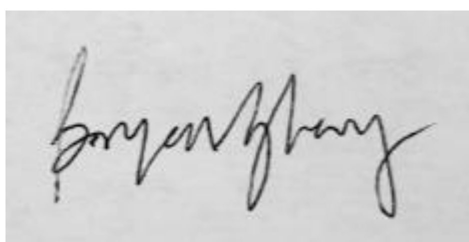
The University of British Columbia

Mini Project 4

ELEC 301 – Electronic Circuits



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Contents

Part A – An Active Filter

1. Component Calculation

   To get a 3dB frequency of 10kHz calculate the capacitance value based on the resistor *R* value of 10kΩ.

$$C = \frac{1}{2\pi f_{3dB} \cdot R} = \frac{1}{2\pi \cdot 10kHz \cdot 10k\Omega} = \underline{1.6nF}$$

From the given transfer function $H(s) = A_M \frac{1/(RC)^2}{s^2 + s\frac{3-A_M}{RC} + \frac{1}{(RC)^2}}$ see that the denominator is of the form $s^2 + 2\zeta\omega_n s + \omega_n^2$ so:

$$\zeta = \frac{3 - A_M}{2} \qquad \omega_n = \frac{1}{RC}$$

A second order Butterworth filter is characterized by having the poles 45˚ from the real axis. Therefor the normalized characteristic equation of a 2nd order filter is $s^2 + \sqrt{2}s + 1$

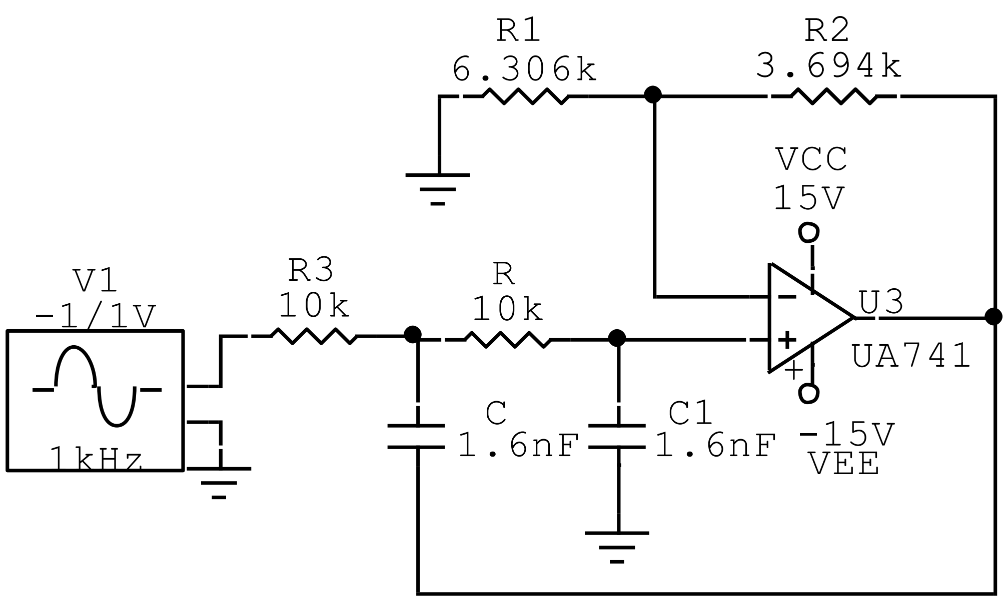
$$\therefore A_M = 3 - \sqrt{2} = \underline{1.568\, {}^{V}/_{V}}$$

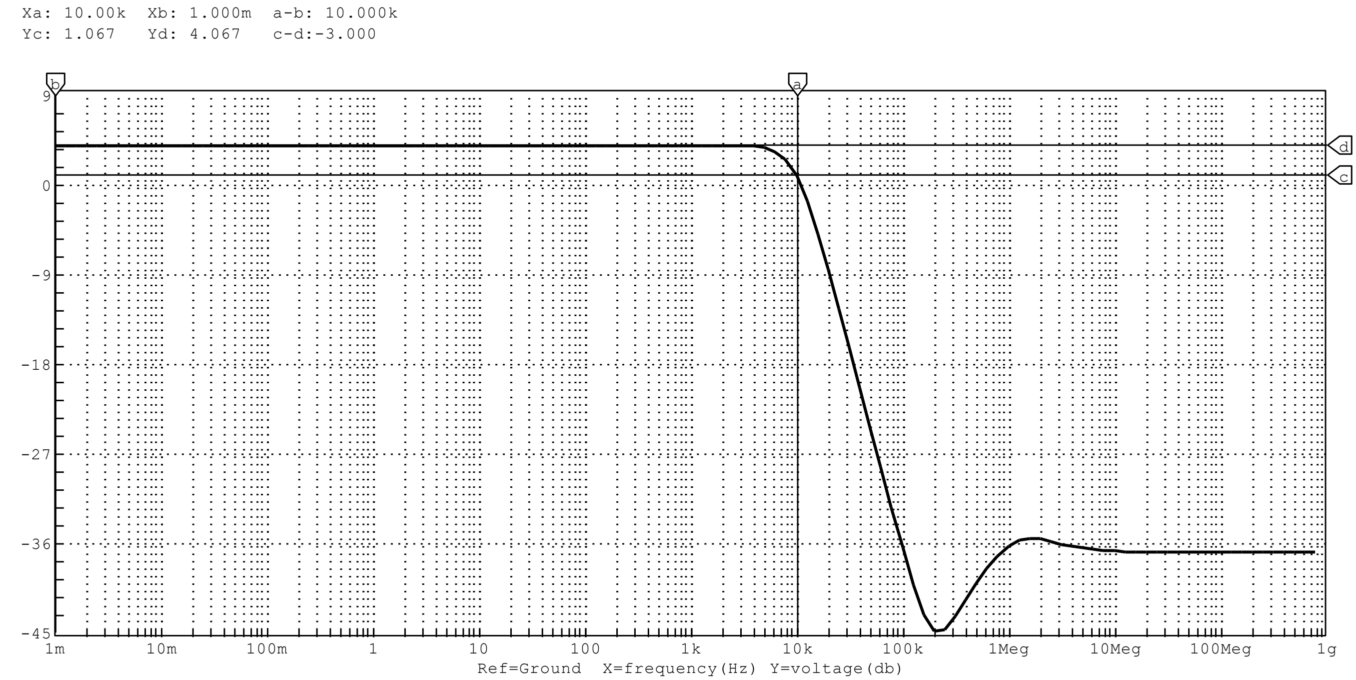
$$A_M = 1 + \frac{R_2}{R_1} \qquad R_1 + R_2 = 10k\Omega$$

