Spatial Regression and GWR – Estonia

GEOG-325: Period III, 2021

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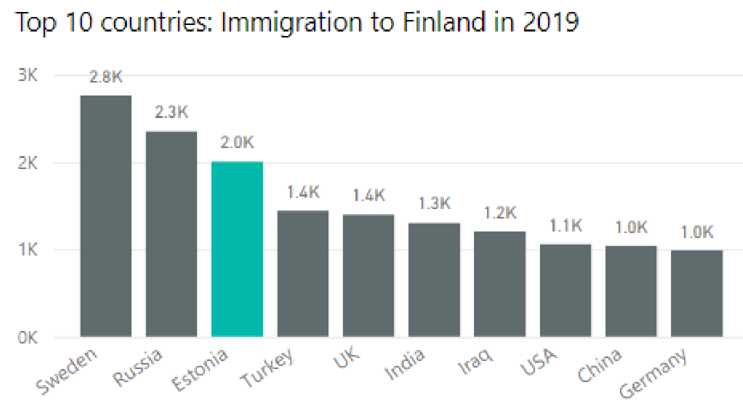
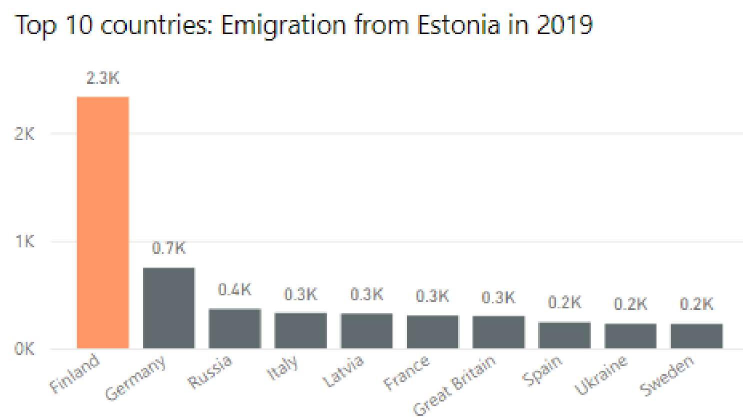
Contents

Introduction

Finland is a key destination for Estonian migrants. In 2019, Finland received almost 33,000 migrants. About 2,000 of those immigrants (6%), arrived from Estonia. In 2019, approximately 13,000 people emigrated from Estonia. About 2,300 (18%) of those migrants moved to Finland.





This project *separately* examines two social processes:

1. Immigration to Finland

2. Emigration from Estonia

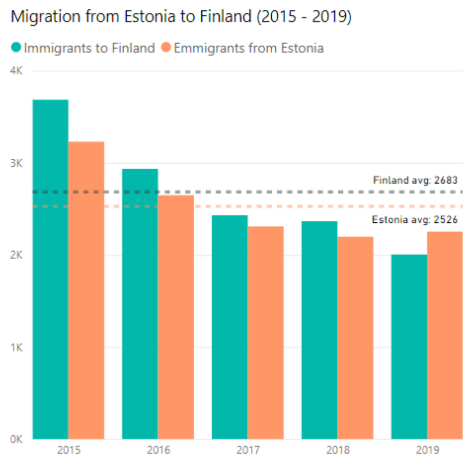
Migration is understood to have important spatial components. Unemployment, education levels, finances, gender, age, and accessibility are also important themes in migration literature. These are explored in our hypotheses.

Hypotheses

Hypothesis 1: In Finland, the unemployment rate, GDP per capita, proximity to Helsinki, age, and gender can be used to model immigration at a regional level.

Hypothesis 2: In Estonia, the unemployment rate, GDP per capita, proximity to Helsinki, age, and education can be used to model emigration at a county level.

Data



Summary

Q1	Median	Q3	Mean	2019
2238	2398	3007	2604	2128

Finland dependent variable: Annual count of Estonian immigrants by region
11ab: Immigration and emigration by country of departure or arrival, origin and region, 1990-2019

Finland independent variables	Description	Geographic division	Year
1. Unemployment	Unemployment rate (%)	By region	2019
2. GDP per capita	GDP per capita (Euros)	By region	2018
3. Proximity	Distance to Helsinki (km)	From centroid of region to point	2020
4. Age	Population over 65 (%)	By region	2019
5. Gender	Proportion of male migrants (%)	By region	2019

Source: Statistics Finland

Estonia dependent variable: Annual count of Estonian emigrants by county
RVR051: External migration by county and country, 2004-2019

Estonia independent variables	Description	Geographic division	Year
1. Unemployment	Unemployment rate (%)	By county	2019
2. GDP per capita	GDP per capita (Euros)	By county	2019
3. Proximity	Distance to Helsinki (km)	From centroid of region to point	2020
4. Age	Population over 65 (%)	By county	2019
5. Education level	Population with higher education (%)	By county	2019

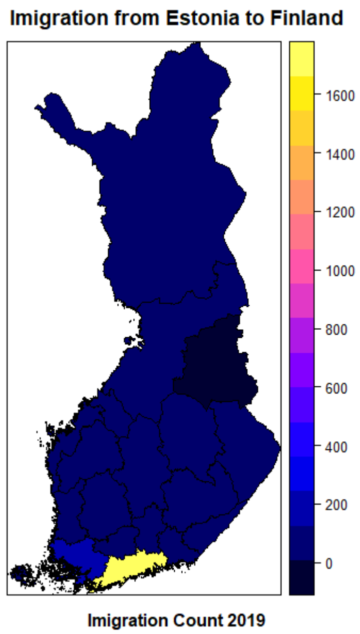
Source: Statistics Estonia

Geographic data: NUTS 3 Europe
Finland's 19 regions or provinces (maakunta) and Estonia's 15 counties (maakond).
Source: Eurostat NUTS Maps

Analysis is performed at the regional level (maakunta) for Finland and at the county level (maakond) for Estonia. Clickable links function in the presentation.

Finland

1. Opening the Shapefile and showing the Dependent Variable



2. OLS

Apply OLS, to understand how much the variables explain the variability of Immigration Rate in Finland.

```
Call:
lm(formula = im_2019 ~ unem_.19 + gdp_eur + proxkm + pop.65 +
    X.male, data = finland)

Residuals:
   Min      1Q Median      3Q     Max
-302.9 -146.3  -25.0  154.5  434.4

Coefficients:
               Estimate  Std. Error t value Pr(>|t|)
(Intercept) -1317.13031   979.30184  -1.345  0.20163
unem_.19       45.65502    34.22833   1.334  0.20516
gdp_eur         0.04602     0.01384   3.326  0.00547 **
proxkm         -0.21568     0.13706  -1.574  0.13961
pop.65        -26.49806    23.74297  -1.116  0.28461
X.male          3.03866     4.85307   0.626  0.54207
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 229.2 on 13 degrees of freedom
Multiple R-squared:  0.7296,	Adjusted R-squared:  0.6256
F-statistic: 7.014 on 5 and 13 DF,  p-value: 0.00221
```

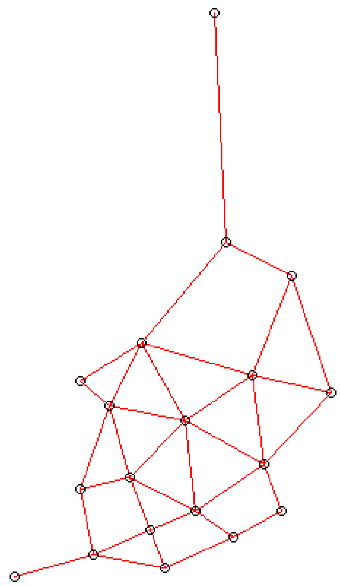
RSquared explains 27.26% of the variation in Immigration Rate.

Now with counts 2019, RSquared explains 72.96% of the variation in Immigrants count 2019

The P-value shows that it is not statistically significant so we keep finding for spatial dependence.

3. Spatial weights

Create the Weights File, in GeoDa



Gabriel Graph

4. Lagrange diagnostics

Apply Lagrange Model for testing spatial dependence

```
        Lagrange multiplier diagnostics for spatial dependence

data:
model: lm(formula = im_2019 ~ unem_.19 + gdp_eur + proxkm + pop.65 + X.male, data = finland)
weights: weights

LMerr = 0.83741, df = 1, p-value = 0.3601
```

It is insignificant, but we keep working. LMError explains 83% of variation.

4. SEM

Apply Spatial Error Model.It gives Nagel Kerke pseudo-P-value 77.98% that explains the variability in Immigration. It increased due to a new variable Lag Error which gives more explanatory power.

```
Residuals:
      Min       1Q   Median       3Q      Max
-331.627  -79.756   10.546   84.239  290.700

Type: error
Coefficients: (asymptotic standard errors)
               Estimate   Std. Error z value   Pr(>|z|)
(Intercept) -149.9307651  677.9443605 -0.2212  0.8249718
unem_.19      85.7981609   21.6055431  3.9711 0.00007154 ***
gdp_eur        0.0292771    0.0097058  3.0164  0.0025577 **
proxkm        -0.3656471    0.0882345 -4.1440 0.00003412 ***
pop.65       -67.3806041   19.6544206 -3.4283  0.0006074 ***
X.male         7.8838485    3.7458946  2.1047  0.0353206 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Lambda: -0.8824, LR test value: 3.9119, p-value: 0.047947
Approximate (numerical Hessian) standard error: 0.26186
    z-value: -3.3697, p-value: 0.00075249
Wald statistic: 11.355, p-value: 0.00075249

Log likelihood: -124.6574 for error model
ML residual variance (sigma squared): 22663, (sigma: 150.54)
Nagelkerke pseudo-R-squared: 0.77989
Number of observations: 19
Number of parameters estimated: 8
AIC: 265.31, (AIC for lm: 267.23)
```

5. SDM

Apply Spatial Durbin Model, to find average effects, interaction effects + Common Factor Hypothesis

```
        Likelihood ratio for spatial linear models

data:
Likelihood ratio = 17.622, df = 5, p-value = 0.00346
sample estimates:
Log likelihood of x Log likelihood of y
          -115.8465           -124.6574
```

If it significant it means our variables are good for explanation. P-value 0.003. The Durbin model, adds Lag variables which help to give more explanatory power.

```
Type: mixed
Coefficients: (numerical Hessian approximate standard errors)
                Estimate   Std. Error z value        Pr(>|z|)
(Intercept)  891.6938665 1217.1861540  0.7326        0.463811
unem_.19      64.1287488   23.0336997  2.7841        0.005367 **
gdp_eur        0.0422322    0.0104769  4.0310 0.00005554738 ***
proxkm        -1.1702625    0.2172535 -5.3866 0.00000007179 ***
pop.65       -52.5552575   22.3633333 -2.3501        0.018770 *
X.male         7.5252708    3.4991365  2.1506        0.031507 *
lag.unem_.19 166.0468367   40.8870003  4.0611 0.00004883877 ***
lag.gdp_eur   -0.0046024    0.0215140 -0.2139        0.830606
lag.proxkm     0.5572931    0.3508625  1.5884        0.112207
lag.pop.65  -145.0562617   30.8944912 -4.6952 0.00000266327 ***
lag.X.male    18.9315480    7.0973630  2.6674        0.007644 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Rho: -0.60097, LR test value: 2.9864, p-value: 0.083964
Approximate (numerical Hessian) standard error: 0.30711
    z-value: -1.9568, p-value: 0.050367
Wald statistic: 3.8292, p-value: 0.050367

Log likelihood: -115.8465 for mixed model
ML residual variance (sigma squared): 10404, (sigma: 102)
Nagelkerke pseudo-R-squared: 0.91293
```

The variables that not helps to the model are Lag Unemployment Rate, and Lag GDP. Anyways, the model explanatory power increased to 91.29%.

