Time Series Decomposition

Time Series Patterns

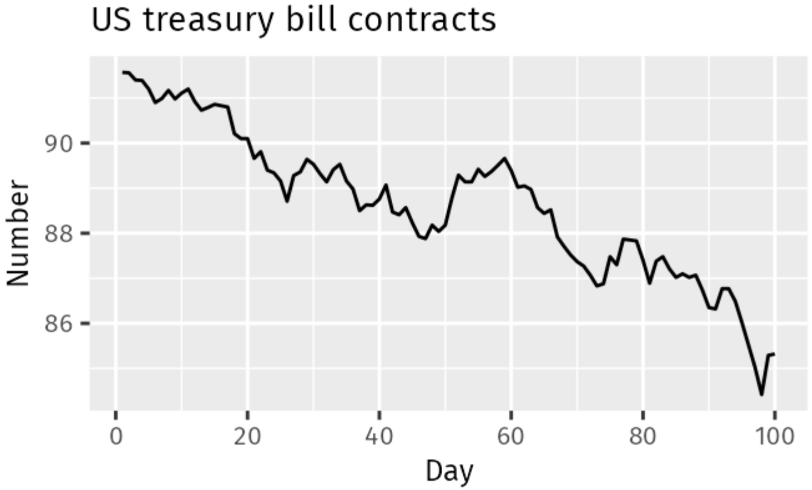
A time series may consist of

- trend,
- seasonality and
- cycles

Trend

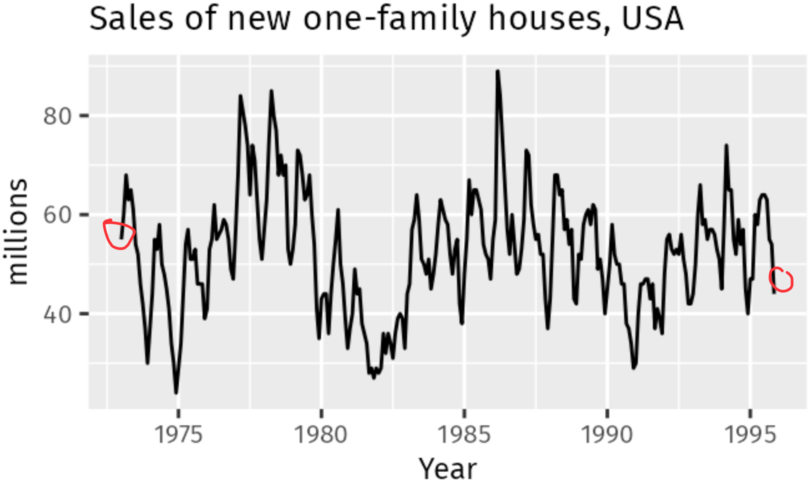
- A trend is a long-term increase or decrease in the data.
- Trend does not have to be linear.
- Sometimes we will refer to a trend as "changing direction", when it might go from an increasing trend to a decreasing trend.

Examples



- The US treasury bill contracts show results from the Chicago market for 100 consecutive trading days in 1981. There is a downward trend

