Autoignition of Methyl Valerate at Low to Intermediate Temperatures and Elevated Pressures in a Rapid Compression Machine

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Abstract

Methyl valerate (C₆H₁₂O₂, methyl pentanoate) is a methyl ester and a relevant surrogate component for biodiesel. In this work, we present ignition delays of methyl valerate measured using a rapid compression machine at a range of engine-relevant temperature, pressure, and equivalence ratio conditions. The conditions we have studied include equivalence ratios from 0.25 to 2.0, temperatures between 680 K and 1050 K, and pressures of 15 bar and 30 bar. The ignition delay data demonstrate a negative temperature coefficient region in the temperature range of 720 K-800 K for both $\phi = 2.0$, 15 bar and $\phi = 1.0$, 30 bar, with two-stage ignition apparent over the narrower temperature ranges of 720 K-760 K for the lower pressure and 740 K-760 K at the higher pressure. In addition, the experimental ignition delay data are compared with simulations using an existing chemical kinetic model from the literature. The simulations with the literature model under-predict the data by factors between 2 and 10 over the entire range of the experimental data. To help determine the possible reasons for the discrepancy between simulations and experiments, a new chemical kinetic model is developed using the Reaction Mechanism Generator (RMG) software. The agreement between the experimental data and the RMG model is improved but still not satisfactory. Directions for future improvement of the methyl valerate model are discussed.

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1. Introduction

For transportation applications, biodiesel is an important constituent in improving environmental friendliness of fuels. This is due to its renewability when produced from sustainable agricultural crops and its ability to reduce emissions relative to conventionally fueled engines [1]. Biodiesel typically consists of long-chain methyl ester molecules, with typical compositions of C₁₄ to C₂₀ [1]. Recognizing that the large molecular size of the methyl esters within biodiesel fuel makes creating and using detailed chemical kinetic mechanisms challenging [2], it is desired to study their combustion chemistry by studying simpler molecules.

A recent review paper summarizes the work on methyl esters relevant to 11 biodiesel combustion [3]; the following summary focuses on ignition delay mea-12 surements, since these are the focus of this paper. Autoignition of methyl bu-13 tanoate (MB, C₅H₁₀O₂) has been well-studied in both shock tube and rapid compression machine experiments [4, 5, 6, 7, 8, 9, 10]. The prevalence of MB data in the literature is largely due to the early identification of MB as a po-16 tential surrogate fuel for biodiesel [11]. However, the experiments have shown 17 that MB may not be an appropriate surrogate for biodiesel, due to its lack of 18 negative temperature coefficient (NTC) behavior, a requirement for a suitable biodiesel surrogate [3].

Larger methyl esters such as methyl valerate (MV, C₆H₁₂O₂, methyl pentanoate) have also been studied as possible biodiesel surrogates. Hadj-Ali et al. [9] used a rapid compression machine (RCM) to study the autoignition of several methyl esters including MV. Although MV exhibited two-stage ignition in this study, little additional research has been done on its oxidation. Korobeinichev et al. [12] studied MV in premixed laminar flames and extended a detailed high temperature chemical kinetic model to include MV and methyl hexanoate. Dmitriev et al. [13] added MV to n-heptane/toluene fuel blends to determine
the resulting intermediate species in premixed flames using a flat burner at
1 atm and an equivalence ratio of 1.75. The addition of MV helped reduce soot
forming intermediates including benzene, cyclopentadienyl, acetylene, propargyl, and vinylacetylene [13]. Hayes and Burgess [14] computationally examined
the peroxy radical isomerization reactions for MV to better understand the low
temperature reaction pathways. Finally, Diévart et al. [15] used diffusion flames
in the counterflow configuration to determine extinction limits for a number of
methyl esters, including MV, and validated a detailed kinetic model with the
experimental data.

This work provides additional data for the autoignition of MV. Data is collected in a RCM under engine relevant conditions spanning from 15 bar to 30 bar, equivalence ratios from 0.25 to 2.0, and temperatures from 682 K to 1048 K. The NTC region of MV is mapped out to provide additional information on the fidelity of using MV as a biodiesel surrogate.

2. Experimental Methods

The RCM used in this study is a single piston arrangement and is pneumatically driven and hydraulically stopped. The device has been described in detail previously [16] and will be described here briefly for reference. The end of compression (EOC) temperature and pressure (T_C and P_C respectively), are independently changed by varying the overall compression ratio, initial pressure (P_0), and initial temperature (T_0) of the experiments. The piston in the reaction chamber is machined with a specially designed crevice to suppress the roll-up vortex effect and promote homogeneous conditions in the reactor during and after compression [17].