6. Direct, Indirect, and Total impacts

```
Impact measures (mixed, exact):
              Direct      Indirect         Total
unem_.19  39.87015132 103.90243136  143.77258268
gdp_eur    0.04793653  -0.02443211    0.02350442
proxkm    -1.41009205   1.02721827   -0.38287378
pop.65   -30.96690214 -92.46546978 -123.43237193
X.male     4.78390629  11.74158732   16.52549361
```

```
Simulated p-values:
         Direct      Indirect   Total
unem_.19 0.186088    0.00851646 0.00000046134
gdp_eur  0.000101    0.07922062 0.0503092
proxkm   0.000032269 0.01252687 0.0083312
pop.65   0.271133    0.00024238 0.00005690437
X.male   0.247685    0.02324785 0.0107115
```

- *Unemployment is Significant, Indirect and Total*
  Increases of Unemployment cause increase of Immigration to Helsinki. So, regions that changes cause Effects in the surrounding regions.

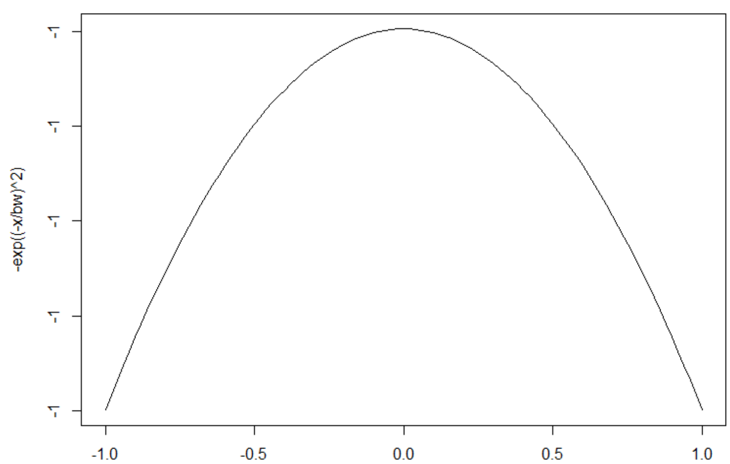
- *GDP is Significant, in Direct.*
  Small changes in GDP causes changes of Immigration, but not relevant.

- *Proximity all Significant. More in Direct.*
  The more proximity (density) less Immigration.

- *Pop over 65 Significant, Indirect and Total.*
  Increases in elderly decreases the Immigration to Finland

- *Male pop Significant in Total.*
  Increases in Males population increases in Immigration.

7. Get the Bandwidth



8. GWR

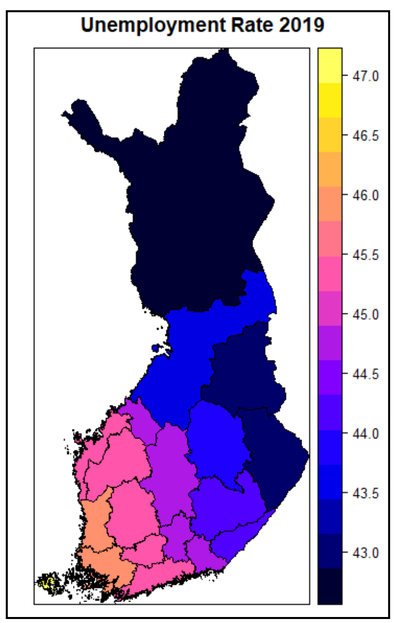
What happen with Migration if distance to Helsinki changes?

Changes in migration in one location will change in another location because of the way the effect travels back and forth. Spatial connectivity causes heterogeneity.

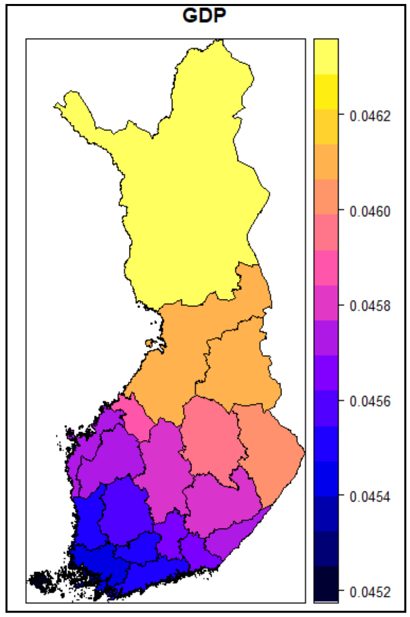
OLS

```
gwr(formula = im_2019 ~ unem_.19 + gdp_eur + proxkm + pop.65 +
    X.male, data = finland, bandwidth = bw, hatmatrix = TRUE)
Kernel function: gwr.Gauss
Fixed bandwidth: 2231982
Summary of GWR coefficient estimates at data points:
                    Min.      1st Qu.       Median      3rd Qu.         Max.     Global
X.Intercept. -1388.045475 -1308.946056 -1287.694022 -1270.534555 -1249.029773 -1317.1303
unem_.19        42.849475    43.919821    44.707462    45.392117    46.934043    45.6550
gdp_eur          0.045247     0.045572     0.045718     0.045894     0.046285     0.0460
proxkm          -0.237887    -0.228633    -0.222577    -0.217043    -0.197288    -0.2157
pop.65         -28.237652   -27.528727   -26.932848   -26.173595   -23.092576   -26.4981
X.male           2.830048     3.017137     3.078543     3.123382     3.150819     3.0387
Number of data points: 19
Effective number of parameters (residual: 2traceS - traceS'S): 6.418765
Effective degrees of freedom (residual: 2traceS - traceS'S): 12.58124
Sigma (residual: 2traceS - traceS'S): 229.1208
Effective number of parameters (model: traceS): 6.218124
Effective degrees of freedom (model: traceS): 12.78188
Sigma (model: traceS): 227.3154
Sigma (ML): 186.4444
AICc (GWR p. 61, eq 2.33; p. 96, eq. 4.21): 278.0285
AIC (GWR p. 96, eq. 4.22): 258.8069
Residual sum of squares: 660469.1
Quasi-global R2: 0.738499
```

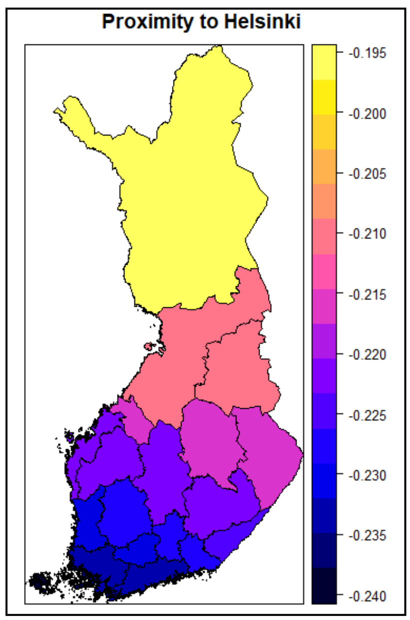
Global is the regression coefficient. But we study the local variation/heterogeneity of the changes by regions.



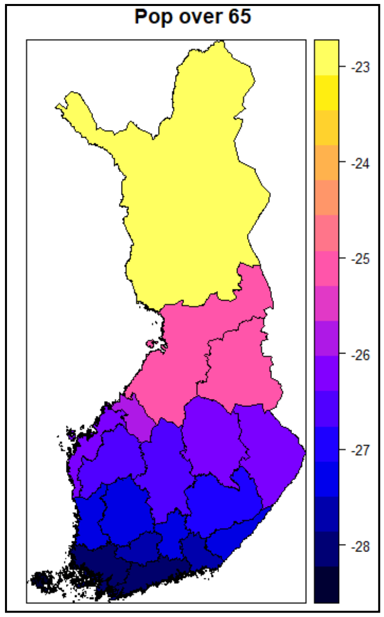
The changes in Unemployment affects more the immigration in south-west of Finland.



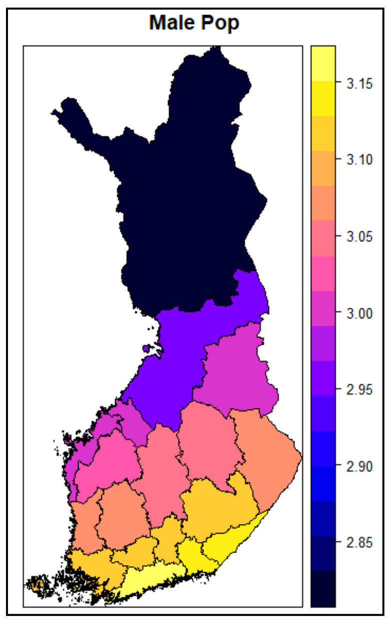
The changes in GDP are not relevant in changes of Immigration. But most likely to be affected in the north of Finland.



