5.7

Introduction

$$I = \int_0^1 \sin^2 \sqrt{100x} dx$$

Part A: Nested Integration

Part B: Romberg Integration

In [6]:

```
%%time
import numpy as np
f = lambda x: np.sin(np.sqrt(100*x))**2

# Part A
print('Part A')
N = 1
a = 0
b = 1
error = 1
maxerror = 1e-6
h = b-a
I = h/2*(f(b)-f(a))
while error > maxerror:
    Iold = I
    I /= 2
    N *= 2
    h /=2
    for k in range(1,N,2):
        I += h*f(a+k*h)
    error = abs((I-Iold)/3/I)
    print(f'N ={N}, I= {I:.8f}, error = {error :.7f}')
    if N > 1e10:
        break

#Part B
print('\nPart B')
R = []
N = 1
h = (b-a)
I = h*(f(a)+f(b))/2
R.append(I)
error = 1
maxerror = 1e-6
i = 1
while error > maxerror:
    N *=2
    i +=1
    h /=2
    I = h*(f(a)+f(b))/2
    for k in range(1,N):
        I += h*f(a+k*h)
    R.append(I)
    m = 1
    while m < i:
        Rim = R[-1]+1/(4**m -1)*(R[-1]-R[-i])
        m+=1
        R.append(Rim)
    error = abs((R[-1]-R[-i-1])/3)
    print(R[-i:-1])
```

```
Part A
N =2, I= 0.32523191, error = 0.1816677
N =4, I= 0.51228285, error = 0.1217107
N =8, I= 0.40299745, error = 0.0903938
N =16, I= 0.43010337, error = 0.0210073
N =32, I= 0.44841467, error = 0.0136119
N =64, I= 0.45391293, error = 0.0040377
N =128, I= 0.45534850, error = 0.0010509
N =256, I= 0.45571127, error = 0.0002653
N =512, I= 0.45580220, error = 0.0000665
N =1024, I= 0.45582495, error = 0.0000166
N =2048, I= 0.45583064, error = 0.0000042
N =4096, I= 0.45583206, error = 0.0000010
N =8192, I= 0.45583241, error = 0.0000003

Part B
[0.3252319078064746]
[0.5122828507233315, 0.5746331650289505]
[0.40299744847824825, 0.3665689810632205, 0.35269803546550516]
[0.43010336929474696, 0.4391386762335799, 0.44397665591160385, 0.4454255229028118]
[0.4484146657874698, 0.4545184312850441, 0.45554374828847505, 0.45572735292937777,
0.4557677522628153]
[0.4539129312153759, 0.45574568635801127, 0.45582750336287575, 0.4558320074116758,
0.45583241782141026, 0.4558324810331]
[0.45534850437280205, 0.45582702875861075, 0.4558324515853174, 0.45583253012853076,
0.45583253217840075, 0.45583253229018666, 0.45583253230270365]
Wall time: 52.5 ms
```

Conclusion

I thought this was a very fun code. The nested integration was fairly simple, but the Romberg Integration was fun. I had to write out the different steps and see patterns. I thought it was so fun to do it.

In Class Nested Integration

Introduction

$$I = \int_1^3 0.95x^5 - 3.6x^4 + 3x^3 - 4.27x^2 + 12x - 3dx$$

In [2]:

```
%%time
import numpy as np
f = lambda x: 0.95*x**5-3.6*x**4+3*x**3-4.27*x**2+12*x-3

a = 1
b = 3
MaxError = [1e-3,1e-6,1e-9]

for maxerror in MaxError:
    N = 1
    error = 1
    print(f'Max Error of {maxerror:.1e}')
    h = b-a
    I = h/2*(f(b)+f(a))
    while error > maxerror:
        Iold = I
        I /= 2
        N *= 2
        h /=2
        for k in range(1,N,2):
            I += h*f(a+k*h)
        error = abs((I-Iold)/3/I)
        print(f'N ={N}, I= {I:.8f}, error = {error :.10f}')
        if N > 1e10:
            break
```

```
Max Error of 1.0e-03
N =2, I= 10.67000000, error = 0.2883473914
N =4, I= 7.25625000, error = 0.1568188343
N =8, I= 6.33367187, error = 0.0485541480
N =16, I= 6.09870605, error = 0.0128423865
N =32, I= 6.03969452, error = 0.0032568720
N =64, I= 6.02492476, error = 0.0008171479
Max Error of 1.0e-06
N =2, I= 10.67000000, error = 0.2883473914
N =4, I= 7.25625000, error = 0.1568188343
N =8, I= 6.33367187, error = 0.0485541480
N =16, I= 6.09870605, error = 0.0128423865
N =32, I= 6.03969452, error = 0.0032568720
N =64, I= 6.02492476, error = 0.0008171479
N =128, I= 6.02123126, error = 0.0002044707
N =256, I= 6.02030782, error = 0.0000511292
N =512, I= 6.02007696, error = 0.0000127830
N =1024, I= 6.02001924, error = 0.0000031958
N =2048, I= 6.02000481, error = 0.0000007990
Max Error of 1.0e-09
N =2, I= 10.67000000, error = 0.2883473914
N =4, I= 7.25625000, error = 0.1568188343
N =8, I= 6.33367187, error = 0.0485541480
N =16, I= 6.09870605, error = 0.0128423865
N =32, I= 6.03969452, error = 0.0032568720
N =64, I= 6.02492476, error = 0.0008171479
N =128, I= 6.02123126, error = 0.0002044707
N =256, I= 6.02030782, error = 0.0000511292
N =512, I= 6.02007696, error = 0.0000127830
N =1024, I= 6.02001924, error = 0.0000031958
N =2048, I= 6.02000481, error = 0.0000007990
N =4096, I= 6.02000120, error = 0.0000001997
N =8192, I= 6.02000030, error = 0.0000000499
N =16384, I= 6.02000008, error = 0.0000000125
N =32768, I= 6.02000002, error = 0.0000000031
N =65536, I= 6.02000000, error = 0.0000000008
Wall time: 65.2 ms
```

Conclusion

Thank you for helping me catch the mistake I had. It runs much better now. Sometimes, it is the little things that make a huge difference.