Moving Average Models

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Simple Moving Average Models - MA(1)

- Today = Mean + Noise + Slope * (Yesterday's Noise)

- Formally

$$Y_t = \mu + \theta_1 \epsilon_{t-1} + \epsilon_t$$

where ϵ $(0, \sigma^2)$

- Three paramters: μ, θ and σ^2

- MA models should not be confused with the MA smoothing
- A MA model is used for forecasting future values
- MA smoothing is used for estimating the trend-cycle of past values.

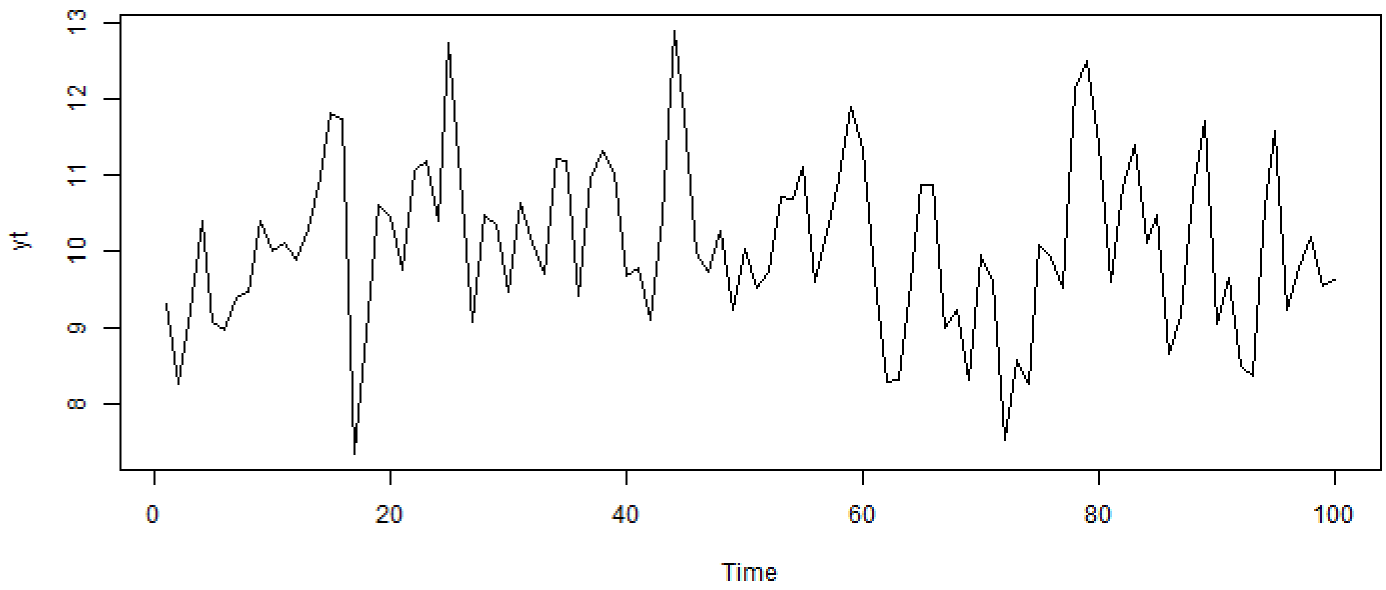
MA(q)

$$Y_t = \mu + \theta_1 \epsilon_{t-1} + \ldots + \theta_q \epsilon_{t-q} + \epsilon_t$$

