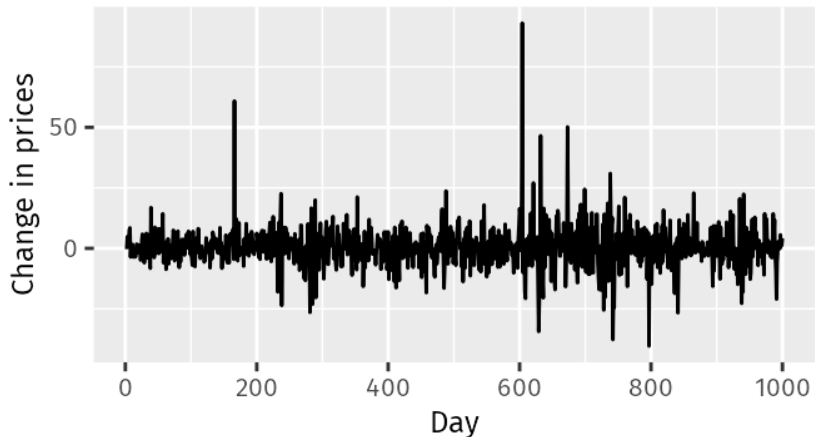
- The monthly housing sales show strong seasonality within each year, as well as some strong cyclic behaviour with a period of about 6–10 years

Examples



The Australian quarterly electricity production shows a strong increasing trend, with strong seasonality. There is no evidence of any cyclic behavior here

Google daily changes in closing stock price



- ▶ No trend, seasonality or cyclic behaviour. There are random fluctuations which do not appear to be very predictable, and no strong patterns that would help with developing a forecasting model.

Time series decomposition

- ▶ When we decompose a time series into components, we usually combine the trend and cycle into a single trend-cycle component (sometimes called the trend for simplicity).
- ▶ Three components: a trend-cycle component, a seasonal component, and a remainder component (containing anything else in the time series).

Two decompositions

- ▶ Additive Decomposition

$$y_t = S_t + T_t + R_t$$

- ▶ Multiplicative decomposition

$$y_t = S_t \times T_t \times R_t$$

Relationship

- ▶ The log transformation will turn a multiplicative model to additive model.

To decompose a time series, Y_t

① Step 1: Estimate the trend of the series: T_t

② Step 2: De-trend the series

$$Y_t - T_t$$

③ Step 3: Estimate the seasonal component

$$S_t$$

④ Step 4: calculate the remainder / Random component.

$$R_t = Y_t - T_t - S_t$$

Central MA vs. MA

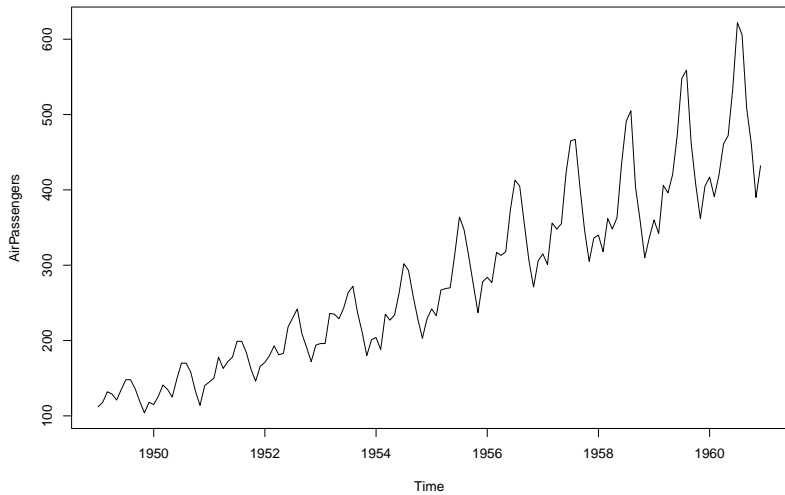
Link.

