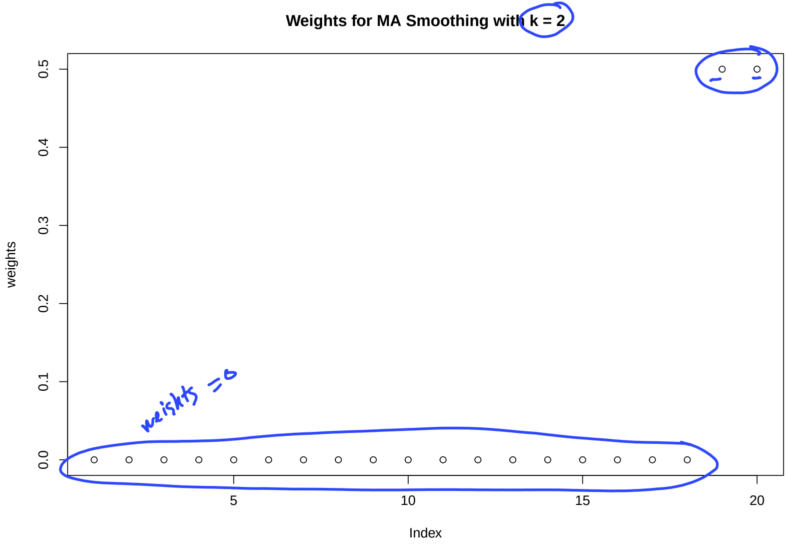
Time Series

MA Weights Distribution

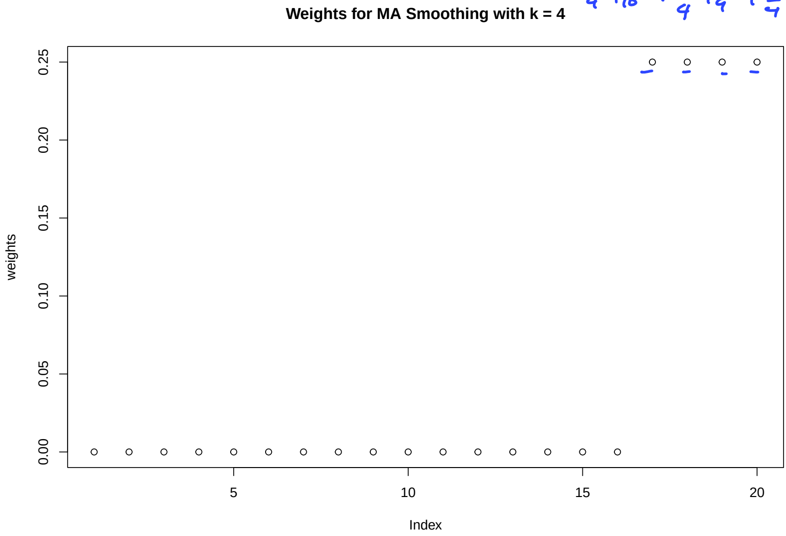
$$s_{10} = \frac{y_{10} + y_9}{2}$$



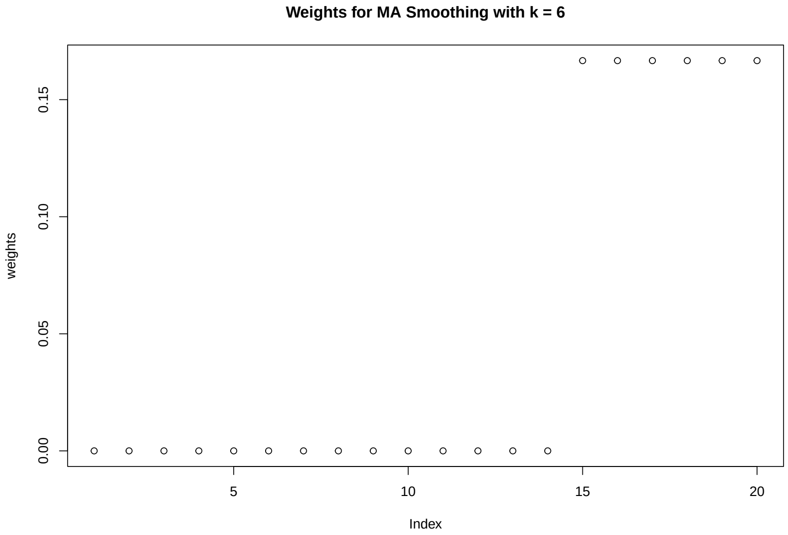
MA Weights Distribution

$$S_{10} = \frac{Y_{10} + Y_9 + Y_8 + Y_7}{4}$$

