

消防站问题

问题建模

设

$$c_{ij} = \begin{cases} 1, & \text{地区}i\text{由消防站}j\text{负责时} \\ 0, & \text{地区}i\text{不由消防站}j\text{负责时} \end{cases}$$
$$x_j = \begin{cases} 1, & \text{消防站}j\text{启用} \\ 0, & \text{消防站}j\text{关闭} \end{cases}$$

其中

$$i = 1, \cdots, 11$$
$$j = 1, 2, 3, 4$$

根据题目得到 c_{ij} 情况如下:

地区\消防站	①	②	③	④
1	1	1	0	0
2	1	1	0	0
3	1	0	0	0
4	1	0	1	0
5	0	0	1	0
6	1	0	1	1
7	1	0	0	1
8	1	1	0	1
9	0	1	0	1
10	0	0	0	1
11	0	0	1	1

则建立如下整数规划问题数学模型:

$$\min z = x_1 + x_2 + x_3 + x_4$$
$$\text{s.t.} \begin{cases} \sum_{j=1}^4 c_{ij}x_j \geq 1, & (i = 1, \cdots, 11) \\ x_j \in \{0, 1\} \end{cases}$$

代码

```
MODEL:
sets:
    num_i/1..11/:b;
    num_j/1..4/:x,c;
    link(num_i,num_j):a;
endsets

data:
    b=1,1,1,1,1,1,1,1,1,1,1,1;
    c=1,1,1,1;
    a=1,1,0,0,
      1,1,0,0,
      1,0,0,0,
```

```

1,0,1,0,
0,0,1,0,
1,0,1,1,
1,0,0,1,
1,1,0,1,
0,1,0,1,
0,0,0,1,
0,0,1,1;
enddata

[OBJ]min=@sum(num_j(j):c(j)*x(j));
@for(num_i(i):@sum(num_j(j):a(i,j)*x(j))>=b(i));
@for(num_j(j):@BIN(x(j)));
END

```

结果

Lingo 18.0 - [Solution Report - Lingo2]

File Edit Solver Window Help

Global optimal solution found.

Objective value:	3.000000
Objective bound:	3.000000
Infeasibilities:	0.000000
Extended solver steps:	0
Total solver iterations:	0
Elapsed runtime seconds:	0.12

Model Class: PILP

Total variables:	4
Nonlinear variables:	0
Integer variables:	4

Total constraints:	12
Nonlinear constraints:	0

Total nonzeros:	25
Nonlinear nonzeros:	0

Variable	Value	Reduced Cost
B(1)	1.000000	0.000000
B(2)	1.000000	0.000000
B(3)	1.000000	0.000000
B(4)	1.000000	0.000000
B(5)	1.000000	0.000000
B(6)	1.000000	0.000000
B(7)	1.000000	0.000000
B(8)	1.000000	0.000000
B(9)	1.000000	0.000000
B(10)	1.000000	0.000000
B(11)	1.000000	0.000000
X(1)	1.000000	1.000000
X(2)	0.000000	1.000000
X(3)	1.000000	1.000000
X(4)	1.000000	1.000000
C(1)	1.000000	0.000000
C(2)	1.000000	0.000000
C(3)	1.000000	0.000000
C(4)	1.000000	0.000000
A(1, 1)	1.000000	0.000000

For Help, press F1

NUM

Ln 24, Col 77 2:10 pm

可以看到解为(1, 0, 1, 1)

从而可以减少消防站的数量，应关闭消防站②