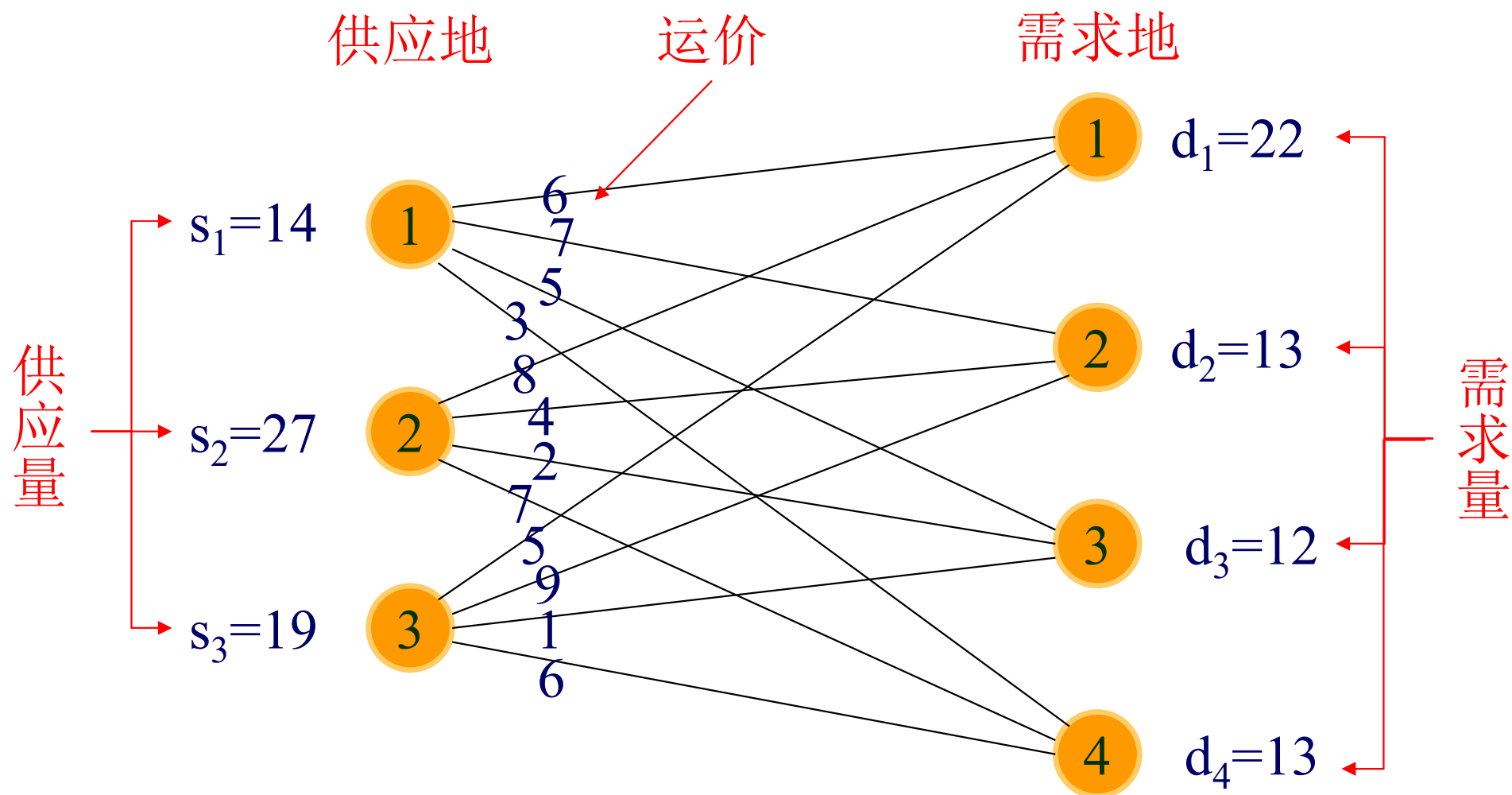


# 第三章 运输规划简介

- 运输问题及其数学模型
- 产销平衡问题的表上作业法
- 产销不平衡问题
- 有转运问题
- 应用举例



# 问题提出



# 数学模型

$$\begin{array}{lcl}
 \min & z = & 6x_{11} + 7x_{12} + 5x_{13} + 3x_{14} + 8x_{21} + 4x_{22} + 2x_{23} + 7x_{24} + 5x_{31} + 9x_{32} + 10x_{33} + 6x_{34} \\
 s.t. & & \\
 & x_{11} + x_{12} + x_{13} + x_{14} & = 14 \\
 & & x_{21} + x_{22} + x_{23} + x_{24} = 27 \\
 & & & x_{31} + x_{32} + x_{33} + x_{34} = 19 \\
 & x_{11} & + x_{21} + x_{31} = 22 \\
 & & x_{12} + x_{22} + x_{32} = 13 \\
 & & & x_{13} + x_{23} + x_{33} = 12 \\
 & & & & x_{14} + x_{24} + x_{34} = 13 \\
 & x_{11} & x_{12} & x_{13} & x_{14} & x_{21} & x_{22} & x_{23} & x_{24} & x_{31} & x_{32} & x_{33} & x_{34} & = & 0
 \end{array}$$