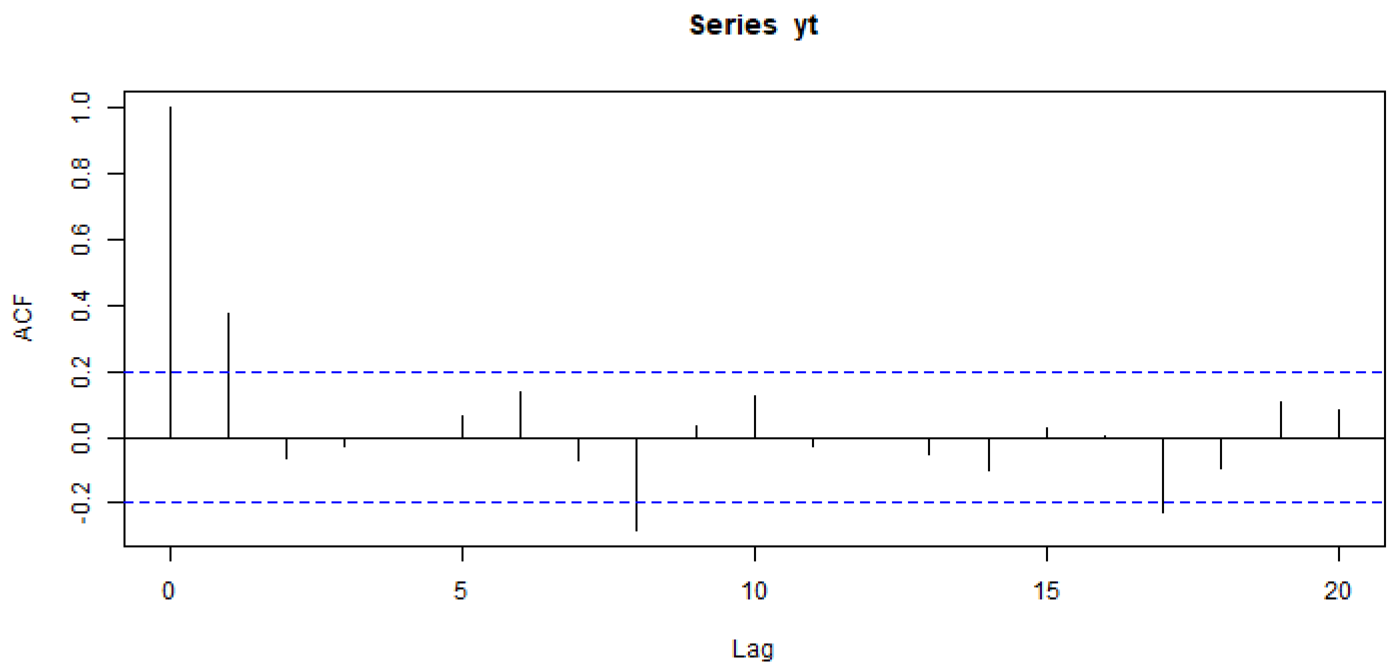
where ϵ $(0, \sigma^2)$

Examples

```
yt ← arima.sim(list(order=c(0,0,1), ma=c(.6)), n=100)
b0 = 10
yt ← yt + b0
plot(yt)
```



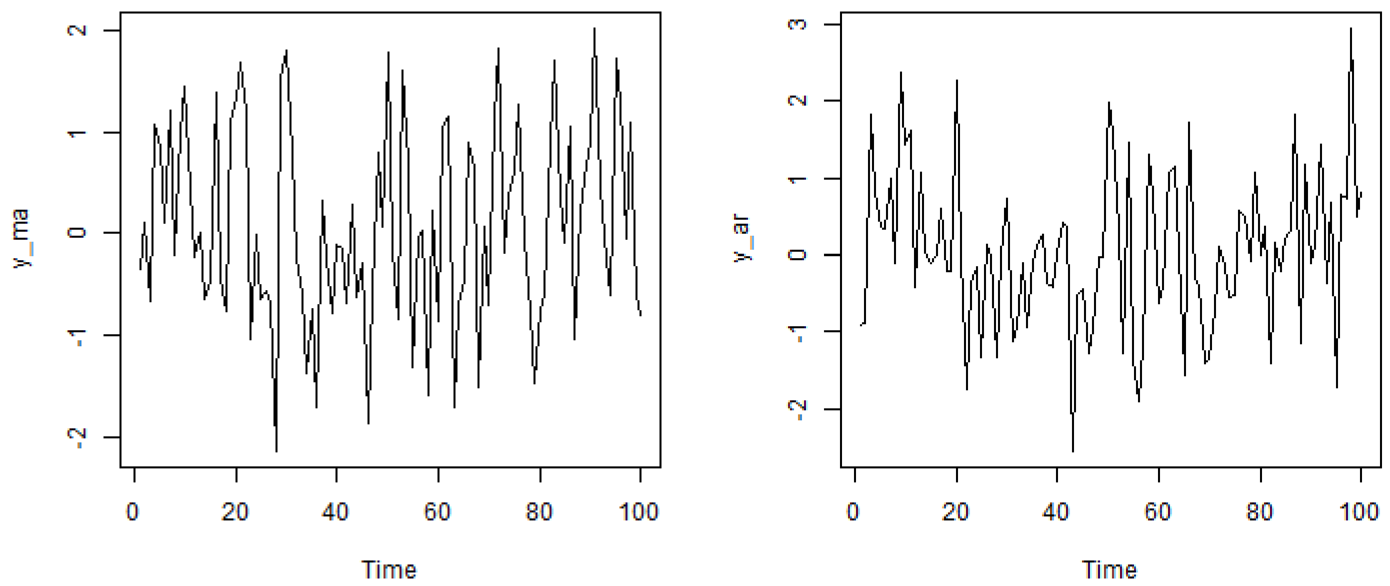
```
acf(yt)
```



AR(1) vs. MA(1)

