ARTICULATED ROBOTS

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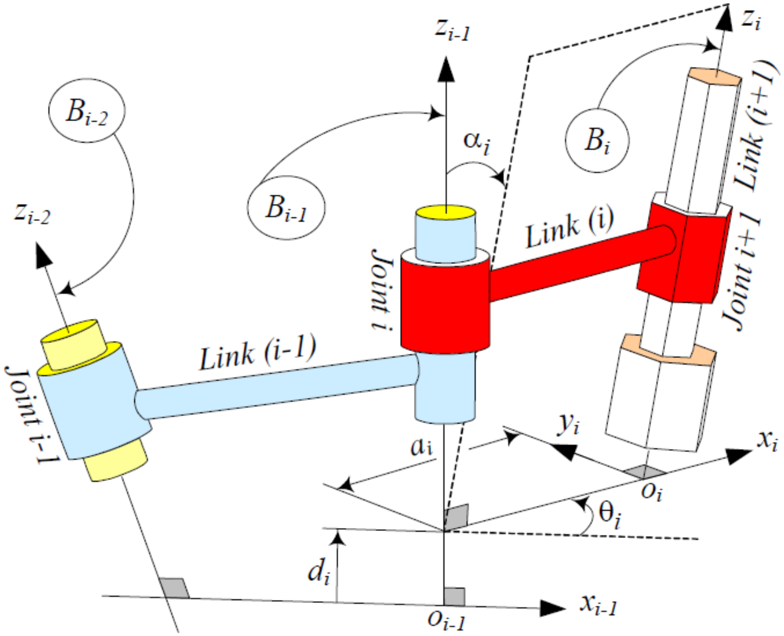
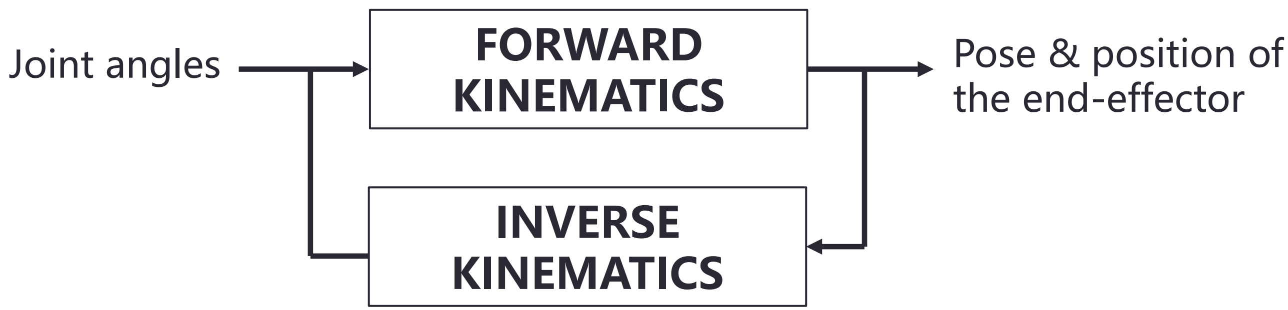
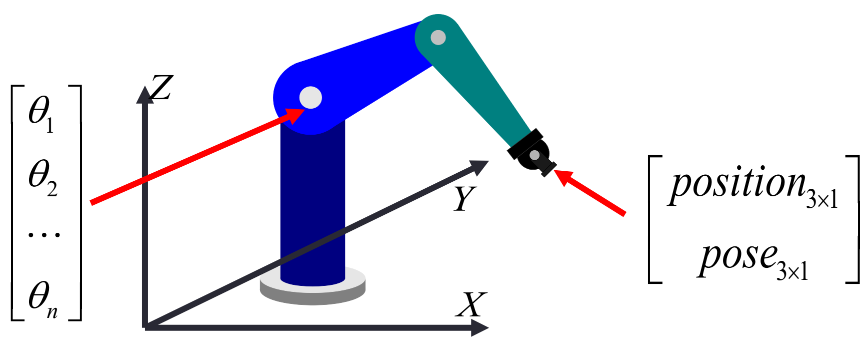
3. FORWARD KINEMATICS II



3.3 Forward Kinematics

✓ The forward (*or direct*) kinematics is the transformation of kinematic information from the robot joint variable space to the Cartesian coordinate space;





- ✓ Finding the end-effector position and orientation for a given set of joint variables is the main problem in forward kinematics;
- ✓ Solving the forward kinematics problem is a process to determining transformation matrices ${}^{0}T_i$ to describe the kinematic information of link (i) in the base link coordinate frame;
- ✓ By usg the Denavit-Hartenberg notations and frames, we have

$$T_n = {}^{0}T_1(\theta_1)\,{}^{1}T_2(\theta_2)\cdots{}^{i-1}T_i(\theta_i)\cdots{}^{n-1}T_n(\theta_n)$$