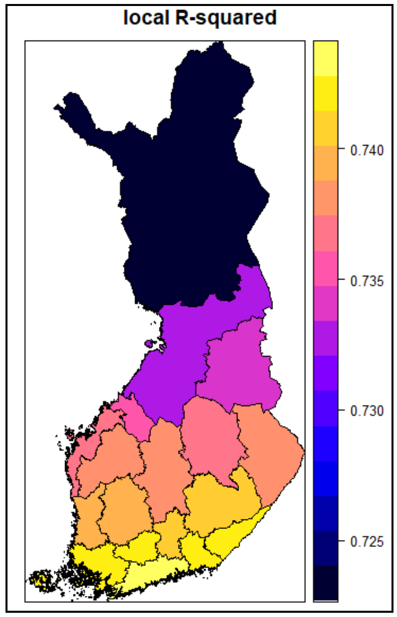
More density/proximity increases the Immigration more likely at the south Finland.



Increase in pop over 65 causes less Immigration at the South of Finland. People normally Migrate to cities more livable with more youngers.



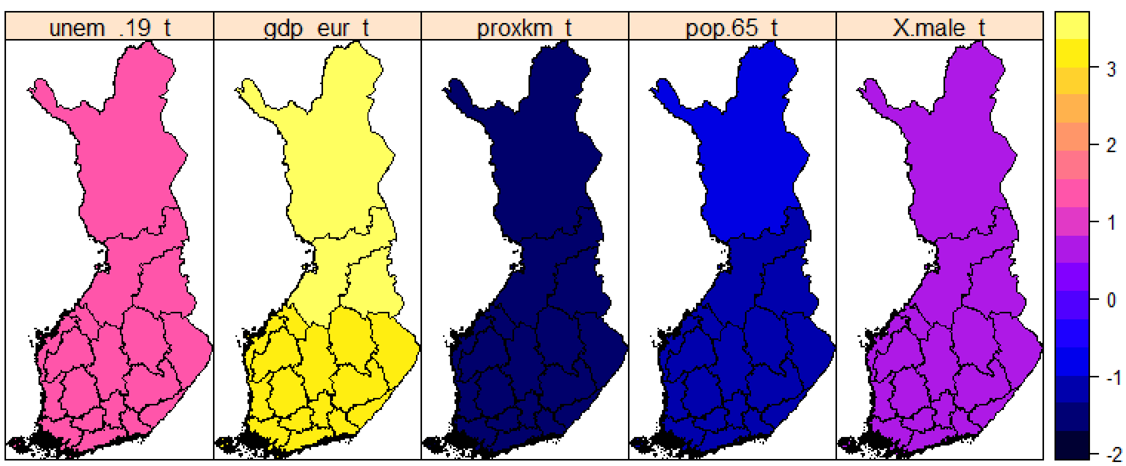
Increases in Male population increases Immigration at South Finland.



Quality of the regression. The set of explanatory variables doesn't explain Immigration the best in Northern Finland. The Southern Finland has the best explanatory power based on our variables to explain Immigration. Over 74%.

9. T-values

T-Values. Higher t values mean higher statistical significance.



Values closer to 0 means no explanatory power.

Better variables to explain Immigration in Finland are GDP, then Unemployment, thenProximity.

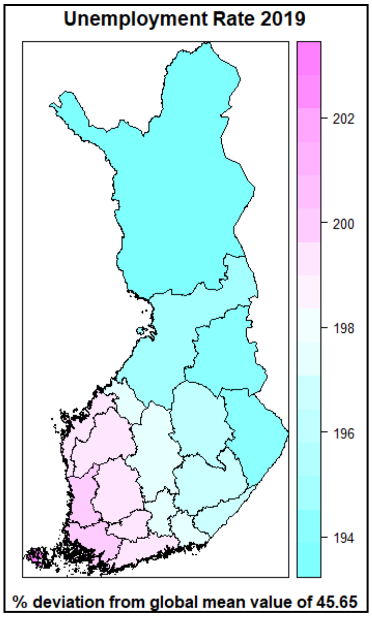
On the other hand, Population over 65 and Male population are bad predictors.

P-values.

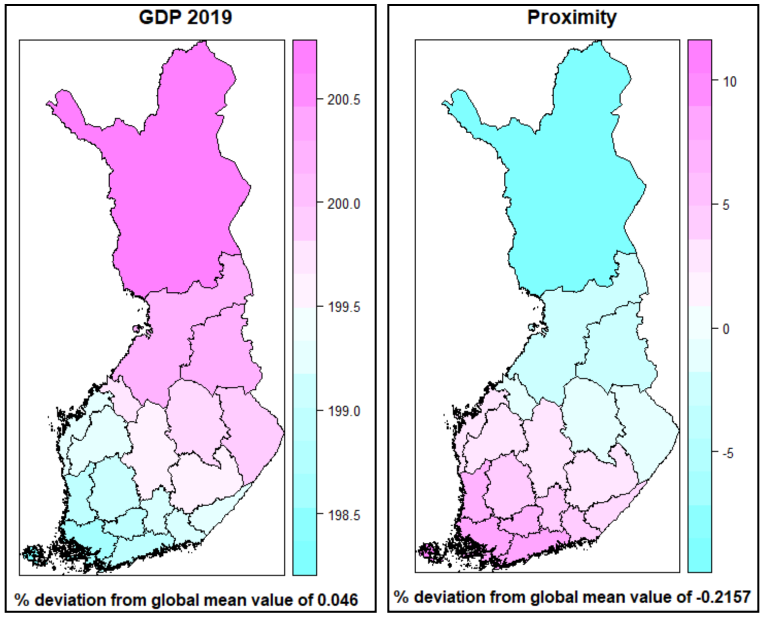
- T-value 3 P-value 0.0105 Significant 99%
- T-value 2 P-value 0.0675 Significant 95%
- T-value 1 P-value 0.336
- T-value 0 P-value 1
- T-value -1 P-value 0.336
- T-value -2 P-value 0.0675 Significant 95%

10. Non-stationary effect.

Anomaly of effect of the Variable in Immigration, at particular location.



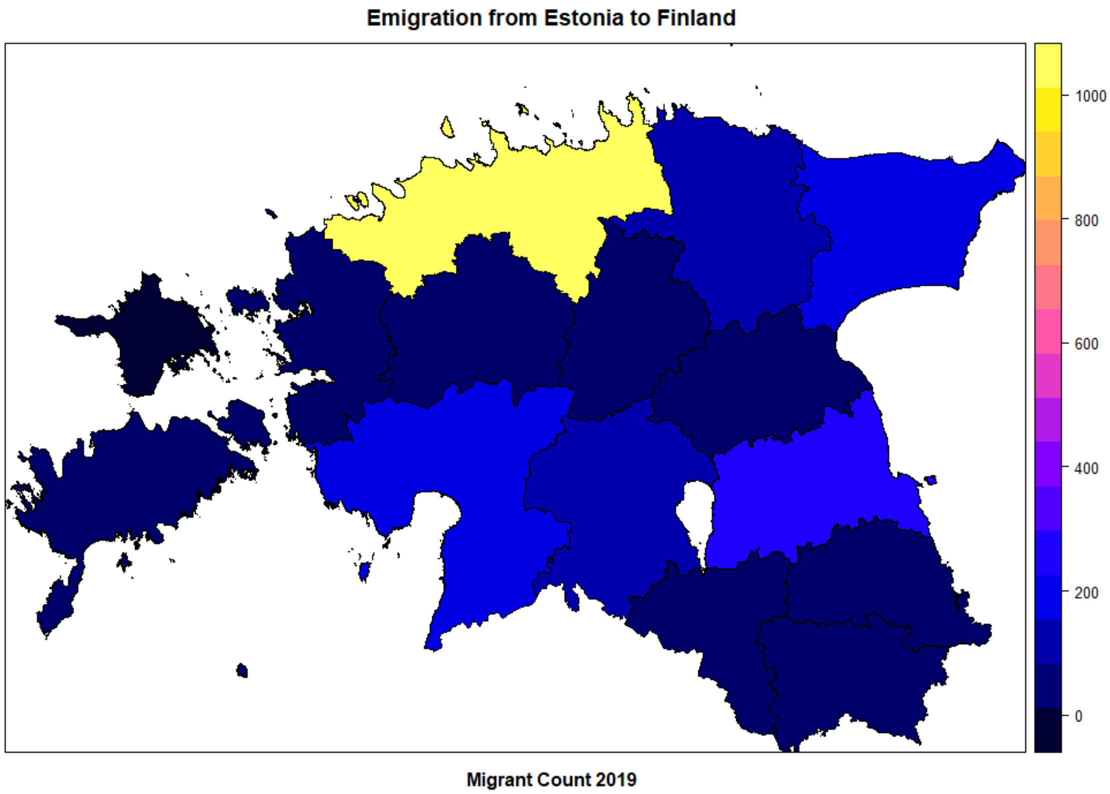
It reveals Niches of changes. Likes South-West. And North-East.2 regions where the effect of Unemployment affects the Immigration sufficiently different. These 2 regions work by different mechanisms of Unemployment which explains Immigration. With other variables, it will give the same regionalization.


GDP 2019
200.5
200.0
199.5
199.0
198.5
% deviation from global mean value of 0.046
Proximity
10
5
0
-5
% deviation from global mean value of -0.2157

Estonia

1. Plotting the dependent variable

Estonia (2019): Map of emigration count by county

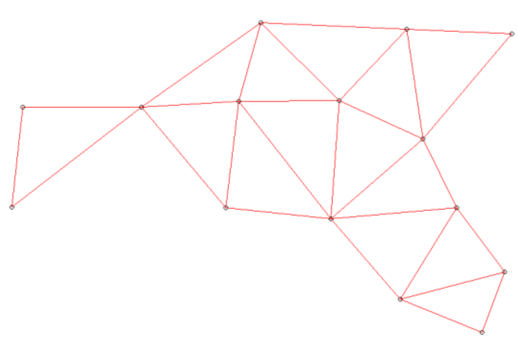


2. OLS results

The residuals are fairly symmetrical, but the median is higher than zero and some non-randomness is present. According to OLS, the model explains 90% of variance in emigration. Although the model is significant (p=0.0002346), most of the independent variables are not significant on their own. Only GDP per capita is significant (p=0.00424) with a 100EUR increase correlated with five more emigrants/year.

