Optimal Cable Design of Wind Farms: The Infrastructure and Losses Cost Minimization Case

Adelaide Cerveira, Amaro de Sousa, E. J. Solteiro Pires, and José Baptista

***Abstract*—Wind power is the source of electrical energy that has grown more over the last years, with annual rate in installed capacity around 20%. Therefore, it is important to optimize the production efficiency of wind farms. In a wind farm, the electrical energy is collected at a central substation from different wind turbines placed nearby. This paper addresses the optimal design of the cable network interconnecting the turbines to the substation aiming to minimize not only the infrastructure cost but also the cost of the energy losses in the cables. Although this problem is nonlinear, different integer linear programming models are proposed considering the wind farm technical constraints. The models are applied to three real cases Portuguese wind farms. The computational results show that the proposed models are able to compute the optimal solutions for all cases.**

***Index Terms*—Distribution networks, wind farm, integer linear programming, capacitated minimum spanning trees.**

I. Introduction

A WIND farm comprises a set of wind turbines with the purpose of generating electric power from the wind to be collected in central substations. Most wind farms are formed by a single substation and several wind turbines, which are connected to the substation by a cable network. The wind farm layout design optimization plays an important role since its power production performance and investment recovery depends on it.

The wind farm layout design considering the energy losses in cables is a highly complex combinatorial, non-differentiable and constrained optimization problem. This is due to the high number of elements in the cable network, and due to constraints characteristics used to model the electrical system behavior [1]. For this reason, meta-heuristics have been very popular in this type of problems although they do not guarantee that the optimal solution is found at the end. Examples of such works consider the optimization of the cable costs [2]–[5], the cost of the losses [2], [3] and the turbines and substation layout [6]–[8], [5].

Some approaches use classical mathematical methods in wind farm studies [9]–[15]. Zhang *et al.* [9] use constraint programming and mixed integer linear programming models that incorporate nonlinearities to solve the wind farm layout. They optimize the wind farm overall power generation capability considering the distance between turbines, farm boundary and turbine number constraints. The proposed models presented a trade-off between reasonable computational capacity and power production accuracy. Bauer and Lysgaard [10] optimize the cable layout for two real-world offshore wind farms using a hop-indexed integer programming formulation. Additionally, they present a heuristic to solve the same problem. The optimization finds both the layout and the type of the cables. Moradzadeh and Tomsovic [16] propose a formulation of a reconfiguration problem for a radial/weakly meshed distribution network or restoration after a fault based on mixed integer programming. In their work, two objectives were considered: the minimization of the active losses and the minimization of the number of switching operations. Kalambe and Agnihotri [14] present a survey of loss minimization techniques used in distribution networks. Recently, Cerveira *et al.* [15] proposed a flow formulation to determine the optimal cable network layout but considering only the cable infrastructure costs. In the present work, the cost of the energy losses in the cables is also considered which invalidates the use of flow formulations to obtain appropriate integer linear programming formulations.

In this paper, different mathematical formulations are proposed to the optimal cable network layout problem aiming to minimize the sum of the infrastructure cost and the costs of energy losses, considering several cable types with different cross sections. Some operating constraints described in [1] were taken into account, such as radial network constraint, node voltage constraint, branch current stability constraint and Kirchoff's current and voltage laws. Moreover, in order to assess the gains of considering the cost of the energy losses in the layout optimization goal, the problem of minimizing only the infrastructure cost is also addressed.

Manuscript received February 18, 2015; revised June 19, 2015 and October 09, 2015; accepted January 09, 2016. Date of publication February 26, 2016; date of current version October 18, 2016. This work was supported in part by the ERDF European Regional Development Fund through the Operational Programme for Competitiveness and Internationalisation—COMPETE 2020 Programme within project POCI-01-0145-FEDER-006961, and in part by National Funds through the FCT Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) as part of project UID/EEA/50014/2013. Paper no. TPWRS-00236-2015.

A. Cerveira is with DMat, Escola de Ciências e Tecnologia, Universidade de Trás-os-Montes e Alto Douro, Vila Real, Portugal, and also with INESC TEC—INESC Technology and Science (formerly INESC Porto, UTAD pole) (e-mail: cerveira@utad.pt).