$$= \begin{bmatrix} R_n & \mathbf{d}_n \\ \mathbf{0} & 1 \end{bmatrix}$$

Orientation of frame B_n

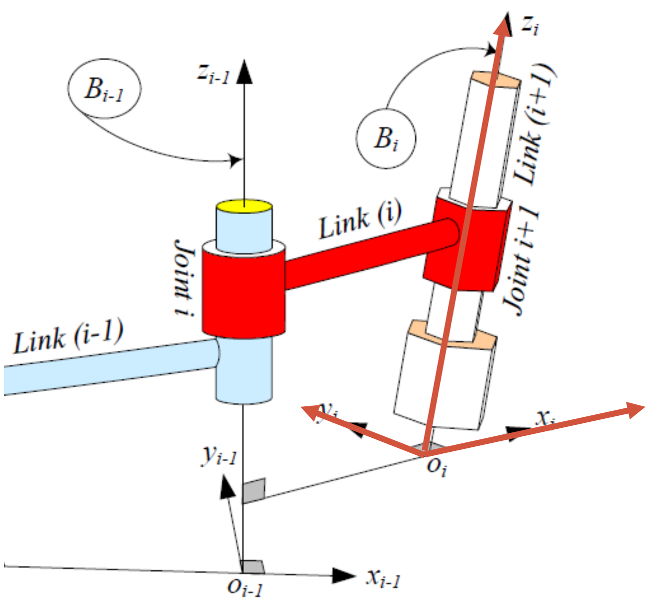
Position of o_n

Transformation of Two Frames

- ✓ The transformation matrix between two adjacent frames attached to link (i) and link (i + 1) is a fundamental block to the forward kinematics problem.

$$\boxed{{}^{i-1}T_i \;:\; B_{i-1} \longrightarrow B_i}$$

- ✓ Initially, frame B_i coincides with B_{i-1}. It becomes the present state after four steps of homogeneous transformations.



B_{i-1} is a global frame and B_i is a local frame

1. B_i translates d_i along z_{i-1} — $D_{z_{i-1}}(d_i)$
2. B_i rotates about z_{i-1} by θ_i — $R_{z_{i-1}}(\theta_i)$

(1 and 2: commutative)

3. B_i translates a_i along x_i — $D_{x_i}(a_i)$
4. B_i rotates about x_i by α_i — $R_{x_i}(\alpha_i)$

(3 and 4: commutative)

Transformation Matrix

- ✓ By pre- or post-multiplications of four homogeneous transformation matrices, the overall transformation from frame $B_{i\text{-}1}$ to B_i can be obtained by

$$
{}^{i-1}T_i = D_{z_{i-1}}(d_i) R_{z_{i-1}}(\theta_i) D_{x_i}(a_i) R_{x_i}(\alpha_i)
= \begin{bmatrix} c\theta_i & -c\alpha_i s\theta_i & s\alpha_i s\theta_i & a_i c\theta_i \\ s\theta_i & c\alpha_i c\theta_i & -s\alpha_i c\theta_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{i-1}T_i^{-1} = {}^{i}T_{i-1} = \begin{bmatrix} c\theta_i & s\theta_i & 0 & -a_i \\ -c\alpha_i s\theta_i & c\alpha_i c\theta_i & s\alpha_i & -d_i s\alpha_i \\ s\alpha_i s\theta_i & -s\alpha_i c\theta_i & c\alpha_i & -d_i c\alpha_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
D_{z_{i-1}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}
\qquad
R_{z_{i-1}} = \begin{bmatrix} \mathrm{c}\theta & \text{-}\mathrm{s}\theta & 0 & 0 \\ \mathrm{s}\theta & \mathrm{c}\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
D_{x_i} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\qquad
R_{x_i} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \mathrm{c}\alpha & \text{-}\mathrm{s}\alpha & 0 \\ 0 & \mathrm{s}\alpha & \mathrm{c}\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Solution to Forward Kinematics Problem

✓ The position of a point P in frame *n*

$$\begin{bmatrix} \mathbf{r}_P \\ 1 \end{bmatrix} = {}^0T_n \begin{bmatrix} {}^n\mathbf{r}_P \\ 1 \end{bmatrix} = \begin{bmatrix} R_n & \mathbf{d}_n \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} {}^n\mathbf{r}_P \\ 1 \end{bmatrix} = \begin{bmatrix} R_n\, {}^n\mathbf{r}_P + \mathbf{d}_n \\ 1 \end{bmatrix}$$

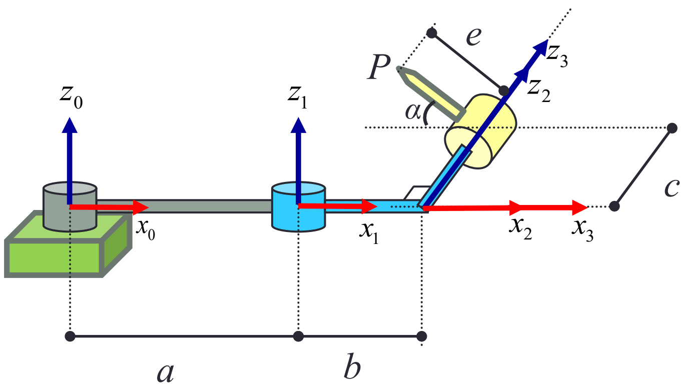
$${}^0T_n = {}^0T_1\, {}^1T_2 \dots {}^{n-1}T_n$$

✓ The pose of the end-effector where B_n is attached

$$R_n = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \begin{bmatrix} n_x & s_x & a_x \\ n_y & s_y & a_y \\ n_z & s_z & a_z \end{bmatrix}$$

Ex 3-3-1

$\alpha=\pi/6$. Find the position of tip point P.