Examples

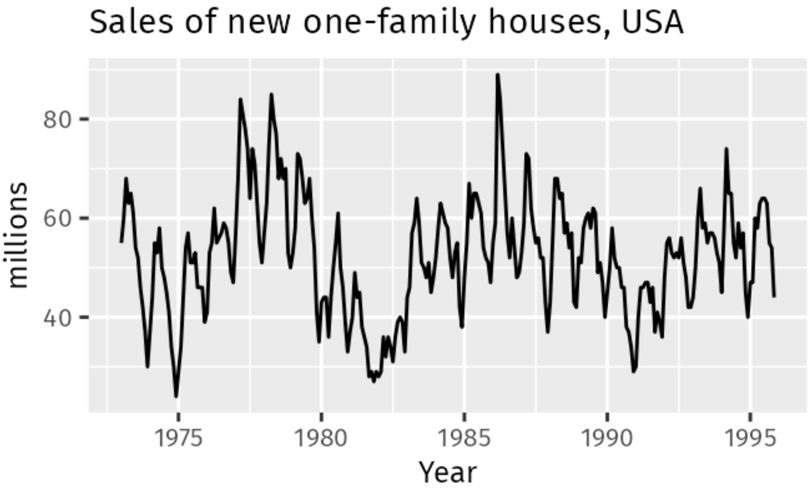


- There is no apparent trend in the data over this period

Cycle and Seasonal

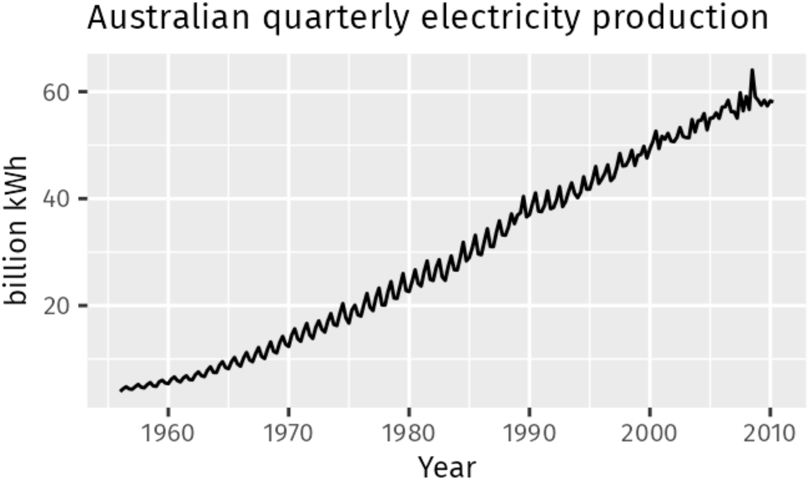
- cycle: repeated events over time that are not equally spaced
- seasonal: repeated events over time that are equally spaced

Examples

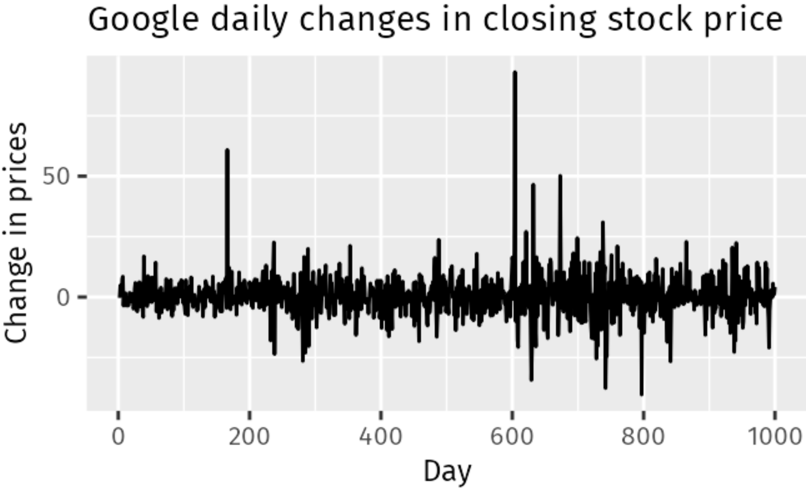


- The monthly housing sales show strong seasonality within each year, as well as some strong cyclic behaviour with a period of about 6–10 years

