

ELEC 341 – Graded Assignments

Assignment A5  
State-Space Representation  
  
10 Marks

Learning Objectives

State-Space Representation	Matlab
Mechanical System	ss()
Electro-Mechanical System	inv()
	eye()
State Transition Matrix	

A State-Space representation of the “Mechanical Circuits” in Assignment #3 allows you to compute angles, positions and component forces, without having to derive an electrical equivalent. It also lets you change you mind about what the output is without starting over.

Derive a state-space representation of “Mechanical Circuit #1”. Configure your matrices so that a **UNIT** step applied to your State-Space object applies the **SPECIFIED INPUT FORCE**.

**Q1      3 mark(s) State Matrices**

Assume all initial conditions are 0.

Use the following state vector, in the following order. Delete any dependent states.

$$\bar{x} = [d0 \ d1 \ d2 \ d3 \ v0 \ v1 \ v2 \ v3]^T$$

All distances in (m), all speeds in (m/s)

Compute the **MECHANICAL** A & B state matrices.

- Q1.A      (mixed)      Matrix
- Q1.B      (mixed)      Matrix

**Q2      2 mark(s) Output Matrices**

Solve for the same two outputs as in Assignment #3.

Use the following output vector.

$$\bar{y} = [d3(m) \ fK20(Nm)]^T$$

Compute the C & D output matrices.

- Q2.C      (mixed)      Matrix
- Q2.D      (mixed)      Matrix

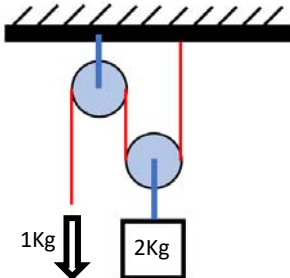
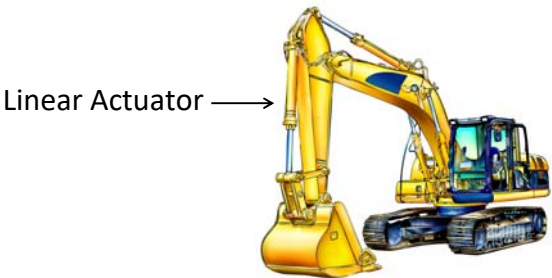
Would you get the same A,B,C,D matrices from the equivalent electric circuit ???

**PROVE IT**, for extra practice.

**COW:** Use **ss()** and **impulse()** to see the response of all outputs. Compare them to what you got in Assignment #3.

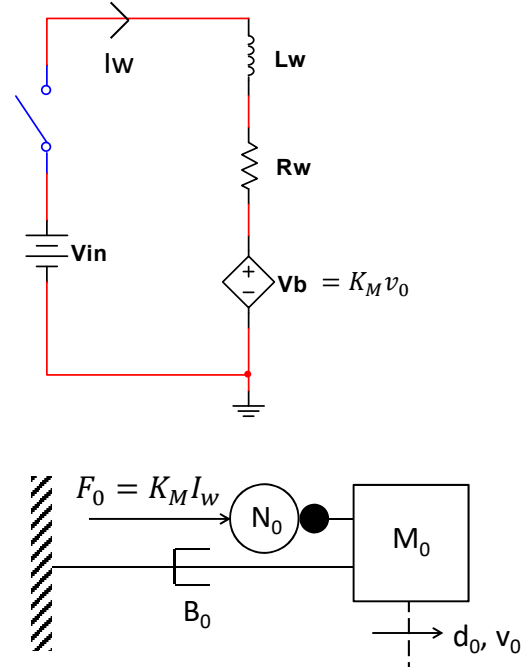
Replace F0 in Assignment #3 with the electric “Liner Actuator Model”. A linear actuator works just like a motor, but the motion is in a straight line. You see linear actuators in excavators, factories, and other heavy equipment. Often, they are hydraulic, but there are electric ones too.

There are also linear transmissions. A pulley system is a good example. It scales speed and force the same way a gearbox scales rotational speed and torque.



### Linear Actuator Model

- $R_w = \#A/3 \text{ } (\Omega)$
- $L_w = \#B \text{ (mH)}$
- $K_M = \#G/2 \text{ (Vs/m or N/A)}$
- $V_{in} = 150 \text{ (V)}$
- $M_0 = \#A + \#B \text{ (Kg)}$  @ actuator
- $B_0 = \#C + \#D \text{ (Ns/m)}$  @ actuator
- $N_0 = 1:2 \text{ (speed ratio)}$  **OVER-DRIVE**



### Q3 3 mark(s) Actuated System

Assume all initial conditions are 0.

$M_0$  and  $B_0$  represent the mechanical impedance of the actuator AND transmission.

Use the following state vector, in the following order. Delete any dependent states.

$$\bar{x} = [d_0 \ d_1 \ d_2 \ d_3 \ v_0 \ v_1 \ v_2 \ v_3 \ I_w]^T$$

All distances in (m), all speeds in (m/s), current in (A)

Do all calculations in **HIGH-SPEED coordinates** (the mechanism when transmission is over-drive)

Compute the **ELECTRO-MECHANICAL** A, B matrices.

Compute the State Transition Matrix **STM**.

Use the following output vector.

$$\bar{y} = d_3 \text{ (m)}$$

Compute the C & D matrices.

Compute the transfer function of the State-Space system from the matrices you computed.

- Q3.G (m/V) LTI

*You can use the Laplace 's' operator in a Matrix and can even invert it using **inv()**.*

*Someone at Mathworks with a sense of humor named the Identity function **eye()**.*

**COW:** Plot the impulse response of G.

*If you set electrical impedance to zero, how different should the response be ???*

Add gravity to **each** mass.       **$g = 9.81 \text{ (m/s}^2\text{)}$**   
Assume the vertical direction is as depicted in Assignment #3.  
Since gravity is an applied force that has nothing to do with motor Voltage (the input), it must be modeled as a 2<sup>nd</sup> input. Include it in the u vector.  
Do you need to change the A, B, C, or D matrix ???

**Q4      2 mark(s) Gravity**  
Assume all initial conditions are 0.  
Use the same state and output vectors from Q3.  
Compute the transfer function that describes to the component due to motor voltage (Gv) and the component due to gravity.  
The units (m/G) means per G-force. On earth, we have 1G.  
• Q4.Gv      (m/V)      LTI  
• Q4.Gg      (m/G)      LTI

**COW:** Plot the impulse response of Gv and compare it to G from Q3.  
Then add gravity. Use superposition.  
If the applied voltage is an impulse, should gravity be an impulse as well ???  
Is the effect reasonable and what you would **EXPECT** ???