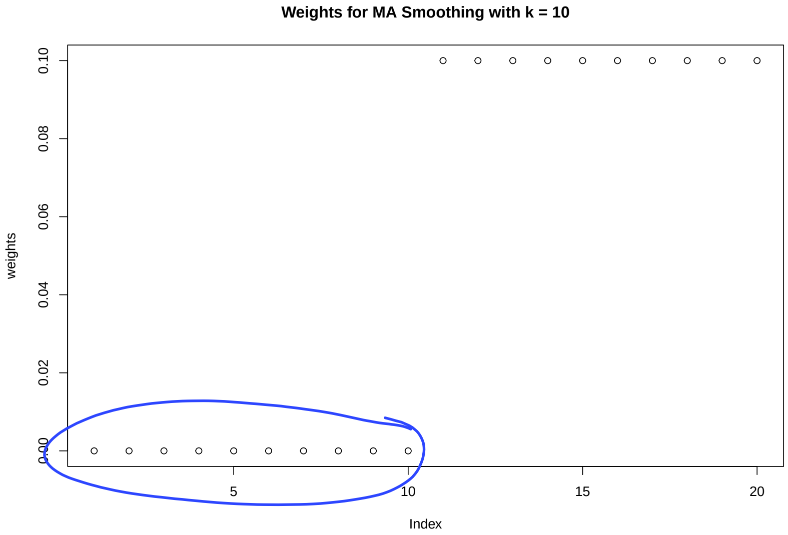
$$= \frac{1}{4}Y_{10} + \frac{1}{4}Y_9 + \frac{1}{4}Y_8 + \frac{1}{4}Y_7$$



MA Weights Distribution



MA Weights Distribution



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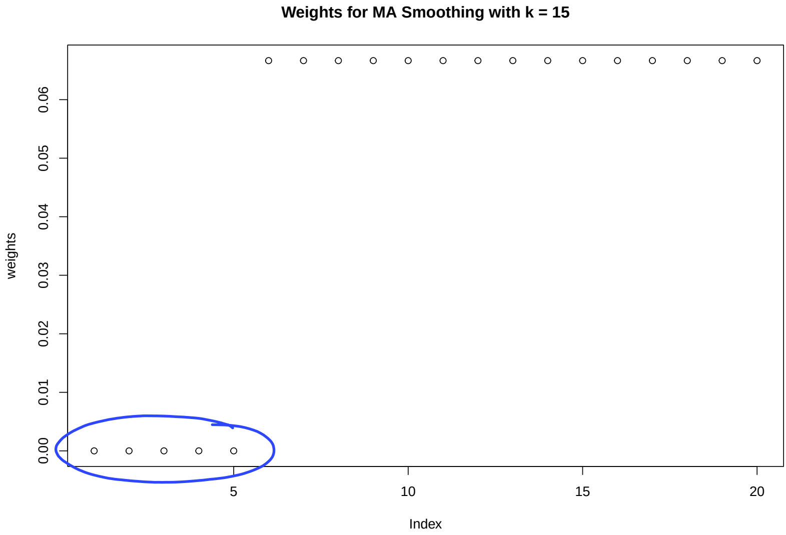
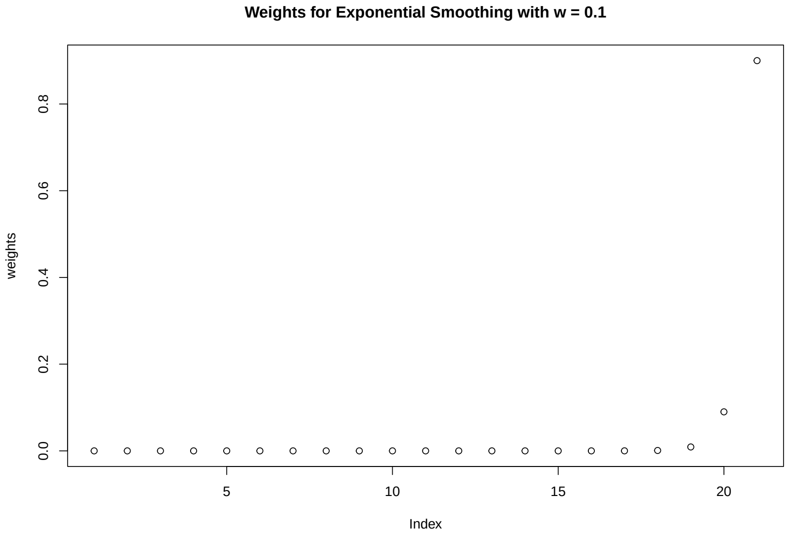


