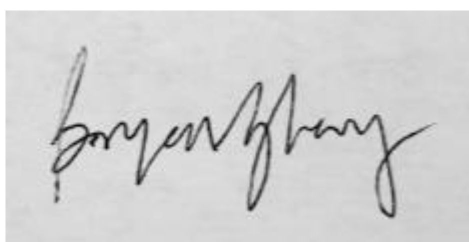
The University of British Columbia

Mini Project 1

ELEC 301 – Electronic Circuits



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10-6-2022

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Part I

A.

The three-pole low-pass filter is to have a transfer function:

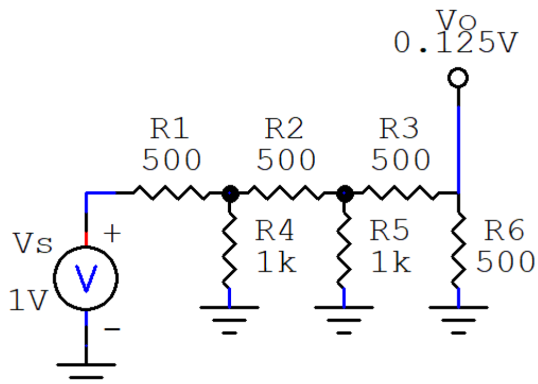
$$T(s) = \frac{V_o(s)}{V_s(s)} = 0.125 \cdot \frac{5\times10^5}{s+5\times10^5} \cdot \frac{5\times10^6}{s+5\times10^6} \cdot \frac{5\times10^7}{s+5\times10^7}$$

Using open circuit time constants, we can derive the equations for values of the capacitors:

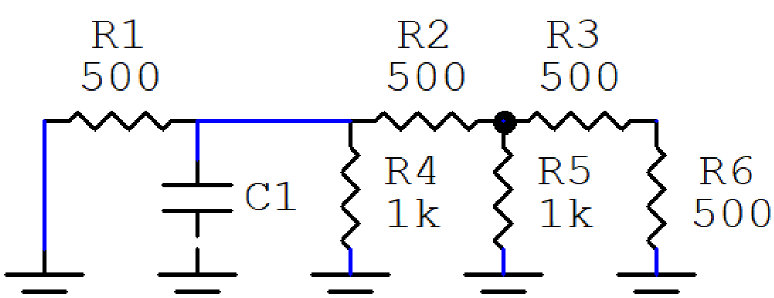
$$\omega_{HP_1} = \frac{1}{C_1 R_{1o}} = 5\times10^5, \omega_{HP_2} = \frac{1}{C_2 R_{2o}} = 5\times10^6, \omega_{HP_3} = \frac{1}{C_3 R_{3o}} = 5\times10^7$$

The conditions of this circuit are $C_1 > C_2 > C_3$. This means that when we are finding the equivalent resistance seen by a capacitor, we *short* any larger capacitors and open smaller ones. There are two $1k\Omega$ and four 500Ω resistors.

The midband gain for this circuit is $\frac{1}{2^3}$ and at the midband high frequency capacitors become open. So, at each of the 3 nodes there must be a voltage reduction of ½. The resistance values can then be calculated using voltage division.



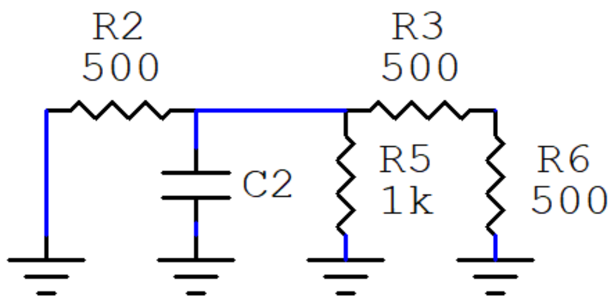
Now the capacitances can be calculated using open circuit time constants. After zeroing the independent sources and opening all other high frequency capacitors, this is the resulting circuit seen by C_1.



$$\omega_{HP_1} = \frac{1}{C_1[(R_4 \,||\, (R_2 + R_5 \,||\, (R_3 + R_6))||R_1]} = 5 \times 10^5$$

$$\omega_{HP_1} = \frac{1}{C_1 \cdot 250} = 5 \times 10^5 \xrightarrow{yields} \underline{C_1 = 8nF}$$

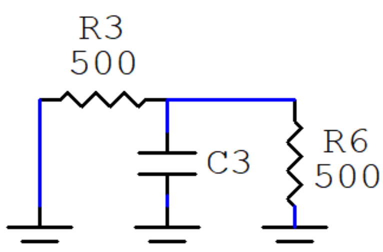
When finding the equivalent circuit seen by C_2 we consider C_1 to be shorted and C_3 to be open.



$$\omega_{HP_2} = \frac{1}{C_2[(R_5 \,||\, (R_3 + R_6)) \,||R_2]} = 5 \times 10^6$$

$$\omega_{HP_1} = \frac{1}{C_2 \cdot 250} = 5 \times 10^6 \xrightarrow{yields} \underline{C_2 = 0.8nF}$$

When finding the equivalent circuit seen by C_3 we consider $C_1$and C_2 to be shorted.

