In Class Activities

Taylor Series Evaluation

Introduction

I did the work by hand to do the series expansion of sine and cosine.

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k$$

$$f(x) = \cos x$$

$$f(0) + f'(0)\,x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{6}x^3 + \frac{f''''(0)}{24}x^4 + \frac{f'''''(0)}{120}x^5 \ldots$$

$$\cos(0) + -\sin(0)\,x + \frac{-\cos(0)}{2}x^2 + \frac{\sin(0)}{6}x^3 + \frac{\cos(0)}{24}x^4 + \frac{-\sin(0)}{120}x^5 \ldots$$

$$1 + 0 - \frac{1}{2}x^2 + 0 + \frac{1}{24}x^4 - \frac{1}{6!}x^6 + \frac{1}{8!}x^8 - \frac{1}{10!}x^{10}$$

$$\cos(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{6!}x^6 + \frac{1}{8!}x^8 - \frac{1}{10!}x^{10} \qquad \cos\left(\frac{3\pi}{4}\right) =$$

$$f(x) = \sin(x)$$

$$f(0) + f'(0)\,x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{6}x^3 + \frac{f''''(0)}{24}x^4 + \frac{f'''''(0)}{120}x^5 \ldots \frac{f^{(n)}}{n!}x^n$$

$$\sin(0) + \cos(0)\,x + \frac{-\sin(0)}{2}x^2 + \frac{-\cos(0)}{6}x^3 + \frac{\sin(0)}{24}x^4 + \frac{\cos(0)}{120}x^5$$

$$0 + x + 0 - \frac{1}{6}x^3 + 0 + \frac{1}{120}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 - \frac{1}{11!}x^{11}$$

$$\sin(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 - \frac{1}{11!}x^{11} \qquad \sin\left(\frac{3\pi}{4}\right) =$$

In [11]:

```
import numpy as np

cosExp = lambda x: 1-1/2*x**2+1/24*x**4-1/np.math.factorial(6)*x**6+1/np.math.factori
sinExp = lambda x: x-1/6*x**3+1/120*x**5-1/np.math.factorial(7)*x**7+1/np.math.factor
print(cosExp(3*np.pi/4),np.cos(3*np.pi/4),(cosExp(3*np.pi/4)-np.cos(3*np.pi/4))/np.cc
print(sinExp(3*np.pi/4),np.sin(3*np.pi/4),(sinExp(3*np.pi/4)-np.sin(3*np.pi/4))/np.si
```

```
-0.7071660809820826 -0.7071067811865475 0.008386257509168273 %
0.7070959900908971 0.7071067811865476 -0.0015260913821798904 %
```

Conclusion

Depending on how precise we need the calculation to be, we could only include so many terms.

Part 2 for In Class

Introduction

This is to help us learn and get comfortable with complex numbers.

Work done by hand

$z_1 = 3+4i \qquad z_2 = -2-5i$

$z_2^* = -2+5i$

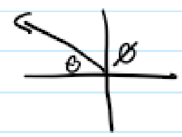
$z_2^* z_2 = (-2-5i)(-2+5i)$

$4 - 10i + 10i - 25i^2$

$4 + 25$

$z_2^* z_2 = 29 \rightarrow \text{real}$

$z_1 = 3+4i \qquad z_2 = -2-5i$



$r = \sqrt{3^2+4^2} = 5$

$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 0.93 \text{ rads}$

$r = \sqrt{2^2+5^2}$

$r = \sqrt{4+25}$

$r = \sqrt{29}$

$\theta = \tan^{-1}\left(\frac{5}{2}\right)$

$\theta = 1.19 \text{ rads}$

$\phi = 1.95 \text{ rads}$

$z_1 = 5e^{i(0.93)}$

$z_2 = \sqrt{29}\, e^{1.95i}$

$z_1 \cdot z_2 = 5e^{.93i} \sqrt{29} e^{1.95i}$

$5\sqrt{29}\, e^{2.88i}$

Conclusion

We can model many real things in the world much more simply with complex numbers.

Exercise 2.7

Introduction

In [15]:

```
import numpy as np
C = 1
n = 0
while C <= 1e9:
    print(C)
    C = (4*n+2)/(n+2)*C
    n +=1
```

```
1
1.0
2.0
5.0
14.0
42.0
132.0
429.0
1430.0
4862.0
16796.0
58786.0
208012.0
742900.0
2674440.0
9694845.0
35357670.0
129644790.0
477638700.0
```

Conclusion

I forgot the n+=1 at first... not a good idea. I put the print statement first so that it doesn't print the last C that is bigger that a billion.

Exercise 2.8

Introduction

This assignment tells me all of what to do. That's nice

In [16]:

```
from numpy import array
a = array([1,2,3,4],int)
b = array([2,4,6,8],int)
print(b/a+1)
print(b/(a+1))
print(1/a)
```

```
[3. 3. 3. 3.]
[1.         1.33333333 1.5        1.6       ]
[1.         0.5        0.33333333 0.25      ]
```

Conclusion

That was easy. It did what I expected.