No.	a_i	α_i	d_i	θ_i
1	a	0	0	θ_1
2	b	-90°	0	θ_2
3	0	0	0	θ_3

$$^{i-1}T_i = D_{z_{i-1}}(d_i)R_{z_{i-1}}(\theta_i)D_{x_i}(a_i)R_{x_i}(\alpha_i)$$

$$= \begin{bmatrix} c\theta_i & -c\alpha_i s\theta_i & s\alpha_i s\theta_i & a_i c\theta_i \\ s\theta_i & c\alpha_i c\theta_i & -s\alpha_i c\theta_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & ac\theta_1 \\ s\theta_1 & c\theta_1 & 0 & as\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ^1T_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & bc\theta_2 \\ s\theta_2 & c\theta_2 & 0 & bs\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ^2T_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

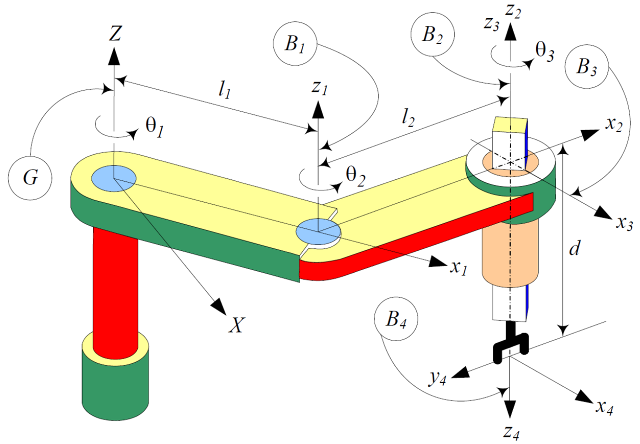
$$^0T_3 = {}^0T_1\,{}^1T_2\,{}^2T_3$$

$$= \begin{bmatrix} c(\theta1+\theta2)c(\theta3), & -c(\theta1+\theta2)s(\theta3), & -s(\theta1+\theta2), & bc(\theta1+\theta2)+ac(\theta1) \\ s(\theta1+\theta2)c(\theta3), & -s(\theta1+\theta2)s(\theta3), & c(\theta1+\theta2), & bs(\theta1+\theta2)+as(\theta1) \\ -s(\theta3), & -c(\theta3), & 0, & 0 \\ 0, & 0, & 0, & 1 \end{bmatrix}$$

$$P = {}^0T_3 \begin{bmatrix} ^3\mathbf{r}_P \\ 1 \end{bmatrix} = \begin{bmatrix} -e\cos\alpha \\ -e\sin\alpha \\ c \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} b*c(\theta1+\theta2)-c*s(\theta1+\theta2)+a*c(\theta1)-e*c(\theta1+\theta2)c(\alpha)c(\theta3)+e*c(\theta1+\theta2)s(\alpha)s(\theta3) \\ c*c(\theta1+\theta2)+b*s(\theta1+\theta2)+a*s(\theta1)-es(\theta1+\theta2)c(\alpha)c(\theta3)+e*s(\theta1+\theta2)s(\alpha)s(\theta3) \\ e*s(\alpha+\theta3) \\ 1 \end{bmatrix}$$

Forward Kinematics - SCARA Arm



$$^0T_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & l_1 c\theta_1 \\ s\theta_1 & c\theta_1 & 0 & l_1 s\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1T_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & l_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & l_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

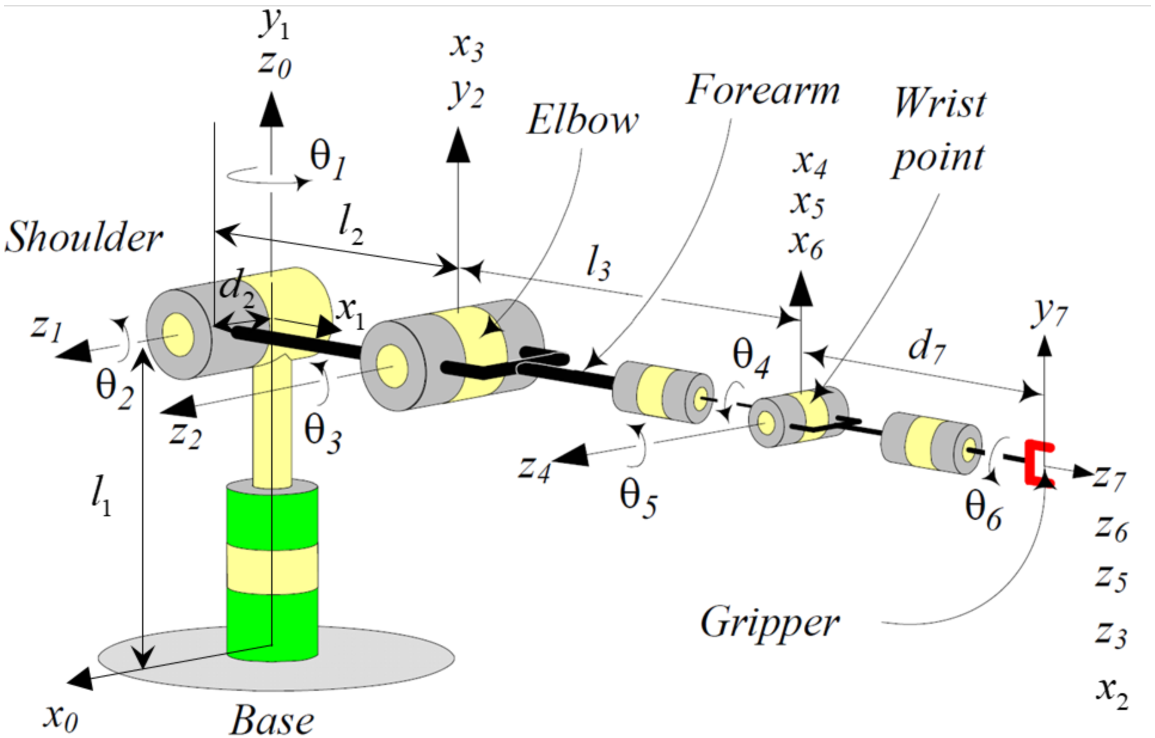
$$^2T_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^3T_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