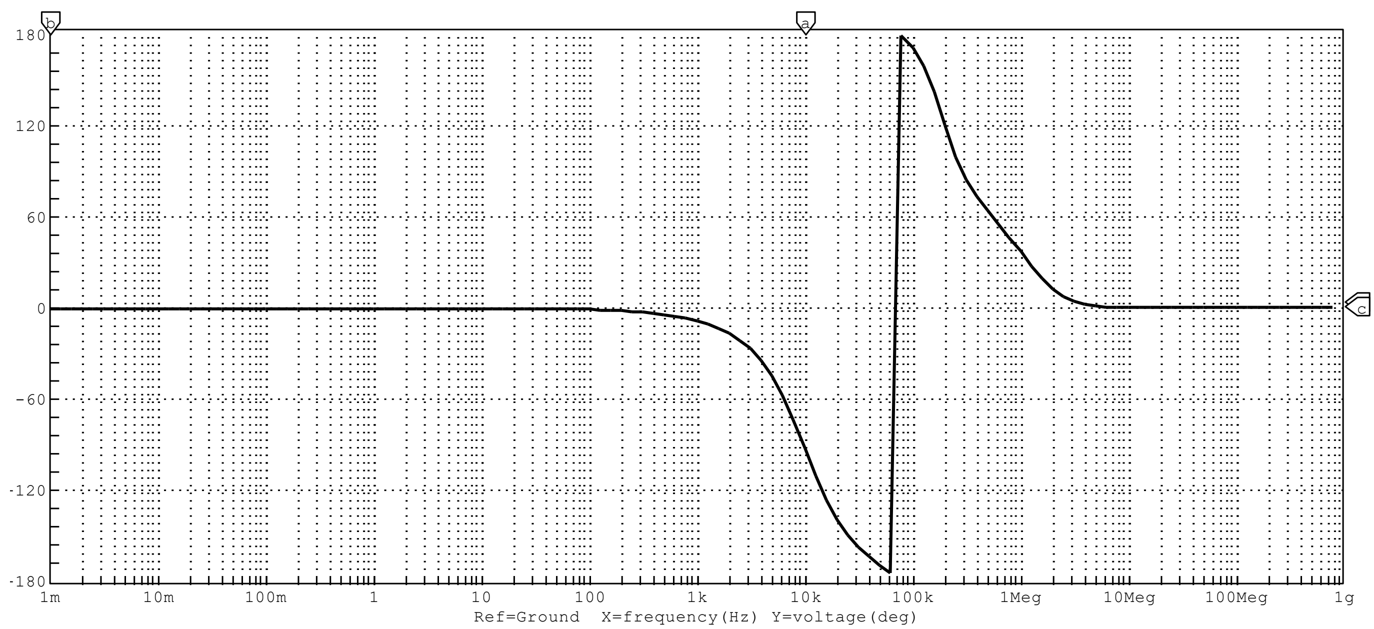
$$\xrightarrow{yields} R_1 = 6.306k\Omega, R_2 = 3.694k\Omega$$

C	A_M	R_1	R_2
1.6nF	1.586 V/V	6.306kΩ	3.694kΩ

Here is the simulated circuit with its magnitude & phase bode plots.





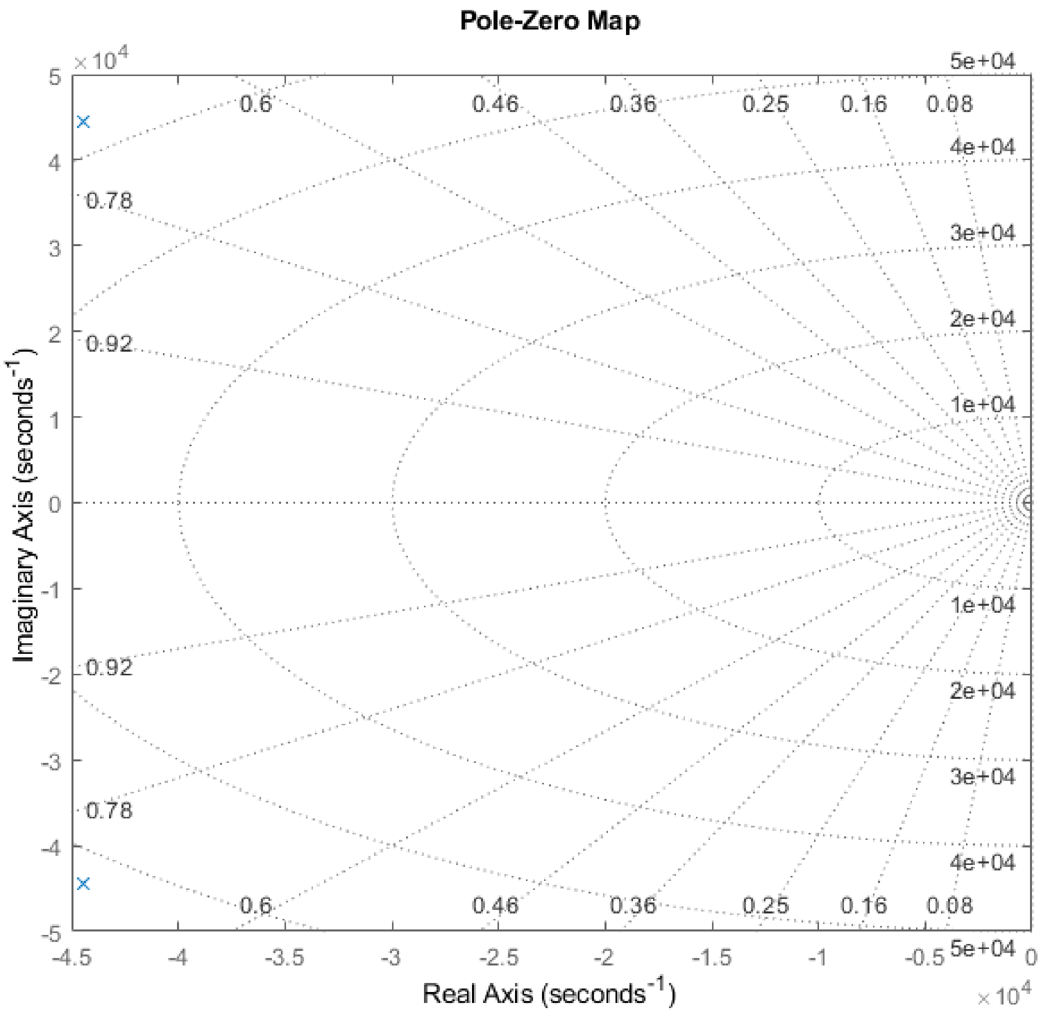


As expected, the cut-off point occurs at 10kHz and the midband gain is

$$4.067dB = 10^{\frac{4.067}{20}} = 1.5972\, {}^{V}/_{V}$$

2. <u>Oscillation midband gain</u>

The poles of the 2nd order Butterworth filter are shown here of the zeros are shown here. They lie on a circle of radius $\omega_c = 2\pi \cdot 10k\, {}^{rad}/_{s}$. The poles complex conjugates located in the left half of the s-plane $\frac{\pi}{4}$ away from the real axis. Therefor the locations of the poles are given as $-\omega_c \cos\frac{\pi}{4} \pm \omega_c \sin\frac{\pi}{4} = \underline{-\pi\sqrt{2} \cdot 10k \pm j\left(\pi\sqrt{2} \cdot 10k\right) = 44.43k \pm j44.43k}.$