3. Spatial weights

Thanks to its condensed, tight form, and representation of Estonia's island, the Sphere of Influence (SOI) graph is selected as the spatial weight. It better represented Estonia's islands than the Delaunay triangulation and Gabriel Graph neighborhoods. Von Neumann (rook) and Moore (queen) spatial weights leave some polygons without neighbors. K-nearest neighbors entails too dense of a connectivity structure.



4. Lagrange Multipliers Testing

Next, we run the Lagrange multiplier (LM) diagnostics. Both the LM error model and LM lag model are not statistically significant, with p-values of 0.593 and 0.4068, respectively. According to the spatial regression workflow, this suggests that we should stop and keep the OLS results or change the variables, then run OLS and LM diagnostics again. Given the tight schedule, we decided to continue with same variables for the sake of the project.

5. Spatial Error Model

After LM testing, we ran a spatial error model (SEM) test. The residuals remained similar, with a slight increase in the interquartile range (IQR). The explanatory power did not change compared to OLS, with an increase in the pseudo R^2 from 90% to only 91%. The model does not correct spatial autocorrelation and would require better variables to do so.

An increased GDP per capita is associated with a slight increase in migration. Perhaps counter intuitively, every 100EUR increase in GDP per capita is associated with five more annual emigrants. This relationship is statistically significant. Other findings, which were not statistically significant, included:

- A 1% increase in unemployment is associated with an 18 more migrants leaving per year.
- An increase of 5km away from Helsinki is associated with one less migrant.
- Counties with aging populations were associated with higher migration numbers. A 1% increase in the population over 65 is associated with 17 people migrating each year.
- Higher education levels, measured through the percentage of those with bachelor's, master's, or doctoral degrees, is associated with decreased migration.

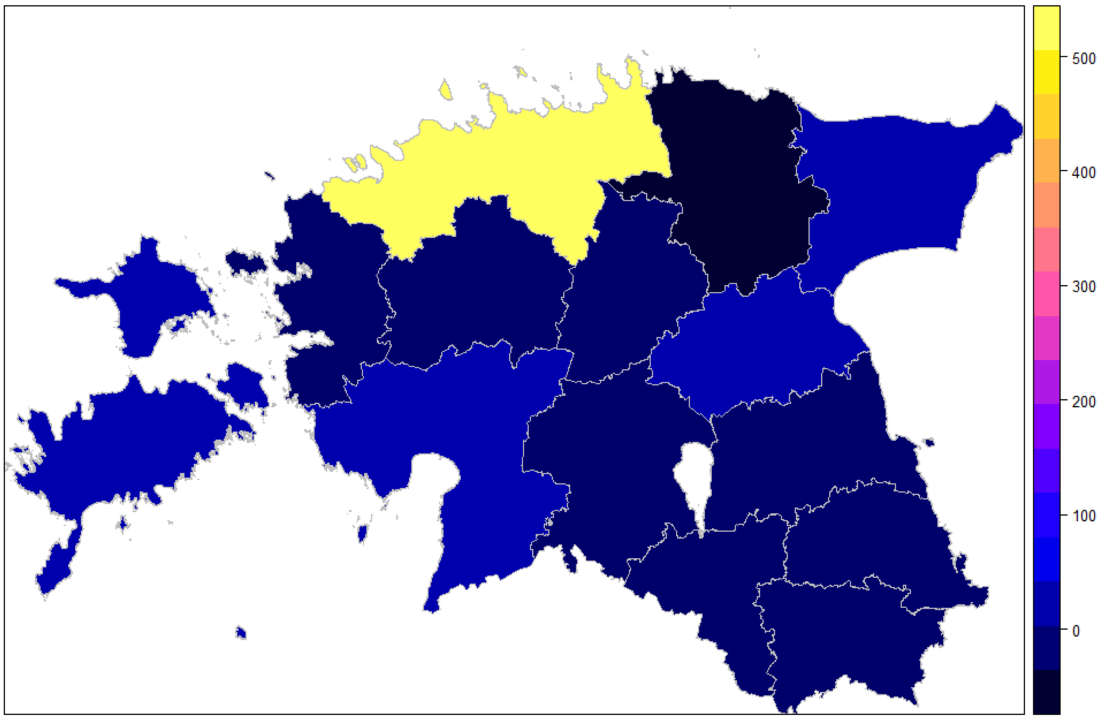
6. Spatial Durbin Model

The p-value of the common factor hypothesis is statistically significant (p=0.000006911), so we retain the results for the Spatial Durbin Model (SDM). According to the Nagelkerke pseudo R-squared value, the model explains almost 99% of variance. In the SDM, all of the variables are highly statistically significant. In this case, it makes sense to replace the SEM with the SDM.

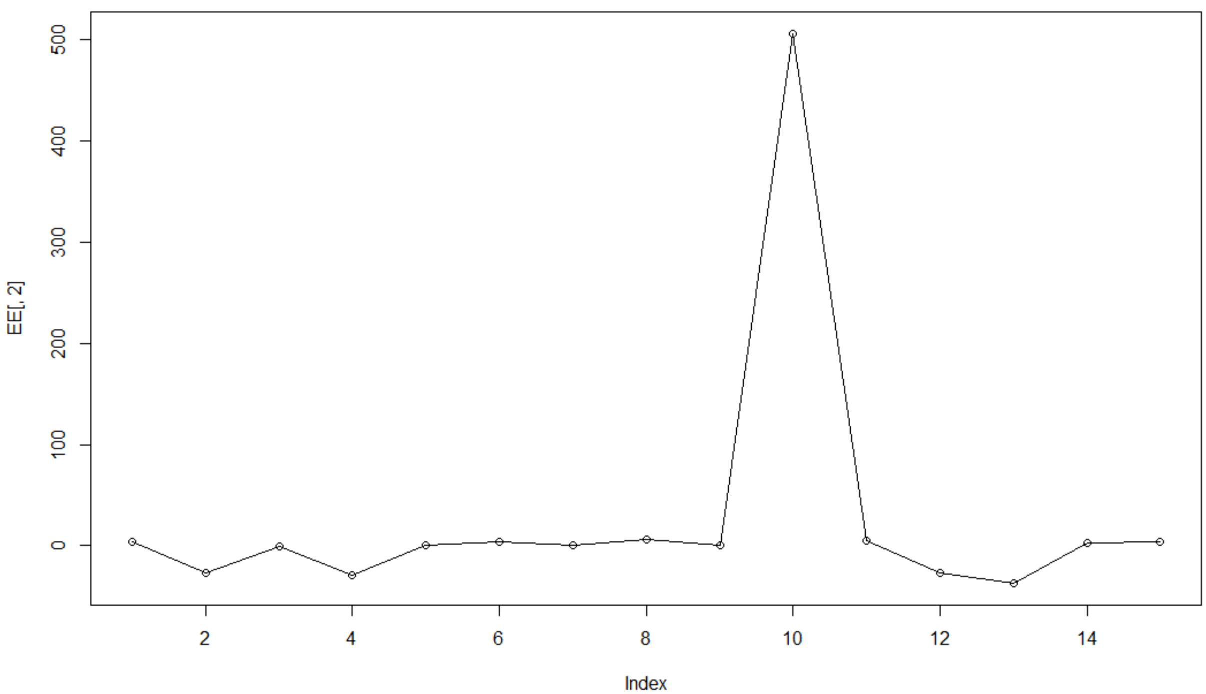
7. Marginal Equilibrium Effects

Now, we look at the final aggregate effect from a change in an independent variable. In this case, we look at unemployment, specifically in Harju county. Harju county has the highest number of emigrants and is home to Estonia's capital, Tallinn. Using marginal equilibrium effects, we can simulate the impact of double unemployment in Harju county, then see the aggregation of indirect effects on top of direct effects. The MEF reveals that following a doubling of unemployment in Harju, emigration would increase dramatically from Harju county. For other counties, emigration would either slightly increase or slightly decrease. For Järva, Lääne, Lääne-Viru, and Tartu counties emigration decreases by about 30 people per 1% increase in unemployment in Harju county. For all other counties, emigration is simulated to increase by 1-6%.

Emigrants in Estonia: 2019



Marginal equilibrium effects for doubling of unemployment in Harju county



8. Geographically weighted regression

Next, geographically weighted regression (GWR) is used as an exploratory tool to complement the rest of the spatial statistical workflow. Using the Akaike information criterion (AIC), a bandwidth of 323998.8 is generated. The GWR shows that there is local variation when compared to the global values. These spatially heterogeneous effects are more easily demonstrated through thematic maps. Generally speaking, three geographic bands emerge across Estonia: north, central, and south.

Proximity to Helsinki

Impact of distance to Helsinki is lower in the areas closest to Helsinki and highest in the areas that are farthest from Helsinki. This suggests that proximity matters, and distance may be a deterrent to emigration.

Education Level

If Estonia were diagonally split into two, higher education levels have a lower impact in the northeastern half and a higher impact in the southeastern half. Those who are more highly educated may be less likely to migrate, despite the proximity of these areas to Helsinki.

GDP per capita

Compared to other variables, the influence of GDP per capita is minimal. On the north-south axis, the influence of GDP per capita slightly decreases as you move south.

Population over 65

Interestingly, the spatial pattern of age is nearly the inverse of GDP per capita. Along the north-south axis, the influence of age becomes stronger as you move south.

