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Teaching Professor
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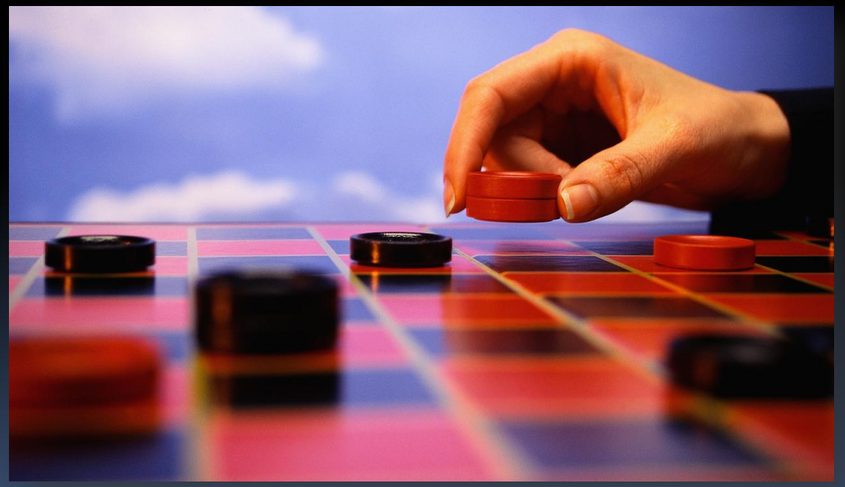
The Beauty and Joy of Computing

Python
Tree Recursion
and Game Theory

Checkers Solved!

A 19-year project led by Prof Jonathan Schaeffer, he used dozens (sometimes hundreds) of computers and AI to prove it is, in perfect play, a … draw! This means that if two Gods were to play, nobody would ever win!



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Game Theory Introduction

What is "Game Theory"?

Combinatorial

- Sprague and Grundy's 1939 Mathematics and Games
- Board games
- Nim, Domineering, dots and boxes
- Film: *Last Year in Marienbad*
- Complete info, alternating moves
- Goal: Last move

Computational

- R. C. Bell's 1988 Board and Table Games from many Civilizations
- Board games
- Tic-Tac-Toe, Chess, Connect 4, Othello
- Film : *Searching for Bobby Fischer*
- Complete info, alternating moves
- **Goal: Varies**

Economic

- von Neumann and Morgenstern's 1944 *Theory of Games and Economic Behavior*
- Matrix games
- Prisoner's dilemma, auctions
- Film : *A Beautiful Mind* (about John Nash)
- **Incomplete** info, simultaneous moves
- Goal: Maximize payoff

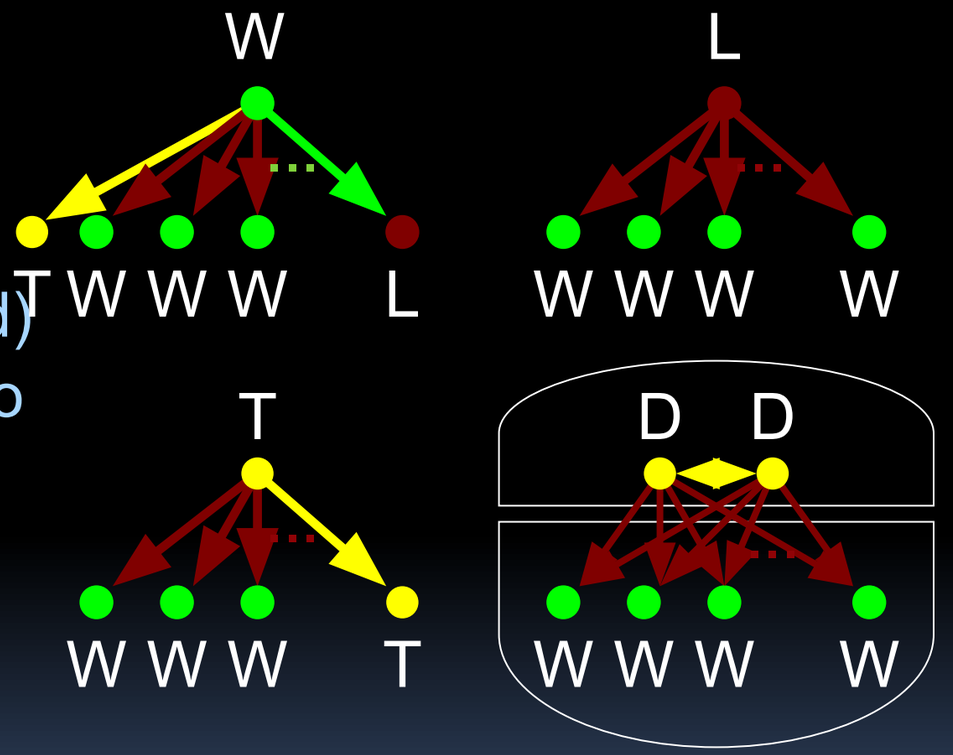
What “Board Games” do you mean?

- No chance, such as dice or shuffled cards
- Both players have complete information
  - No hidden information, as in Stratego & Magic
- Two players (Left & Right) usually alternate moves
  - Repeat & skip moves ok
  - Simultaneous moves not ok
- The game can end in a pattern, capture, by the absence of moves, or …

Game Theory Basics

What's in a Strong Solution?

- For every position
  - Assuming alternating play
  - Value … (for player whose turn it is)
    - Winning (∃ losing child)
    - Losing (All children winning)
    - Tieing (!∃ losing child, but ∃ tieing child)
    - Drawing (can't force a win or be forced to lose)
  - Remoteness
    - How long before game ends?