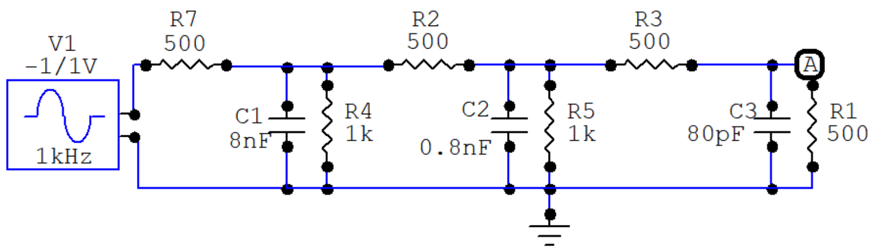


$$\omega_{HP_3} = \frac{1}{C_3[R_3||R_6]} = 5 \times 10^6$$

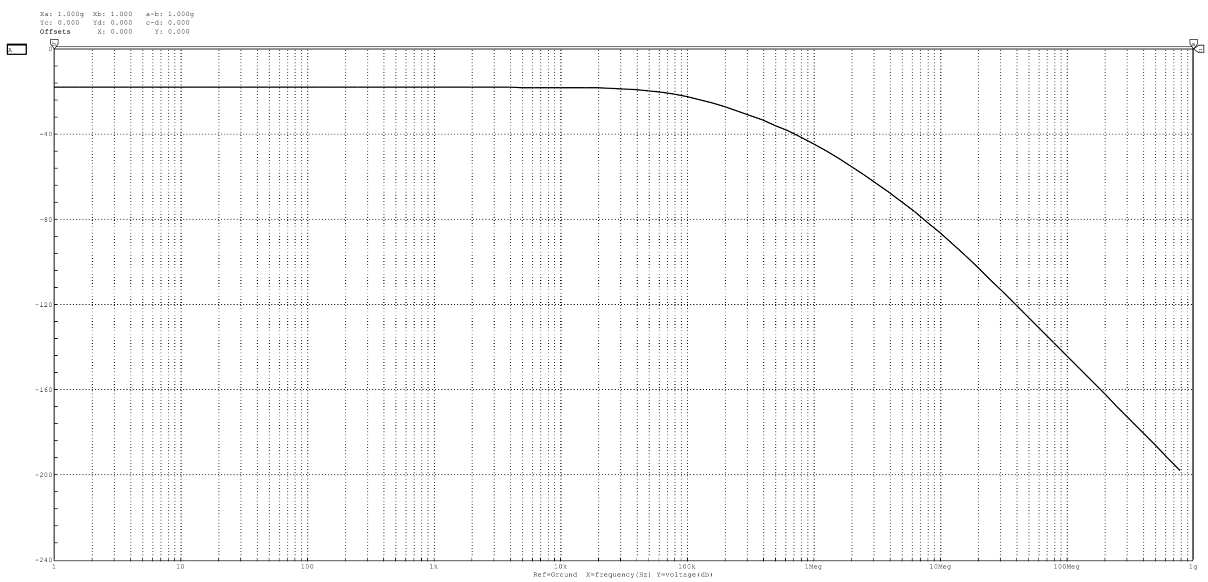
$$\omega_{HP_1} = \frac{1}{C_3 \cdot 250} = 5 \times 10^7 \xrightarrow{yields} \underline{C_3 = 80pF}$$

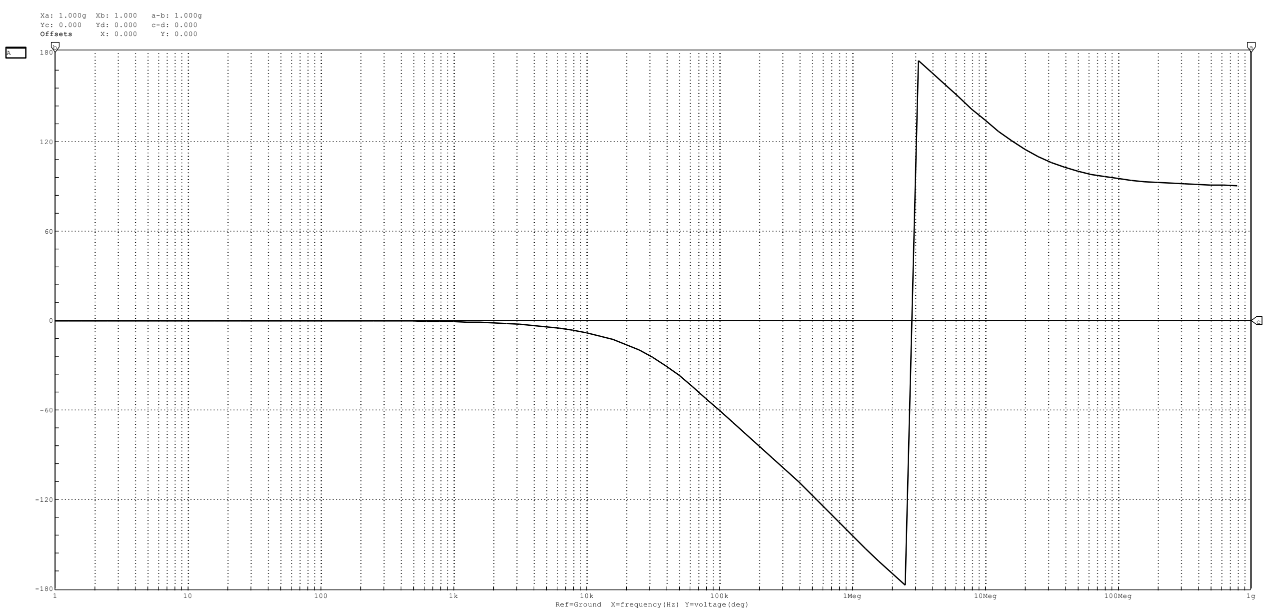
AC simulation

This is the resulting circuit to be simulated.