# 产销平衡问题的一般模型

$$\min z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_{j=1}^n x_{ij} = a_i$$

$$\sum_{i=1}^m x_{ij} = b_j$$

$$x_{ij} \geq 0 \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n$$

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$$

产销平衡约束

# 运输问题的特点

## 1 .A矩阵

$$P_{ij} = [0, \dots, 0, 1, 0, \dots, 0, 1, 0, \dots, 0]^T$$

第*i*个

第*m+j*个

## 2 .存在有界最优解

$$x_{ij} = \frac{a_i b_j}{\sum_{i=1}^m a_i} = \frac{a_i b_j}{\sum_{j=1}^n b_j}$$

可行解

# 单纯形法解的特点

可以证明： $\text{rank}(A) = m+n-1$

基变量个数： $m+n-1$

单纯形法变量数目： $m \times n + (m+n-1)$

需要寻找新的方法

# 运输表

	1	2	3	4	
1	6 $x_{11}$	7 $x_{12}$	5 $x_{13}$	3 $x_{14}$	14
2	8 $x_{21}$	4 $x_{22}$	2 $x_{23}$	7 $x_{24}$	27
3	5 $x_{31}$	9 $x_{32}$	10 $x_{33}$	6 $x_{34}$	19
	22	13	12	13	

# 表上作业法

	1	2	3	4	
1	6	7	5	3	14
2	8	4	2	7	27
3	5	9	10	6	19
	22	13	12	13	

The table illustrates a transportation problem solution. The top row (1-4) and left column (1-3) represent supply and demand nodes. The bottom row (22, 13, 12, 13) represents the total supply and demand for each column. The cells contain the following values:

- Row 1: (1,1)=6, (1,2)=7, (1,3)=5, (1,4)=3
- Row 2: (2,1)=8, (2,2)=4, (2,3)=2, (2,4)=7
- Row 3: (3,1)=5, (3,2)=9, (3,3)=10, (3,4)=6

The rightmost column contains the total supply/demand for each row: 14, 27, 19. The bottom row contains the total supply/demand for each column: 22, 13, 12, 13. The cells (1,2), (1,3), (1,4), (2,4), (3,1), (3,2), (3,3), and (3,4) contain circled values: 5, 5, 7, 9, -11, -3, 6, and 13 respectively. The cells (1,1), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), and (3,4) contain red values: 14, 8, 13, 6, -11, -3, 6, and 13 respectively. Yellow lines connect the cells (2,2) and (3,2), (2,3) and (3,3), and (3,2) and (3,3).

$$\sigma_{32} = c_{32} - c_{22} + c_{23} - c_{33} = 9 - 4 + 2 - 10 = -3$$



# 运输问题的对偶问题

$$\max w = \sum_{i=1}^m a_i u_i + \sum_{j=1}^n b_j v_j$$

$$\text{s.t.} \quad u_i + v_j \leq c_{ij} \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n$$

$$u_i, v_j \text{ free}$$

$$\text{检验数:} \quad \sigma_{ij} = c_{ij} - \mathbf{y}^T \mathbf{p}_{ij} = c_{ij} - (u_i + v_j)$$

$$\text{基变量部分:} \quad u_i + v_j = c_{ij}$$

# 产销不平衡问题的数学模型

$$\sum_{i=1}^m a_i > \sum_{j=1}^n b_j \quad \text{产大于销问题}$$

$$\min z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_{j=1}^n x_{ij} \leq a_i$$

$$\sum_{i=1}^m x_{ij} = b_j$$

$$x_{ij} \geq 0 \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n$$

# 标准化

思路：化为平衡问题

方法：增加一个假象的销地  $n+1$ ,

$$\min z = \sum_{i=1}^m \sum_{j=1}^{n+1} c_{ij} x_{ij} \quad c_{i,n+1} = 0 \quad i = 1, \dots, m$$

$$\text{s.t.} \quad \sum_{j=1}^{n+1} x_{ij} = a_i$$

$$\sum_{i=1}^m x_{ij} = b_j \quad b_{n+1} = \sum_{i=1}^m a_i - \sum_{j=1}^n b_j$$