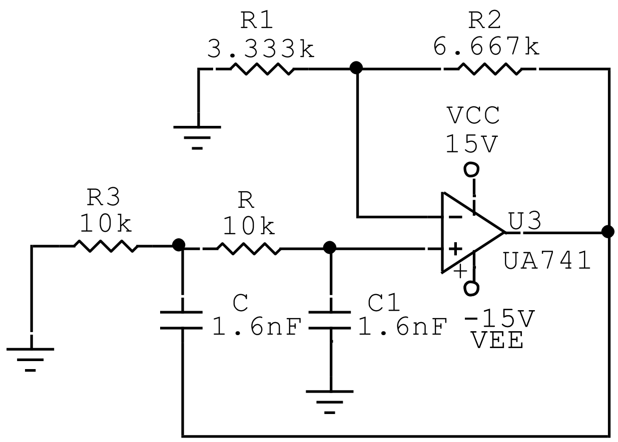


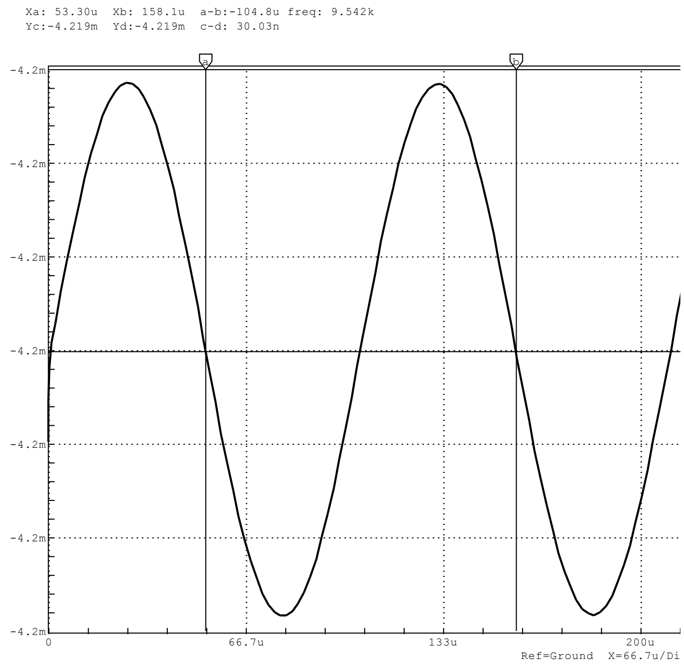
The circuit will begin oscillating when the poles reach $j\omega$ axis of the imaginary axis. This happens when the s term in the denominator of the transfer function is equal to zero, which is when $A_M = 3$.

$$A_M = 3 = 1 + \frac{R_2}{R_1} \qquad\qquad \boldsymbol{R_1 + R_2 = 10k\Omega}$$

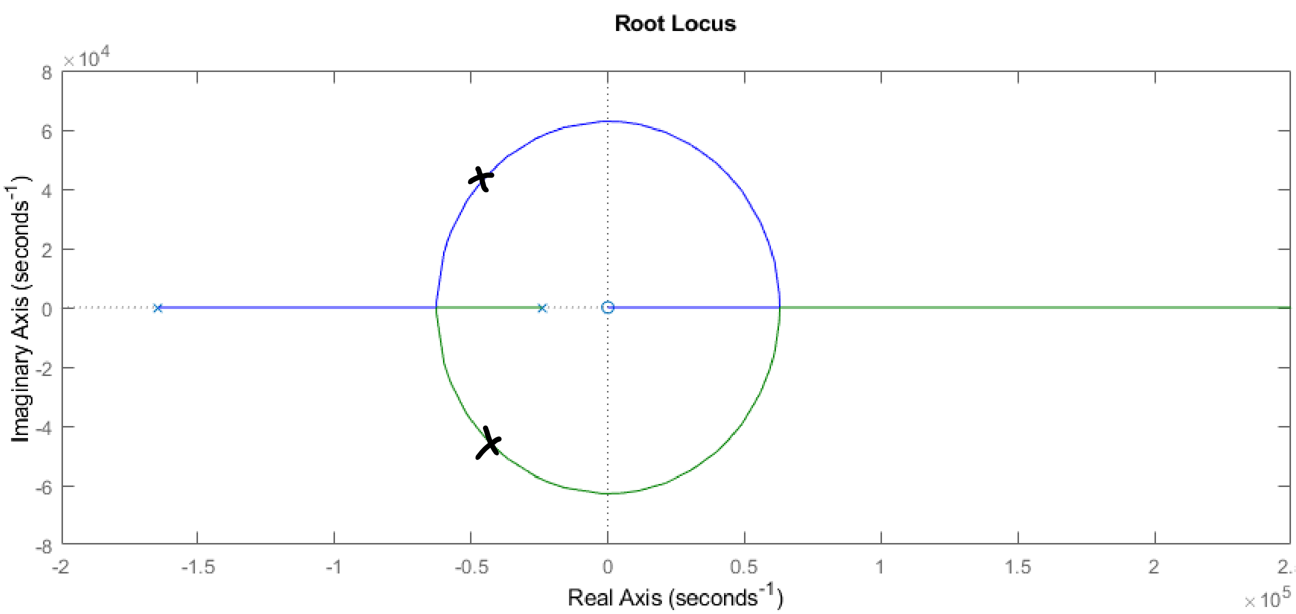
$$\xrightarrow{yields} R_1 = 3.333k\Omega, R_2 = 6.667k\Omega$$

The simulated circuit with input grounded and its oscillatory transient response is shown.