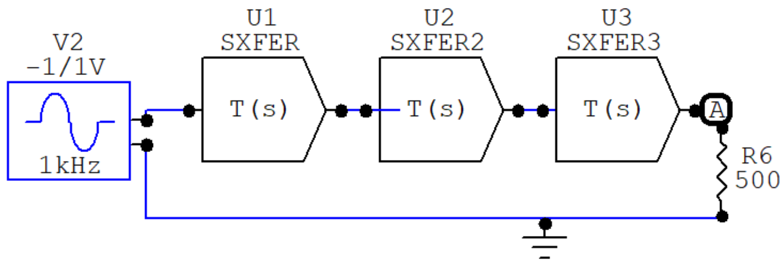
The first plot shows the magnitude bode plot and the second plot shows the phase bode plot of the circuit (both plots simulated from 1 to 1GHz).

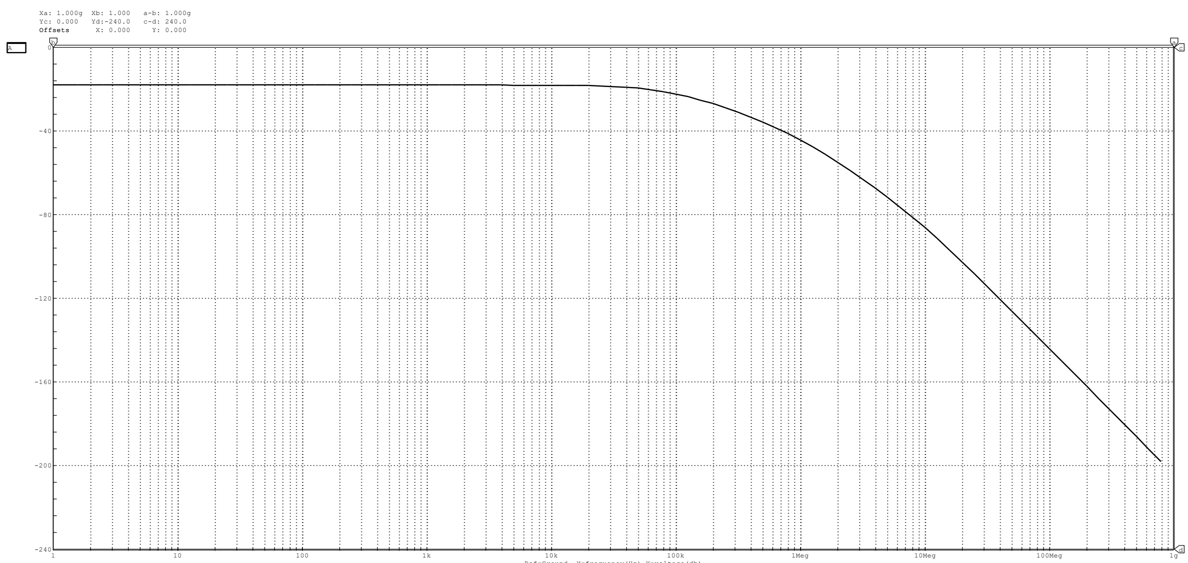


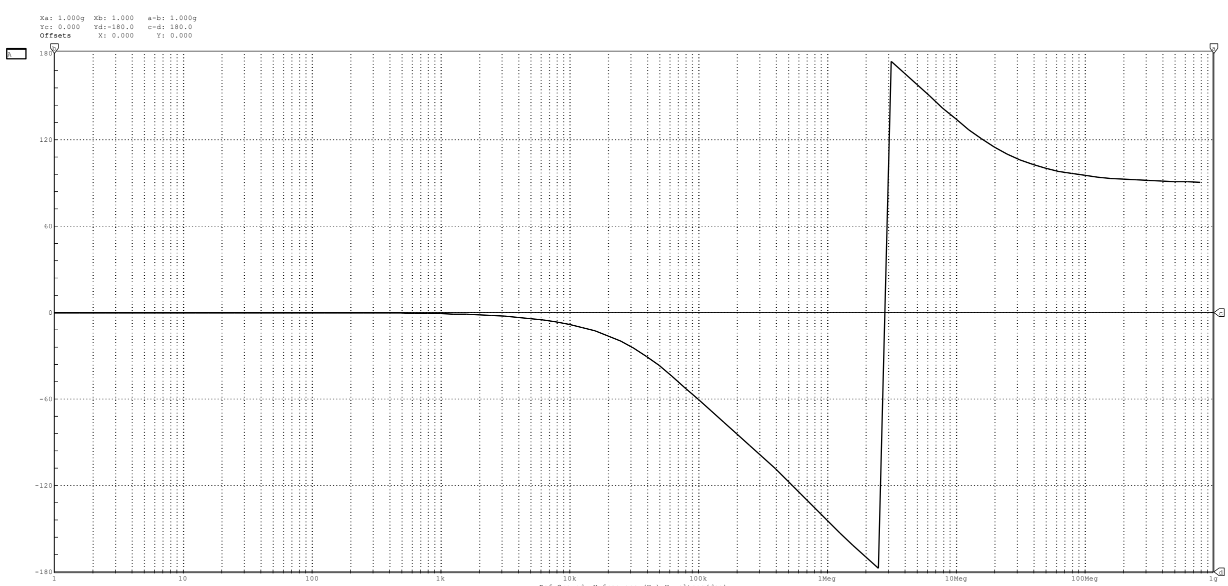


B.