Exponential Weights Distribution



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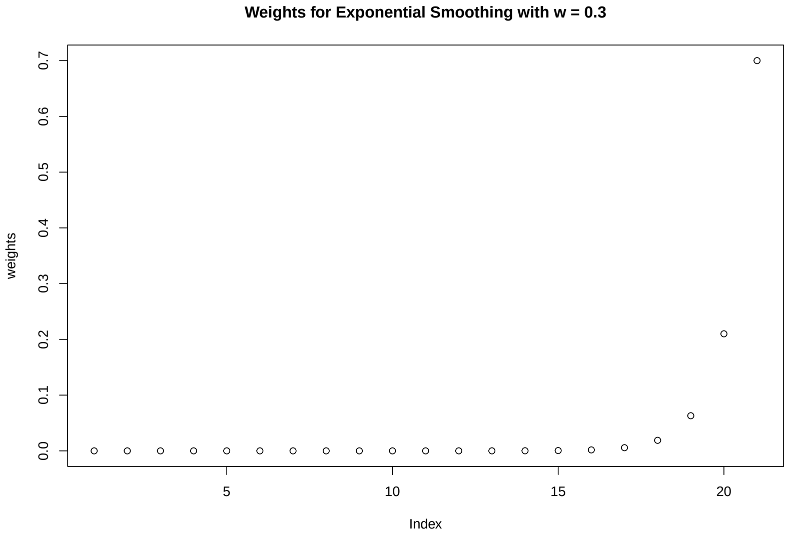
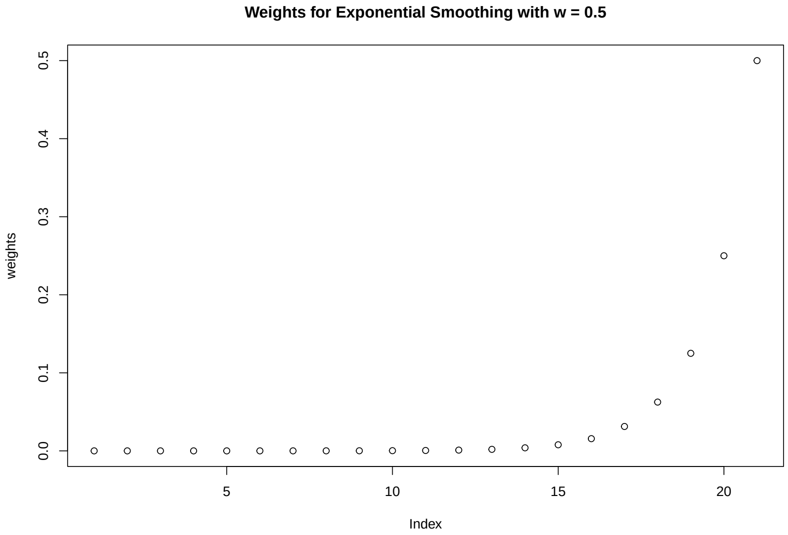
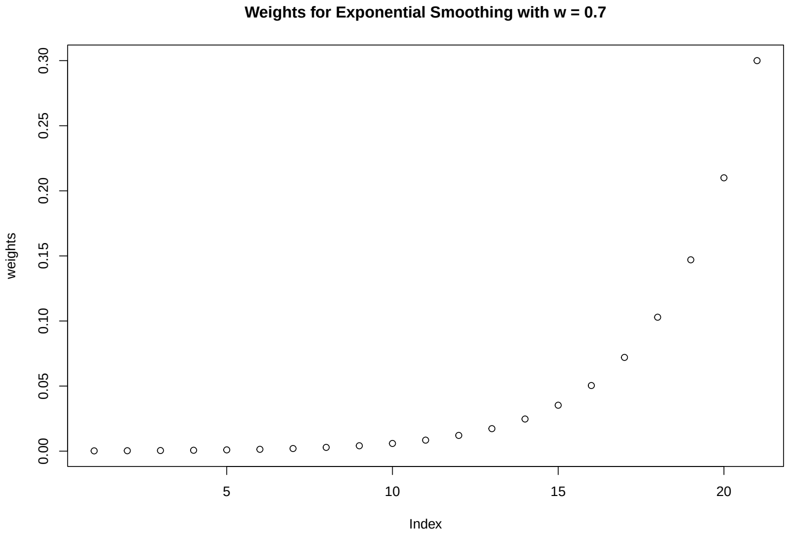


