EKF Example 1

State variable represents location in cartesian coordinates.

$$x = \begin{bmatrix} x \\ y \end{bmatrix}$$

Measurements are taken in polar coordinates.

$$z = \begin{bmatrix} r \\ \theta \end{bmatrix}$$

Measurement function maps the state to the measurements.

$$h(x) = \begin{bmatrix} \sqrt{x^2 + y^2} \\ \tan^{-1} \frac{y}{x} \end{bmatrix}$$

Residual equation:

$$y = z - h(x)$$

Jacobian/linearization of measurement function:

$$\begin{aligned} H &= \begin{bmatrix} \frac{\partial r}{\partial x} & \frac{\partial r}{\partial y} \\ \frac{\partial \theta}{\partial x} & \frac{\partial \theta}{\partial y} \end{bmatrix} \\ &= \begin{bmatrix} \frac{x}{\sqrt{x^2+y^2}} & \frac{y}{\sqrt{x^2+y^2}} \\ -\frac{y}{x^2+y^2} & \frac{x}{x^2+y^2} \end{bmatrix} \end{aligned}$$

EKF Example 2: Quadrotor

State variable represents roll angle, y velocity, and y position of a quadrotor.

$$x = \begin{bmatrix} \phi \\ \dot{y} \\ y \end{bmatrix}$$

Measurements to wall are taken using a range finder. Measurement function for the range finder:

$$h(x) = \begin{bmatrix} \frac{wall - y}{cos\phi} \end{bmatrix}$$

Jacobian/linearization of measurement function:

$$\begin{aligned} H &= \begin{bmatrix} \frac{\partial h}{\partial \phi} & \frac{\partial h}{\partial \dot{y}} & \frac{\partial h}{\partial y} \end{bmatrix} \\ &= \begin{bmatrix} \frac{sin\phi}{cos^2\phi}(wall - y) & 0 & -\frac{1}{cos\phi} \end{bmatrix} \end{aligned}$$