To check the result, we simulate a circuit with the exact desired transfer function. The first plot shows the magnitude bode plot and the second plot shows the phase bode plot both from 1Hz – 1GHz. We see that these plots are like the ones from part A.



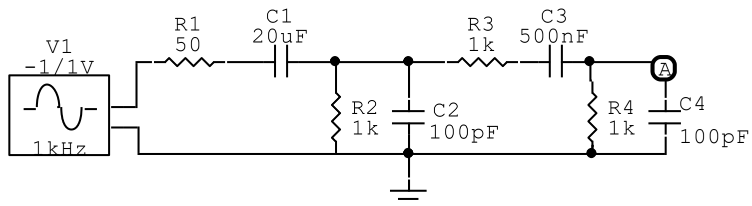


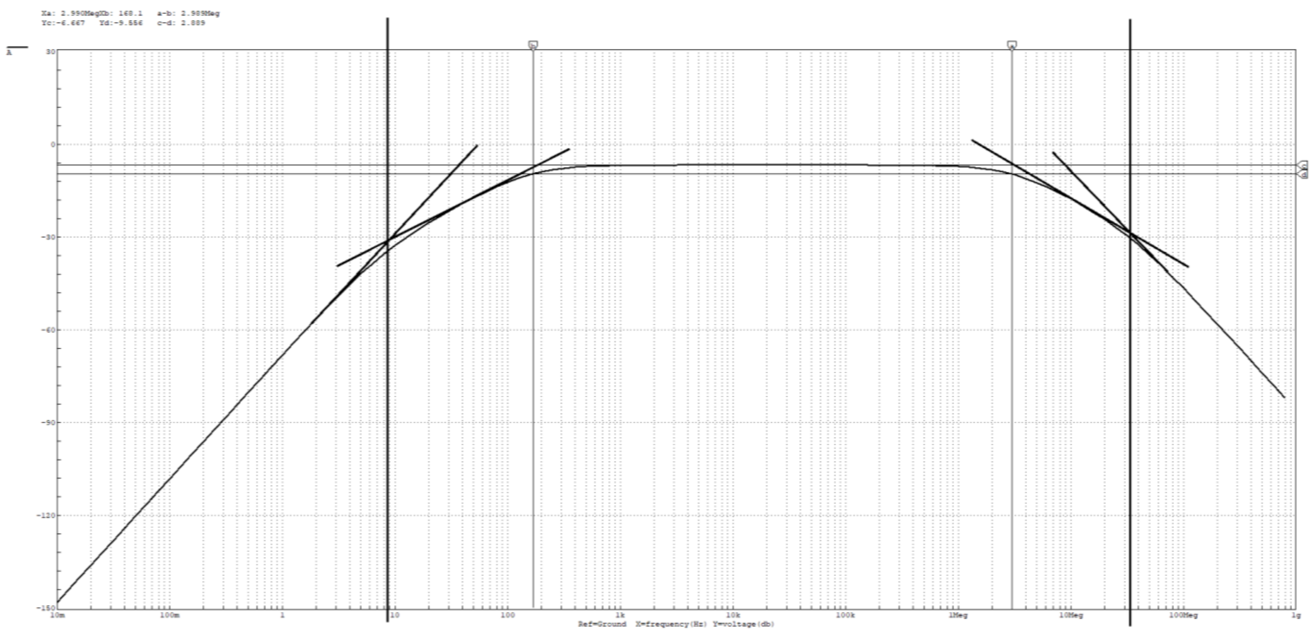


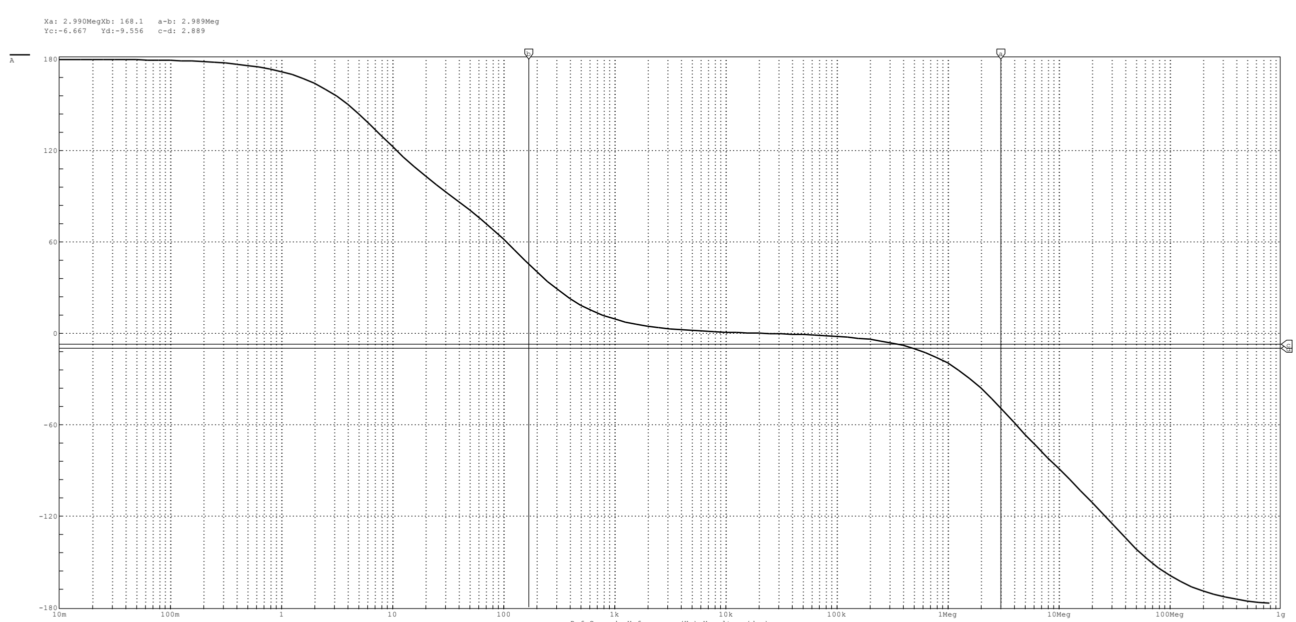
Part II

A.

Here is the simulated circuit along with its magnitude and phase bode plots from 10mHz to 1GHz.





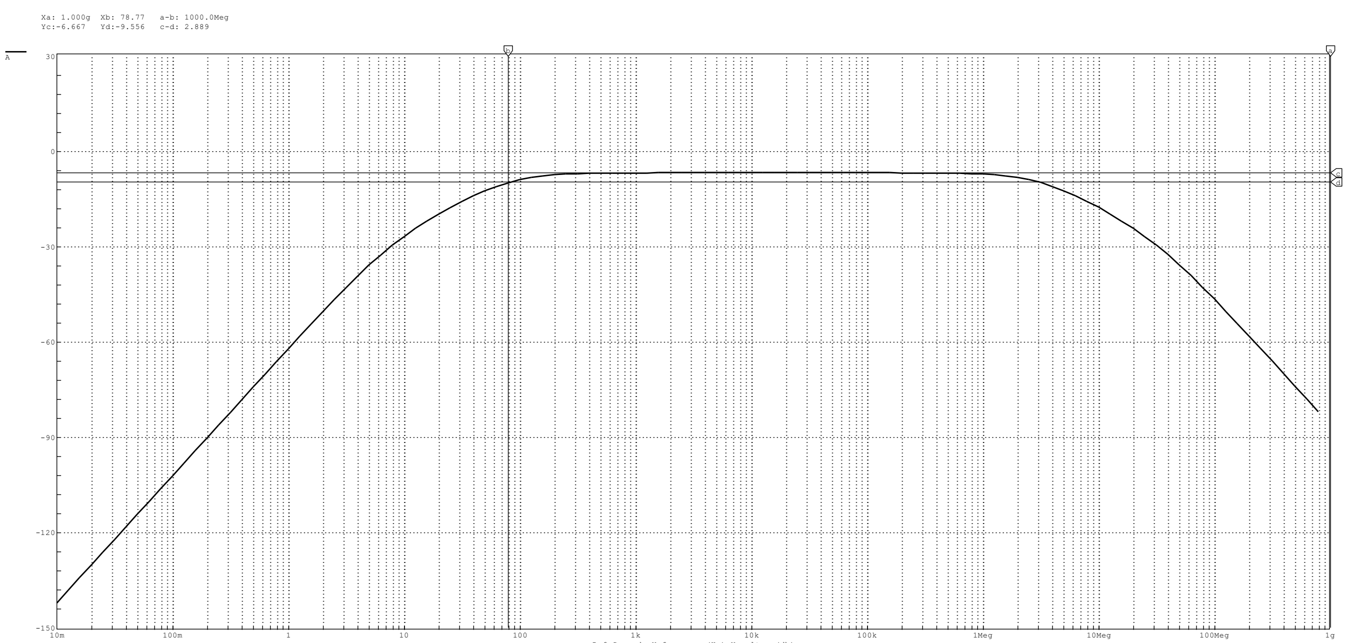


f_{LP_1} may be graphically identified by observing when the slope changes from 40dB/decade to 20dB/decade, which is at the point where the 40dB/decade tangent intercepts the 20dB/decade tangent. We can find the other poles using a similar method from 20dB/decade to 0dB/decade to -20dB/decade to -40dB/decade.

Pole	Frequency
f_{LP_1}	8.750 Hz
f_{LP_2}	175.3 Hz
f_{HP_1}	2.989 MHz
f_{HP_2}	33.50 MHz

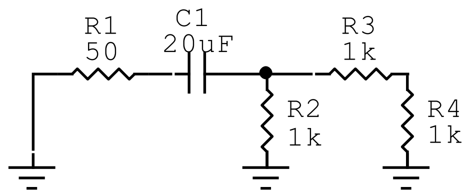
B.

To find the low-frequency 3-dB point from simulation, we can use the method from part A or simply use the cursors in the bode plot. The 'c' and 'd' cursors are horizontal. The 'c' cursor is set to the height of the midband, the 'd' cursor is set to 3dB below the 'c' cursor. The 'a' cursor is vertical, it is set to the point where the 'd' cursor intersects the bode plot. The x value of the 'a' cursor is then identified as the low-frequency 3-dB point. For example (plotted below) when $C_3 = 1\mu F$, we see $\omega_{L3dB} = 78.77Hz$.