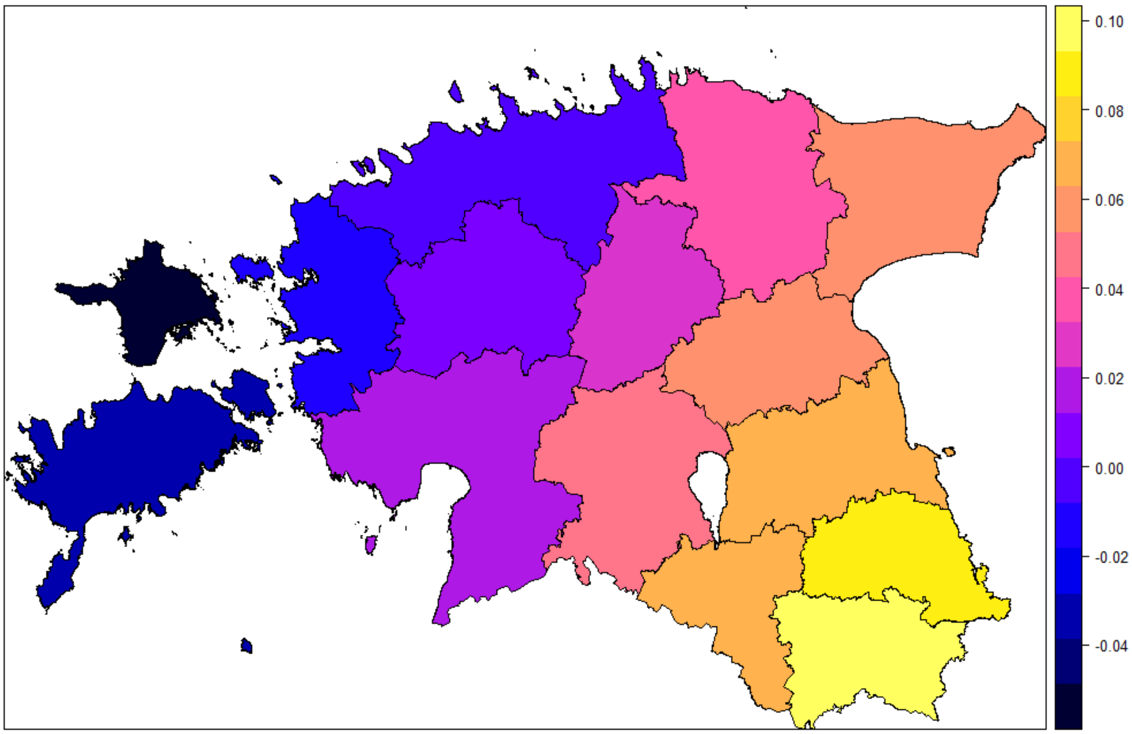
Unemployment

The influence of unemployment on emigration appears to be strongest for Estonia’s islands. This is followed by the capital area, central Estonia, and is weakest in southern Estonia.

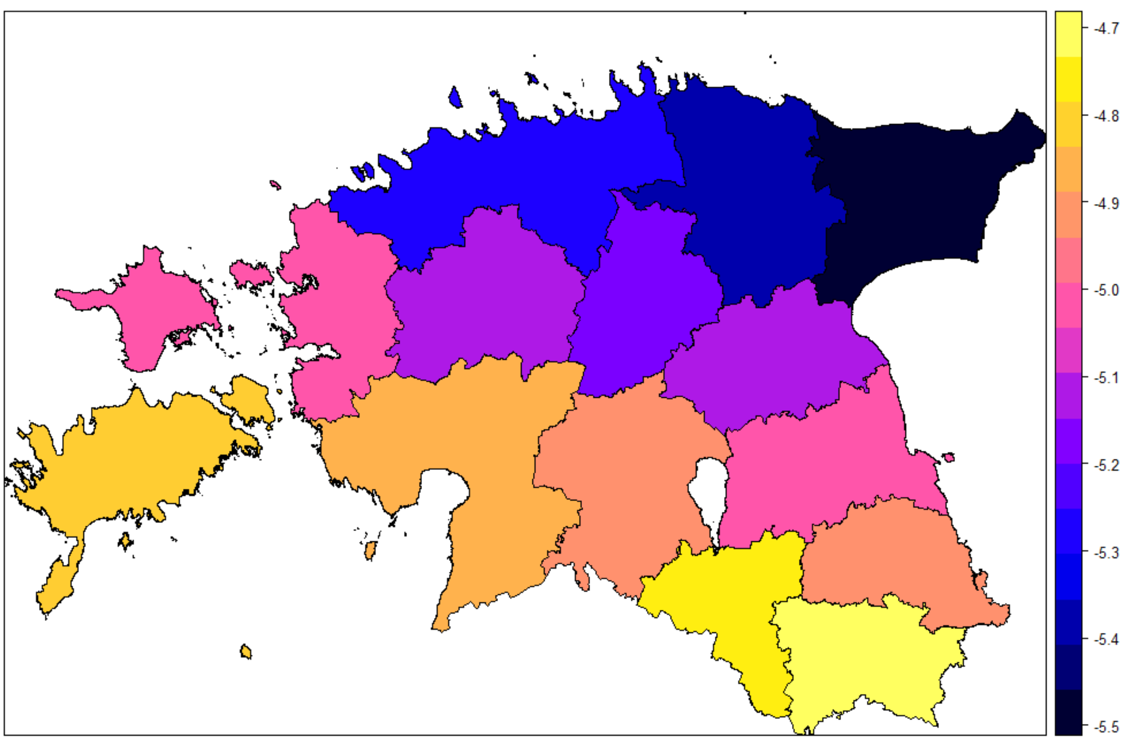
R-squared

Based on the local R-squared, the model seems best explain counties in northern Estonia. Counties in central Estonia fair slightly worse and the model has the least explanatory power for counties in southern Estonia.