Examples



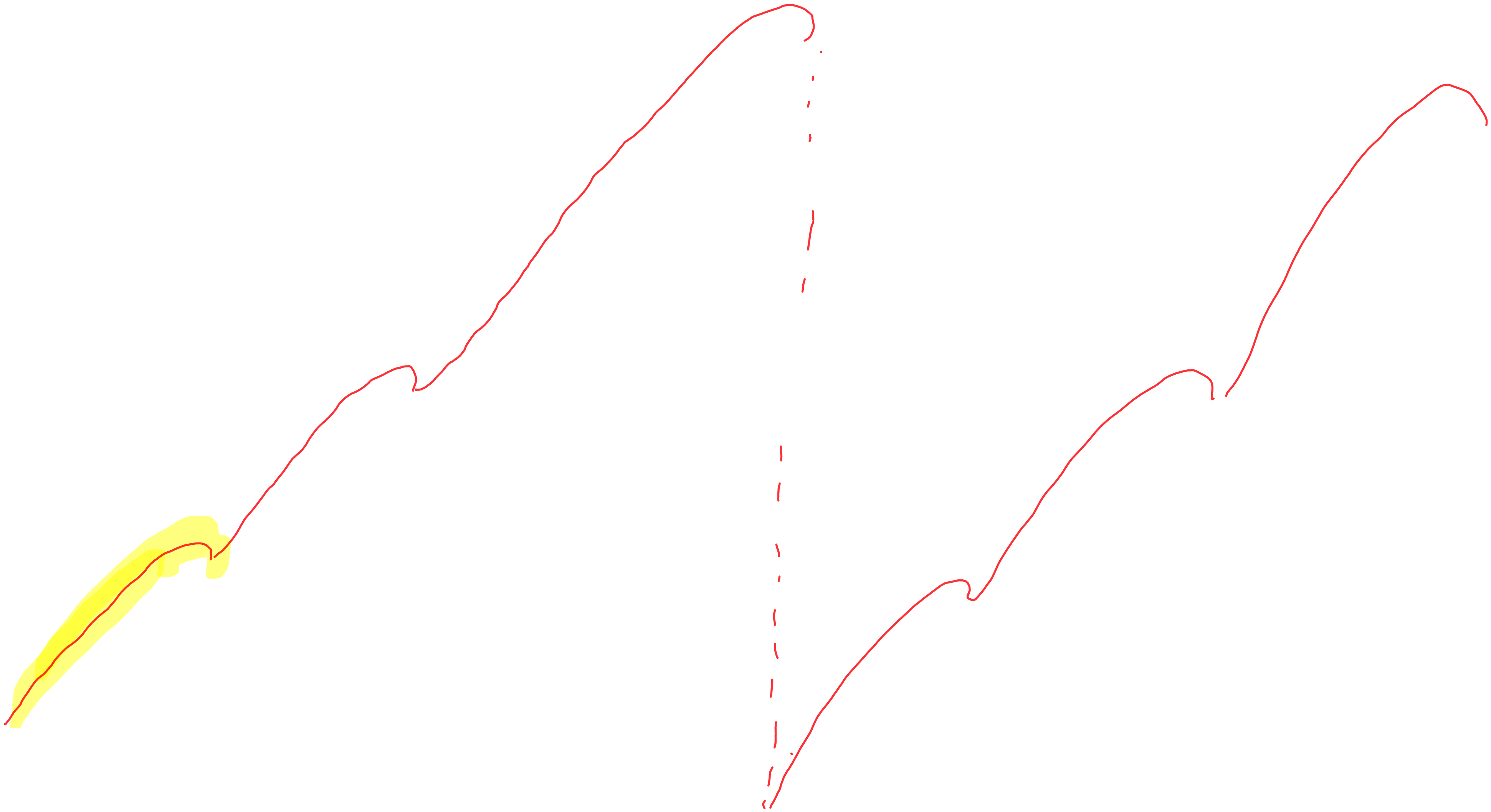
The Australian quarterly electricity production shows a strong increasing trend, with strong seasonality. There is no evidence of any cyclic behavior here



- No trend, seasonality or cyclic behaviour. There are random fluctuations which do not appear to be very predictable, and no strong patterns that would help with developing a forecasting model.