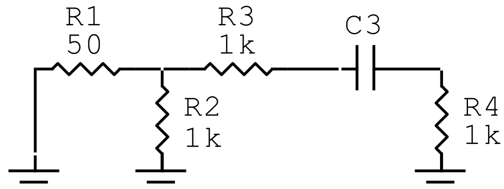
This process may be repeated for all values of C_3 to graphically identify ω_{L3dB}

Now to estimate the low-frequency 3-dB points using the OC and SC time constants. Starting with finding $\tau_{C_1}^{sc}$. After zeroing the independent sources, opening all high frequency capacitors, and shorting C_3 we are left with the following circuit. $\tau_{C_1}^{sc}$ remains constant when C_3 is changed.



$$\tau_{C_1}^{sc} = C_1[R_1 + R_2||(R_3 + R_4)] = \underline{1.433 \times 10^{-2}\ Hz}$$

To find $\tau_{C_3}^{sc}$ we zero all independent sources, open all high frequency capacitors, and short C_1. The following circuit results. $\tau_{C_3}^{sc}$ will change whenever C_3 changes.



$$\tau_{C_3}^{sc} = C_3[R_4 + R_1||R_2 + R_3]$$

The low-frequency 3-dB point can be then calculated as

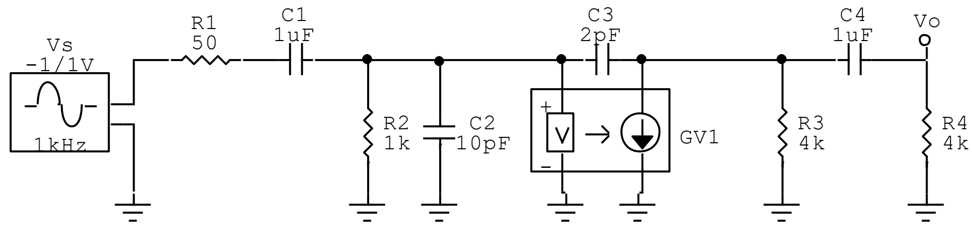
$$\boxed{f_{L3dB} = \frac{1}{2\pi}\left(\frac{1}{\tau_{C_1}^{sc}} + \frac{1}{\tau_{C_3}^{sc}}\right)}$$

C_3	$500nF$	$1\mu F$	$2\mu F$	$5\mu F$	$10\mu F$
f_{L3dB} (simulated)	158.9 Hz	78.77 Hz	43.68 Hz	23.55 Hz	15.67 Hz
f_{L3dB} (calculated)	166.6 Hz	88.83 Hz	49.97 Hz	26.65 Hz	18.88 Hz
Percentage error	4.85%	12.8%	14.4%	13.1%	20.4%

Part III

A.

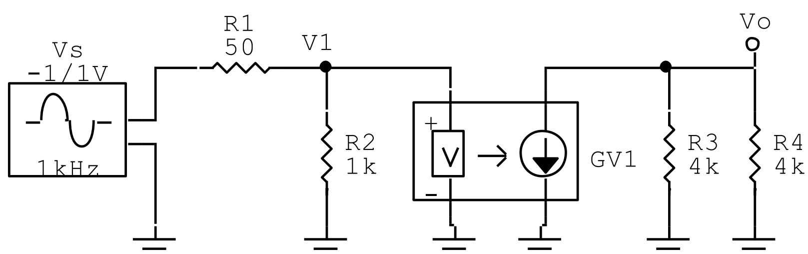
Using Miller's theorem and the method of OC and SC time constants, the mid band gain and location of all the poles of the circuit below will be calculated. ($G = 0.1\mho$)



The first step is to simplify the circuit by applying Miller's theorem to C_3.

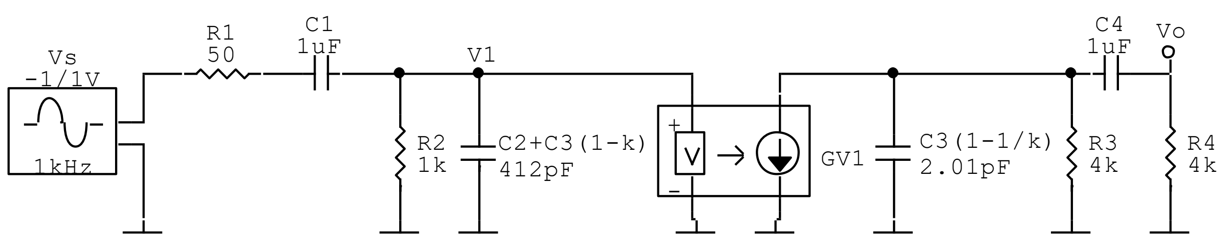
$$k = \frac{V_o}{V_1} = \frac{-GV_1 \cdot (R_3||R_4)}{V_1} = -200$$

To find the midband gain the circuit is analyzed at the midband by shorting all low frequency capacitors and opening all high frequency capacitors, yielding the following circuit. $\frac{V_o}{V_1}$ is simply k and the ratio of $\frac{V_1}{V_s}$ can be determined using voltage division.

