浙江大学
ZHEJIANG UNIVERSITY

空间站机械臂运动仿真

Simulation of Robotic Arm in Space Station

日期：2023.12

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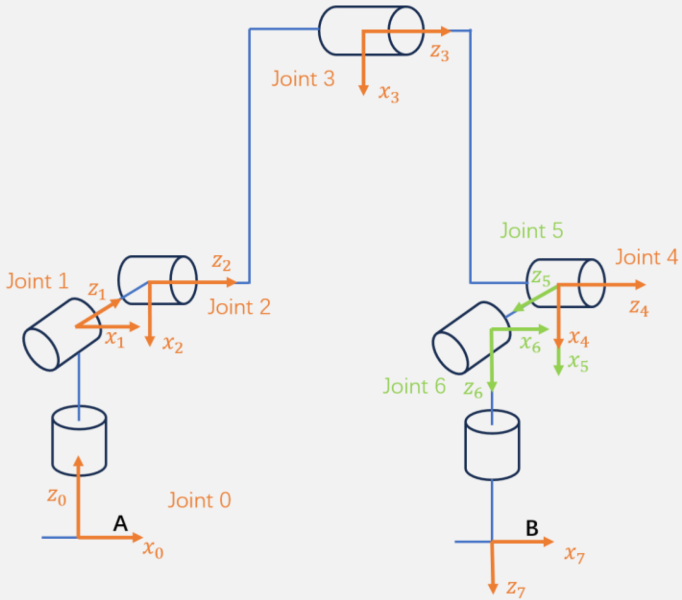
DH 参数表构建

DH parameters

取A为基座



$No.$	a_i	α_i	d_i	θ_i
1	0	-90°	120mm	θ_1
2	0	90°	100mm	$\theta_2 + 90°$
3	-400mm	0	150mm	θ_3
4	400mm	0	150mm	θ_4
5	0	90°	0	θ_5
6	0	90°	100mm	$\theta_6 + 90°$
7	0	0	120mm	θ_7

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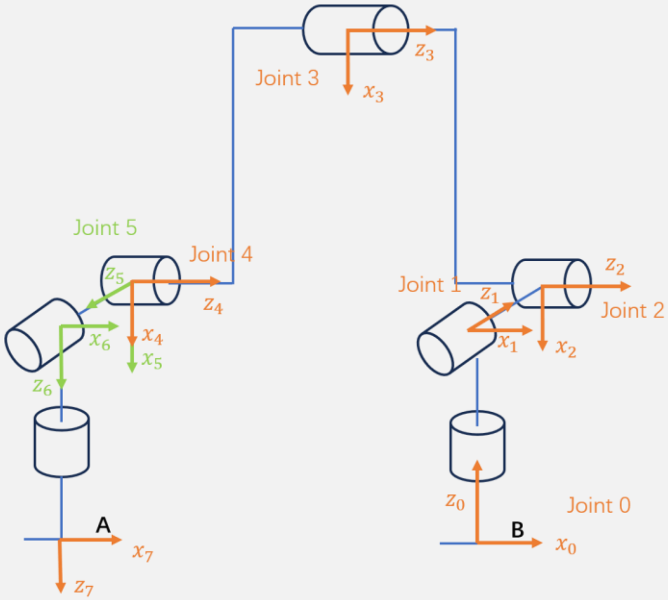
DH 参数表构建

DH parameters

取B为基座



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正逆运动学求解

Forward & Inverse Kinematics

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逆运动学求解

Inverse Kinematics

求解析解

首先根据机械臂运动的特征

我们可以假设第二个关节角$\theta_2 = 0$

根据逆运动学基本公式

$$^0T_7 = {}^0T_1 \cdot {}^1T_2 \cdot {}^2T_3 \cdot {}^3T_4 \cdot {}^4T_5 \cdot {}^5T_6 \cdot {}^6T_7$$

$$^0T_7 = \begin{pmatrix} n_x & o_x & a_x & d_x \\ n_y & o_y & a_y & d_y \\ n_z & o_z & a_z & d_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^1T_7 = {}^1T_2 \cdot {}^2T_3 \cdot {}^3T_4 \cdot {}^4T_5 \cdot {}^5T_6 \cdot {}^6T_7 = {}^0T_1^{-1} \cdot {}^0T_7$$

取第(3,1)(3,2)(3,3)号元素相等可以得到等式

$$\begin{cases} a_y c_1 + a_x s_1 = s_{345} + s_6 \\ n_y c_1 + n_x s_1 = s_{345} c_6 c_7 - c_{345} s_7 \\ o_y c_1 + o_x s_1 = -s_{345} c_6 s_7 - c_{345} c_7 \end{cases}$$