When poll is active, respond at **pollev.com/ddg**

Text **DDG** to **22333** once to join

L21 Select the true statement about Game Theory

It doesn't matter how you play at the start, as long as you play perfectly at the end, you can always win

You can always win from any game but you have to play perfectly the whole time

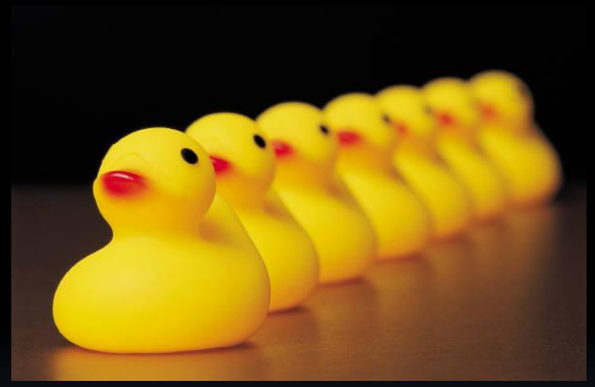
All of the above

None of the above

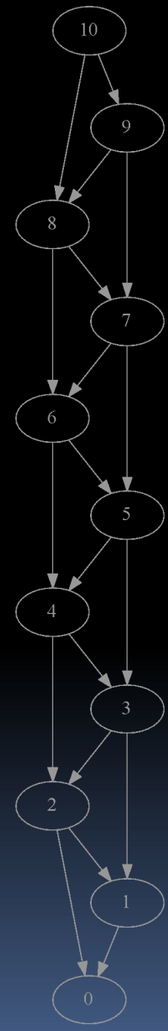
10□0 (by 1 or 2)
Let’s Solve It!

Strong Solving Example: 10…0 (by 1 or 2)

- Initial Position
  - 10 pieces on table
- Rules (on your turn):
  - Subtract 1 or 2 from table
- Goal
  - Be the FIRST to get to 0
- Example
  - Ana: “taking 2 makes it 8”
  - Bob: “taking 1 makes it 7”
  -
  - Ana: “taking 2 makes it 5”
  - Bob: “taking 2 makes it 3”
  - Ana: “taking 1 makes it 2”
  - Bob: “taking 2 makes it 0”
    - I WIN!



7 ducks (out of 10)



Coding 10…0 (by 1 or 2) in Python

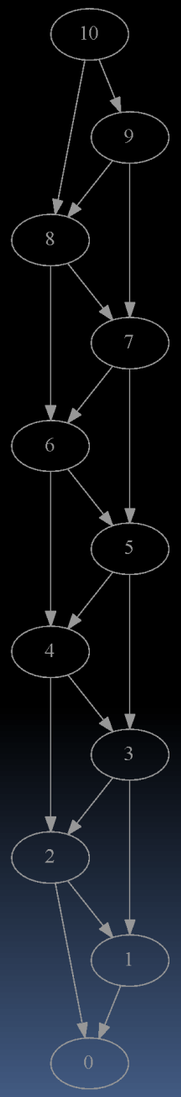
```
LOSE          = "Lose"
WIN           = "Win"
TIE           = "Tie"
NOT_PRIMITIVE = "Not Primitive"
```

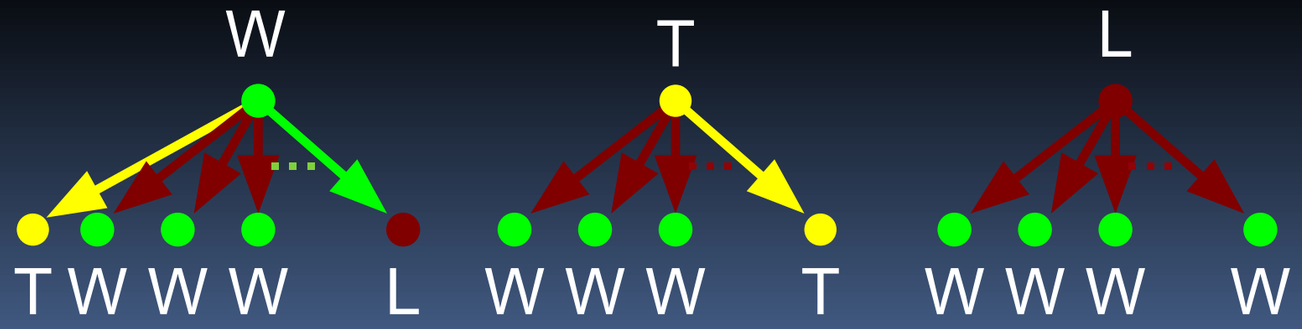
```
### 10-to-0-by-1-or-2
def primitive_value(position):
    return LOSE if position == 0 else NOT_PRIMITIVE

def generate_moves(position):
    return (1,) if position == 1 else (1,2)

def do_move(position, move):
    return position - move
```

Coding the Solver!

```
def children(position):
    return [do_move(position, move) for move in
                                    generate_moves(position)]

def value(position):
    if primitive(position) != NOT_PRIMITIVE:
      return primitive(position)
    else:
      values = [value(child) for child in children(position)]
      if LOSE in values:
         return WIN
      elif TIE in values:
         return TIE
      else:
         return LOSE
```

```
10's value:Win
9's value:Lose
8's value:Win
7's value:Win
6's value:Lose
5's value:Win
4's value:Win
3's value:Lose
2's value:Win
1's value:Win
0's value:Lose
```





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