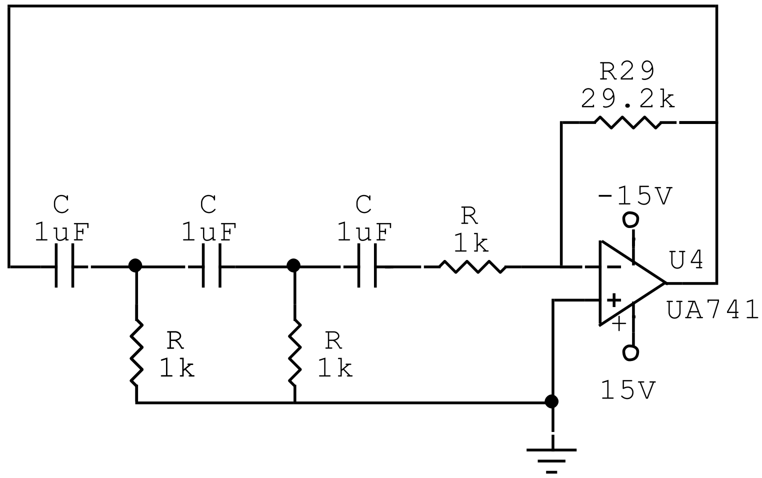


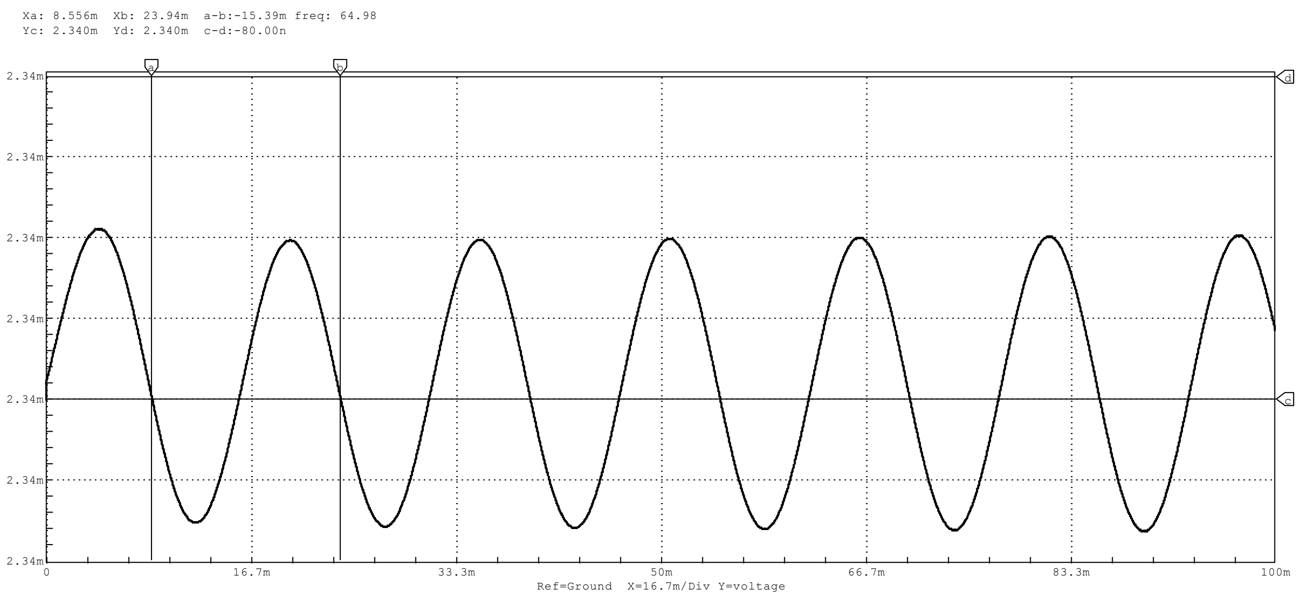
The oscillation frequency is measured to be <u>9.5kHz</u> which is close to the expected value of 10kHz. The poles will start approximately in the positions marked by 'X' on the root locus as shown from the pole-zero map above. As the midband gain A_M is increased the poles will move towards the $j\omega$ axis and at A_M=3 the system will be the poles will be on the $j\omega$ axis causing undamped oscillation. Once the poles cross into the right half plane the filter will become unstable.



Part B – A Phase Shift Oscillator

Using 29R as a 29.2kΩ resistor is when the circuit starts stable oscillations at about 65Hz.



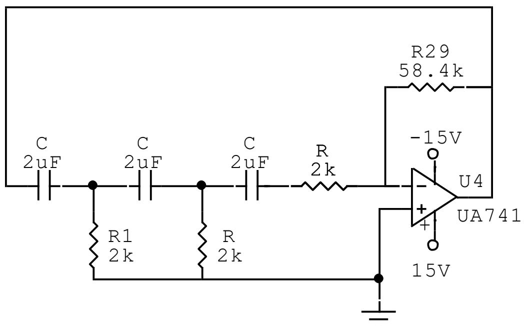


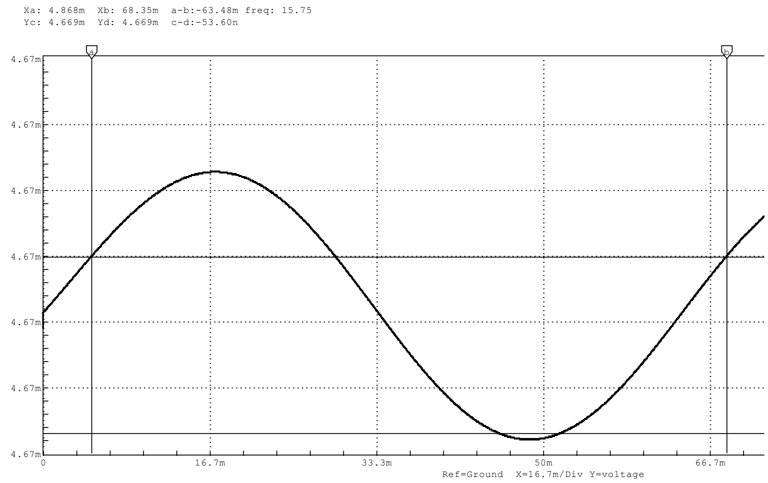
The oscillation frequency of this is calculated by $f = \frac{1}{2\pi\sqrt{6}RC} = \frac{1}{2\pi\sqrt{6}\cdot 1k\Omega\cdot 1\mu F} = 64.97Hz$ which is closely approximated by the simulated result oof 64.98Hz.

Increasing the resistance and capacitance by a factor of 2 leads to the oscillation frequency of

$$f = \frac{1}{2\pi\sqrt{6}RC} = \frac{1}{2\pi\sqrt{6}\cdot 2k\Omega\cdot 2\mu F} = 16.24Hz$$

To simulate this, double the values of all resistors and capacitors, so *29R* becomes 58.4kΩ.