The primary diagnostic on the RCM is the in-cylinder pressure measured by a Kistler 6125C dynamic transducer that is compensated for thermal shock. The transducer is coupled to a Kistler 5010B charge amplifier. The voltage output of the charge amplifier is recorded by a National Instruments 9125 analog input $_{57}\,$ device connected to a cDAQ 9178 chassis. The voltage is sampled at a rate

of either 50 kHz or 100 kHz by a LabView VI and processed by a Python

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package called UConnRCMPy [18]. Version 3.0.0 of UConnRCMPy [19], 3.6.1

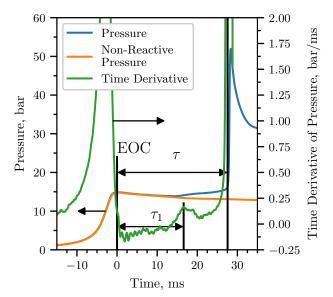
of Python, 2.3.0 of Cantera [20], 1.13 of NumPy [21], 0.19.0 of SciPy [22], and

61 2.0.1 of Matplotlib [23] were used in the analysis in this paper.

The compression stroke of the RCM brings the fuel/oxidizer mixture to the EOC conditions, and for suitable thermodynamic states, the mixture will ignite after a delay period. The definitions of the ignition delays are shown in Fig. 1. The time of the EOC is defined as the maximum of the pressure trace prior to the start of ignition and the ignition delays are defined as the time from the EOC until local maxima in the first time derivative of the pressure. Each experimental condition is repeated at least five times to ensure repeatability of the data. As there is some random scatter present in the data, the standard deviation (σ) of

the ignition delays from the runs at a given condition is computed. In all cases,

 σ is less than 10% of the mean value of the overall ignition delay.



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Figure 1: Definition of the ignition delays used in this work. The experiment in this figure was conducted for a $\phi=2.0$ mixture with Ar/(N₂ + AR) = 0.5, $P_0=0.7806$ bar, $T_0=373$ K, $P_C=14.92$ bar, $T_C=720$ K, $\tau=(27.56\pm0.89)$ ms, $\tau_1=(16.60\pm0.46)$ ms.

In addition to the reactive experiments, non-reactive experiments are conducted to determine the influence of machine-specific behavior on the experimental conditions and permit the calculation of the EOC temperature via the isentropic relations between pressure and temperature [24]. The EOC temperature is calculated by the procedure described in Section 3.

The mixtures considered in this study are shown in Table 1. Four equivalence ratios of MV in "air" are considered. The ratio of Ar: N₂ in the oxidizer is varied to adjust the temperatures reached at the EOC for a given mixture. Two P_C conditions are studied in this work, 15 bar and 30 bar, representing engine-relevant conditions. For the $\phi = 2.0$ condition, only $P_C = 15$ bar is considered because we could not achieve T_C values low enough that the ignition was long enough to be measured in our apparatus (the typical lower limit of ignition delay on the present RCM is approximately 5 ms).

Mixtures are prepared in stainless steel mixing tanks, approximately 17L and 15 L in size. The proportions of reactants in the mixture are determined by specifying the absolute mass of the fuel, the equivalence ratio (ϕ) , and the ratio of Ar: N₂ in the oxidizer. Mixtures are made by first vacuuming the mixing tanks to an ultimate pressure less than 5 torr. Since MV is a liquid with a relatively small vapor pressure at room temperature and pressure, it is measured gravimetrically in a syringe to within 0.01 g of the specified value. The fuel is injected into the mixing tank through a septum. Proportions of O₂, Ar, and N₂ are added manometrically at room temperature and the total pressure is measured by an Omega Engineering MMA type static pressure transducer. The same transducer is used to measure the pressure of the reactants prior to an experiment.

