GraphSLAM

Maximum Likelihood Estimation

x_0 is the robot's initial position
z_1 is the robot's distance from a landmark m_1
Gaussian probablility density function of location given a single measurement of 1.8 m:

$$p(z_1) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\frac{(z_1 - (x_0 + 1.8))^2}{\sigma^2}\right)$$

Since $x_0 = 0$:

$$p(z_1) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\frac{(z_1 - 1.8)^2}{\sigma^2}\right)$$

Adding another measurement of 2.2 m:

$$p(z_1) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\frac{(z_1 - 1.8)^2}{\sigma^2}\right) * \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\frac{(z_1 - 2.2)^2}{\sigma^2}\right)$$

Remove scaling factors:

$$= \exp\left(-\frac{1}{2}\frac{(z_1 - 1.8)^2}{\sigma^2}\right) * \exp\left(-\frac{1}{2}\frac{(z_1 - 2.2)^2}{\sigma^2}\right)$$

Simplify:

$$= \exp\left(-\frac{1}{2}\frac{(z_1 - 1.8)^2}{\sigma^2} - \frac{1}{2}\frac{(z_1 - 2.2)^2}{\sigma^2}\right)$$

Take natural log; log-likelihood:

$$= -\frac{1}{2}\frac{(z_1 - 1.8)^2}{\sigma^2} - \frac{1}{2}\frac{(z_1 - 2.2)^2}{\sigma^2}$$

Note that the funtion now returns a negative value for the probability, which is between 0 and 1.
Remove scaling factors:

$$= (z_1 - 1.8)^2 + (z_1 - 2.2)^2$$

Simplify:

$$= 2z_1^2 - 8z_1 + 8.08$$

Find the value that minimizes the function:

$$\frac{\partial}{\partial z_1}(2z_1^2 - 8z_1 + 8.08) = 4z_1 - 8 = 0 \therefore z_1 = 2$$

MLE Example

The robot starts at $x_0 = 0$. It takes a measurement to landmark z_1 of 7 m. It then moves 10 m. In its new location, it measures 4 m to landmark z_1. The resulting sum of constraints:

$$\begin{aligned} J_{graphSLAM} &= \frac{(z_1 - (x_0 + 7))^2}{\sigma^2} + \frac{(z_1 - (x_1 - 4))^2}{\sigma^2} + \frac{(x_1 - (x_0 + 10))^2}{\sigma^2} \\ &= \frac{(z_1 - 7)^2}{\sigma^2} + \frac{(z_1 - (x_1 - 4))^2}{\sigma^2} + \frac{(x_1 - 10)^2}{\sigma^2} \end{aligned}$$

Find values that optimize the function:

$$
\begin{aligned}
\frac{\partial}{\partial x_1} J_{graphSLAM} &= \frac{2}{\sigma^2}(z_1 - x_1 + 4)(-1) + \frac{2}{\sigma^2}(x_1 - 10) = 0 \\
2x_1 - z_1 - 14 &= 0 \\
\frac{\partial}{\partial z_1} J_{graphSLAM} &= \frac{2}{\sigma^2}(z_1 - 7) + \frac{2}{\sigma^2}(z_1 - (x_1 - 4)) = 0 \\
2z_1 - x_1 - 3 &= 0 \\
x_1 &= \frac{31}{3} = 10.33 \\
z_1 &= \frac{20}{3} = 6.67
\end{aligned}
$$

Now, the motion variance is 0.02 and the measurement variance is 0.1.

$$
\begin{aligned}
\frac{\partial}{\partial x_1} J_{graphSLAM} &= \frac{2}{0.1}(z_1 - x_1 + 4)(-1) + \frac{2}{0.02}(x_1 - 10) = 0 \\
60x_1 - 10z_1 - 540 &= 0 \\
6x_1 - z_1 - 54 &= 0 \\
\frac{\partial}{\partial z_1} J_{graphSLAM} &= \frac{2}{(0.1)^2}(z_1 - 7) + \frac{2}{(0.1)^2}(z_1 - (x_1 - 4)) = 0 \\
2z_1 - x_1 - 3 &= 0 \\
x_1 &= 10.09 \\
z_1 &= 6.55
\end{aligned}
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