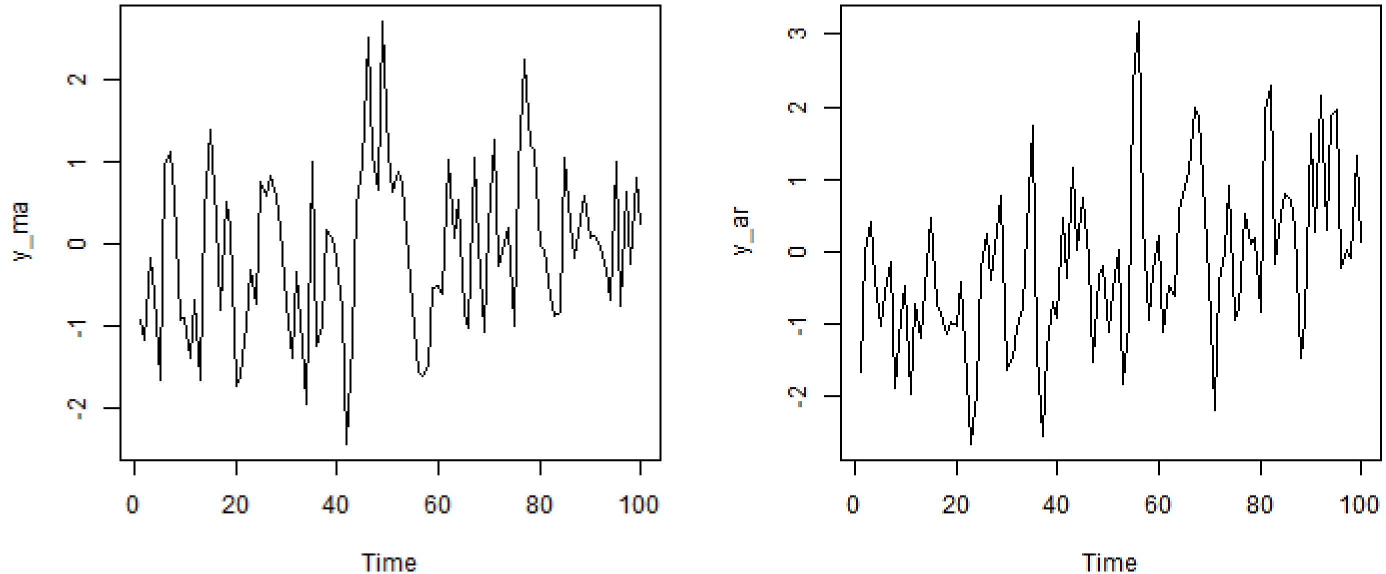
Both AR(1) and MA(1) are stationary so it is not easy to tell the different looking at the series plots

```
y_ma ← arima.sim(list(order=c(0,0,1), ma=c(.1)), n=100)
y_ar ← arima.sim(list(order=c(1,0,0), ar=c(.1)), n=100)

par(mfrow = c(1, 2))
plot(y_ma)
plot(y_ar)
```