Table 1: Mixtures considered in this work

•	φ		$Ar/(N_2 + Ar)$			
		MV (100%)	$O_2 (99.994\%)$	Ar (99.999%)	$N_2 (99.999\%)$	-
	0.25	0.0065	0.2087	0.7848	0.0000	1.0
101	0.5	0.0130	0.2074	0.7798	0.0000	1.0
	1.0	0.0256	0.2047	0.7697	0.0000	1.0
	1.0	0.0256	0.2047	0.3849	0.3848	0.5
	2.0	0.0499	0.1996	0.0000	0.7505	0.0
	2.0	0.0499	0.1996	0.3752	0.3753	0.5

The RCM is equipped with heaters to control the initial temperature of the mixture. After filling in the components to the mixing tanks, the heaters are switched on and the system is allowed 1.5 h to come to steady state. The mixing tanks are also equipped with magnetic stir bars so the reactants are well mixed for the duration of the experiments.

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The initial temperature is chosen such that the saturated vapor pressure (P_{sat}) of the fuel at the initial temperature is at least twice the partial pressure of the fuel in the mixing tank. The Antoine equation

$$\log_{10} P_{\text{sat}} = A - \frac{B}{T - C} \tag{1}$$

is used to model the saturated vapor pressure of MV as a function of temper-108 ature, where A, B, and C are substance-specific coefficients. Coefficients for 109 Eq. (1) are given in the literature by Ortega et al. [25], Camacho et al. [26], 110 and Stephenson et al. [27]. Unfortunately, the values of the coefficients are dif-111 ferent among all three authors and, more importantly, the temperature ranges 112 provided in those three fits do not cover the entire range of interest for this 113 study. Therefore, coefficients for use in Eq. (1) are determined in this work by 114 least squares fitting of the data of Ortega et al. [25], van Genderen et al. [28], and 115 Verevkin and Emel'yanenko [29] using the curve_fit() function of SciPy [22] 116 version 0.19.0. Figure 2 shows that the coefficients fit with this procedure give good agreement with the experimental data; values for the coefficients computed in this work and in the literature works are given in Table 2. The data used to calculate the coefficients are provided in the Supplementary Material.

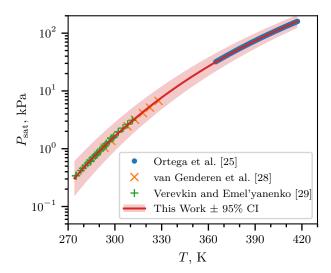


Figure 2: Saturated vapor pressure of MV as a function of temperature, plotted using the Antoine equation, Eq. (1), with A = 6.4030, B = 1528.69, and C = 52.881.

Table 2: Antoine Equation coefficients computed in this work and from the literature. The 2σ confidence interval is estimated by taking the square root of the diagonals of the covariance matrix returned from curve_fit()

	A	B	C	T_{\min} , K	$T_{ m max},{ m K}$
This Work	6.4030	1528.69	52.881	274.9	417.18
2σ Confidence Interval	0.0919	53.47	4.934	_	_
Ortega et al. [25]	6.23175	1429.00	62.30	364.75	417.18
Camacho et al. [26]	5.9644	1281.06	75.94	281	547
Stephenson et al. [27]	6.62646	1658.4	42.09	297	411

5 3. Computational Methods

3.1. RCM Modeling

The Python 3.6 interface of Cantera [20] version 2.3.0 is used for all sim-127 ulations in this work. Detailed descriptions of the use of Cantera for these 128 simulations can be found in the work of Weber and Sung [18] and Dames et al. 129 [30]; a brief overview is given here. As mentioned in Section 2, non-reactive 130 experiments are conducted to characterize the machine-specific effects on the 131 experimental conditions in the RCM. This pressure trace is combined with the 132 reactive pressure trace and used to compute a volume trace by assuming that 133 the reactants undergo a reversible, adiabatic, constant composition (i.e., isen-134 tropic) compression during the compression stroke and an isentropic expansion 135 after the EOC. The volume trace is applied to a simulation conducted in an 136 IdealGasReactor in Cantera [20] using the CVODES solver from the SUNDI-137 ALS suite [31]. The ignition delay from the simulations is defined in the same 138 manner as in the experiments. The time derivative of the pressure in the sim-139 ulations is computed by second order Lagrange polynomials, as discussed by Chapra and Canale [32]. 141

To the best of our knowledge, there are three mechanisms for MV combus-142 tion available in the literature. The first two, by [12] and [13], were developed 143 to simulate flames, and do not include the low-temperature chemistry necessary to simulate the conditions in these experiments. The third model was devel-145 oped by [15] and includes low-temperature chemistry of MV, although it was 146 only validated by comparison with flame extinction limits. In converting this 147 mechanism for use in Cantera, we found that there were many species in the 148 thermodynamic database with multiple data entries. For most of these species the thermodynamic data is identical. However, some species are not exact du-150 plicates. For these species, it is not clear from the thermodynamic database file 151 which data set should be preferred. Since Cantera (and CHEMKIN) choose the 152 first instance of a duplicate species to be used, we retained the first entry for all duplicated species. The detailed [15] model includes 1105 species and 7141

reactions, and the CHEMKIN and Cantera formatted input files are available in the Supplementary Material. 156