A. de Sousa is with the Instituto de Telecomunicações, Universidade de Aveiro, Aveiro, Portugal (e-mail: asou@ua.pt).

E. J. Solteiro Pires and J. Baptista are with DEng, Escola de Ciências e Tecnologia, Universidade de Trás-os-Montes e Alto Douro, Vila Real, Portugal, and alsow tih INESC TEC—INESC Technology and Science (formerly INESC Porto, UTAD pole) (e-mail: epires@utad.pt; baptista@utad.pt).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TPWRS.2016.2521700

The paper is organized in 5 sections, including the Introduction. Section II presents the line model used in the optimization models, emphasizing some concepts related with the transmission lines, relationships which allow to calculate the voltage drop and the energy losses in the network branches. The wind farm layout mathematical formulations are presented in Section III and the computational results are discussed in Section IV. Section V draws the main conclusions of this work.

II. Branch Line Model

In onshore wind farms, the distance between turbines and substation is usually of the order of hundreds of meters. Therefore, the short line model can be used, where the transmission line model considers only the series impedance and neglects the shunt admittance. In order to obtain a feasible network layout, it is important to ensure that voltage drop is within regulatory limits and to reduce energy losses to a minimum. In the following subsections, some inherent concepts of these aspects are presented.

A. Voltage Drop

Consider a cable connecting node i to node l such that node i is in the substation side of the cable network. The power injected in node l, $\overline{S}_l$, in the short line model, is given by $\overline{S}_l = P_l + \mathrm{j}Q_l$ and the voltage at node i, $\overline{V}_i$, is given by

$$\overline{V}_i = V_l + \frac{R_{il}P_l + X_{il}Q_l}{V_l} + \mathrm{j}\frac{R_{il}P_l - X_{il}Q_l}{V_l} \tag{1}$$

where P_l is the active power, Q_l is the reactive power and R_{il} and X_{il} are the resistance and the reactance of the line section (i,l), respectively. For low values of the angle between $\overline{V}_i$ and $\overline{V}_l$ (normal situation for short and medium line problems), the voltage drop ΔV in the line section is given by

$$\Delta V = V_i - V_l \approx \frac{R_{il}P_l + X_{il}Q_l}{V_l}. \tag{2}$$

Alternatively, (2) can be rewritten obtaining (3), where $\cos\varphi$ is the power factor of the load.

$$\Delta V = R_{il}I_{il}\cos\varphi + X_{il}I_{il}\sin\varphi \tag{3}$$

B. System Constraints

Onshore wind farm cable networks have a radial structure (as illustrated in Fig. 1), where the sum of the currents flowing to a node is equal to the sum of the current flowing out of that node, and the voltage in a node depends on the voltage drop across the path until this node. In this work, it is assumed a radial cable network structure where these constraints, known by Kirchoff's current and voltage laws, and the branch current stability constraint are guaranteed.

Most wind turbines allow to control the power factor between 0.95(*capacitive*) to 0.95(*indutive*), over the entire range of power. Therefore, the power electronics is an alternative to capacitors banks, since it allows to regulate the power factor up to a certain level. In this work, it is assumed that the turbines

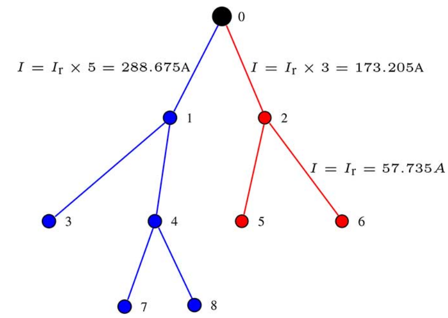

Fig. 1. Wind farm layout example.

TABLE I
CHARACTERISTICS OF UNIPOLAR CABLES (LXHIOV) 12/20 kV

Type k	Section (mm^2)	Max. Current, I_z (A)	Inductance, L (mH/km)	Electrical Resistance, R (Ω/km)	Price, C (€/m)
1	25	122	0.60	1.200	4.50
2	35	144	0.60	0.868	5.30
3	50	170	0.58	0.641	6.80
4	70	209	0.55	0.443	7.12
5	95	249	0.53	0.320	7.98
6	120	283	0.52	0.253	8.70
7	150	316	0.51	0.206	12.77
8	185	357	0.50	0.164	13.23
9	240	413	0.47	0.125	14.89
10	300	463	0.46	0.100	17.50
11	400	526	0.45	0.0778	21.09
12	500	592	0.44	0.0605	23.77

have unity power factor, i.e., $\tan(\varphi) = 0$, meaning that the current and voltage drawn by the turbines into the network are in phase, $\varphi = 0$. The rated current drawn by each turbine, defined by I_{r}, is given by

$$I_{\mathrm{r}} = \frac{P_{\mathrm{r}}}{\sqrt{3}\cdot U \cdot \cos\varphi} \tag{4}$$

where P_{r} is the rated power of the wind turbines and U is the voltage of the interconnection grid.