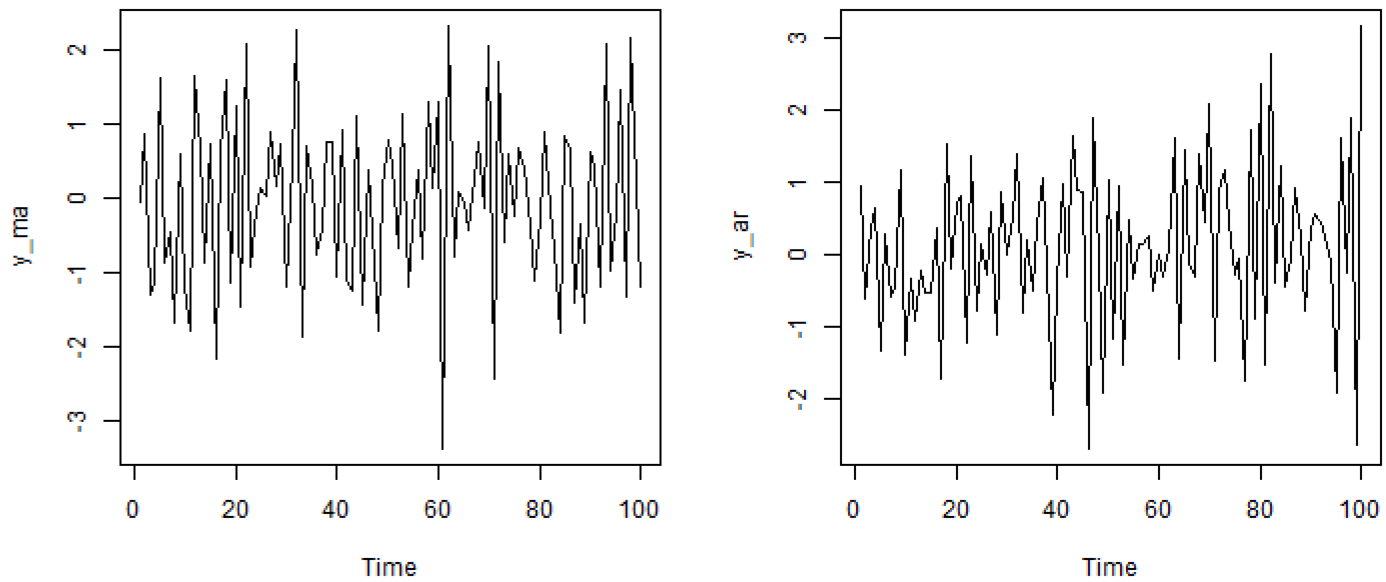
AR(1) vs. MA(1)

```
y_ma ← arima.sim(list(order=c(0,0,1), ma=c(.5)), n=100)
y_ar ← arima.sim(list(order=c(1,0,0), ar=c(.5)), n=100)

par(mfrow = c(1, 2))
plot(y_ma)
plot(y_ar)
```



AR(1) vs. MA(1)

```r
y_ma <- arima.sim(list(order=c(0,0,1), ma=c(-.5)), n=100)
y_ar <- arima.sim(list(order=c(1,0,0), ar=c(-.5)), n=100)

par(mfrow = c(1, 2))
plot(y_ma)
plot(y_ar)
```