对三个式子做完全平方和

可以得到θ_1

$$(n_y^2 + o_y^2 + a_y^2) c_1^2 + (n_x^2 + o_x^2 + a_x^2) s_1^2 \\ -2(n_x n_y + o_x o_y + a_x a_y) s_1 c_1 = 1$$

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逆运动学求解

Inverse Kinematics

求解析解

利用公式

$$ {}^{1}T_5 = {}^{1}T_2 \cdot {}^{2}T_3 \cdot {}^{3}T_4 \cdot {}^{4}T_5 = {}^{0}T_1^{-1} \cdot {}^{0}T_7 \cdot {}^{6}T_7^{-1} \cdot {}^{5}T_6^{-1} $$

取第(2,2)(2,3)号元素相等可以得到等式

$$ \begin{cases} o_z c_6 s_7 - n_z c_6 c_7 - a_z s_6 = 0 \\ o_z c_7 + n_z s_7 = 0 \end{cases} $$

容易得到

$$ \begin{cases} \theta_7 = -atan\dfrac{o_z}{n_z} \\ \theta_6 = \dfrac{o_z s_7 - n_z c_7}{a_z} \end{cases} $$

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逆运动学求解

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取第(1,1)号元素相等可以得到等式

$$a_x c_1 s_6 + a_y s_1 s_6 + n_x c_1 c_6 c_7 + n_y c_6 c_7 s_1 - o_x c_1 c_6 s_7 - o_y c_6 s_1 s_7 = c_{345}$$

即可得到

$$\theta_3 + \theta_4 + \theta_5 = acos(a_x c_1 s_6 + a_y s_1 s_6 + n_x c_1 c_6 c_7 + n_y c_6 c_7 s_1 - o_x c_1 c_6 s_7 - o_y c_6 s_1 s_7)$$

逆运动学求解

Inverse Kinematics

求解析解

利用公式

$$^{1}T_4 = {}^{1}T_2 \cdot {}^{2}T_3 \cdot {}^{3}T_4 = {}^{0}T_1^{-1} \cdot {}^{0}T_7 \cdot {}^{6}T_7^{-1} \cdot {}^{5}T_6^{-1} \cdot {}^{4}T_5^{-1}$$

取第(2,1)号元素相等可以得到

$$\theta_5 = atan\left(\frac{o_z c_6 s_7 - n_z c_6 c_7 - a_z s_6}{o_z c_7 + n_z s_7}\right)$$

取第(2,4)号元素相等可以得到等式

$$d_y c_1 - d_x s_1 + a_x d_7 s_1 - a_y d_7 c_1 + d_6 n_x s_1 s_7 - d_6 o_y c_1 c_7 - d_6 n_y c_1 s_7 + d_6 o_x c_7 s_1 = a_4 s_{34} + a_3 s_3$$

配合前述θ_5 与$\theta_3 + \theta_4 + \theta_5$的值，即可求得$\theta_3$与$\theta_4$，进而求得所有解析解

正运动学求解

——逆运动学合法性的检查

Forward Kinematics

1. 齐次变换矩阵求解

$$T = \begin{pmatrix} c\theta & -c\alpha s\theta & s\alpha s\theta & ac\theta \\ s\theta & c\alpha c\theta & -s\alpha c\theta & as\theta \\ 0 & s\alpha & c\alpha & d \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T = \prod^{i-1} T_i$$

2. 反解RPY方位角，并与T中的位置合并

	$\psi(Roll)$	$\phi(Pitch)$	$\theta(Yaw)$
$\phi \neq \frac{\pi}{2}$	$-tan^{-1}\frac{R_{23}}{R_{33}}$	$\pi^{\delta} + (-)^{\delta}$ $sin^{-1}R_{13}$	$-tan^{-1}\frac{R_{12}}{R_{11}}$
$\phi = \frac{\pi}{2}$	0	$\pm\frac{\pi}{2}$	$-tan^{-1}\frac{R_{21}}{R_{32}}$

轨迹规划

Trajectory Planning

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轨迹规划

Trajectory Planning

1. quinticCurvePlanning()

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

利用上述多项式进行五次规划,
得到系数矩阵$\{\boldsymbol{a}_i\}$

2. quinticCurveExcute()