$$x_{ij} \geq 0 \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n+1$$

# 产小于销问题

思路：如果产小于销问题如何处理？

方法：增加一个假象的产地  $m+1$ ,

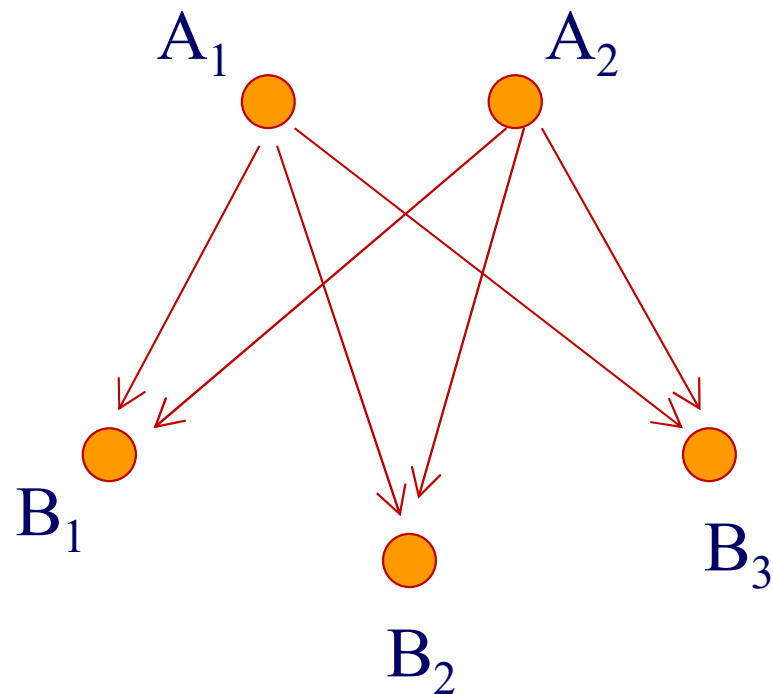
$$\min z = \sum_{i=1}^{m+1} \sum_{j=1}^n c_{ij} x_{ij} \quad c_{m+1,j} = 0 \quad i = j, \dots, n$$

$$\text{s.t.} \quad \sum_{j=1}^{n+1} x_{ij} = a_i \quad a_{m+1} = \sum_{j=1}^n b_j - \sum_{i=1}^m a_i$$

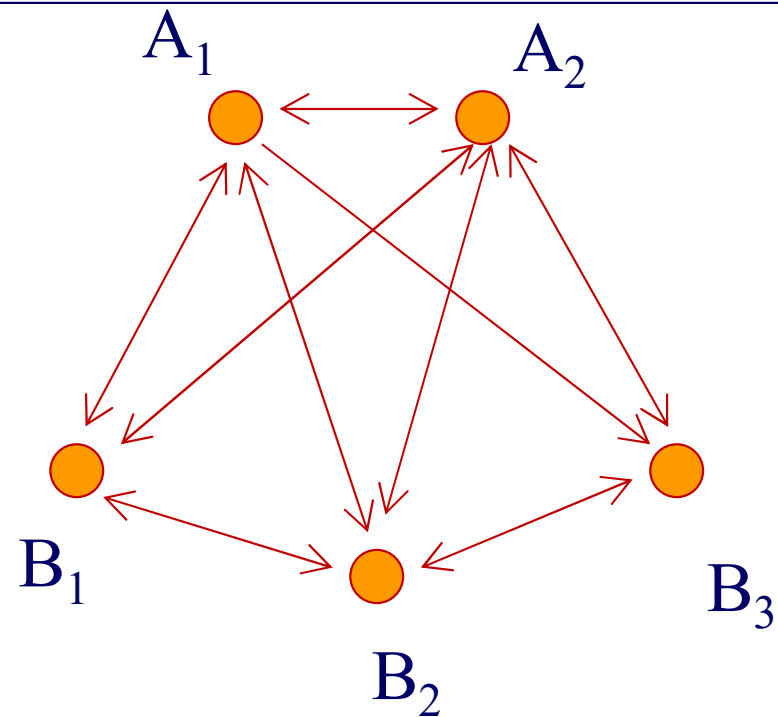
$$\sum_{i=1}^m x_{ij} = b_j$$

$$x_{ij} \geq 0 \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n+1$$

# 有转运问题



无转运问题



有转运问题

新增“产地”： $a_{m+j} = 0 \quad j = 1, 2, \dots, n$

新增“销地”： $b_i = 0 \quad i = 1, 2, \dots, m$

# 有转运问题

$$\min z = \sum_{\substack{i=1 \\ i \neq j}}^{m+n} \sum_{j=1}^{m+n} c_{ij} x_{ij} + \sum_{i=1}^{m+n} c_i t_i$$

$$\text{s.t.} \quad \sum_{j=1, j \neq i}^{m+n} x_{ij} = a_i + t_i \quad i = 1, 2, \dots, m+n$$

$$\sum_{i=1, i \neq j}^{m+n} x_{ij} = b_j + t_j \quad j = 1, 2, \dots, m+n$$

$$x_{ij} \geq 0, t_i \geq 0 \quad i, j = 1, 2, \dots, m+n; i \neq j$$

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$$

产销平衡约束