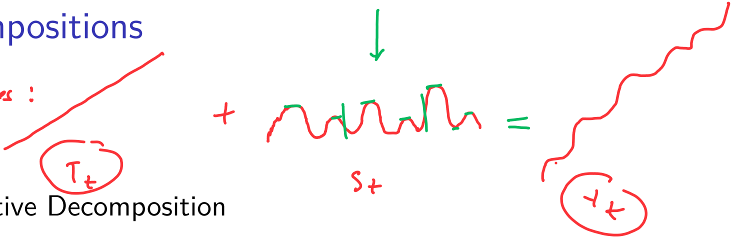
Time series decomposition

- When we decompose a time series into components, we usually combine the trend and cycle into a single trend-cycle component (sometimes called the trend for simplicity).
- Three components: a trend-cycle component, a seasonal component, and a remainder component (containing anything else in the time series).



Two decompositions

additive series :



- Additive Decomposition

$$y_t = S_t + T_t + R_t$$

- Multiplicative decomposition

$$\Rightarrow \qquad y_t = S_t \times T_t \times R_t$$

$$Y_t = S_t \cdot T_t \cdot R_t$$

If we apply the additive model to Y_t, it would not be most appropriate.

we can convert Y_t to $\log Y_t$ then apply the additive model.

$$\log Y_t = \log(S_t \cdot T_t \cdot R_t)$$

$$= \underline{\log S_t} + \underline{\log T_t} + \underline{\log R_t}$$

Relationship

- The log transformation will turn a multiplicative model to additive model.

Central MA vs. MA

Link.

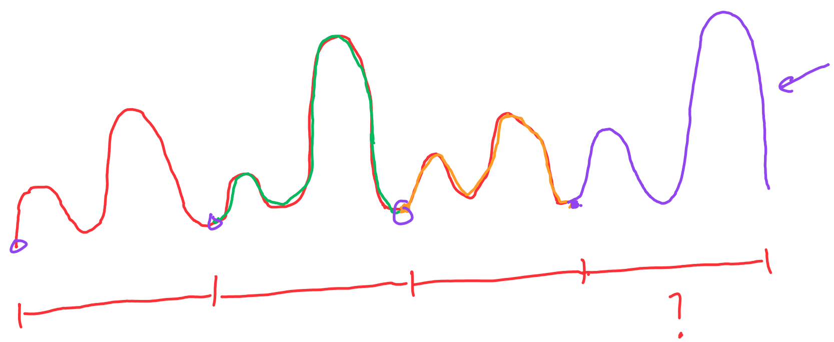
Classical Decomposition

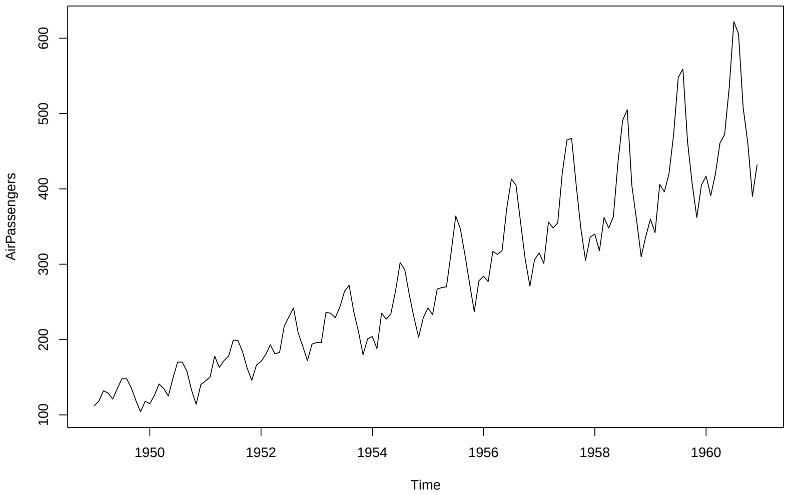
- In classical decomposition, we assume that the seasonal component is constant from year to year