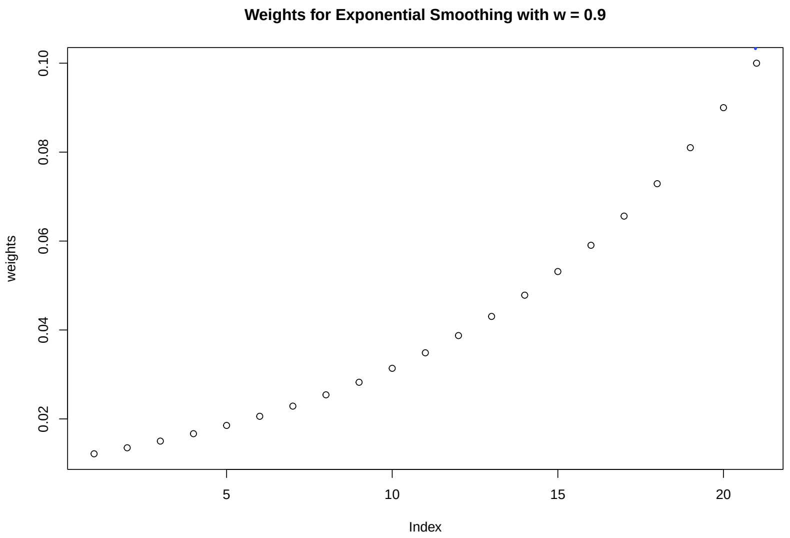
Exponential Weights Distribution



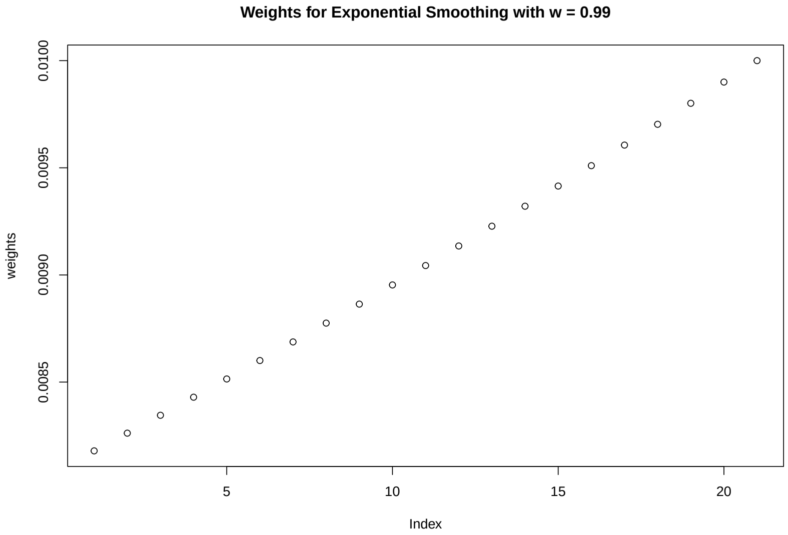
Exponential Weights Distribution



Exponential Weights Distribution



Exponential Weights Distribution



Exponential Smoothing

- MA distributes the weight equally to the recent observations