No.	a_i	α_i	d_i	θ_i
1	l_1	0	0	θ_1
2	l_2	0	0	θ_2
3	0	0	0	θ_3
4	0	-180º	$d(d_0)$	0

$$^0T_4 = {}^0T_1\,{}^1T_2\,{}^2T_3\,{}^3T_4$$

$$= \begin{bmatrix} c(\theta_1+\theta_2+\theta_3) & s(\theta_1+\theta_2+\theta_3) & 0 & l_1 c\theta_1 + l_2 c(\theta_1+\theta_2) \\ s(\theta_1+\theta_2+\theta_3) & -c(\theta_1+\theta_2+\theta_3) & 0 & l_1 s\theta_1 + l_2 s(\theta_1+\theta_2) \\ 0 & 0 & -1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward Kinematics – 6R Manipulator



No.	a_i	α_i	d_i	θ_i
1	0	90º	l_1	θ_1(90º)
2	l_2	0	d_2	θ_2
3	0	90º	0	θ_3(90º)
4	0	-90º	l_3	θ_4
5	0	90º	0	θ_5
6	0	0	0	θ_6

$$
{}^{i-1}T_i = D_{z_{i-1}}(d_i)R_{z_{i-1}}(\theta_i)D_{x_i}(a_i)R_{x_i}(\alpha_i)
$$

$$
= \begin{bmatrix} c\theta_i & -c\alpha_i s\theta_i & s\alpha_i s\theta_i & a_i c\theta_i \\ s\theta_i & c\alpha_i c\theta_i & -s\alpha_i c\theta_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Forward Kinematics – 6R Manipulator

$$^{0}T_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ^{1}T_2 = \begin{bmatrix} c_2 & -s_2 & 0 & l_2 c_2 \\ s_2 & c_2 & 0 & l_2 s_2 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ^{2}T_3 = \begin{bmatrix} c_3 & 0 & s_3 & 0 \\ s_3 & 0 & -c_3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ^{3}T_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{4}T_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ^{5}T_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ^{6}T_7 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