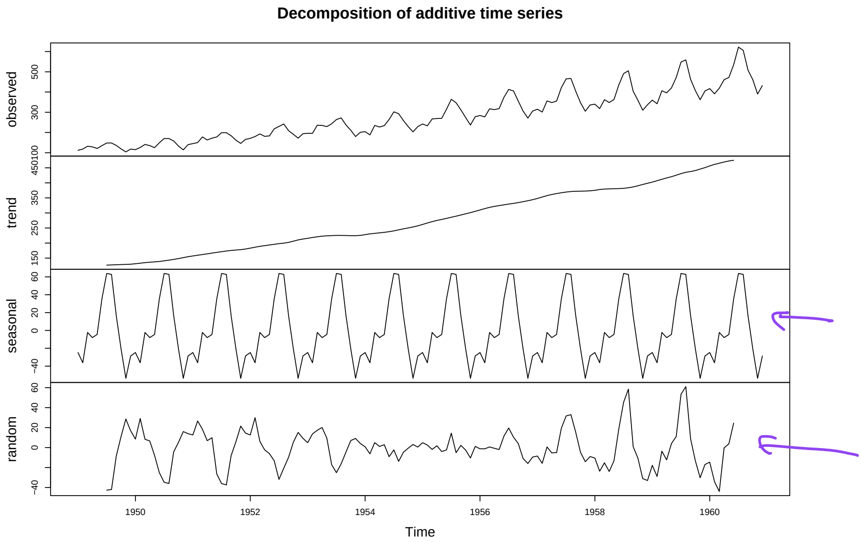
Classical Decomposition

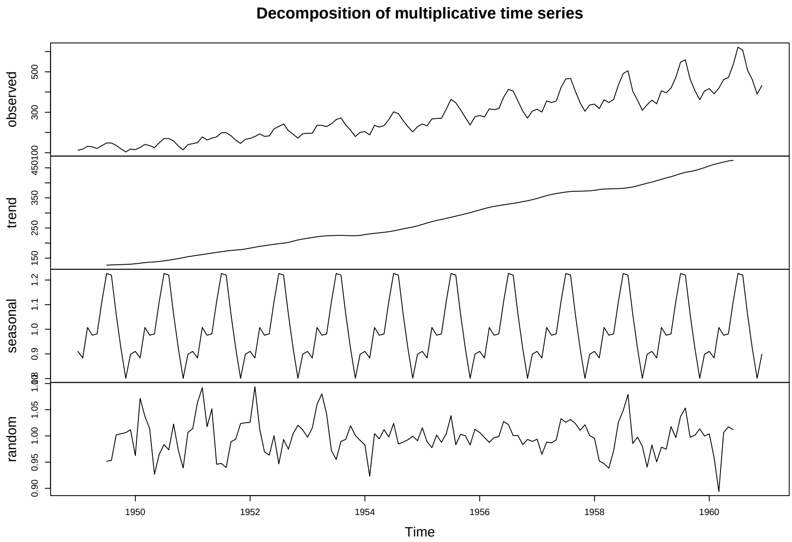
- Trend Estimate: Smooth the data using centred moving average (CMA) of the order equal to the periodicity of the data
- Detrend the data: calculate the detrended series by subtracting/dividing the trend estimate
- Seasonal Estimate: Average value for each period is calculated based on the de-trended series.
- Random/Remainder: subtract from the series the trend and seasonal component estimations

Seasonal component:




AirPassengers
100
200
300
400
500
600
1950
1952
1954
1956
1958
1960
Time


Decomposition of additive time series
observed
trend
seasonal
random
100
300
500
150
250
350
450
-40
0
20
40
60
1950
1952
1954
1956
1958
1960
Time