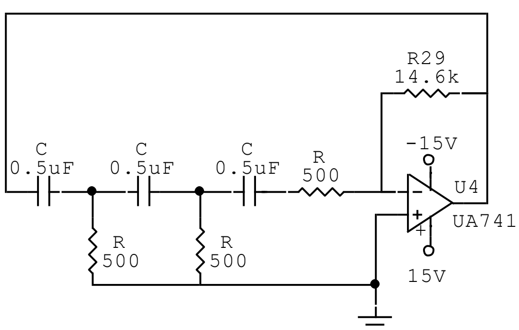


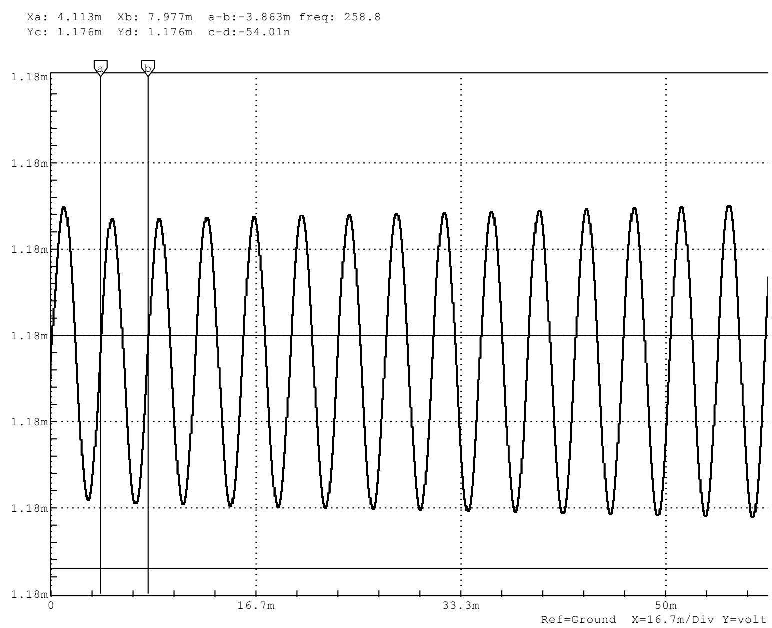


The simulated frequency is 15.75Hz which matches closely with the calculated frequency of 16.24Hz. Decreasing the resistance and capacitance by a factor of 2 results in a frequency of:

$$f = \frac{1}{2\pi\sqrt{6}RC} = \frac{1}{2\pi\sqrt{6}\cdot\frac{1}{2}k\Omega\cdot\frac{1}{2}\mu F} = 260Hz$$

To simulate this, halve the values of all resistors and capacitors, so *29R* becomes 14.6kΩ.





The simulated frequency is 258.8Hz which matches closely with the calculated frequency of 260Hz.

Circuit		Oscillation frequency (Hz)		Percent error
R	C	calculated	simulated	
1kΩ	1μF	64.97	65	0.046%
2kΩ	2μF	16.24	15.75	3.11%
500Ω	0.5μF	260	258.8	0.46%

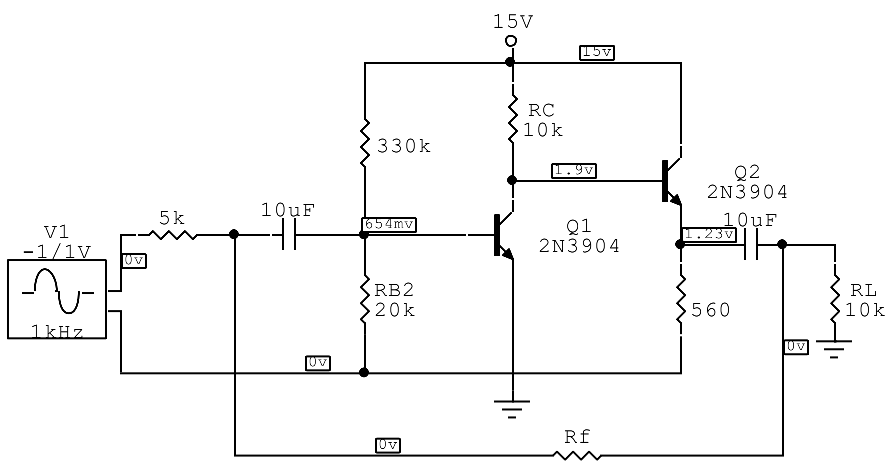
Phase-Shift Oscillator Explanation

The phase-shift oscillator functions by applying a $-\pi$ phase shift through the feedback network of an inverting op amp which creates a positive feedback loop. A non-zero output regardless of the input is enabled by the oscillator having a pair of complex poles on the $j\omega$ axis. The value of *29R* needed to be increased slightly which would increase the gain by moving the poles closer to the right half plane and avoid a decaying oscillation. This adjustment may be because in the calculations an ideal amplifier was assumed but a non-ideal one was used in the simulation. Nevertheless, the calculated results are extremely close to the simulated results.

Part C – Feedback Circuit

The feedback network senses the output voltage and then converts it into current that is mixed with the input signal at the base of Q1. So, since the input is adding current it must be connected in shunt and since the output voltage is being sensed, it must also be done in shunt. The feedback circuit is therefor shunt-shunt topology.

The first step is to vary R_{B2} to obtain the greatest open loop gain by opening the feedback resistor and measuring the output for several values of R_{B2}. The optimal R_{B2} is found to be about 20kΩ. The circuit is constructed like below.



1. d.c. operating point

   The d.c. operating points are measured for both transistors.

	V_C	V_B	V_E	I_C	I_B	I_E
Q_1	1.90V	0.654V	0V	1.295mA	10.77µA	1.306mA
Q_2	15.0V	1.90V	1.235V	2.184mA	15.34µA	2.199mA

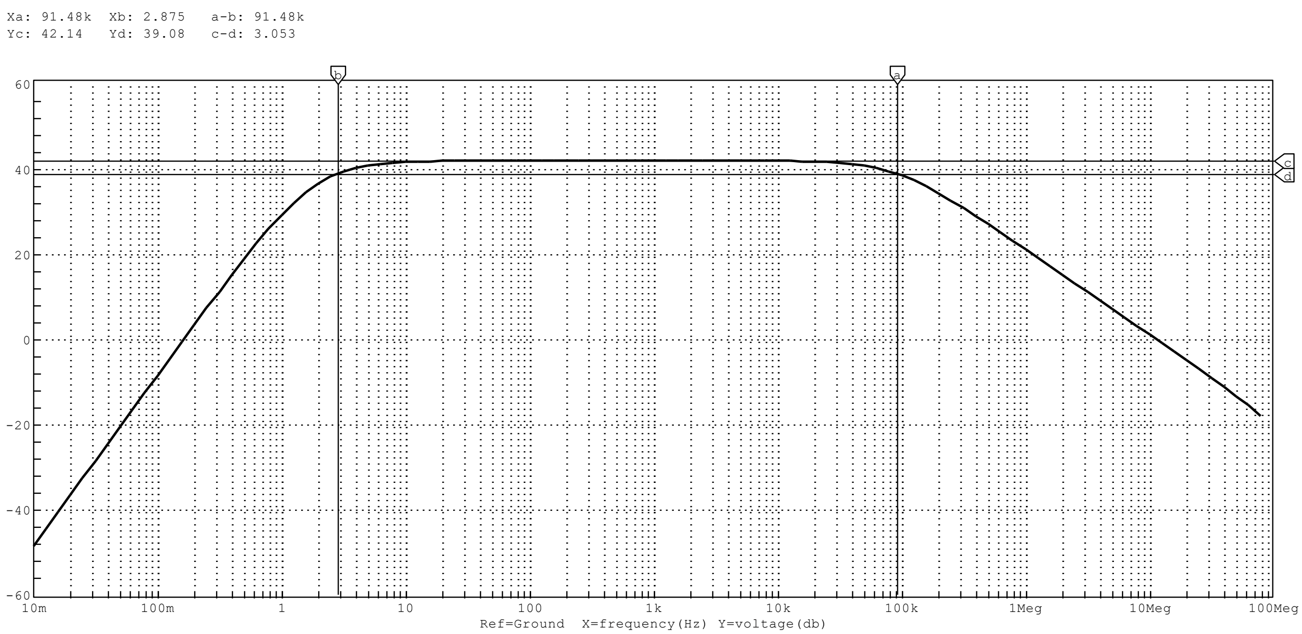
The small signal parameters g_m, r_π, h_{FE} can be calculated from the d.c. operating point.