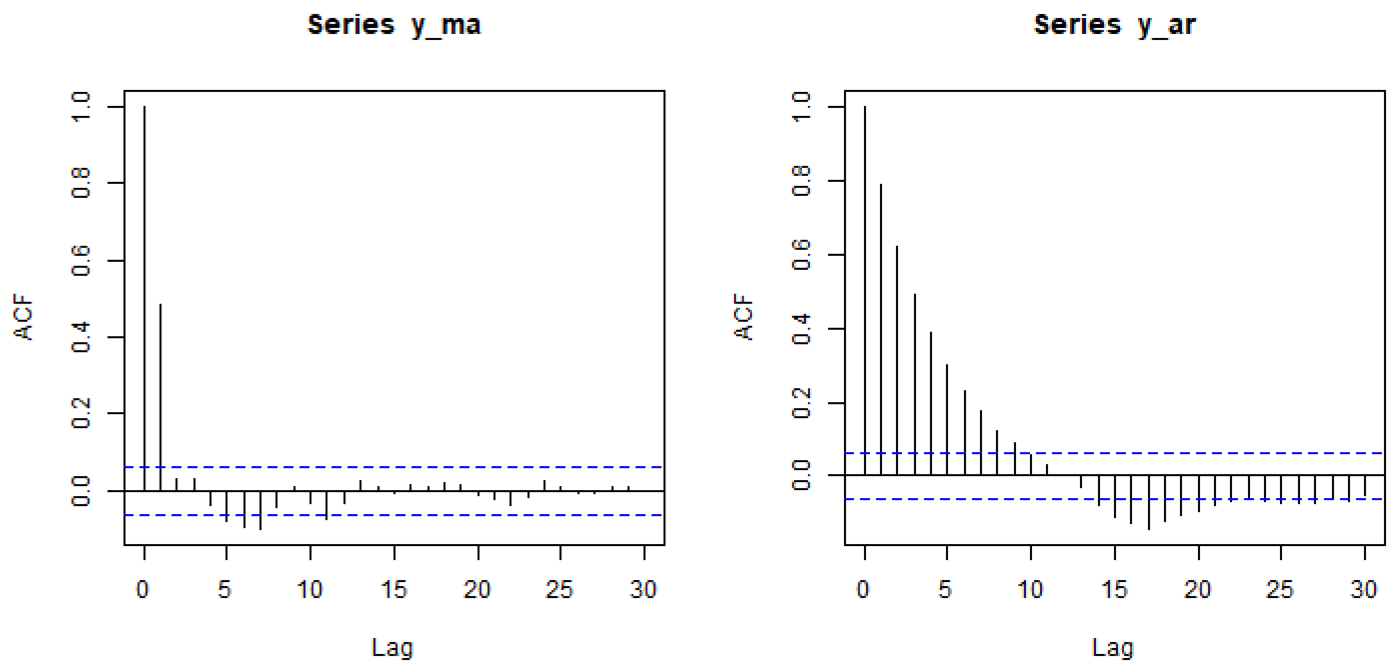


AR(1) vs. MA(1)

- MA(1) has autocorrelation at lag 1 only
- AR(1) has autocorrelation at many lags

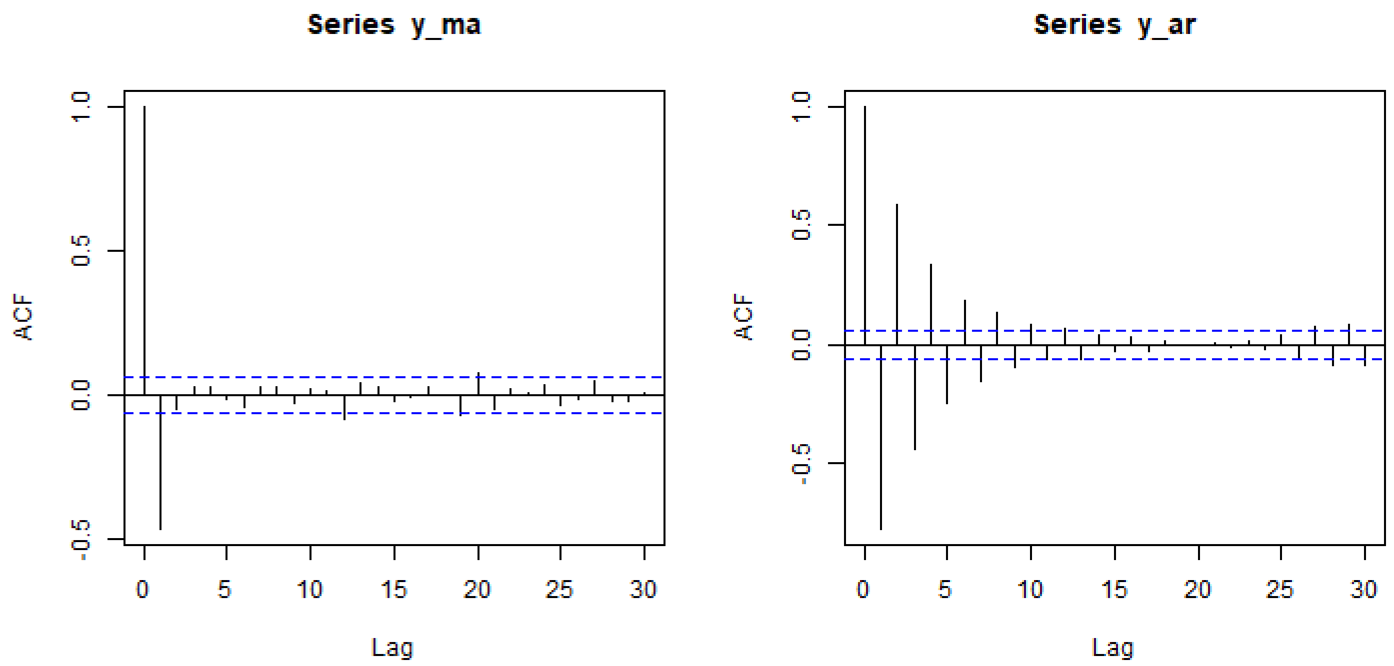
AR(1) vs. MA(1)

```r
y_ma ← arima.sim(list(order=c(0,0,1), ma=c(.8)), n=1000)
y_ar ← arima.sim(list(order=c(1,0,0), ar=c(.8)), n=1000)

par(mfrow = c(1, 2))
acf(y_ma)
acf(y_ar)
```