3.2. Reaction Mechanism Generator 157

In addition to using a mechanism from the literature, we investigate the use 158 of an automatic mechanism generator, the open-source Reaction Mechanism 159 Generator (RMG) [33] version 2.1.0. The Python version of RMG is used, which requires Python 2.7, and version 2.1.0 of the RMG database is used. 161 The final RMG model contains 427 species and 13640 reactions. Note that the 162 number of species is much lower than the Diévart et al. [15] model because 163 the RMG model focuses on only one fuel (MV), but the number of reactions is 164 substantially higher. The input file used to generate the model is available in the Supplementary Material. 166

4. Experimental Results 167

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Figure 3 shows the ignition delay results measured in this study. Filled mark-168 ers denote the overall ignition delay and hollow markers indicate the first-stage 169 ignition delay. Vertical error bars are drawn on the symbols to represent the 170 uncertainty in the ignition delay; for many of the experiments, the uncertainty 17 is approximately the same size as the data point, so the error bar is hidden. 172 Horizontal error bars are shown on the first and last points of each equivalence 173 ratio indicating the estimated uncertainty in the EOC temperature of $\pm 1\%$ [34]. 174 Fig. 3a shows the results for a compressed pressure of 15 bar, while Fig. 3b shows 175 the results for a compressed pressure of 30 bar. Note that $\phi = 2.0$ results were not collected for 30 bar, so there are no red data points in Fig. 3b. 177 It can be seen from Fig. 3 that the ignition delays for the $\phi = 0.25$ and 0.5 178 mixtures do not show an NTC region of the ignition delay for both of the 179

pressures studied in this work. However, the $\phi = 1.0$ mixture shows an NTC

shows an NTC region of ignition delay at 15 bar from approximately 720 K to 780 K, with measured first-stage ignition delays between 720 K and 750 K.

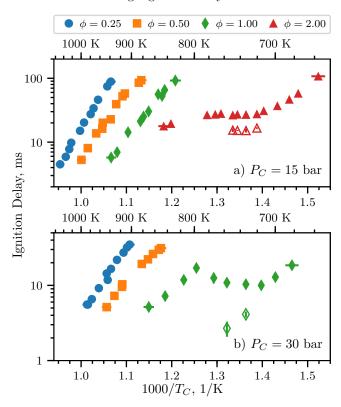


Figure 3: Ignition delays of MV as a function of inverse temperature. Filled points are the overall ignition delays and hollow points are the first stage ignition delays. a) 15 bar. b) 30 bar

Figure 4a shows the pressure traces for selected experiments at $\phi = 1.0$, $P_C = 30$ bar. The three reactive pressure traces shown are at the low-temperature end of the NTC (blue, 700 K), one case with two-stage ignition (orange, 734 K), and one case near the high-temperature limit of the NTC region (green, 775 K). Also shown is the non-reactive pressure trace for the 700 K case (red). By comparing the 700 K pressure trace with the non-reactive pressure trace, it can be seen that there is substantial heat release prior to main ignition as measured by the deviation of the reactive pressure trace from the non-reactive trace. However, there is only one peak in the time derivative of the pressure, so no first-stage ignition delay is defined for this case. It can also be seen in Fig. 4 that the 775 K

case shows some heat release prior to ignition, although again there is only one peak in the time derivative of the pressure. Furthermore, the heat release at 775 K appears to be more gradual than at the lowest temperature.