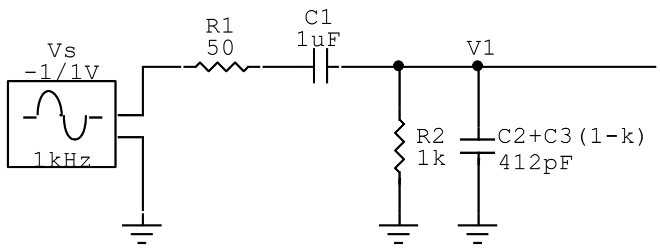


$$A_m = \frac{V_o}{V_1} \cdot \frac{V_1}{V_s} = k\frac{R_2}{R_1 + R_2} = -190.5$$

Here is the circuit after being simplified through applying Miller's theorem.



Now the poles of the circuit can be determined by analyzing the first and second stages separately. Starting with the first stage:



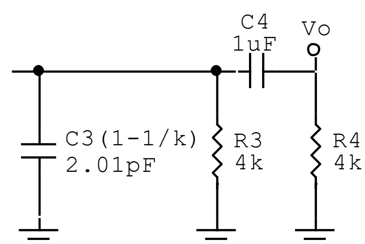
To find ω_{LP_1} the independent sources are zeroed, and the high frequency capacitors are opened.

$$\tau_{C_1}^{oc} = (R_1 + R_2)C_1 = 1.05 \times 10^{-3}s \xrightarrow{yields} \omega_{LP_1} = \frac{1}{\tau_{C_1}^{oc}} = \underline{952.3\, {}^{rad}/_{s}}$$

To find ω_{HP_2} the independent sources are zeroed, and the low frequency capacitors are shorted.

$$\tau_{C_2}^{sc} = (R_1||R_2)\big(C_2 + C_3(1-k)\big) = 1.96 \times 10^{-8}s \xrightarrow{yields} \omega_{HP_1} = \frac{1}{\tau_{C_2}^{sc}} = \underline{51.0M\, {}^{rad}/_{s}}$$

Repeating the same process with the second stage of the circuit:



$$\tau_{C_4}^{oc} = (R_3 + R_4)C_4 = 8.00 \times 10^{-3}s \xrightarrow{yields} \omega_{LP_2} = \frac{1}{\tau_{C_4}^{oc}} = \underline{125\, {}^{rad}/_{s}}$$

$$\tau_{C_3}^{sc} = (R_3||R_4)\left(C_3\left(1 - \frac{1}{k}\right)\right) = 4.02 \times 10^{-9}s \xrightarrow{yields} \omega_{HP_2} = \frac{1}{\tau_{C_3}^{sc}} = \underline{248.8M\, {}^{rad}/_{s}}$$

Pole	Frequency
$f_{LP_1} = \frac{1}{2\pi}\omega_{LP_1}$	151.6 Hz
$f_{LP_2} = \frac{1}{2\pi}\omega_{LP_2}$	19.89 Hz
$f_{HP_1} = \frac{1}{2\pi}\omega_{HP_1}$	8.112 MHz
$f_{HP_2} = \frac{1}{2\pi}\omega_{HP_2}$	39.59 MHz

Midband gain: $A_m = -190.5$

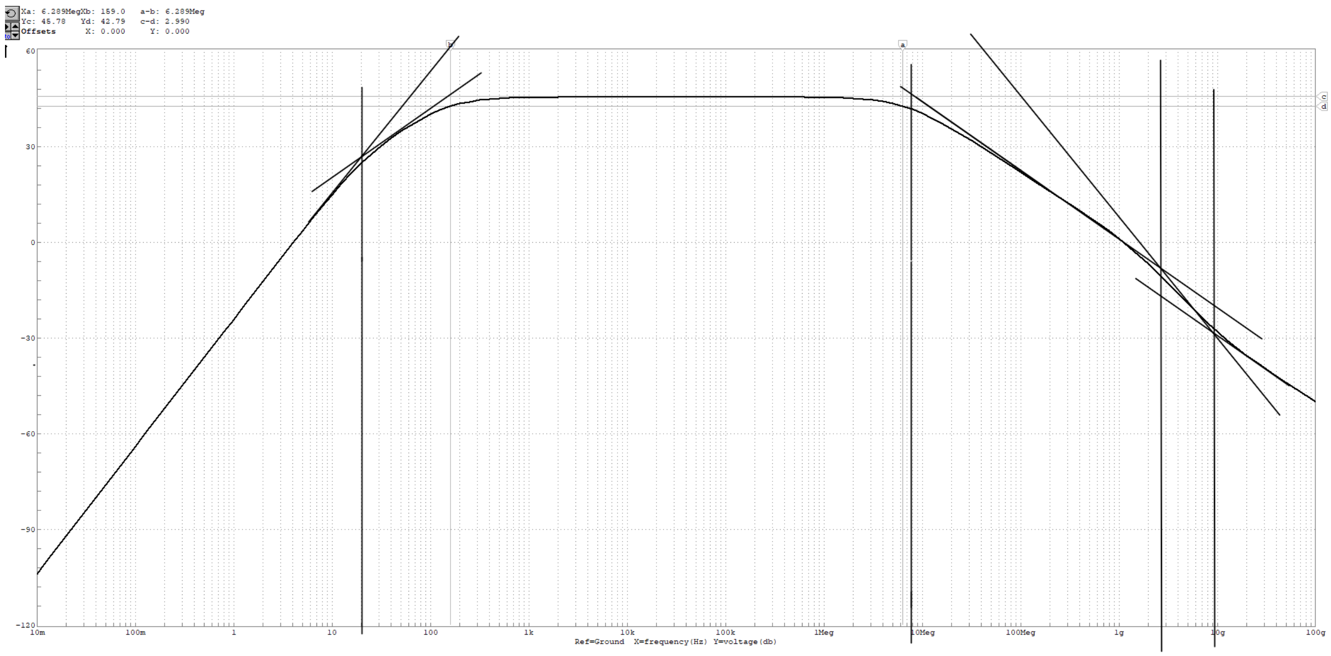
B.