Given P_{r} and U of a wind farm, the resulting value of I_{r} constrains the number of wind turbines that can be connected on the same branch due to the maximum current that each cable type can safely carry continuously, defined as I_{z}. The cable sizing has in account the protection against damage from thermal overload. The device protection must be selected to exceed the full load current, but not to exceed the cable's installed current rating ($I_{\mathrm{r}} \leq I_{\mathrm{p}} \leq I_{\mathrm{z}}$, where I_{p} is the device protection rating). These values are presented in Table I, which summarizes the characteristics of all cable types considered in this work.

The maximum allowable voltage drop, 5%, in the entire path between any wind turbine and the substation must be satisfied, i.e., $\Delta V_{\mathrm{p}} \leq 0.05$. This constraint was not explicitly considered in the reported computational results since in Portuguese wind farms (which were the motivation for this work) the distances between the wind turbines and the substation are small enough (the computational results will show that the optimal solutions are always compliant with the maximum allowable voltage drop). Nevertheless, in Section III, devoted to the mathematical formulations, we also describe the required variables and constraints that, added to the proposed mathematical formulations, guarantee the maximum allowable voltage drop constraint.

C. Line Losses

There are two types of power losses: active and reactive. For a given cable connection (i, l), the active loss, defined as $P_{\text{loss}\,il}$, is caused by the line resistance R_{il} which is given by $R_{il} I_{il}^2$ and the reactive loss, defined as $Q_{\text{loss}\,il}$, is caused by the line reactance X_{il} which is given by $X_{il} I_{il}^2$. On the other hand, the resistance of the cable connection (i, l) is $R_{il} = \ell_{il}.R_{k_{il}}$, the product of the resistance per unit of length $R_{k_{il}}$ and the connection length ℓ_{il}, where k_{il} is the cable type used in connection (i, l). Equivalently, the reactance of the cable connection (i, l) is $X_{il} = \ell_{il}.L_{k_{il}}$, the product of the inductance by the length. The current generated by each turbine is given by $I_{\text{r}}.l_{\text{f}}$, where I_{r} is the generated rated current and l_{f} is the load factor. The value of l_{f} reflects the real operating conditions and is the ratio between the generated current and the maximum current that can be generated (in the case studies presented in this paper it is assumed that $l_{\text{f}} = 0.5$). Thus, the total loss of cable connection (i, l) is given by

$$P_{\text{loss}\,il} + Q_{\text{loss}\,il} = \ell_{il} \cdot R_{k_{il}} \cdot (I_{il} \cdot l_f)^2 + \ell_{il} \cdot L_{k_{il}} \cdot (I_{il} \cdot l_f)^2 \quad (5)$$

where I_{il} is the current crossing connection (i, l) which is the sum of the currents generated by all turbines that are supported by (i, l). Therefore, the total loss of a wind farm is the sum of all cable losses.

Fig. 1 shows a wind farm layout example with 8 wind turbines, nodes 1 to 8, and one substation, node 0. In this example, assuming that $P_{\text{r}} = 2$ MW and $U = 20$ kV, the rated current drawn by each turbine is $I_{\text{r}} = 57.735$ A (by (4)). This layout has 2 branches: one starting in cable connection (0, 1) and the other starting in cable connection (0, 2). In a branch, the total current reaching the substation is the sum of the currents drawn by all turbines connected through this branch. For example, the branch starting in connection (0,1) supports 5 wind turbines (including turbine 1) and so, the current crossing this cable is $I_{01} = I_{\text{r}} \times 5 = 288.675$ A.

III. Mathematical Formulation

This section presents alternative Integer Linear Programming (ILP) formulations addressing two different objective functions. In both cases, the substation location and the wind turbine locations are given and the aim is to design the cable network connecting them.

The main objective function (named **Global cost function**) is the minimization of the sum of the infrastructure cost, C_{c}, with the costs of active losses, C_{p}, and reactive losses, C_{q}, these last two taking into account the expected wind farm lifetime. The infrastructure cost accounts for the cables cost and their installation. In order to assess the gain of considering the losses cost in the design goal, a second objective function (named **Infrastructure cost function**) is also addressed which considers only the minimization of the infrastructure cost, C_{c}.

The following subsections present, separately, the basic ILP formulations for the two objective functions, then, the alternative more efficient ILP formulations and, finally, the variables and constraints that, added to the previous formulations, guarantee the maximum allowable voltage drop value.

