

Question 1. Calculation

Given the following time series y_t .

t	y_t	$\hat{s}_t^{(1)}$	$\hat{s}_t^{(2)}$
1	1		
2	3	$\frac{1+3}{2} = 2$	
3	5	$\frac{3+5}{2} = 4$	$\frac{2+4}{2} = 3$
4	8	$\frac{5+8}{2} = 6.5$	$\frac{4+6.5}{2} = 5.25$
5	12	$\frac{8+12}{2} = 10$	$\frac{6.5+10}{2} = 8.25$

- Calculate the moving average series with $k = 2$
- Calculate the moving average series with $k = 3$
- Calculate the double moving average series with $k = 3$

Question 2. Application of MA: Forecasting Linear Trend Time Series

We assume the series in question 1 has linear trend. Use double smoothing average to estimate the linear trend (slope) of the time series. Use the estimated linear trend to predict the next value (y_6)

$$b_1 = \hat{\beta}_1 = \frac{2}{k-1} \left(\hat{s}_T^{(1)} - \hat{s}_T^{(2)} \right)$$

$$T = 5$$

$$= \frac{2}{2-1} \left[\hat{s}_5^{(1)} - \hat{s}_5^{(2)} \right]$$

$$= 2 \left[10 - 8.25 \right] = 3.5$$

$$\hat{y}_6 = \hat{s}_5^{(1)} + 3.5 = 10 + 3.5 = \boxed{13.5}$$