CIS 256 Lab #7

1) $T(n) = 2^n + 1$ and $f(n) = 4^n - 16$

$c = 1,\ n = 3$

$2^3 + 1 \leq 4^3 - 16 \Rightarrow T(n) \leq cf(n)$

$9 \leq 48$

2) $f(n) = O(g(n))$

$f(n) \leq c' \cdot g(n)$ for all $n \geq N'$

$g(n) \leq O(h(n))$

$g(n) \leq c'' \cdot h(n)$ for all $n \geq N''$

so $f(n) \leq c' \cdot c'' h(n)$ and $c' \cdot c'' = c'''$

$f(n) \leq c''' h(n)$

so $f(n) = O(h(n))$

3) $T(n) = 0.01n^2 - 1$

$f(n) = O(n)$

$$\lim_{n\to\infty} \frac{cf(n)}{T(n)} = \lim_{n\to\infty} \frac{cn}{0.01n^2 - 1}$$

$$= \lim_{n\to\infty} \frac{c\cancel{n}}{\cancel{n}(0.01n - \frac{1}{n})} = \lim_{n\to\infty} \frac{c}{\underbrace{0.01n}_{\to\infty} - \underbrace{\frac{1}{n}}_{\to 0}}$$

$$= \lim_{n\to\infty} \frac{c}{\infty - 0} = \boxed{\lim_{n\to\infty} \frac{c}{\infty} = 0}$$

so $0.01n^2 - 1$ is NOT in $O(n)$