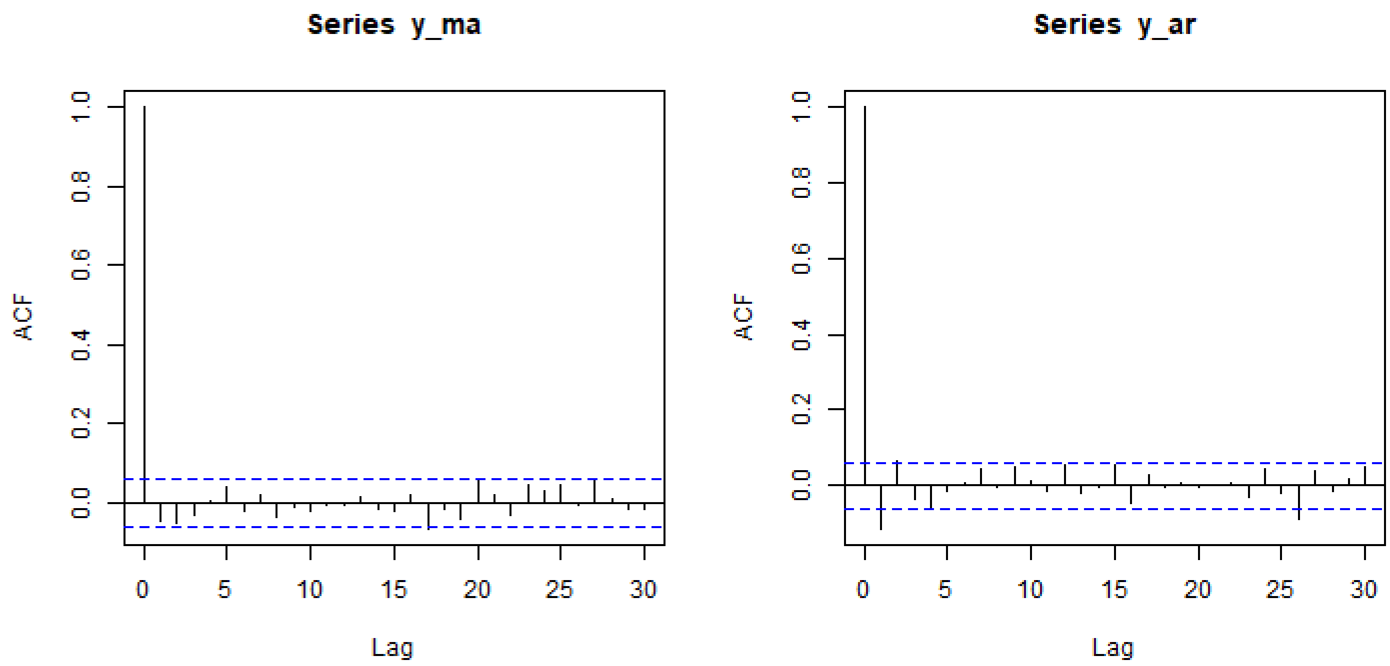
AR(1) vs. MA(1)

```r
y_ma <- arima.sim(list(order=c(0,0,1), ma=c(-.8)), n=1000)
y_ar <- arima.sim(list(order=c(1,0,0), ar=c(-.8)), n=1000)

par(mfrow = c(1, 2))
acf(y_ma)
acf(y_ar)
```



AR(1) vs. MA(1)

```
y_ma ← arima.sim(list(order=c(0,0,1), ma=c(-.1)), n=1000)
y_ar ← arima.sim(list(order=c(1,0,0), ar=c(-.1)), n=1000)

par(mfrow = c(1, 2))
acf(y_ma)
acf(y_ar)
```



Forecasting with MA Models

- Inflation Dataset: monthly observations in the US from 1950-2 to 1990-12

A time series containing :

pai1: one-month inflation rate (in percent, annual rate)

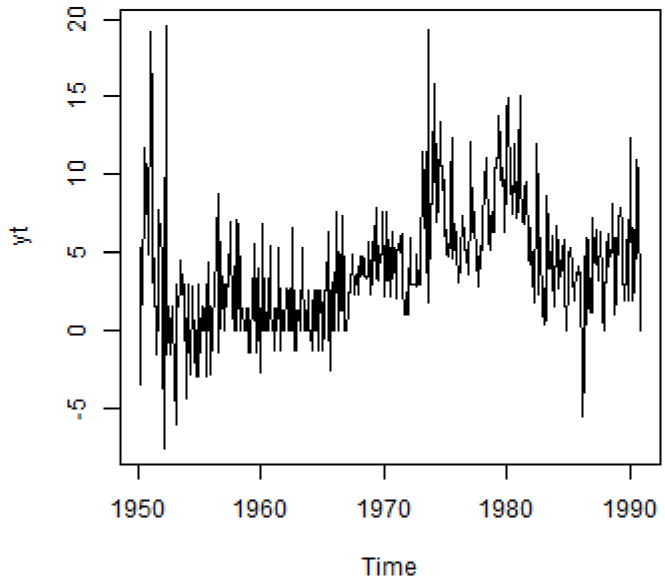
pai3: three-month inflation rate (in percent, annual rate)

