

## 2.2.4 Exercises

1) a) expand  $A_{ij}y_j$

$$A_{ij}y_j \rightsquigarrow \left( \sum_{j=1}^3 A_{ij}y_j \right)_{i=1,2,3} = Ay \quad \text{with } A = (A_{ij}) \\ y = (y_j)$$

b) same as a)

c) expand

$$A_{\ell m}y_\ell \rightsquigarrow \left( \sum_{\ell=1}^3 A_{\ell m}y_\ell \right)_{m=1,2,3} = y^t A$$

d) expand

$$A_{ij}y_i y_j \rightsquigarrow \sum_{i=1}^3 \sum_{j=1}^3 A_{ij}y_i y_j = y^t A y$$

e) same as d)

2) a) expand

$$B_{ij} B_{ij} \rightsquigarrow \sum_{i=1}^3 \left( \sum_{j=1}^3 \underbrace{B_{ij} B_{ij}}_{B_{ij}^2} \right) = (B_{11}^2 + B_{12}^2 + B_{13}^2) + (B_{21}^2 + B_{22}^2 + B_{23}^2) + (B_{31}^2 + B_{32}^2 + B_{33}^2)$$

b) expand

$$B_{ij} B_{ij} \text{ where } B_{ij} = B_{ji} \text{ (i.e. } B \text{ is symmetric!)}$$

$$B_{ij} y_i y_j \rightsquigarrow \text{as in a) then apply } B_{ij} = B_{ji}$$

$$= (B_{11} y_1^2 + B_{22} y_2^2 + B_{33} y_3^2) + 2(B_{12} y_1 y_2 + B_{13} y_1 y_3 + B_{23} y_2 y_3)$$

c) expand

$B_{ij} B_{ij}$  where  $B_{ij} = -B_{ji}$  for  $i \neq j$  (i.e.  $B$  is skew symmetric!)

$$B_{ij} y_i y_j \rightsquigarrow \text{as in b)} \text{ then apply } B_{ij} = -B_{ji}, i \neq j$$

$$= B_{11} y_1^2 + B_{22} y_2^2 + B_{33} y_3^2$$

3) a) simplify

$$\delta_{ij} y_j \rightsquigarrow \left( \sum_{j=1}^3 \delta_{ij} y_j \right)_{i=1,2,3} = (y_i) \rightsquigarrow y_i$$

b) simplify

$$\delta_{ij} c_{ij} \rightsquigarrow \sum_{i=1}^3 \left( \sum_{j=1}^3 \delta_{ij} c_{ij} \right) = (\delta_{11} c_{11} + \delta_{12} c_{12} + \delta_{13} c_{13}) +$$

$$+ (\delta_{21} c_{21} + \delta_{22} c_{22} + \delta_{23} c_{23}) + (\delta_{31} c_{31} + \delta_{32} c_{32} + \delta_{33} c_{33})$$

$$= c_{11} + c_{22} + c_{33}$$

c) simplify

$$\delta_{ij} y_i y_j \rightsquigarrow \text{use b)} \quad y_1^2 + y_2^2 + y_3^2$$

d) simplify

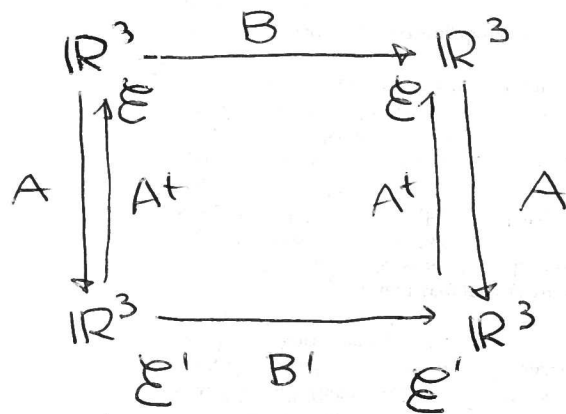
$$\delta_{ij} \delta_{ij} \rightsquigarrow \text{use b)} \quad 3$$

e) simplify

$$\delta_{ij} \delta_{ik} \delta_{jk} = \delta_{ii} \delta_{ii} \delta_{ii} \rightsquigarrow 3$$

4] Verify  $B_{ij} = A_{ei} A_{ej} B'_{ke}$  of Theorem 2.4

In the lecture we saw that



$$B = A^t B' A$$

and

$$B' = A B A^t$$

we translate  $B = A^t B' A$  into index notation:

$$B_{ij} = A_{ei} B'_{ke} A_{ej} = A_{ei} A_{ej} B'_{ke}$$