Q_1: $g_m = \frac{I_C}{V_T} = \frac{1.295mA}{25mV} = \underline{0.0518S}$ $r_\pi = \frac{V_T}{I_B} = \frac{25mV}{10.77\mu A} = \underline{2.32k\Omega}$ $h_{FE} = r_\pi \frac{I_C}{V_T} = \underline{120}$

Q_2: $g_m = \frac{I_C}{V_T} = \frac{2.184mA}{25mV} = \underline{0.0874S}$ $r_\pi = \frac{V_T}{I_B} = \frac{25mV}{15.34\mu A} = \underline{1.63k\Omega}$ $h_{FE} = r_\pi \frac{I_C}{V_T} = \underline{142}$

2. Open-loop & predicted Closed-loop frequency response

Open R_f and plot the open loop amplitude response. The 3dB points are identified.



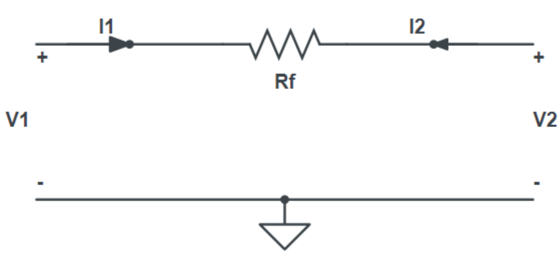
$f_{L3dB} = 2.875Hz$ $f_{H3dB} = 91.48kHz$

$$A_M = 42.14dB = (-)127.4\, {}^{V}/_{V}$$

To measure the input/output impedances a test source of 1mV and 1kHz is used. Replace either the 5kΩ source resistor or the 10kΩ load resistor with the test source and then short the other resistor. Use the AC multimeter to probe the voltage & current at the test source.

$R_{in} = \frac{706.2\mu V}{274.7nA} = 2.567k\Omega$ $R_{out} = \frac{706.2\mu V}{11.29\mu A} = 62.55\Omega$

To predict closed-loop frequency response use y-parameters to represent the feedback network because it uses shunt-shunt topology. The related diagram and equations are shown.



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

For R_f=100kΩ, the feedback gain can be calculated as $\beta = y_{12} = -10\mu S$. From the open-loop amplitude bode plot, the midband gain is -127.4 V/V. Since this is shunt-shunt topology, calculate the open loop current to voltage amplification A_i, where R_S = 5kΩ. Then A_f can be calculated using A_i.

$$\boldsymbol{y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} = \frac{1}{R_f}} \qquad \boldsymbol{y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0} = -\frac{1}{R_f}}$$

$$\boldsymbol{y_{21} = 0} \qquad y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0} = \frac{1}{R_f}$$

$$A_i = \frac{V_{out}}{I_{in}} = A_M \cdot R_s = -637\, {}^{kV}/_{A}$$

$$A_f = \frac{A_i}{1 + A_i\beta} = -86.43\, {}^{kV}/_{A}$$

$$A = \frac{A_f}{R_s} = -17.28\, {}^{V}/_{V}$$

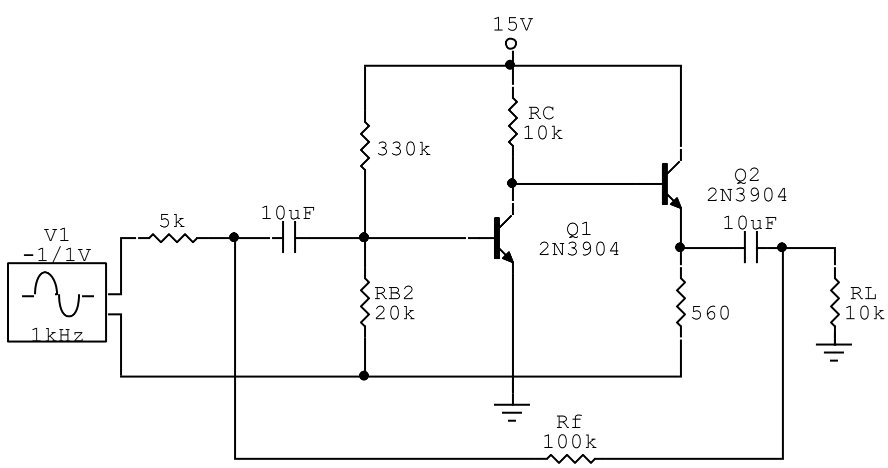
Then to calculate the 3dB frequencies, apply the bandwidth extension of (1+$A_i\beta$) to the open loop 3dB points f_{L3dB} = 2.875Hz and f_{H3dB} = 91.48kHz.

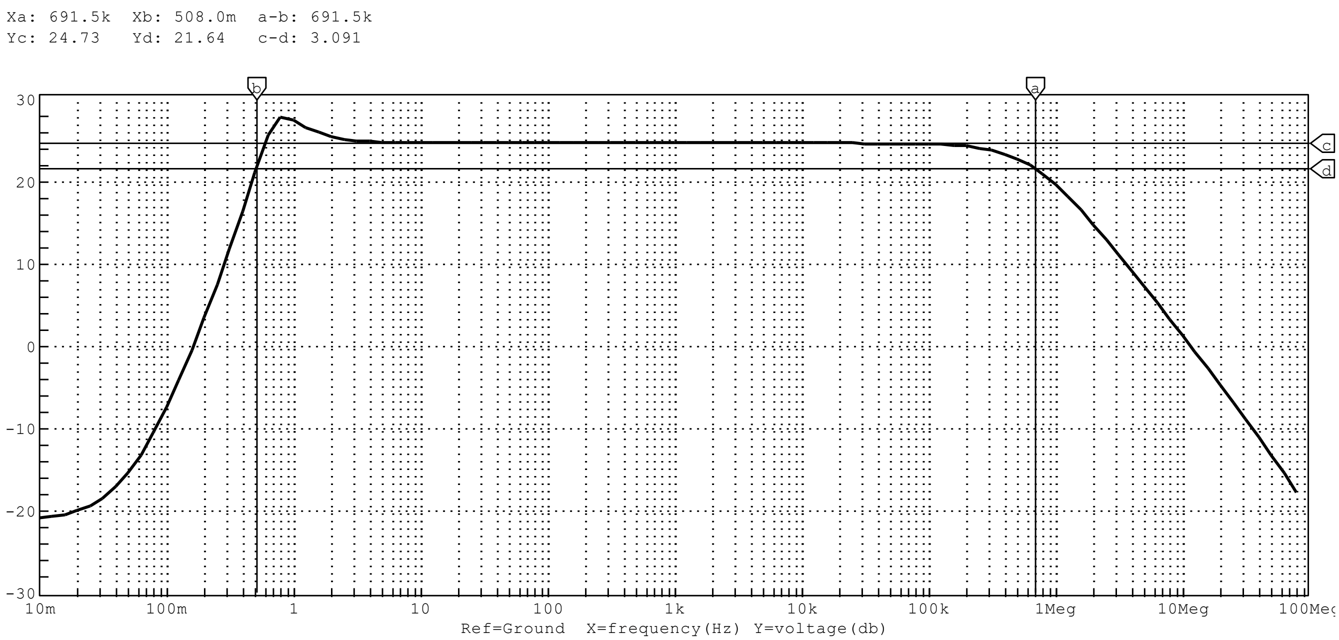
$$\boldsymbol{f_{L3dB_f} = \frac{2.875Hz}{(1 + A_i\beta)} = 390mHz} \qquad \boldsymbol{f_{H3dB_f} = 91.48kHz \cdot (1 + A_i\beta) = 674.2kHz}$$