利用前述系数矩阵$\{\boldsymbol{a}_i\}$,
输入当前时刻
即可得到当前时间各关节角的位置

Note: 不同过程需要进行不同基座间的变换

仿真模拟

Simulation

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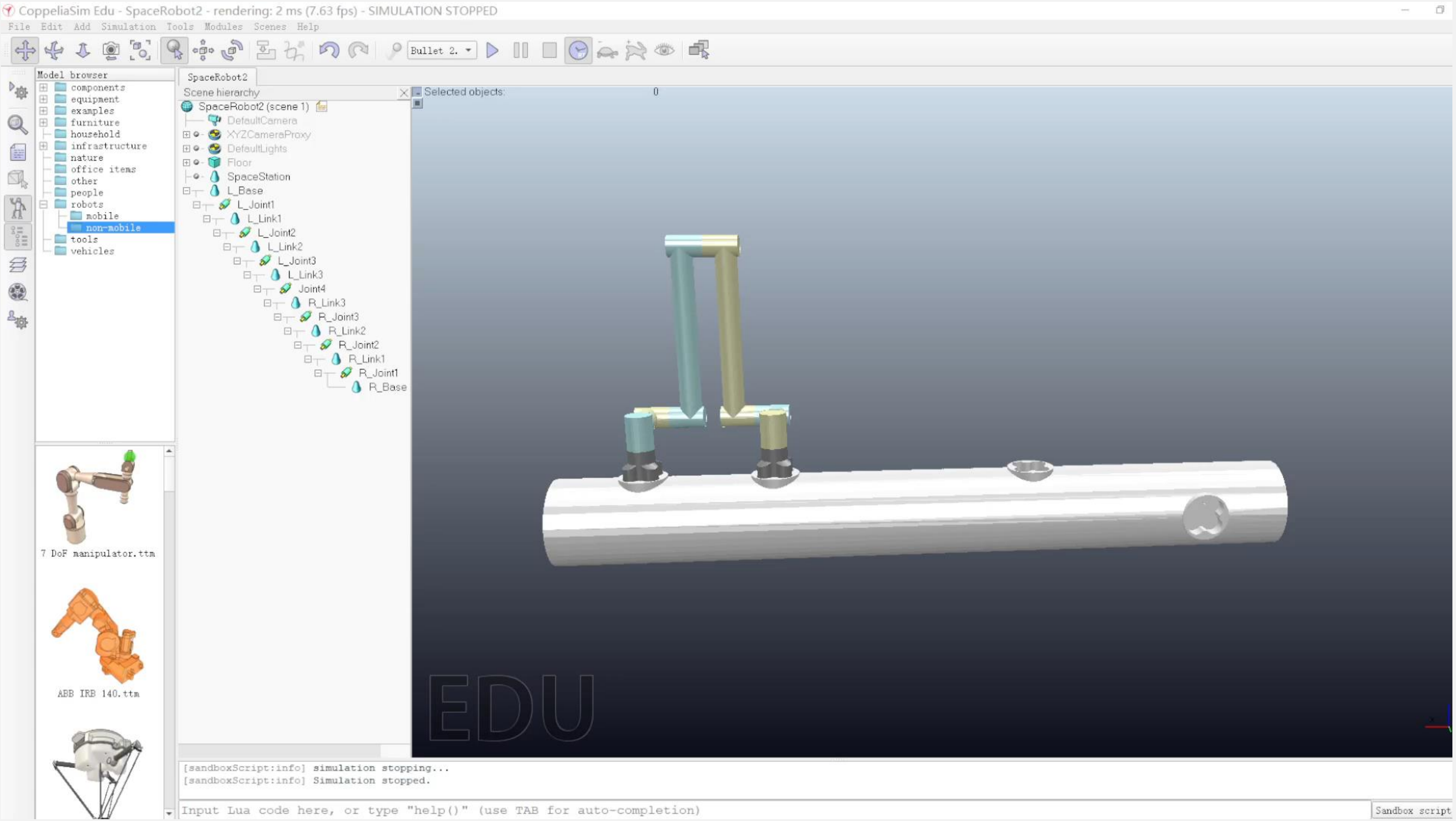
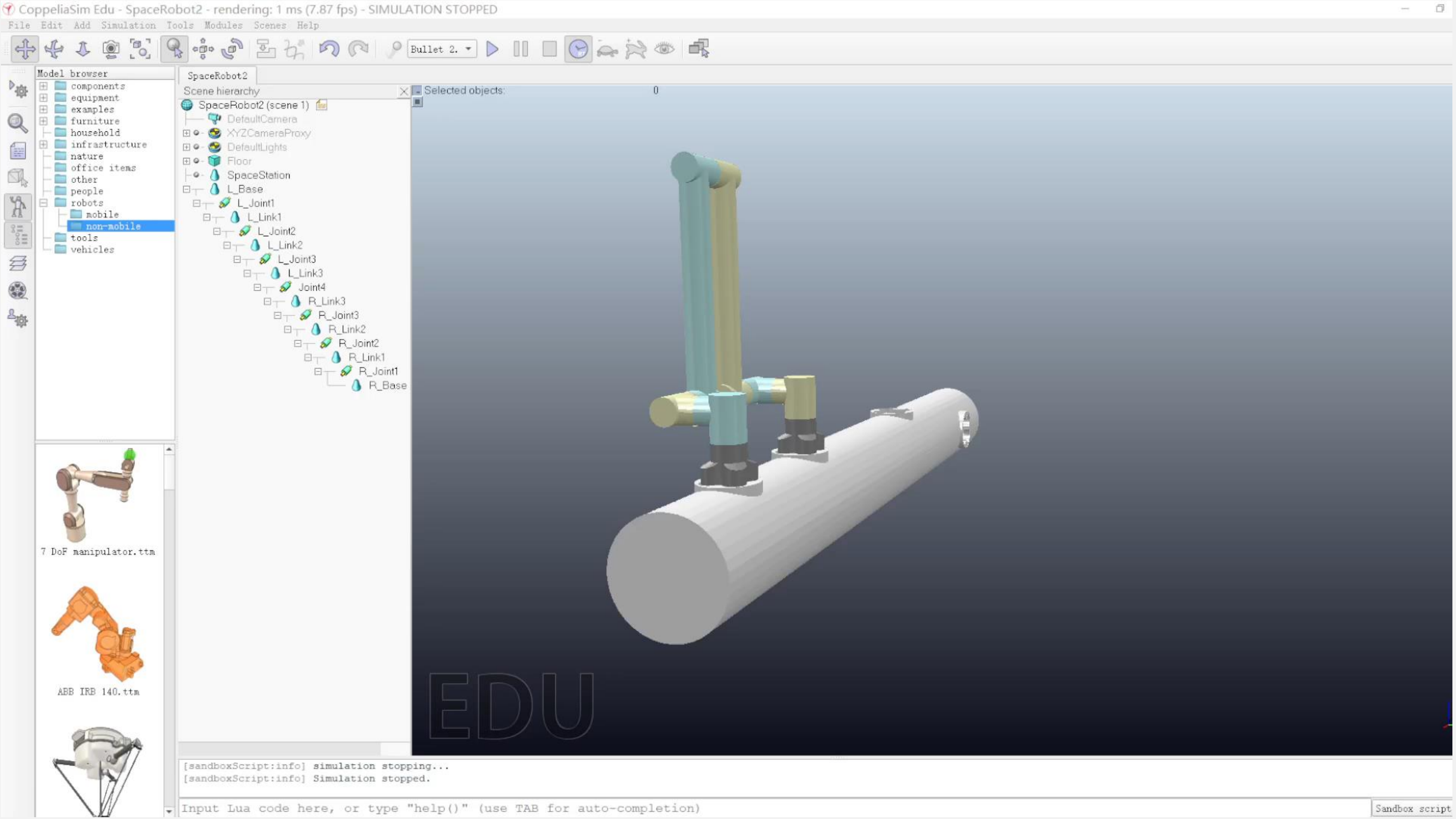