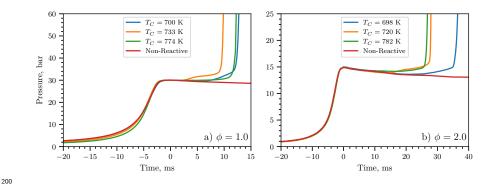


Figure 4: Selected pressure traces around the NTC region of ignition delay. a) $\phi=1.0$ b) $\phi=2.0$

5. Computational Results

?? compares experimentally measured overall ignition delays with ignition delays computed with the detailed model of [15] for the $\phi=1.0$ experiments. Results for the other equivalence ratios are similar to these results, so are not shown here. It is important to note that the model of [15] was not validated for MV ignition delays, only for extinction strain rates. At 15 bar, the model tends to under-predict the ignition delay and predicts an NTC region that is not present in the experiments. At 30 bar, the model predicts the low-temperature ignition delays well, but does not predict the NTC region found experimentally. To understand the underlying reasons for the disagreement between the [15] model and the data, we constructed an additional model using RMG (see Section 3.2). As can be seen in ??, the agreement between the RMG model and the experimental data is similar to the [15] model for the 30 bar data. At 15 bar, the RMG model predicts a somewhat longer ignition delay than the model of [15], but still predicts an NTC region where none is present in the experimental data.

In general, there could be three likely sources of error in the models: missing reaction pathways, incorrect values of the reaction rates, and incorrect values for thermodynamic properties of the species. We have noted in Section 3.2 that the RMG model has many more reactions than the [15] model and the algorithm used in RMG considers a substantial number of the possible pathways. This reduces the possibility of missing reaction pathways affecting the model. Further detailed studies are required to ensure that the RMG model includes all of the relevant reaction pathways.

The second source of error may be incorrect reaction rate parameters, either 226 because the rates are specified incorrectly in the model (e.g., typos) or because 227 the rates are not well estimated by the typical analogy based-rules. It should 228 be noted that errors of this type may affect the model generated by RMG—if 229 the rates are not estimated correctly, reactions that are important in reality may not be included in the model. Determining the accuracy of the reaction 231 rates used in the RMG and [15] models requires further detailed studies of the 232 models. Another related source of error could be incorrect estimation of the 233 pressure dependence of the reaction rates, which may be particularly important 234 for the isomerization reactions prevalent in low-temperature chemistry. 235

The third source of error may lie in the estimation of the thermodynamic 236 properties of the species, particularly their heats of formation. We have begun to 237 analyze the possibility of this source of error by conducting a reaction pathway 238 analysis to determine which radicals are formed from the breakdown of the fuel. The following analysis is conducted for a constant volume simulation at 700 K, 240 30 bar, where the rates of production of the species have been integrated until 241 the time of 20% fuel consumption. The results of this analysis are shown in 242 Fig. 5 and Table 3 for the two models. The percentages shown in the Table 3 243 are the percent of the fuel destroyed to form a particular fuel radical by all the reactions that can form that radical.

At the relatively low temperature and high pressure condition of this analysis, all of the fuel is destroyed by H-atom abstractions to form the fuel radicals shown. It can be seen that the two models have quite different distributions of

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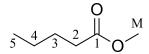


Table 3: Percent of MV destroyed to form fuel radical species with a hydrogen atom missing at the location indicated in the first column

Figure 5: Structure of MV with carbon atoms labeled according to the convention used in Table 3

Radical Site	[15] [%]	RMG Model [%]
2	29.3	7.4
3	17.5	36.0
4	17.5	41.1
5	9.4	3.7
M	26.3	11.8

products from the first H-abstraction reactions. The model of [15] predicts that
H-abstraction from the second carbon is the most prevalent, followed closely by
abstraction from the methyl group. This is in line with the bond energies of the
C-H bonds for those carbon atoms; we expect that the presence of the oxygen atoms will cause hydrogen abstraction at the nearby carbons to be favored.
However, the RMG model predicts that radicals in the middle of the carbon
chain will be primarily formed. The cause of this discrepancy is under investigation, but it may be caused by the estimation of thermodynamic properties of
the radicals.

6. Conclusions

In this study, we have measured ignition delays for methyl valerate over a 259 wide range of engine-relevant pressures, temperatures, and equivalence ratios. 260 An NTC region of the ignition delay and two-stage ignition were recorded for 261 pressures of 15 bar at $\phi = 2.0$ and 30 bar at $\phi = 1.0$. A detailed chemical ki-262 netic model available in the literature was unable to reproduce the experimental 263 results, so a new model was constructed using the Reaction Mechanism Generator software. Although the new model contains many more reactions than 265 the literature model, it is still unable to predict the experimental ignition de-266 lays satisfactorily. Possible reasons for the discrepancy include missing reaction

- 268 pathways, incorrect rate estimates, and incorrect thermodynamic property esti-
- 269 mates. Future work will include investigation of the discrepancies between mod-
- els and experiments to further understand the autoignition kinetics of methyl
- valerate.

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