Due to the shunt-shunt topology, the open-loop input and output impedances are reduced by a factor of (1+$A_i\beta$). Recall the open loop impedances R_i=2.567kΩ & R_o=62.55Ω.

$$\boldsymbol{R_{i_f} = \frac{R_i}{(1 + A_i\beta)} = 348.6\Omega} \qquad \boldsymbol{R_{o_f} = \frac{R_o}{(1 + A_i\beta)} = 8.487\Omega}$$

Now connect the feedback resistor and simulate the closed loop circuit to verify the results.





The midband gain is 24.73dB = (-)17.25V/V and the 3dB points are 508mHz and 691.5kHz.

Remeasure the input/output impedances like before but now with the feedback resistor R_f connected.

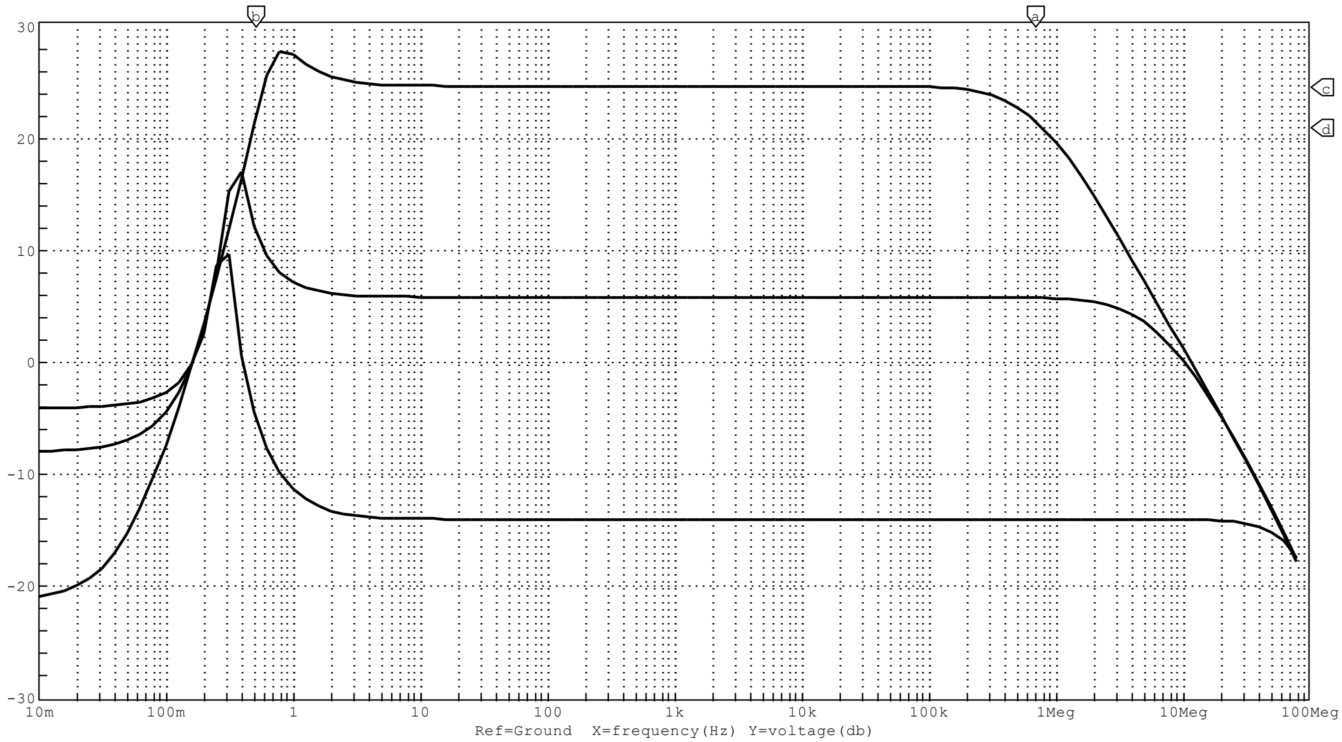
$$\boldsymbol{R_{i_f} = 243\Omega} \qquad\qquad \boldsymbol{R_{o_f} = 8.649\Omega}$$

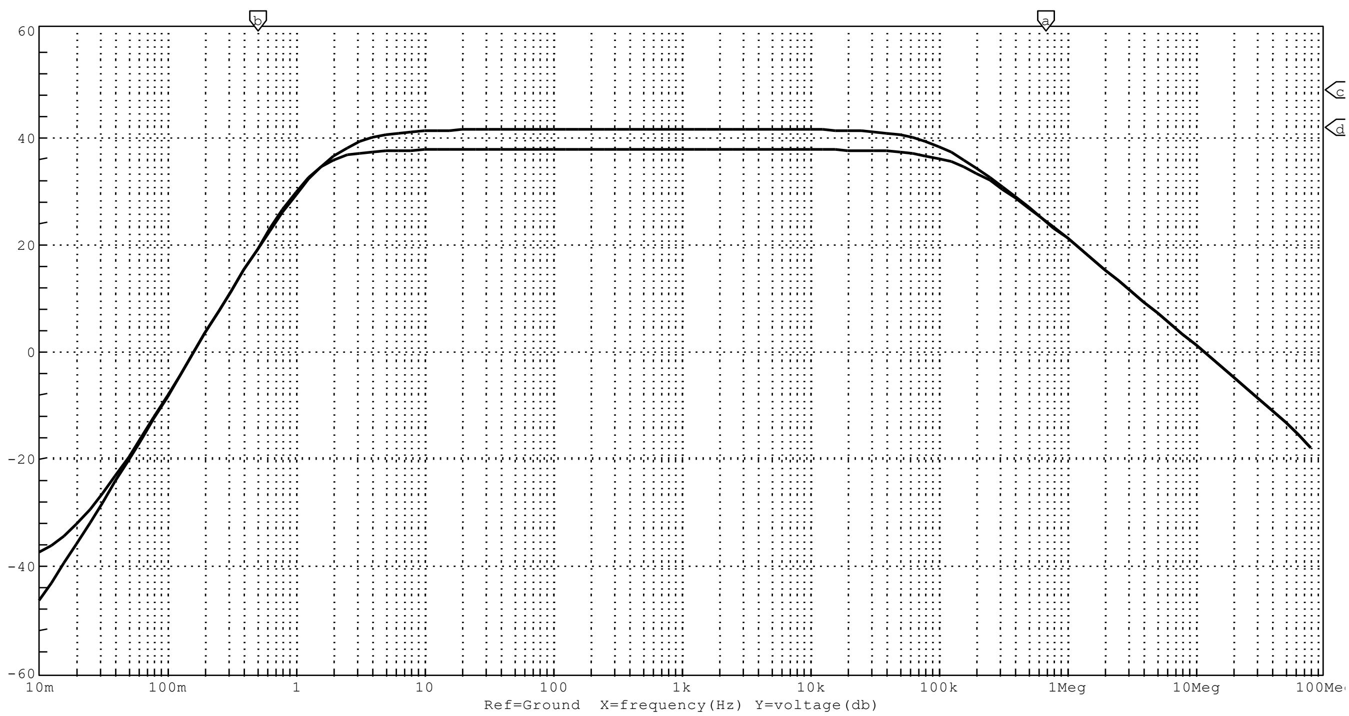
Here are the calculated and simulated results of the closed loop amplifier. The midband gain prediction is excellent. The low 3dB point prediction is inaccurate which could be due to the overshoot at the low frequency cut-off. The input impedance has a higher error but is of similar magnitude, and the output is accurately calculated.