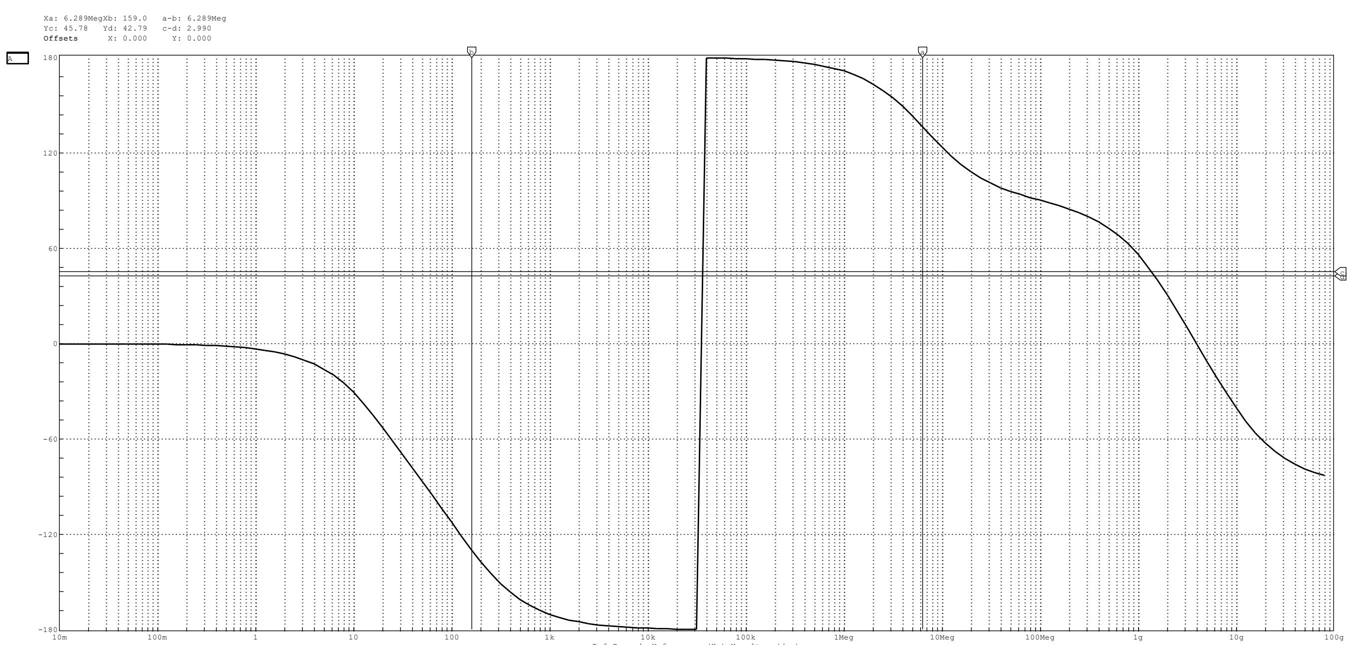
Location of the 3dB points may be calculated from the frequencies found in part A.

$$f_{L3dB} = \frac{1}{2\pi}\sqrt{\omega_{LP_1}^2 + \omega_{LP_2}^2} = \underline{152.9\ Hz}$$

$$\tau_{H3dB} = \sqrt{\tau_{HP_1}^2 + \tau_{HP_2}^2} = 2 \times 10^{-8} s \xrightarrow{yields} f_{H3dB} = \frac{1}{2\pi} \cdot \frac{1}{\tau_{H3dB}} = \underline{7.955\ MHz}$$

The original circuit is simulated from 10 mHz to 1 GHz for the magnitude and phase bode plots.





Pole/Zero (simulated)	Graphically determined frequency
f_{LP_1}	20 Hz
f_{LP_2}	175 Hz
f_{HP_1}	7.9 MHz
f_{HP_2}	2.9 GHz
f_{zero}	9.1 GHz

The only value that is significantly off from the calculated value is f_{HP_2}. The cursor method is used to determine the simulated 3dB points

3dB point	Calculated value	Simulated value	Percent error
f_{L3dB}	152.9 Hz	159.0 Hz	3.84%
f_{H3dB}	7.955 MHz	6.289 MHz	26.5%

We see that although the second high frequency pole is very off, our calculated 3dB points are like the simulated ones. The reason that the second high frequency pole is off is because when applying miller's theorem, it causes there to be a zero at zero instead of where it was in the simulation.

References

1. ELEC 301 Course Notes
2. A. Sedra and K. Smith, "Microelectronic Circuits,"5 th (or higher) Ed., Oxford University Press, New York
3. CircuitMaker™ User's Manual