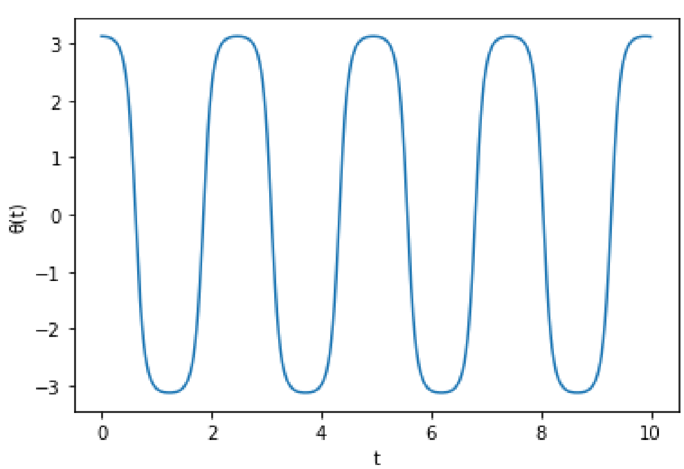
8.4a

Introduction

A) Motion of a nonlinear pendulum. $\ell = 10$ cm, $\theta = 179^{\circ}$. Plot θ vs t

In [39]:

```
from math import sin,pi
from numpy import arange,array
from pylab import plot,xlabel,ylabel,show

def f(r,t):
    theta = r[0]
    omega = r[1]
    dtheta = omega
    domega = -(g/l)*sin(theta)
    return array([dtheta,domega],float)

a = 0.0
b = 10.0
N = 10000
h = (b-a)/N
g = 9.81
l = 0.1

tpoints = arange(a,b,h)
thetapoints = []
theta = 179*pi/180

r = array([theta,0])

for t in tpoints:
    thetapoints.append(r[0])
    k1 = h*f(r,t)
    k2 = h*f(r+0.5*k1,t+0.5*h)
    k3 = h*f(r+0.5*k2,t+0.5*h)
    k4 = h*f(r+k3,t+h)
    r += (k1+2*k2+2*k3+k4)/6

plot(tpoints,thetapoints)
xlabel("t")
ylabel("θ(t)")
show()
```



Conclusion

I hope this is correct. I messed up in the for loop at first. I had thetapoints.append(theta) instead and it was just a straight line.

8.5

Introduction

Now make it have an oscillating force to gice it an acceleration.

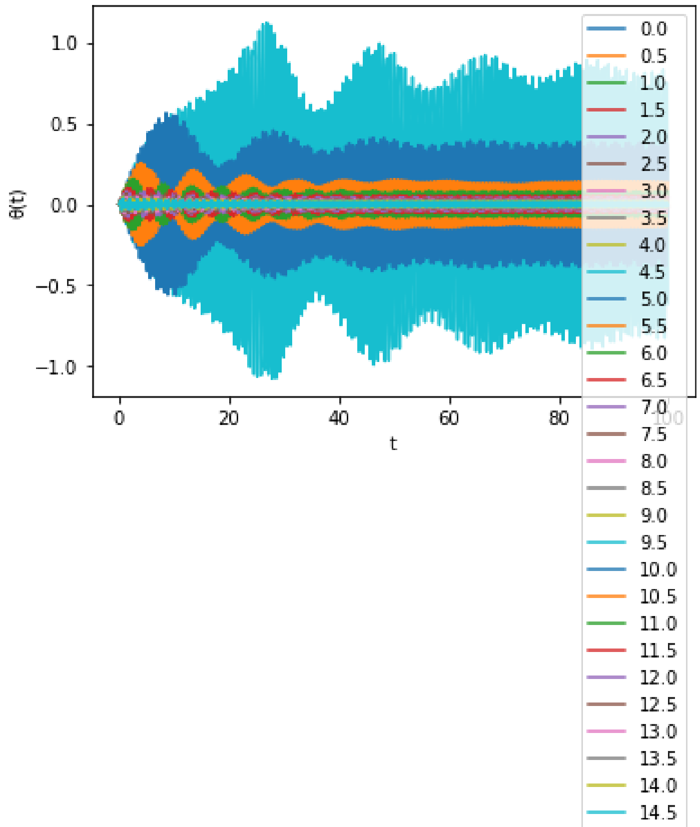
$$\frac{d^2\theta}{dt^2} = -\frac{g}{\ell}\sin\theta + C\cos\theta\sin\Omega t$$

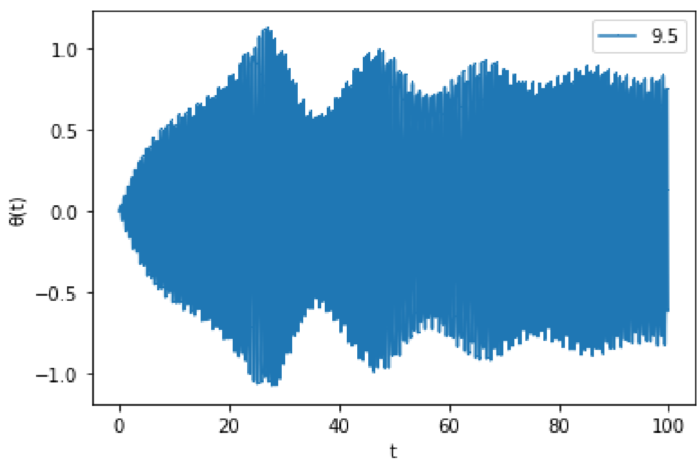
A) Plot with RK4 for θ with $\ell = 10$cm $C = 2s^{-2}$ and $\Omega = 5s^{-1}$ starts at $\theta = 0$ and at rest.