	A (V/V)	f_{L3dB_f}	f_{H3dB_f}	R_{if}	R_{of}
Calculated	-17.28 V	390mHz	674.2kHz	348.6 Ω	8.487 Ω
Simulated	-17.25	508mHz	691.5kHz	243 Ω	8.649 Ω
error	0.17%	23.2%	2.50%	43.5%	1.87%

3. Closed loop frequency response while varying R_f

Here are the results of varying R_f from 1kΩ - 100kΩ. The plots for R_f = 1MΩ and 10MΩ are shown on a separate plot. 1kΩ corresponds to the bottom curve of the first plot, and 10MΩ corresponds to the top curve of the second plot. In the first plot notice that there is overshoot. It may be because at lower frequencies the coupling capacitors have not yet started conducting so the gain is a voltage division between the feedback and load resistance. Another possible explanation is that as R_f is increased, the poles are moved causing a shift in resonance frequency of the peaks.





To measure the feedback factor β first measure the midband gain (with feedback) in dB and then convert to V/V. A_f is determined by converting this value to V/A by multiplying by R_s. Then solve $A_f = \frac{A}{1+A\beta}$ where A is the open-loop midband gain previously determined as -637kV/A. The theoretical value of β is calculated by $\frac{-1}{R_f}$.

R_f	A_f(dB)	A_f(V/V)	A_f(kV/A)	Simulated β	Theoretical β
1kΩ	-14.07	-0.1979	-0.9895	-1.009 mS	-1 mS
10kΩ	5.845	-1.960	-9.8	-100.5 µS	-100 µS
100kΩ	24.73	-17.24	-86.2	-10.03 µS	-10 µS
1MΩ	37.82	-77.80	-389	-1.001 µS	-1 µS
10MΩ	41.57	-119.8	-599	-99.59 nS	-100 nS

From these results, the theoretical and simulated feedback factor β are very similar.

4. Estimate feedback values with I/O Impedances

   Measure the input and output impedances while varying the feedback resistor between 10kΩ, 100kΩ, 1MΩ using the same techniques as before. Now with these closed-loop input/output impedances, open-loop input/output impedances, and open-loop gain the feedback gain β can be used to determine the "amount of feedback", $a_{theoretical}$, using the calculated β values for each feedback resistor from part 3 where A is the open loop gain (637 kV/A).

$$R_{if} = \frac{R_i}{1 + A_i\beta} \qquad R_{of} = \frac{R_o}{1 + A_i\beta}$$

$$a_{theoretical} = 1 + A_i\beta$$

   The amount of feedback seen from the input and output a_{in} and a_{out} can be averaged to find the measured "amount of feedback". R_{in} and R_{out} is the open loop impedance respectively.

$$a_{in} = \frac{R_{in}}{R_{if}} \qquad a_{out} = \frac{R_{out}}{R_{of}}$$

R_f	R_{if} (measured)	R_{of} (measured)	$a_{theoretical}$	a_{in}	a_{out}	$\frac{a_{in} + a_{out}}{2}$	Percentage error
10kΩ	26.3Ω	1.21Ω	64.7	97.68	51.70	74.68	17.8%
100kΩ	239Ω	8.34Ω	7.37	10.75	7.500	9.124	19.2%
1MΩ	1.31kΩ	38.1Ω	1.637	1.961	1.642	1.801	9.1%

This method of averaging ends up being close to the theoretical value but with significant error.

5. Desensitivity Factor

   The Desensitivity factor is given by taking the derivative of the feedback gain with respect to the open loop gain. d denotes the Desensitivity factor. A = 637kV/A

$$\frac{dA_f}{dA} = \frac{1}{(1 + A\beta)^2} \xrightarrow{yields} d = 1 + A\beta$$

   When $R_f = \infty$, β = 0 because there is no feedback so $d = 1$. This table summarizes the gains when $R_f = \infty$ and R_c is varied.

R_C(kΩ)	A (V/V)
9.9	-126.765187
10	-127.49701
10.1	-128.085509

Change R_f to 100kΩ and repeat the measurements. The gains don't change appreciably so it is desensitized by the feedback network.

R_C(kΩ)	A_f (V/V)
9.9	-17.198873
10	-17.258379
10.1	-17.298164

The theoretical desensitization factor can be calculated as $1 + 637\,{}^{kV}\!/_{A}\,(10\mu S) = \underline{7.37}$

To measure the desensitization factor, use the formula below. Use 10k as the reference point and then for R_C = 9.9k and 10.1k determine the open and closed-loop gains like in previous parts.

$$d = \sqrt{\frac{dA}{dA_f}}$$

$$d_{9.9k} = \sqrt{\frac{A_{10k} - A_{9.9k}}{A_{f10k} - A_{f9.9k}}} \qquad d_{10.1k} = \sqrt{\frac{A_{10.1k} - A_{10k}}{A_{f10.1k} - A_{f10k}}}$$

$$d = \frac{d_{9.9k} + d_{10.1k}}{2} = \underline{7.91}$$

The calculated and measured desensitization factors are similar.

References

1. ELEC 301 Course Notes
2. A. Sedra and K. Smith, "Microelectronic Circuits,"5 th (or higher) Ed., Oxford University Press, New York
3. CircuitMaker™ User's Manual
4. 2N3904 datasheet [https://datasheetspdf.com/pdf-file/1114626/Motorola/2N3904/1]