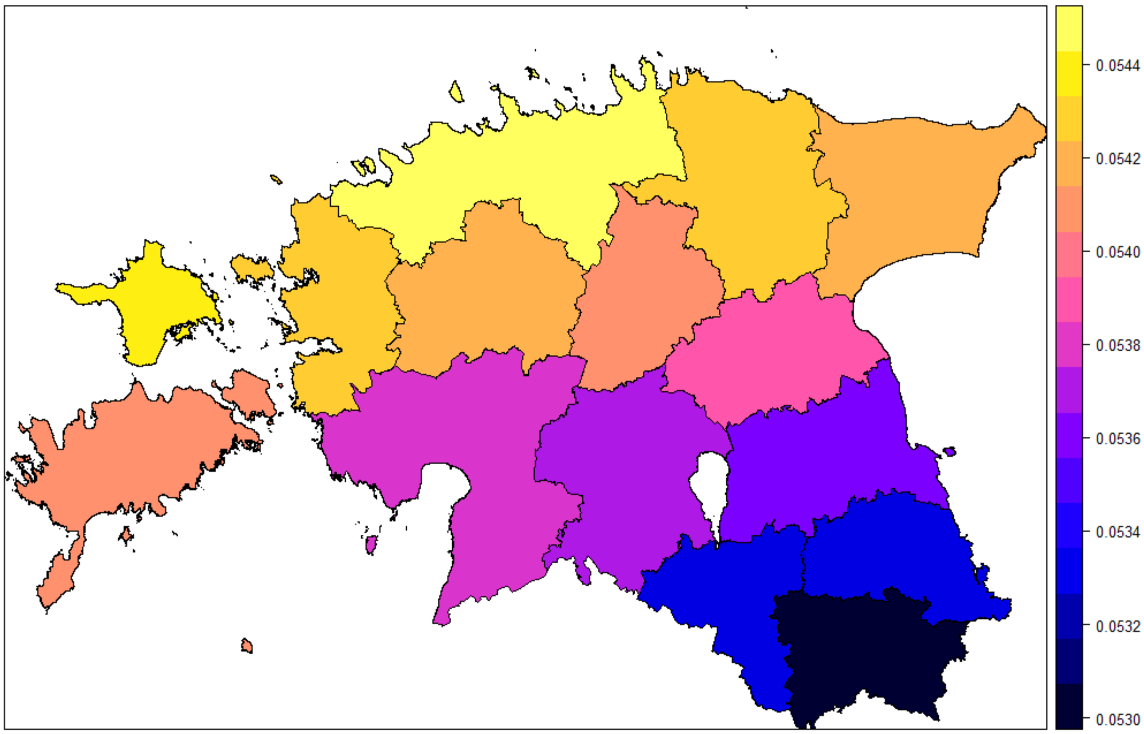
Proximity to Helsinki



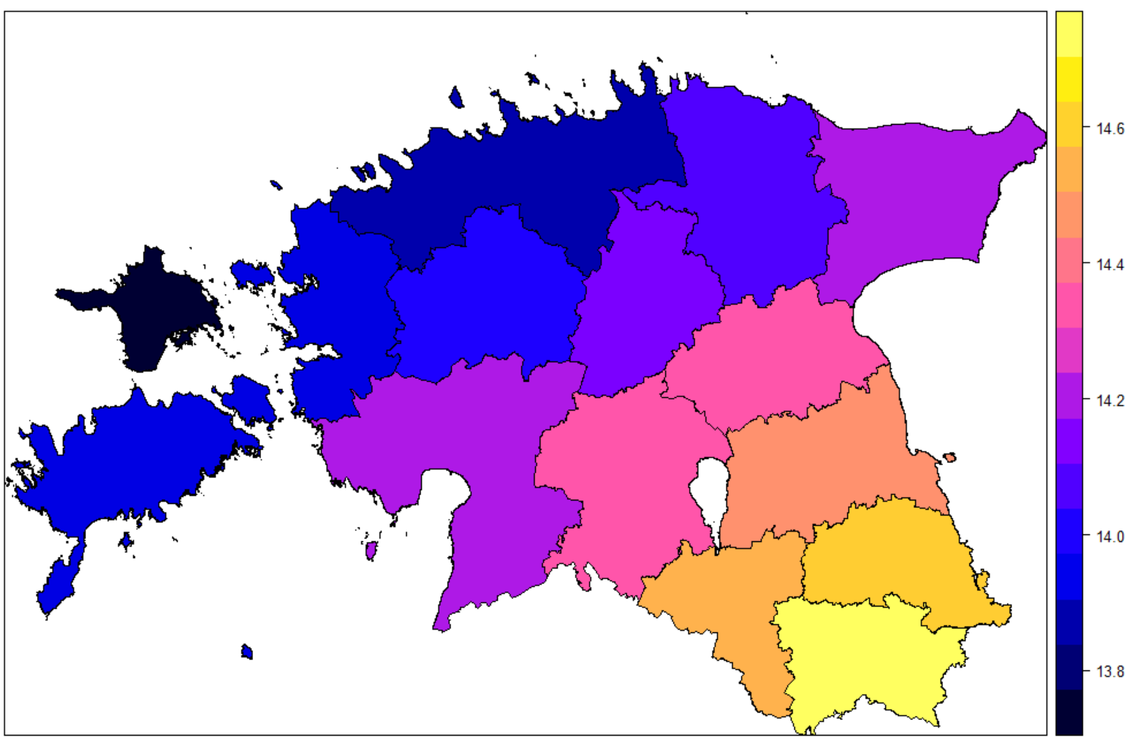
Education level

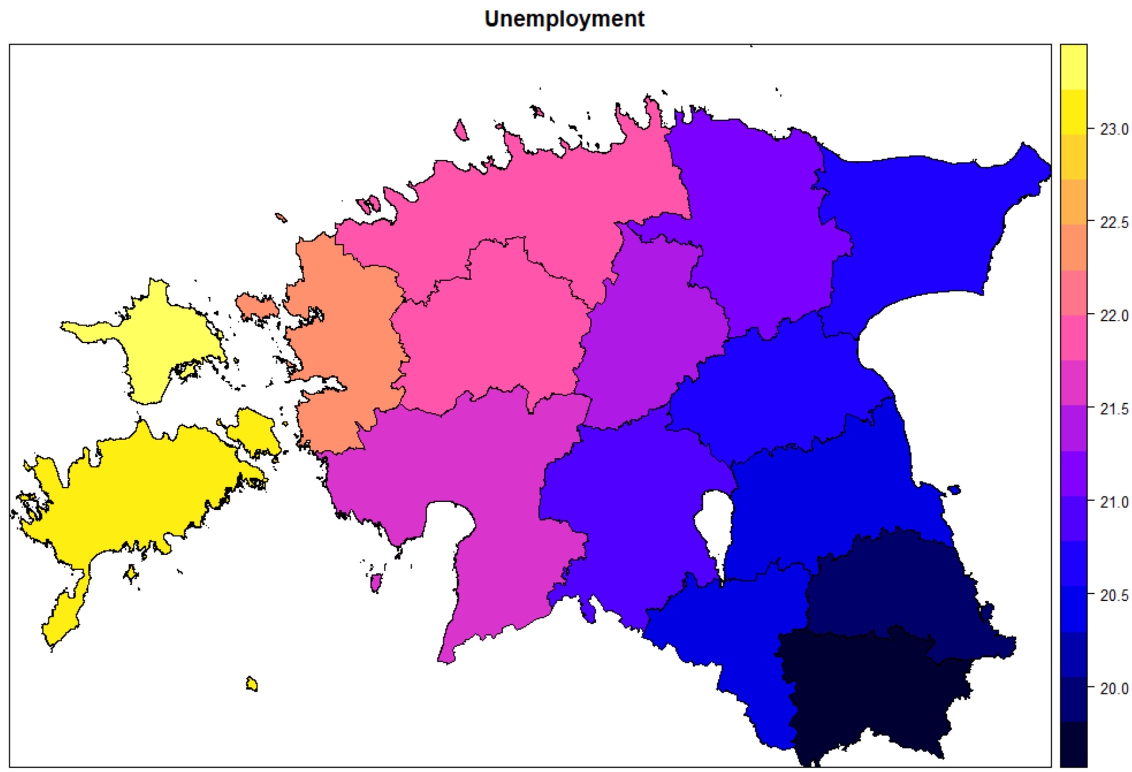


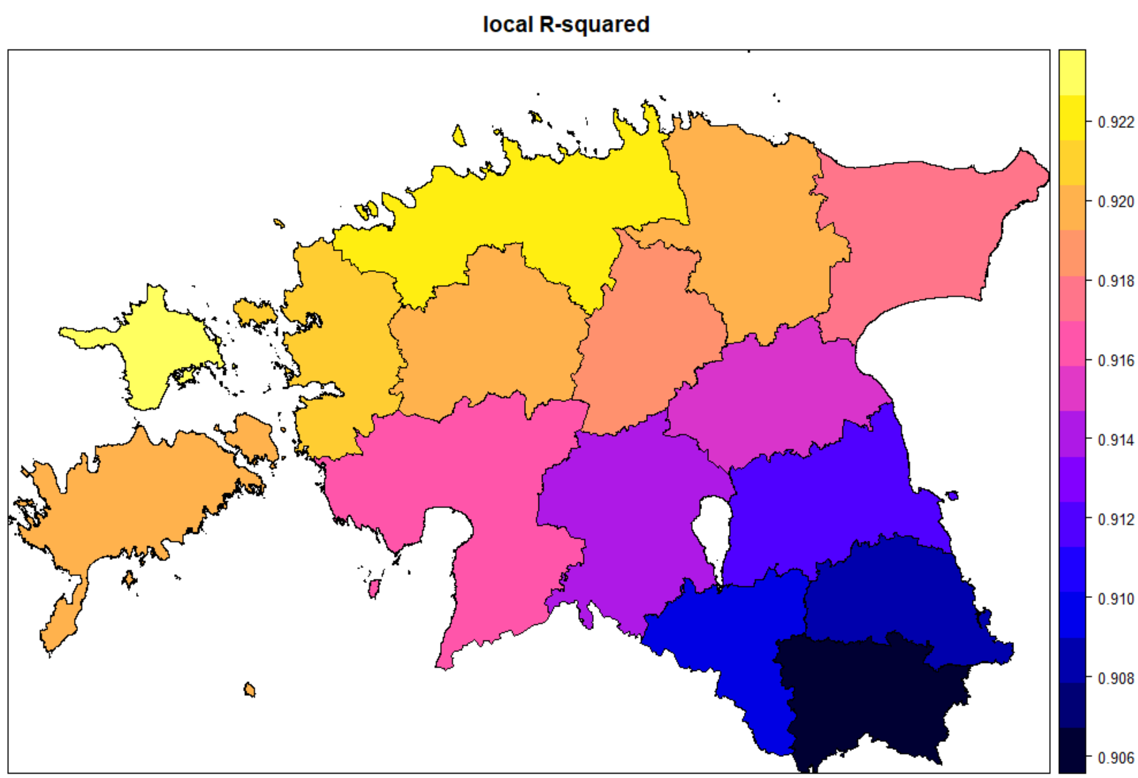
GDP per Capita



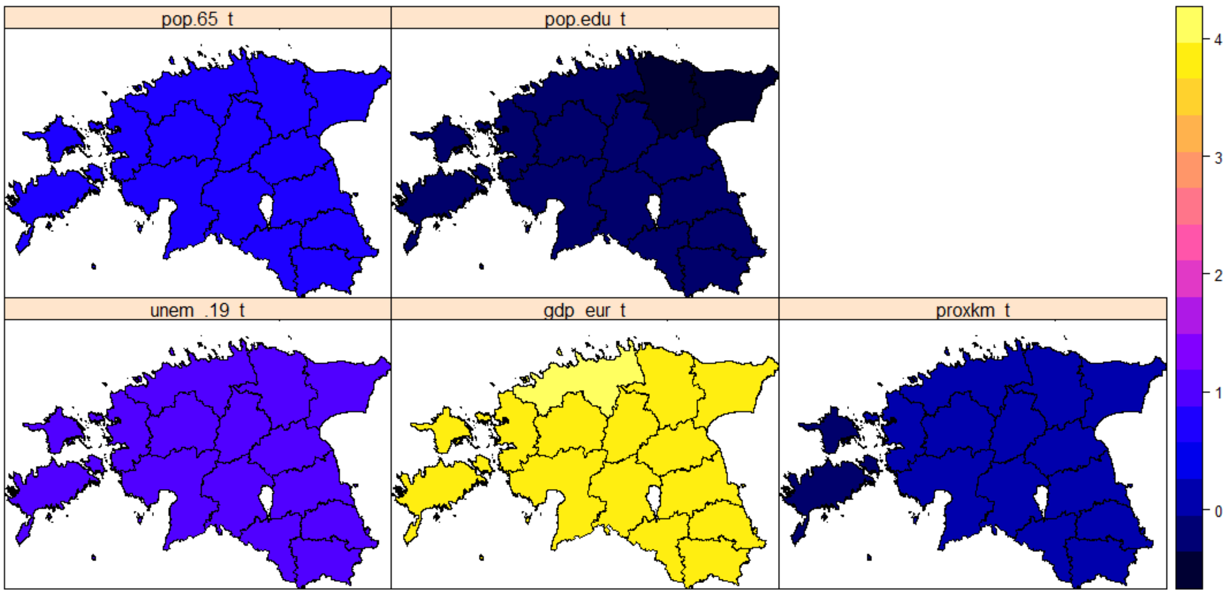
Population over 65




Unemployment
23.0
22.5
22.0
21.5
21.0
20.5
20.0


local R-squared
0.922
0.920
0.918
0.916
0.914
0.912
0.910
0.908
0.906

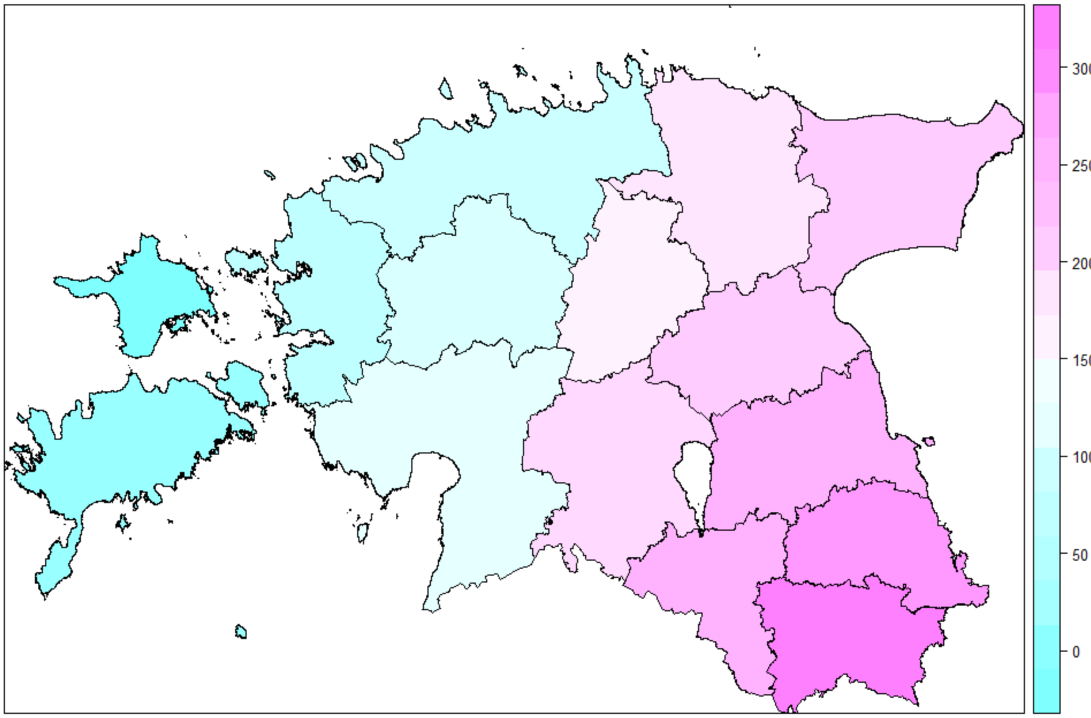
Next, we used another exploratory technique to understand the t-values of the variables. First, five subplots, one for each variable, were created. Values close to zero have little to no explanatory power. According to the subplots, the GDP per capita has the strongest explanatory power. The percent of the population over 65, the unemployment rate, and proximity has lower explanatory power. The education level has very little to no explanatory power.



