Multivariate Gaussians

1. You are tracking the position and velocity of a robot in two dimensions, x and y. The state is represented as so,

$$x = \begin{bmatrix} p_x \\ p_y \\ v_x \\ v_y \end{bmatrix}$$

Find the state update function, F, that will advance the state from time t to time $t+1$ based on the state transition equation below.

$$x' = Fx$$

$$F = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. You are tracking the position, velocity, and acceleration of a quadrotor in the vertical dimension, z. The state of the quadrotor can be represented as so,

$$x = \begin{bmatrix} z \\ \dot{z} \\ \ddot{z} \end{bmatrix}$$

Find the state update function, F, that will advance the state from time t to time $t+1$ based on the state transition equation below.

$$x' = Fx$$

$$F = \begin{bmatrix} 1 & \Delta t & \frac{1}{2}(\Delta t)^2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix}$$

EKF Example

1. State variable represents roll angle, y velocity, and y position of a quadrotor.

$$x = \begin{bmatrix} \phi \\ \dot{y} \\ y \end{bmatrix}$$

Assuming no roll, what is the measurement function of the range finder?

$$h(x) = wall - y$$

2. More generally, what is the measurement function for the range finder:

$$h(x) = \begin{bmatrix} \frac{wall-y}{cos\phi} \end{bmatrix}$$

3. What is the Jacobian/linearization of the measurement function?

$$\begin{aligned} H &= \begin{bmatrix} \frac{\partial h}{\partial \phi} & \frac{\partial h}{\partial \dot{y}} & \frac{\partial h}{\partial y} \end{bmatrix} \\ &= \begin{bmatrix} \frac{sin\phi}{cos^2\phi}(wall - y) & 0 & -\frac{1}{cos\phi} \end{bmatrix} \end{aligned}$$