B) Change Ω until it resonates.

In [40]:

```
from math import sin,pi,cos
from numpy import arange,array
from pylab import plot,xlabel,ylabel,show,legend

def f(r,t):
    theta = r[0]
    omega = r[1]
    dtheta = omega
    domega = -(g/l)*sin(theta)+C*cos(theta)*sin(Ω*t)
    return array([dtheta,domega],float)

a = 0.0
b = 100.0
N = 1000
h = (b-a)/N
g = 9.81
l = 0.1
C = 2
omegas = arange(0,15,.5)
for Ω in omegas:

    tpoints = arange(a,b,h)
    thetapoints = []

    r = array([0,0],float)

    for t in tpoints:
        thetapoints.append(r[0])
        k1 = h*f(r,t)
        k2 = h*f(r+0.5*k1,t+0.5*h)
        k3 = h*f(r+0.5*k2,t+0.5*h)
        k4 = h*f(r+k3,t+h)
        r += (k1+2*k2+2*k3+k4)/6

    plot(tpoints,thetapoints,label=f'{Ω}')
xlabel("t")
ylabel("θ(t)")
legend()
show()

Ω = 9.5
tpoints = arange(a,b,h)
thetapoints = []

r = array([0,0],float)

for t in tpoints:
    thetapoints.append(r[0])
    k1 = h*f(r,t)
    k2 = h*f(r+0.5*k1,t+0.5*h)
    k3 = h*f(r+0.5*k2,t+0.5*h)
    k4 = h*f(r+k3,t+h)
    r += (k1+2*k2+2*k3+k4)/6

plot(tpoints,thetapoints,marker=',',label=f'{Ω}')
xlabel("t")
ylabel("θ(t)")
legend()
show()
```





Conclusion

I found it resonated the most when $\Omega = 9.5$. To find it, I plotted a variety of values for Ω and looked at the highest. I thought it was a pretty graph.

Exercise 8.7

Introduction

Make a plot of the trajectory of a ball thrown taking into account air resistence.

A)

$$\frac{d^2x}{dt^2} = -\frac{\pi R^2 \rho C}{2m} v_x \sqrt{v_x^2 + v_y^2}$$

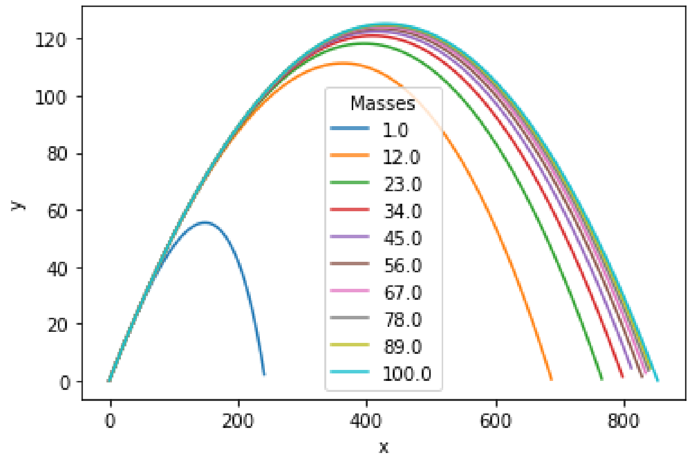
$$\frac{d^2 y}{dt^2} = -\frac{\pi R^2 \rho C}{2m} v_y \sqrt{v_x^2 + v_y^2} - g$$

B) 4th Order RK to find and plot trajectory. mass of 1kg, radius of 8 cm, shot at 30 degrees, velocity of 100 m/s, density of air is 1.22 kg/m^3, C = 0.47

C) How does mass change it?

In [61]:

```
from numpy import arange, sin,cos, sqrt,pi,linspace
from pylab import plot,xlabel,ylabel,show,legend

def f(r,t):
    x = r[0][0]
    vx = r[0][1]
    y = r[1][0]
    vy = r[1][1]
    dx = vx
    dy = vy
    dvx = -pi*R*R*ρ*C/2/m*vx*sqrt(vx*vx+vy*vy)
    dvy = -pi*R*R*ρ*C/2/m*vy*sqrt(vx*vx+vy*vy)-g
    return array([[dx,dvx],[dy,dvy]])

a = 0.0
b = 1000
N = 10000
h = (b-a)/N
masses = linspace(1,100,10)
R = 8e-2
ρ=1.22
C = 0.47

for m in masses:
    tpoints = []
    xpoints = []
    ypoints = []
    x = 0.0
    y = 0.0
    v = 100
    θ = 30*pi/180
    vx = v*cos(θ)
    vy = v*sin(θ)
    r = array([[x,vx],[y,vy]])
    t = 0
    while r[1][0] >= 0:
        xpoints.append(r[0][0])
        ypoints.append(r[1][0])
        tpoints.append(t+h)
        t = tpoints[-1]
        k1 = h*f(r,t)
        k2 = h*f(r+0.5*k1,t+0.5*h)
        k3 = h*f(r+0.5*k2,t+0.5*h)
        k4 = h*f(r+k3,t+h)
        r += (k1+2*k2+2*k3+k4)/6

    plot(xpoints,ypoints,label=m)
xlabel("x")
ylabel("y")
legend(title="Masses")
show()
```



Conclusion

The mass makes the effect of drag have less of an effect. However, the higher the mass, the less of the change in the effect. I messed up my indexing at first, but I enjoyed this a lot.