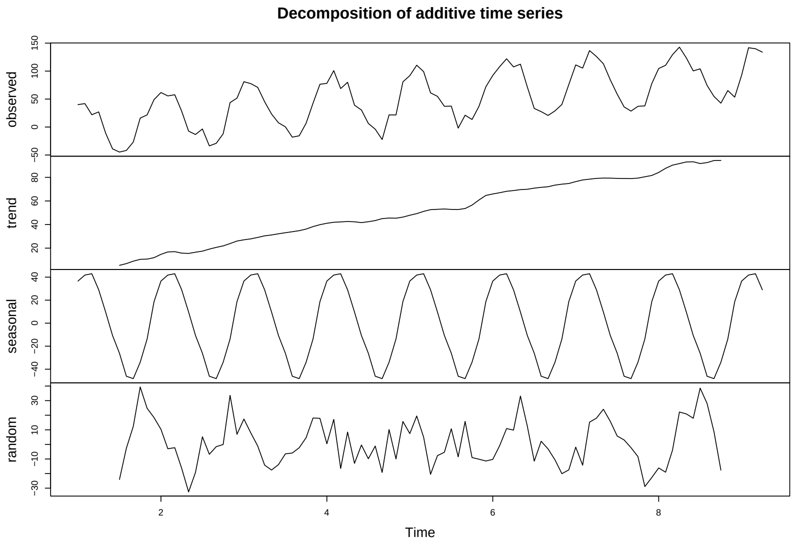

Decomposition of multiplicative time series
observed
100
300
500
trend
150
250
350
450
seasonal
0.8
0.9
1.0
1.1
1.2
random
0.90
0.95
1.00
1.05
1.10
1950
1952
1954
1956
1958
1960
Time

sequence: 1, 2, 3, .. 100 (trend)

seasonal

```
y <- ts(c(1:100) +
        50*cos(seq(0,16*pi,length.out=100)) +
        rnorm(100,0,10),frequency=12)
```

random normal with mean 0 sd = 10

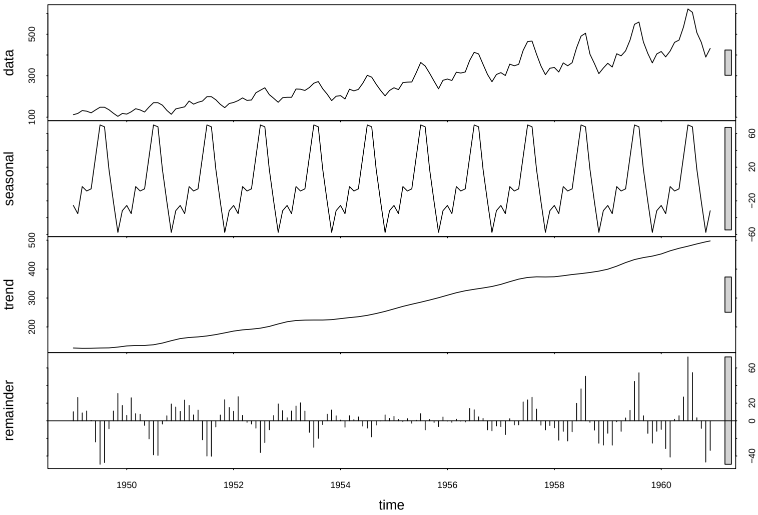

Decomposition of additive time series
observed
trend
seasonal
random
Time
-50
0
50
100
150
20
40
60
80
-40
-20
0
20
40
-30
-10
10
30
2
4
6
8

STL Decomposition

- Classical Decomposition uses $\underbrace{\text{Central Moving Average}}_{\text{CMA}}$ to estimate the trend
- STL uses LOESS (locally estimated scatterplot smoothing) to estimate the trend
- Link
- The seasonal component is estimated using moving averages of the de-trend series

- STL will handle any type of seasonality
- STL is a versatile and robust method for decomposing time series
- STL can be implemented using `stl` function

```r
data("AirPassengers")
stl_Decomposition <- stl(AirPassengers,
                         s.window = "periodic")
plot(stl_Decomposition)
```


data
seasonal
trend
remainder
100
300
500
60
20
−20
−60
200
300
400
500
60
20
0
−40
1950
1952
1954
1956
1958
1960
time

Other Decomposition Methods

- X-11 decomposition: originated in the US Census Bureau and Statistics Canada
- SEATS decomposition: developed at the Bank of Spain, and is now widely used by government agencies around the world