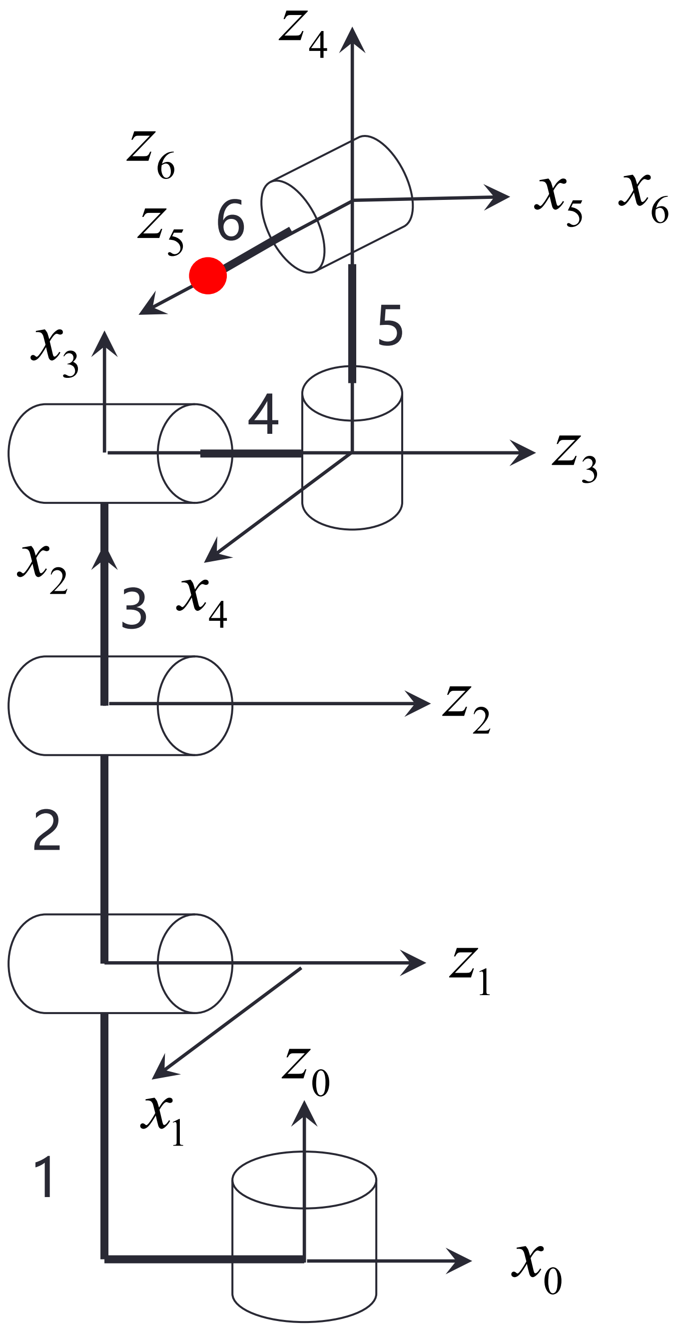
$$^{0}T_3 = {^{0}T_1}\,{^{1}T_2}\,{^{2}T_3} = \begin{bmatrix} c_1 c_{23} & s_1 & c_1 s_{23} & l_2 c_1 c_2 + d_2 s_1 \\ s_1 c_{23} & -c_1 & s_1 s_{23} & l_2 s_1 c_2 - d_2 c_1 \\ s_{23} & 0 & -c_{23} & l_1 + l_2 s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{3}T_6 = {^{3}T_4}\,{^{4}T_5}\,{^{5}T_6} = \begin{bmatrix} c_4 c_5 c_6 - s_4 s_6 & -s_4 c_6 - c_4 c_5 s_6 & c_4 s_5 & 0 \\ c_4 s_6 + s_4 c_5 c_6 & c_4 c_6 - s_4 c_5 s_6 & s_4 s_5 & 0 \\ -s_5 c_6 & s_5 s_6 & c_5 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_6 = {^{0}T_1}\,{^{1}T_2}\,{^{2}T_3}\,{^{3}T_4}\,{^{4}T_5}\,{^{5}T_6} = {^{0}T_3}\,{^{3}T_6} \qquad ^{0}T_7 = {^{0}T_3}\,{^{3}T_6}\,{^{6}T_7} = {^{0}T_6}\,{^{6}T_7}$$

Code Session

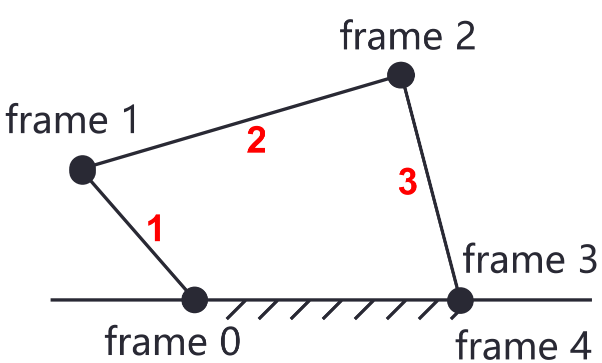
Ch3_3.m



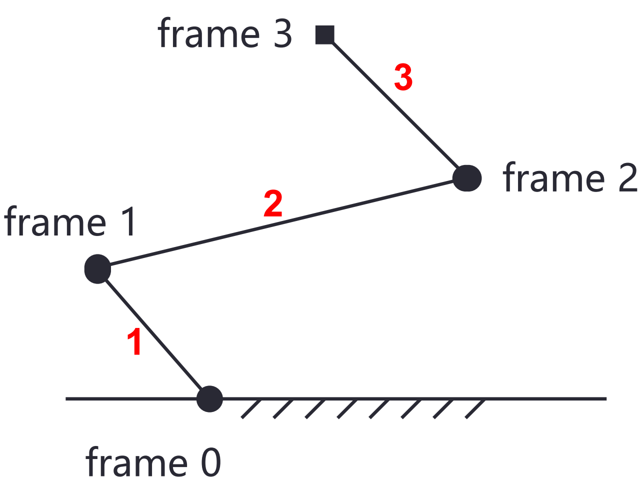
No.	a_i	α_i	d_i	θ_i
1	0	-90º	*1*	θ_1(90º)
2	*2*	0	d_1	θ_2
3	3	0	0	θ_3(90º)
4	0	-90º	*d2*	θ_4
5	0	-90º	d3	θ_5
6	0	0	0	θ_6

3.4 Non-standard DH Parameters

✓ The standard DH parameters will be inefficient if the mechanism is a closed chain where the base binary link connects both link 1 and link *n*, respectively. It will cause the ambiguity because the base link will have two different attached coordinate frames.