9. Niches

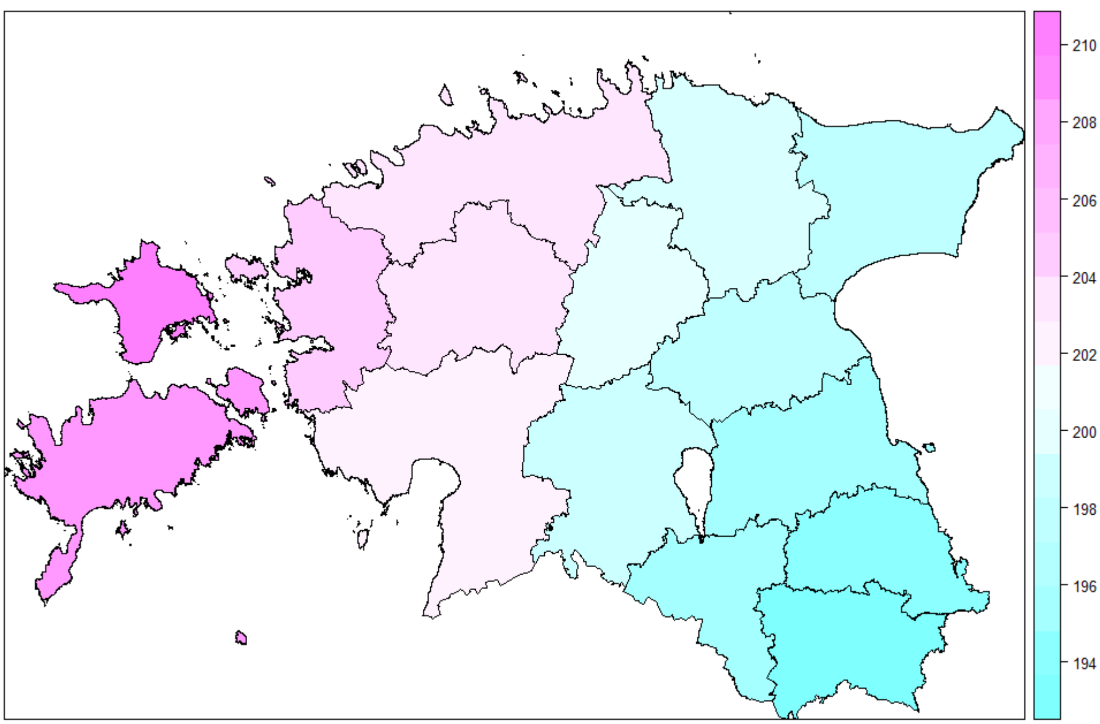
Finally, visualizations are created which compare local values to the global ones. This helps to show regionalization or where certain niches are in the data. Depending on the variable, two patterns emerge: an east-west regionalization (e.g. proximity to Helsinki and unemployment rate) and a north-south regionalization (e.g. GDP per capita and population over 65). The variable education cuts the country along a diagonal, like a mix of the two other patterns.

Proximity to Helsinki 2019



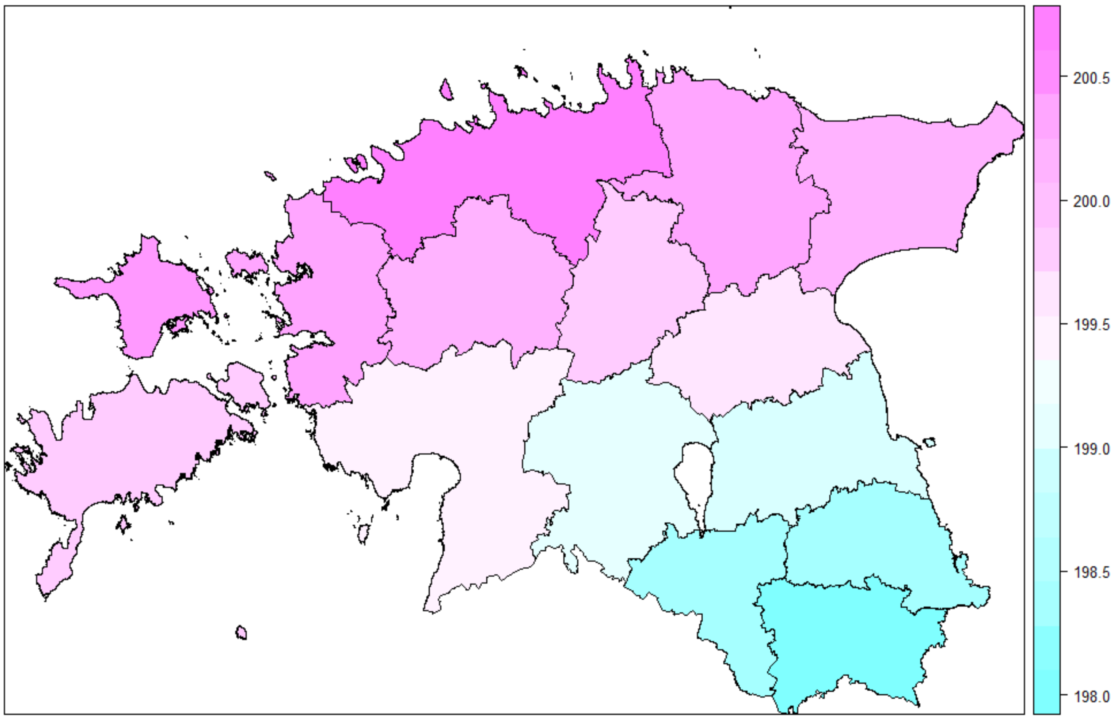
% deviation from global mean value of 0.0445

Unemployment Rate 2019



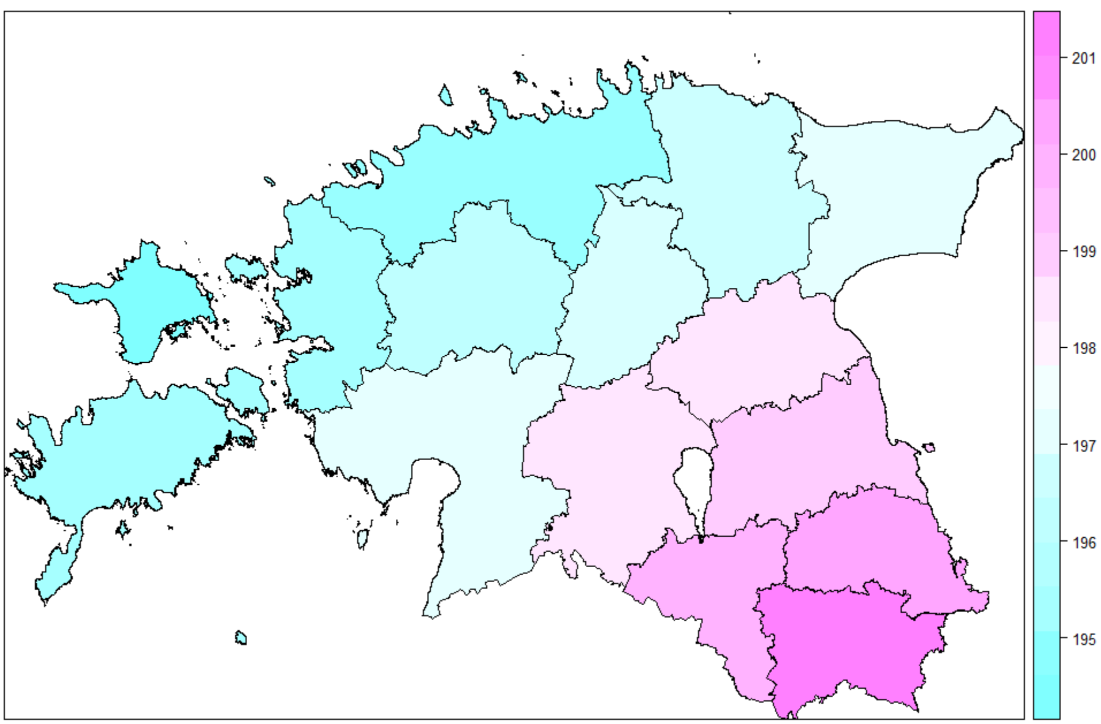
% deviation from global mean value of 21.1508

