$(k=2)$

t	y_t	MA $(k=2)$	weights of y_t
1	y_1		0
2	y_2		0
3	y_3		0
4	y_4		$1/2$
5	y_5	$\dfrac{y_4 + y_5}{2}$	$1/2$

① For MA

$$S_t = \frac{y_4 + y_5}{2} = \frac{1}{2} y_5 + \frac{1}{2} y_4 + 0 \cdot y_3 + 0 \cdot y_2 + 0 \cdot y_1$$

② For Exp. smoothing

$$S_t = .7 \ y_5 + .21 \cdot y_4 + .063 \ y_3 + .018 \ y_2 + .00567 \ y_1$$

$$\hat{s}_t = \frac{y_t + w y_{t-1} + w^2 y_{t-2} + \ldots + w^t y_0}{1/(1-w)}$$

$$\hat{s}_t = (1-w)\, y_t + w(1-w)\, y_{t-1} + w^2(1-w)\, y_{t-2} + \cdots +$$

$$w^t (1-w)\, y_0$$

Another formula for exp. smoothing

$$\hat{s}_t = \hat{s}_{t-1} + (1-w)(y_t - \hat{s}_{t-1})$$
$$= (1-w)y_t + w\hat{s}_{t-1}$$

$$\hat{s}_0 = y_0$$
$$\hat{s}_1 = (1-w)\, y_1 + w \cdot \hat{s}_0$$
$$\hat{s}_2 = (1-w)\, y_2 + w \cdot \hat{s}_1$$
$$\vdots$$
$$\hat{s}_T = (1-w) \cdot y_T + w\, \hat{s}_{T-1}$$

Example

t	1	2	3	4	5
y_t	①	③	5	8	13
	y_1	y_2			

$w = .8$

ⓐ calculate the (single) exp. smoothing S_t

$$S_1 = y_1 = 1$$

$$\boxed{S_2 = (1-w) \cdot y_2 + w \cdot S_1}$$

$$= .2 \cdot y_2 + .8 \cdot = 1.4$$

$$S_3 = .2 \cdot y_3 + .8 \cdot S_2 = .2 \cdot 5 + .8 \cdot 1.4$$

and so on ....