CoppeliaSim Edu - SpaceRobot2 - rendering: 2 ms (7.63 fps) - SIMULATION STOPPED
File Edit Add Simulation Tools Modules Scenes Help
Bullet 2.
Model browser
components
equipment
examples
furniture
household
infrastructure
nature
office items
other
people
robots
mobile
non-mobile
tools
vehicles
SpaceRobot2
Scene hierarchy
SpaceRobot2 (scene 1)
DefaultCamera
XYZCameraProxy
DefaultLights
Floor
SpaceStation
L_Base
L_Joint1
L_Link1
L_Joint2
L_Link2
L_Joint3
L_Link3
Joint4
R_Link3
R_Joint3
R_Link2
R_Joint2
R_Link1
R_Joint1
R_Base
Selected objects: 0
7 DoF manipulator.ttm
ABB IRB 140.ttm
EDU
[sandboxScript:info] simulation stopping...
[sandboxScript:info] Simulation stopped.
Input Lua code here, or type "help()" (use TAB for auto-completion)
Sandbox script


CoppeliaSim Edu - SpaceRobot2 - rendering: 1 ms (7.87 fps) - SIMULATION STOPPED
File Edit Add Simulation Tools Modules Scenes Help
Bullet 2.
Model browser
components
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感谢各位批评指正

Thank you for the criticism of the experts

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