GDP per capita 2019

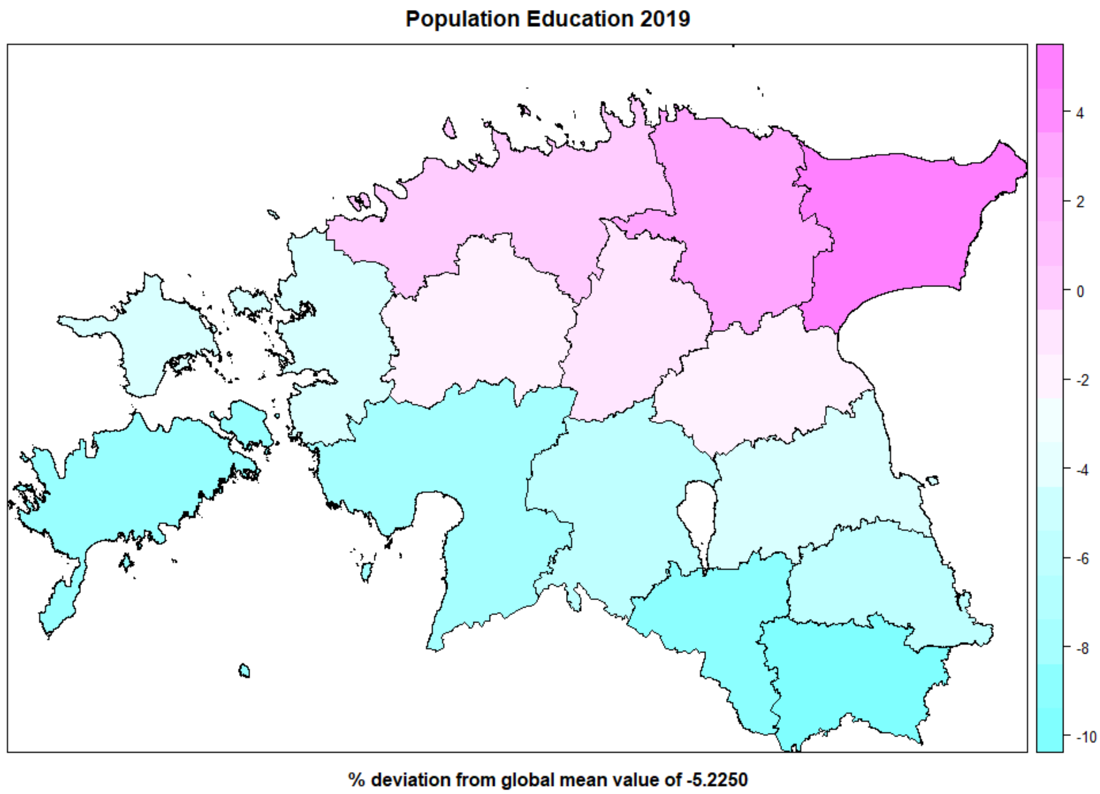


% deviation from global mean value of 0.0541

Population over 65 2019



% deviation from global mean value of 14.5553


Population Education 2019
4
2
0
-2
-4
-6
-8
-10
% deviation from global mean value of -5.2250

Appendix A: Estonia

Estonia Counties and ID

County	PolyID
Harju county	10
Hiiu county	8
Ida-Viru county	11
Järva county	2
Jõgeva county	1
Lääne county	4
Lääne-Viru county	13
Pärnu county	8
Põlva county	9
Rapla county	3
Saare county	6
Tartu county	12
Valga county	7
Viljandi county	14
Võru county	5

OLS Results

```
Call:
lm(formula = em_2019 ~ unem_.19 + gdp_eur + proxkm + pop.65 +
    pop.edu, data = estonia)

Residuals:
    Min      1Q  Median      3Q     Max
-116.52  -56.98   12.89   38.47  121.05

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -943.75580  636.41258  -1.483  0.17224
unem_.19      21.15083   23.05139   0.918  0.38278
gdp_eur        0.05412    0.01425   3.797  0.00424 **
proxkm         0.04446    0.57261   0.078  0.93981
pop.65        14.55532   22.27294   0.653  0.52978
pop.edu       -5.22505   15.31744  -0.341  0.74084
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 95.46 on 9 degrees of freedom
Multiple R-squared:  0.9046,    Adjusted R-squared:  0.8516
F-statistic: 17.06 on 5 and 9 DF,  p-value: 0.0002346
```

Lagrange Multipliers Testing

```
        Lagrange multiplier diagnostics for spatial dependence

data:
model: lm(formula = em_2019 ~ unem_.19 + gdp_eur + proxkm + pop.65 + pop.edu, data = estonia)
weights: weights

LMerr = 0.28564, df = 1, p-value = 0.593


        Lagrange multiplier diagnostics for spatial dependence

data:
model: lm(formula = em_2019 ~ unem_.19 + gdp_eur + proxkm + pop.65 + pop.edu, data = estonia)
weights: weights

LMlag = 0.68828, df = 1, p-value = 0.4068


        Lagrange multiplier diagnostics for spatial dependence

data:
model: lm(formula = em_2019 ~ unem_.19 + gdp_eur + proxkm + pop.65 + pop.edu, data = estonia)
weights: weights

RLMerr = 0.06189, df = 1, p-value = 0.8035


        Lagrange multiplier diagnostics for spatial dependence

data:
model: lm(formula = em_2019 ~ unem_.19 + gdp_eur + proxkm + pop.65 + pop.edu, data = estonia)
weights: weights

RLMlag = 0.46453, df = 1, p-value = 0.4955


        Lagrange multiplier diagnostics for spatial dependence

data:
model: lm(formula = em_2019 ~ unem_.19 + gdp_eur + proxkm + pop.65 + pop.edu, data = estonia)
weights: weights

SARMA = 0.75017, df = 2, p-value = 0.6872
```

Spatial Error Model

```
Residuals:
     Min       1Q   Median       3Q      Max
-101.307  -66.277   14.972   32.395  110.436

Type: error
Coefficients: (asymptotic standard errors)
              Estimate  Std. Error z value     Pr(>|z|)
(Intercept) -942.862911  434.286711 -2.1711      0.02993 *
unem_.19      18.701616   15.512555  1.2056      0.22798
gdp_eur        0.053097    0.010181  5.2154 0.0000001834 ***
proxkm        -0.215631    0.506012 -0.4261      0.67001
pop.65        17.225023   15.306007  1.1254      0.26043
pop.edu       -4.448801   11.134704 -0.3995      0.68949
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Lambda: 0.36309, LR test value: 0.59684, p-value: 0.43979
Approximate (numerical Hessian) standard error: 0.44913
    z-value: 0.80844, p-value: 0.41884
Wald statistic: 0.65357, p-value: 0.41884

Log likelihood: -85.53582 for error model
ML residual variance (sigma squared): 5045, (sigma: 71.028)
Nagelkerke pseudo-R-squared: 0.9083
Number of observations: 15
Number of parameters estimated: 8
AIC: 187.07, (AIC for lm: 185.67)
```

Common Factor Hypothesis

```
        Likelihood ratio for spatial linear models

data:
Likelihood ratio = 31.668, df = 5, p-value = 0.000006911
sample estimates:
Log likelihood of x Log likelihood of y
          -69.70171           -85.53582
```

Spatial Durbin Model

```
Residuals:
     Min       1Q   Median       3Q      Max 
-59.4844 -11.7663  -2.8034  13.1741  41.0363 

Type: mixed 
Coefficients: (numerical Hessian approximate standard errors) 
                Estimate    Std. Error  z value          Pr(>|z|)    
(Intercept)  10183.0451889 1014.6478338  10.0360          < 2.2e-16 ***
unem_.19       107.9792921   10.0964111  10.6948          < 2.2e-16 ***
gdp_eur          0.0546788    0.0045028  12.1434          < 2.2e-16 ***
proxkm          -5.7488214    0.6695438  -8.5862          < 2.2e-16 ***
pop.65         -73.7685798   10.8644998  -6.7899 0.0000000000112232 ***
pop.edu        -65.0131030    6.9948634  -9.2944          < 2.2e-16 ***
lag.unem_.19   205.9578756   28.3480529   7.2653 0.0000000000003721 ***
lag.gdp_eur     -0.1515476    0.0215758  -7.0240 0.0000000000021565 ***
lag.proxkm       4.7044253    0.5491746   8.5664          < 2.2e-16 ***
lag.pop.65    -310.1032182   28.1082008 -11.0325          < 2.2e-16 ***
lag.pop.edu     29.1666903   11.4492030   2.5475            0.01085 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Rho: -0.30117, LR test value: 1.0028, p-value: 0.31664
Approximate (numerical Hessian) standard error: 0.28697
    z-value: -1.0495, p-value: 0.29396
Wald statistic: 1.1014, p-value: 0.29396

Log likelihood: -69.70171 for mixed model
ML residual variance (sigma squared): 621.46, (sigma: 24.929)
Nagelkerke pseudo-R-squared: 0.9889 
Number of observations: 15 
Number of parameters estimated: 13 
AIC: 165.4, (AIC for lm: 164.41)
```

Marginal Equilibrium Effects

```
     Current Unemployment MEF(Unemployment Doubles in Harju) Unemployment Doubles
[1,]                  6.6                         3.41712143            10.017121
[2,]                  5.1                       -26.94320287           -21.843203
[3,]                  4.7                        -0.42471280             4.275287
[4,]                  5.7                       -29.60608442           -23.906084
[5,]                  2.9                         0.01378343             2.913783
[6,]                  4.4                         3.87473895             8.274739
[7,]                  6.9                        -0.14760387             6.752396
[8,]                  6.8                         5.50352673            12.303527
[9,]                  7.9                         0.05607092             7.956071
[10,]                 4.6                       505.79768213           510.397682
[11,]                10.7                         5.01015214            15.710152
[12,]                 4.7                       -27.53087378           -22.830874
[13,]                 6.1                       -36.68851267           -30.588513
[14,]                 5.0                         2.31527239             7.315272
[15,]                 5.8                         3.87473895             9.674739
```

Geographically weighted regression

```
Call:
gwr(formula = em_2019 ~ unem_.19 + gdp_eur + proxkm + pop.65 + 
    pop.edu, data = estonia, bandwidth = bw, hatmatrix = TRUE)
Kernel function: gwr.Gauss 
Fixed bandwidth: 323998.8 
Summary of GWR coefficient estimates at data points:
                   Min.      1st Qu.       Median      3rd Qu.         Max.    Global
X.Intercept. -945.4480921 -938.8589461 -934.8657795 -929.8614739 -926.2342127 -943.7558
unem_.19       19.8041370   20.4921596   21.1548751   21.8970093   23.2068784   21.1508
gdp_eur         0.0530694    0.0536359    0.0540799    0.0542255    0.0544283    0.0541
proxkm         -0.0490365    0.0042383    0.0365157    0.0659999    0.0932239    0.0445
pop.65         13.7695885   13.9712501   14.1796339   14.3954260   14.7033407   14.5553
pop.edu        -5.4610638   -5.1532504   -5.0107767   -4.8735417   -4.7331897   -5.2250
Number of data points: 15 
Effective number of parameters (residual: 2traceS - traceS'S): 6.593901 
Effective degrees of freedom (residual: 2traceS - traceS'S): 8.406099 
Sigma (residual: 2traceS - traceS'S): 93.39694 
Effective number of parameters (model: traceS): 6.311414 
Effective degrees of freedom (model: traceS): 8.688586 
Sigma (model: traceS): 91.86611 
Sigma (ML): 69.91724 
AICc (GWR p. 61, eq 2.33; p. 96, eq. 4.21): 202.7811 
AIC (GWR p. 96, eq. 4.22): 176.2989 
Residual sum of squares: 73326.31 
Quasi-global R2: 0.9146973 
```