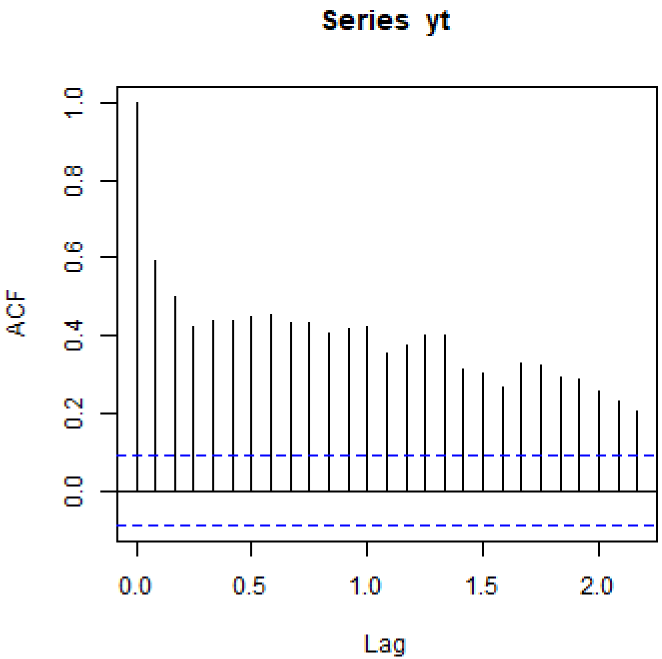
tb1: one-month T-bill rate (in percent, annual rate)

tb3: three-month T-bill rate (in percent, annual rate)

cpi: CPI for urban consumers, all items (the 1982-1984 average is set to 100)

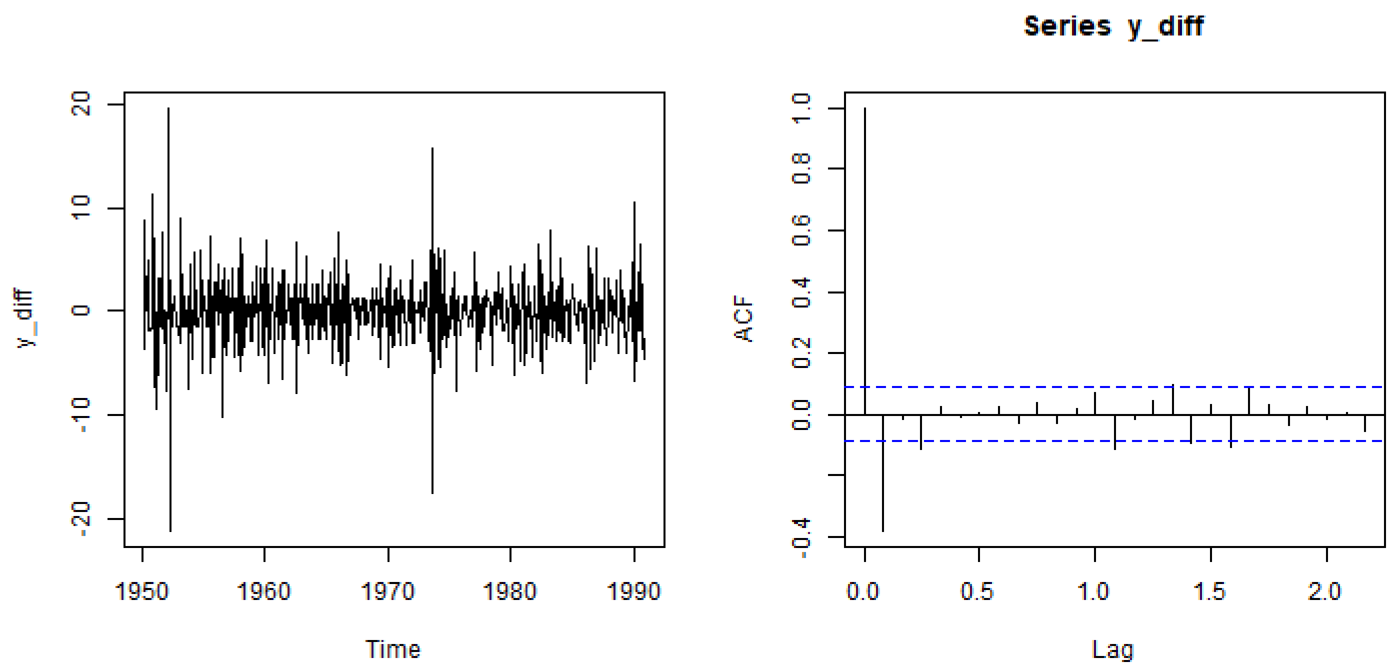
```r
# import the data
df = read.csv('inflation.csv')
# define the series
yt = ts(df$pai1, frequency = 12, start = c(1950, 2))
# check for stationary
par(mfrow = c(1, 2))
plot(yt)
acf(yt)
```