- Exponential Smoothing controls the weights of the recent observations by w

$$s_t = \frac{y_t + wy_{t-1} + w^2 y_{t-2} + ... + w^t y_0}{1/(1-w)}$$

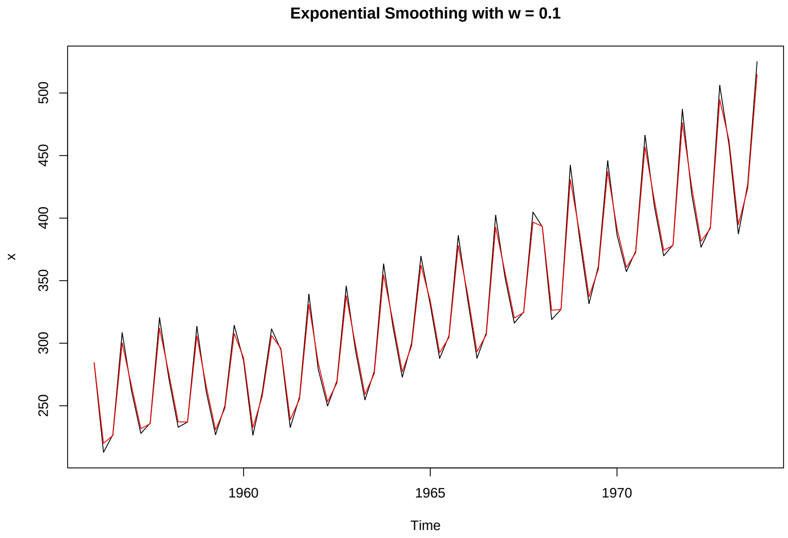
- Smaller w smooths the series more lightly.
- Greater w smooths the series more strongly.

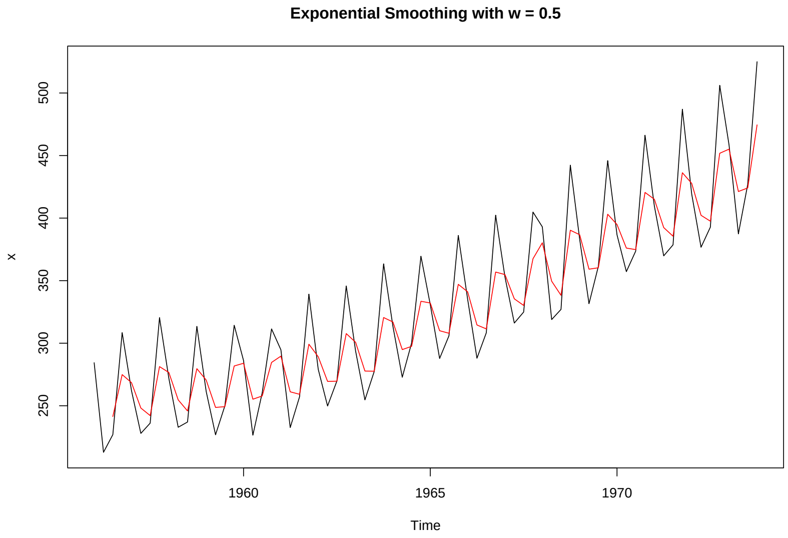
Another Formula:

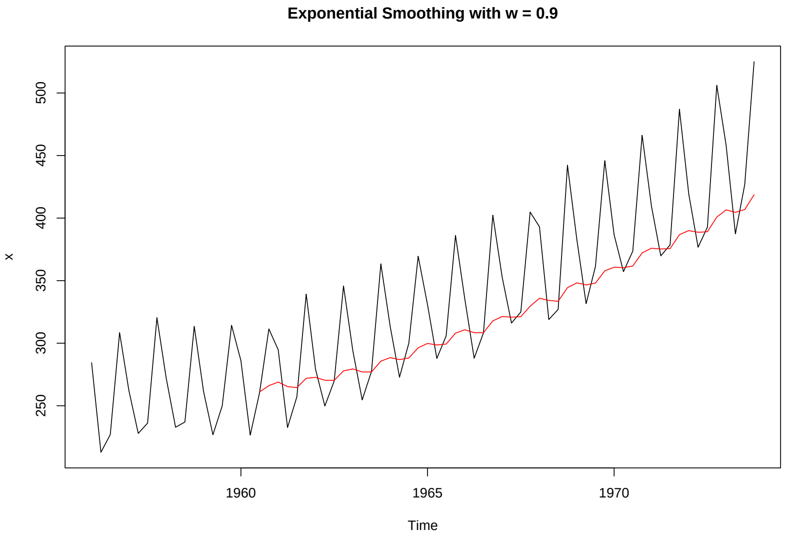
- Exponential Smoothing can be calculated by

$$
\begin{aligned}
s_1 &= y_1, \text{and} \\
s_t &= s_{t-1} + (1-w)(y_t - s_{t-1}) \\
&= (1-w)y_t + ws_{t-1}
\end{aligned}
$$

- Notice that: when $w \to 0$, $s_t \to y_t$, or little smoothing has taken.


Exponential Smoothing with w = 0.1
x
250
300
350
400
450
500
1960
1965
1970
Time


Exponential Smoothing with w = 0.5
x
250
300
350
400
450
500
1960
1965
1970
Time


Exponential Smoothing with w = 0.9
x
Time
250
300
350
400
450
500
1960
1965
1970

Double Exponential Smoothing

t	y_t
1	1
2	3
3	5
4	8
5	12

t	y_t
1	1
2	3 $\leftarrow y_2$
3	5 $\leftarrow y_3$
4	8
5	12

$$s_1 = y_1, \text{ and}$$

$$s_t = s_{t-1} + (1-w)(y_t - s_{t-1})$$

$$= \boxed{(1-w)y_t + \underline{ws_{t-1}}}$$

$$s_1 = y_1 = 1$$

$$s_2 = (1-w)\, y_2 + .2 \cdot s_1$$

$$= .8\, y_2 + .2\, s_1$$

$$= .8 * 3 + .2 * 1 = 2.6$$

$$s_3 = (1-.2) \cdot y_3 + .2 * s_2$$

$$= .8 * 5 + .2 * (2.6)$$

$$= 4 + .52 = \boxed{4.52}$$

$$s_4 = .8 * y_4 + .2 * s_3$$

$$= .8 * 8 + .2 * 4.52$$

$$= 7.3$$

$$s_5 = .8 * y_5 + .2 * s_4$$

$$= .8 * 12 + .2 * 7.3$$

$$= 11.06.$$

Example

You are given the following time series

t	1	2	3	4	5
y_t	1	3	5	8	13

- Assume that this is a linear trend time series. Using double exponential smoothing with $w = .8$ to estimate the trend (slope) and forecast y_6.