St		
MA		
k=2		
t		Control (C_t)
1	—	
3	2	
7	5	
5	6	—
10	7.5	10.67
20	15	—

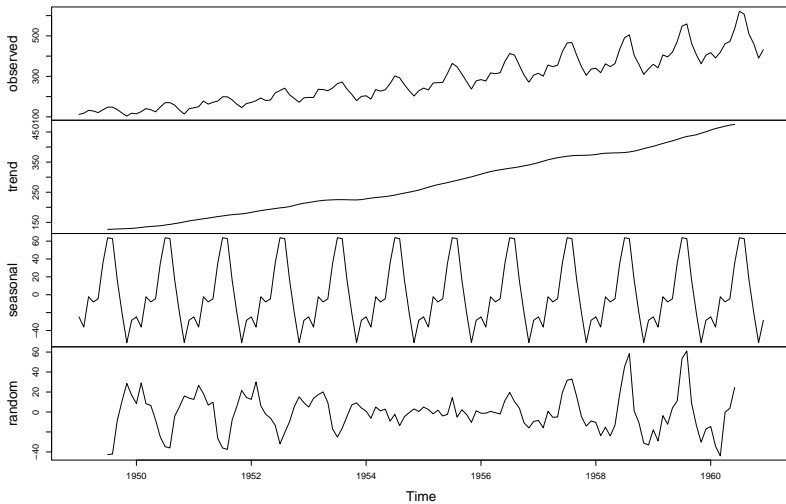
-13 -14 -15 -16 -17
 -14 -15 -16
 -15

Classical Decomposition

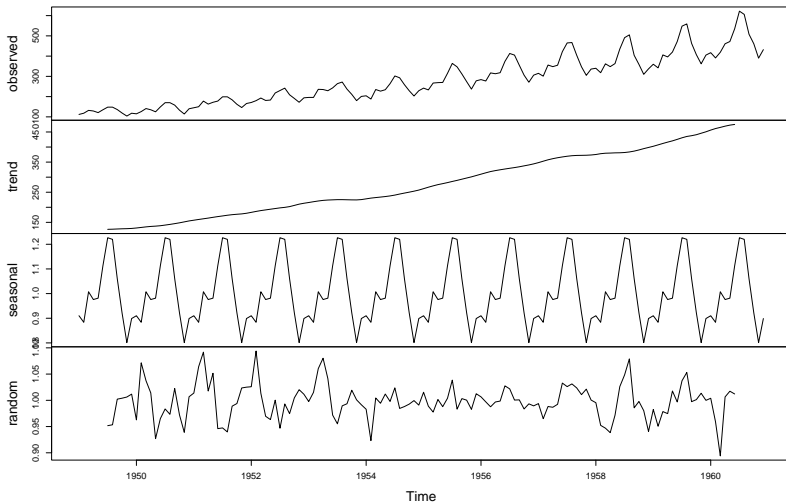
- ▶ Trend Estimate: Smooth the data using centred moving average (CMA) of the order equal to the periodicity of the data
- ▶ Detrend the data: calculate the detrended series by subtracting/dividing the trend estimate
- ▶ Seasonal Estimate: Average value for each period is calculated based on the de-trended series.
- ▶ Random/Remainder: subtract from the series the trend and seasonal component estimations



Decomposition of additive time series



Decomposition of multiplicative time series



construct a series

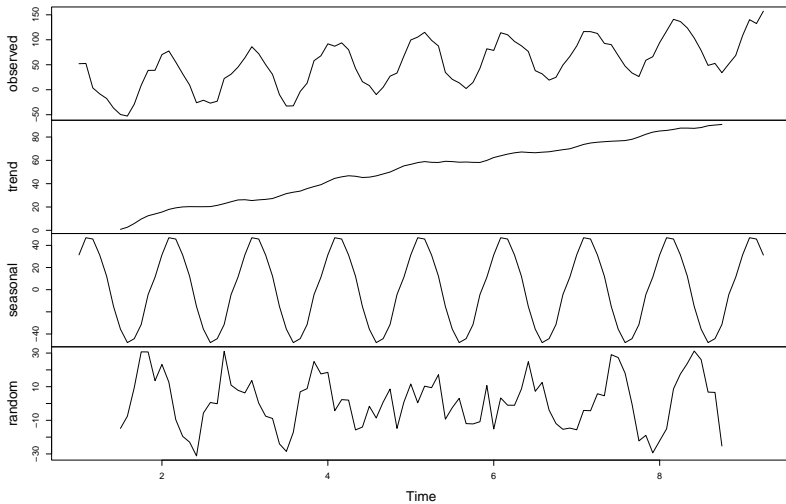
trend

seasonal

```
y <- ts(c(1:100)) +  
      50*cos(seq(0,16*pi,length.out=100)) +  
      rnorm(100,0,10),frequency=12)
```

random

Decomposition of additive time series

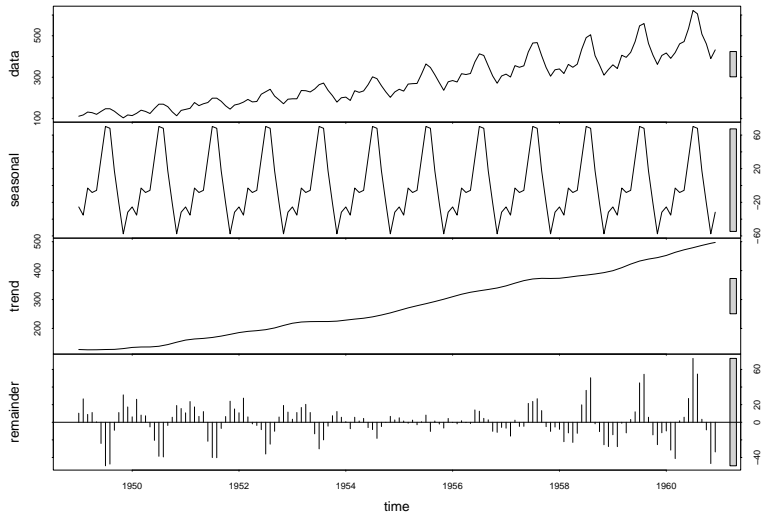


STL Decomposition

- ▶ Classical Decomposition uses Central Moving Average to estimate the trend
- ▶ STL uses LOESS (locally estimated scatterplot smoothing) to estimate the trend
- ▶ Link
- ▶ The seasonal component is estimated using moving averages of the de-trend series

- ▶ STL is a versatile and robust method for decomposing time series
- ▶ STL can be implemented using `stl` function

```
data("AirPassengers")
stl_Decomposition <- stl(AirPassengers,
                          s.window = "periodic")
plot(stl_Decomposition)
```

Other Decomposition Methods

- ▶ X-11 decomposition: originated in the US Census Bureau and Statistics Canada
- ▶ SEATS decomposition: developed at the Bank of Spain, and is now widely used by government agencies around the world