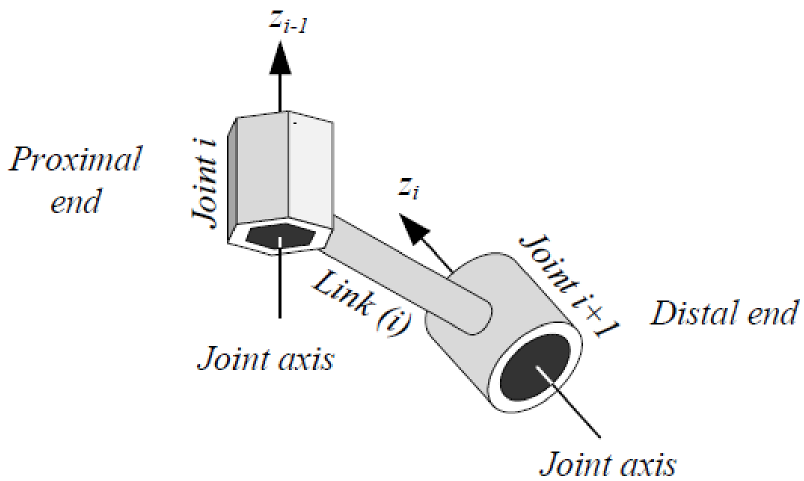
Closed chain
3 active links
4 joints



Open chain
3 active links
3 joints

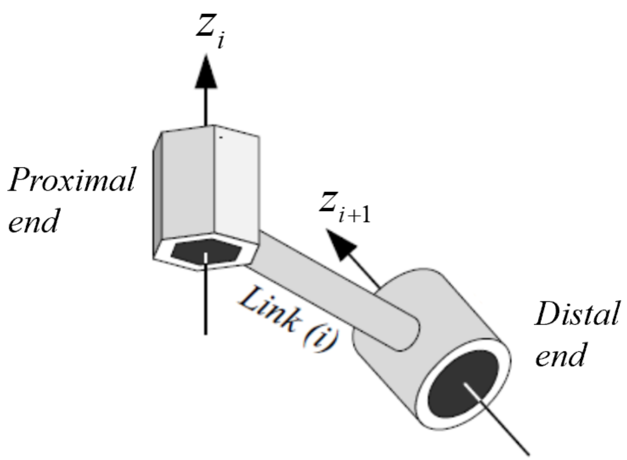
3.4 Non-standard DH Parameters

- ✓ In order to cope with the closed chain mechanism, standard DH parameters can be modified in the way that changing the location of the body attached coordinated frames.



Standard

Local frame *i* is setup at the distal end of the binary link



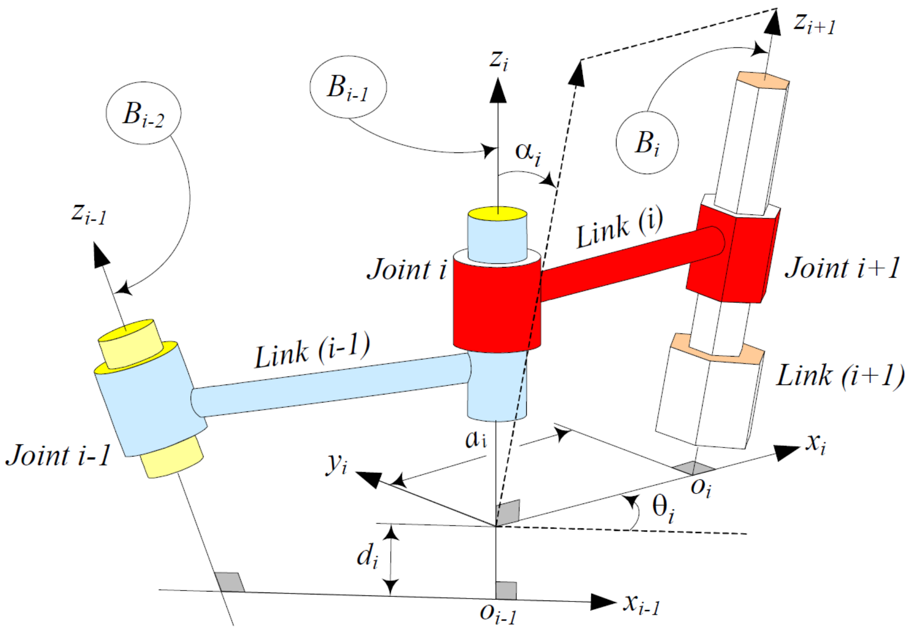
Non-standard

Local frame *i* is setup at the proximal end of the binary link

3.4 Non-standard DH Parameters

✓ With non-standard configurations of frames, the homogeneous transformation matrix can be derived in a similar manner. Initially, frame B_i coincides with $B_{i\text{-}1}$. It becomes the present state after four steps of homogeneous transformations.

$B_{i\text{-}1}$ is a global frame and B_i is a local frame



1. B_i rotates $\alpha_{i\text{-}1}$ about x_i
2. B_i translates $a_{i\text{-}1}$ along x_i
3. B_i translates d_i along z_i
4. B_i rotates θ_i about z_i