- The ACF does not die out to zero (coming into the blue strip) quickly indicates non-stationary

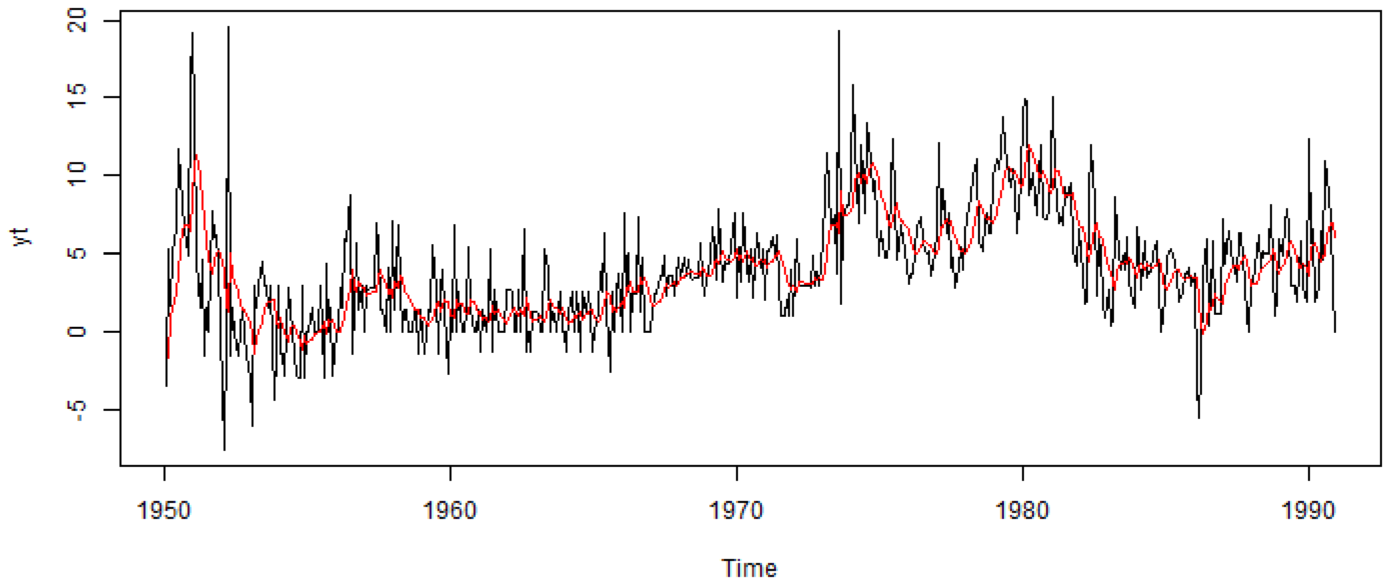
```r
# create the differenced series for stationary
y_diff = diff(yt)
# check for stationary
par(mfrow = c(1, 2))
plot(y_diff)
acf(y_diff)
```



- The ACF dies out to zero (coming into the blue strip) quickly indicates stationary

```
# fit the MA(1) model to the differenced series
y_ma = arima(y_diff, order = c(0,0,1))
```

```r
# plot the fitted series
plot(yt)
lines(yt-y_ma$residuals, col = "red")
```



```r
# make predictions
d_n = forecast(y_ma, h = 1)
y_next = d_n$mean + yt[length(yt)]
y_next = as.numeric(y_next)
y_next
```

```
## [1] 4.831632
```

AR(1) vs. MA(1)

```
# fit the MA(1) model to the data
y_ma = arima(y_diff, order = c(0,0,1))
y_ar = arima(y_diff, order = c(1,0,0))

# plot the fitted series
plot(yt)
lines(yt-y_ma$residuals, col = "red")
lines(yt-y_ar$residuals, col = "blue")
```