Homogeneous Transformation Matrix

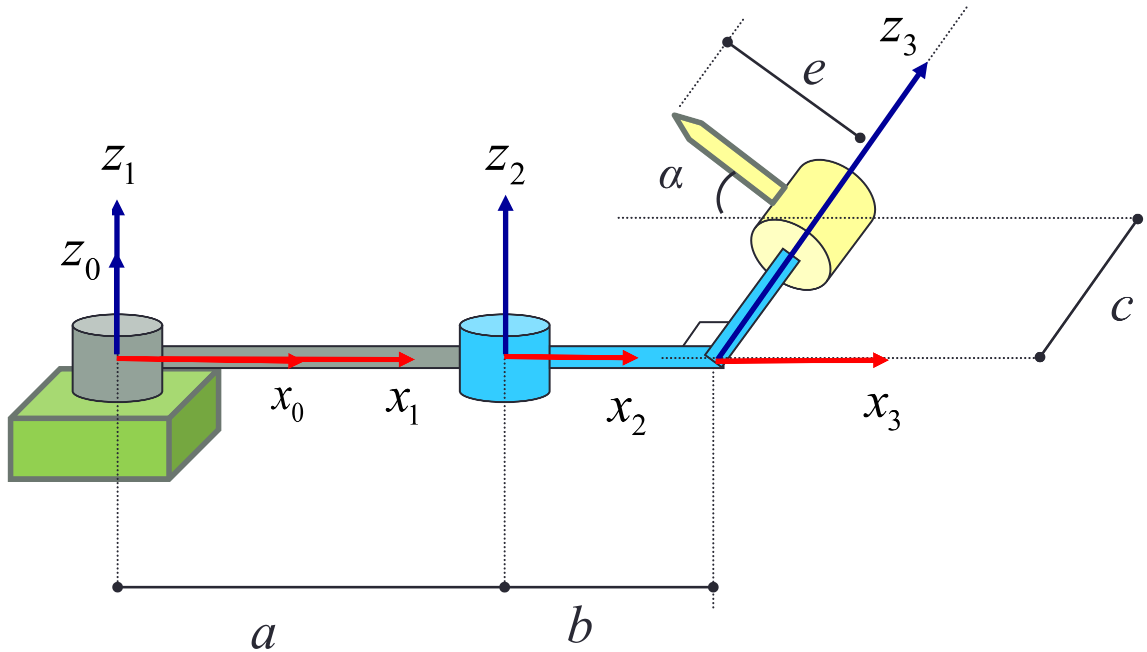
- ✓ By pre- or post-multiplications of four homogeneous transformation matrices, the overall transformation from frame B_{i-1} to B_i can be obtained by

$$
\begin{aligned}
{}^{i-1}T_i &= R_{x_i}(\alpha_{i-1}) D_{x_i}(a_{i-1}) D_{z_i}(d_i) R_{z_i}(\theta_i) \\
&= \begin{bmatrix} \mathrm{c}\theta_i & -\mathrm{s}\theta_i & 0 & a_{i-1} \\ \mathrm{s}\theta_i \mathrm{c}\alpha_{i-1} & \mathrm{c}\alpha_{i-1}\mathrm{c}\theta_i & -\mathrm{s}\alpha_{i-1} & -d_i \mathrm{s}\alpha_{i-1} \\ \mathrm{s}\alpha_{i-1}\mathrm{s}\theta_i & \mathrm{s}\alpha_{i-1}\mathrm{c}\theta_i & \mathrm{c}\alpha_{i-1} & d_i \mathrm{c}\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

- ✓ An advantage of the non-standard DH method is that the rotation θ_i is around the z_i-axis and the joint number is the same as the coordinate number;
- ✓ A disadvantage is that the transformation matrix is a mix of $i-1$ and i indices.

Ex 3-4-1

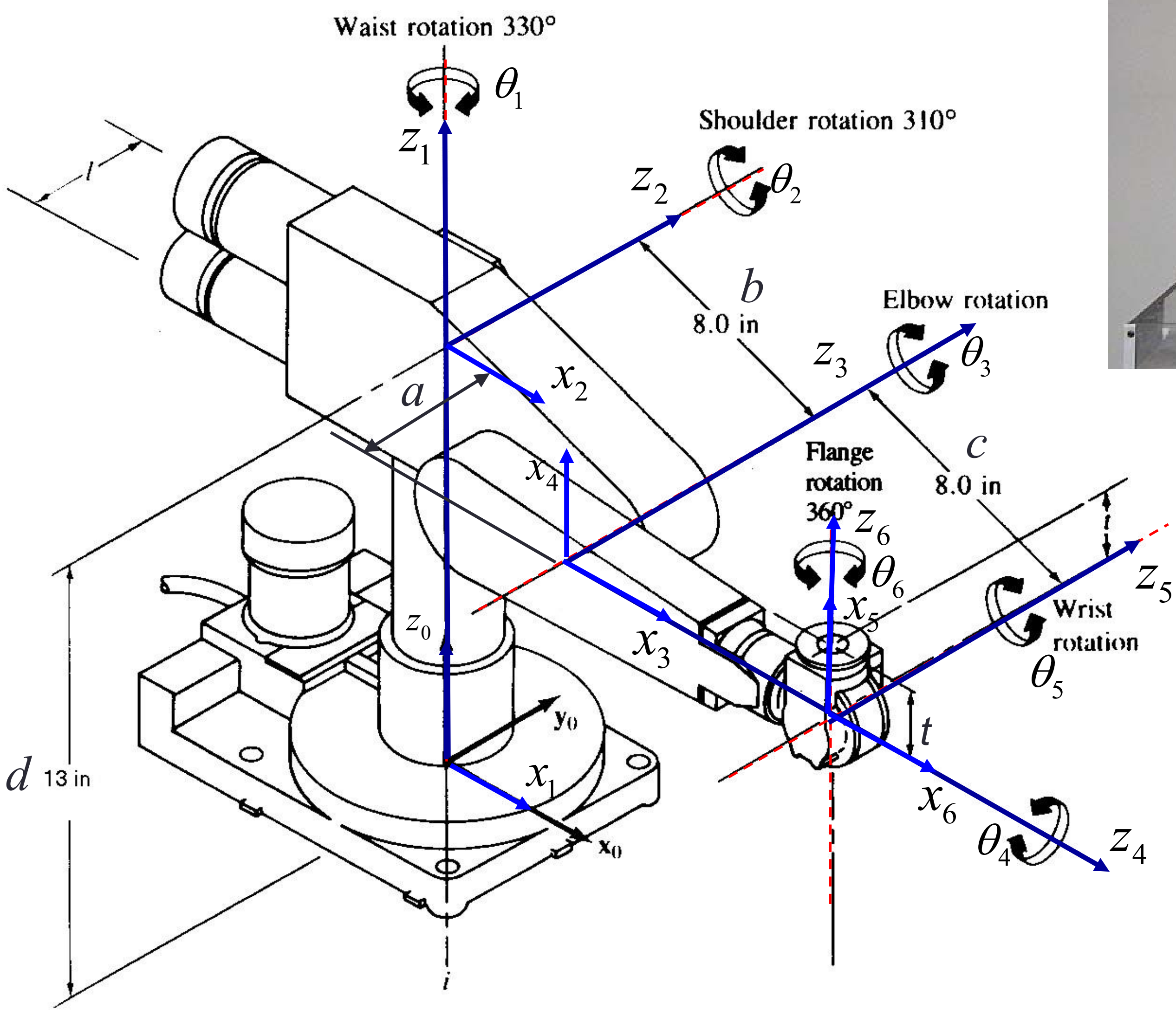
Fill in the table of non-standard DH parameters for the 3R robotic arm



Frame No.	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	a	0	0	θ_2
3	b	-90°	0	θ_3

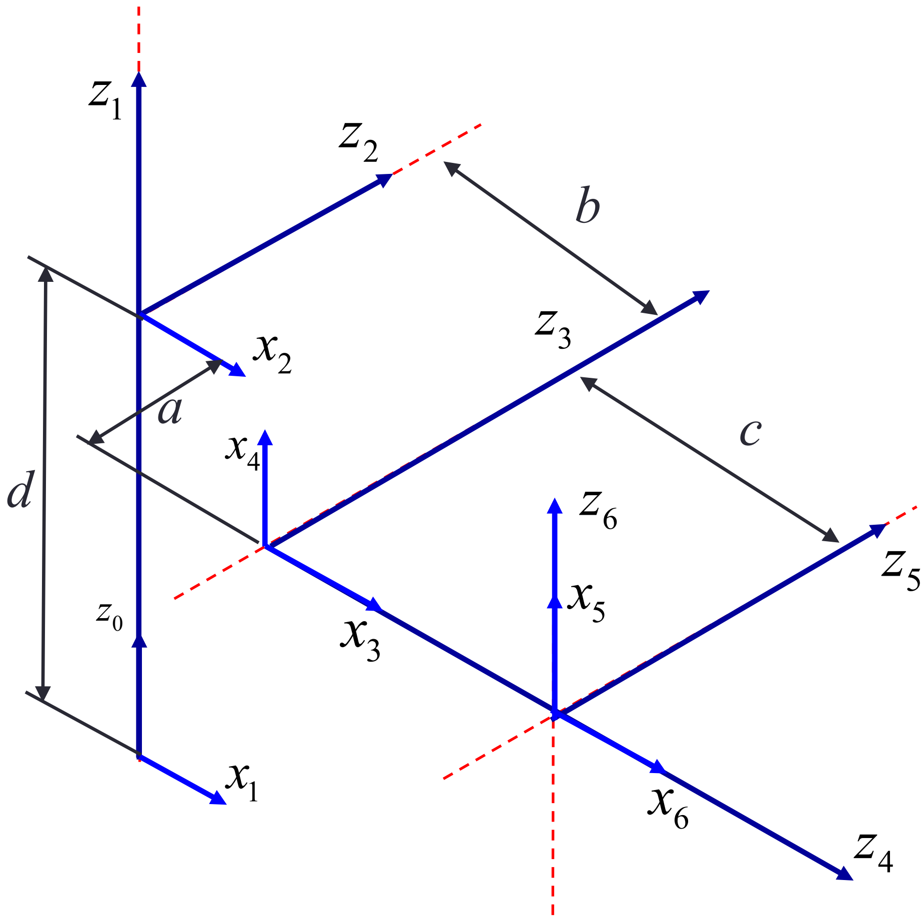
Ex 3-4-2

Fill in the table of non-standard DH parameters for PUMA200 robot



Ex 3-4-2

Fill in the table of DH parameters for the PUMA200 robot



No.	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	-90º	d	θ_2
3	b	0	-a	θ_3
4	0	-90º	0	θ4(-90º)
5	0	90º	c	θ_5
6	0	90º	0	θ_6(90º)

Code Session

Ch3_4.m