A. Basic ILP Formulations

Consider node sets $\mathcal{N} = \{1, \ldots, n\}$ and $\mathcal{N}_0 = \{0, 1, \ldots, n\}$ where node 0 represents the substation and the other nodes represent the wind turbines. For all pairs of nodes i and j, their distance is given by ℓ_{ij} (in this notation, j does not represent the imaginary number). Consider a set of cable types $k = 1, \ldots, K$ such that each type k is defined by a cost per unit of length, C_{k}, a resistance per unit of length, R_{k}, an inductance per unit of length, L_{k} and a maximum current intensity, I_{z_k}, that it can support (see Table I). Cable types k are ordered in an increasing way of their value of I_{z_k} and (since cables supporting higher current intensities are also more expensive) of their value of C_{k}. In addition, consider a digging cost per unit of length D which is equal for all cable types. In order to derive appropriate ILP formulations, the nonlinear relation between the current intensity supported by each cable and its active and reactive losses, as defined by (5), must be modeled in a linear way. This is possible by considering the binary variables x_{ij}^{kt} that, when equal to 1, indicate that nodes i and j are connected by a cable of type k such that i is the node on the substation side and this cable supports the current of t downstream wind turbines (including the one located in j). Being 0 the root node, arcs $(i, 0)$ are not considered in the formulations.

To understand these variables, consider again the example shown in Fig. 1 and assume that each connection is implemented with the cheapest cable type. The cross section of the cable connection (0,1) should be at least 150 mm^2 (cable type $k = 7$) to support the current passing through it. Therefore, this solution is defined by $x_{01}^{75} = 1$. Analogously, the other cable connections shown in Fig. 1 are defined by $x_{02}^{43} = 1$, $x_{13}^{11} = 1$, $x_{14}^{43} = 1$, $x_{25}^{11} = 1$, $x_{26}^{11} = 1$, $x_{47}^{11} = 1$, $x_{48}^{11} = 1$ and all remaining variables are equal to zero.

Note that when one variable x_{ij}^{kt} is equal to 1, the active and reactive loss of the represented connection is independent of the other connections. Therefore, the associated cost values can be computed in advance for all possible k and t index values and used as parameters in the objective function. Consider parameters p_{ij}^{kt} and q_{ij}^{kt} as the active loss and reactive loss costs, respectively, of a cable type k between i and j that supports t downstream wind turbines, during the wind farm lifetime. These parameter are given by:

$$p_{ij}^{kt} = 3 \cdot \frac{h \cdot c_{\text{e}} \cdot l_{\text{f}}^2 \cdot I_{\text{r}}^2}{1000} \cdot \ell_{ji} \cdot R_{\text{k}} \cdot t^2, \quad (6)$$

$$q_{ij}^{kt} = 3 \cdot \frac{h \cdot 0.5\, c_{\text{e}} \cdot l_{\text{f}}^2 \cdot I_{\text{r}}^2}{1000000} \cdot \omega \cdot \ell_{ji} \cdot L_{\text{k}} \cdot t^2 \quad (7)$$

where h is the number of hours expected during the wind farm lifetime, c_{e} is the energy cost and ω is the angular frequency. According to the Portuguese market price, half of the cost was considered for the reactive losses $(0.5 c_{\text{e}})$.

Since each cable type k has a maximum current intensity value that it can support, I_{z_k}, the maximum number of turbines m_k that can be supported by each cable type $k = 1, \ldots, K$ is given by[1] $m_k = \left\lfloor \frac{I_{\mathrm{z_k}}}{I_{\mathrm{r}}} \right\rfloor$, where I_{r} is the current generated by each wind turbine, as defined in (4). Note that the variables x_{ij}^{kt} such that $t > m_k$ cannot be 1 and, therefore, are not considered in the formulations.

Consider the parameter Q that specifies the maximum number of wind turbines that can be supported by any cable type, given by $Q = \max_{k=1,\ldots,K} m_k$. Note that variables x_{ij}^{kQ} can be equal to 1 only for connections in the form $(0, j)$, with $j \in \mathcal{N}$. We introduce the parameters t_{ki} that specify the maximum number of wind turbines that can be supported by a connection outgoing from node i with a cable of type k as follows:

$$t_{ki} = \begin{cases} \min\{m_k, Q\} & , i = 0 \\ \min\{m_k, Q-1\} & , i \in \mathcal{N} \end{cases}.$$

Finally, consider the infrastructure cost parameter $c_{ij}^k = (D + 3 \cdot C_k) \cdot \ell_{ij}$ as the cost associated to the installation of a type k cable between i and j, which includes the digging cost, D, and the three-phase cable cost, $3 \cdot C_k$.

An appropriate ILP model defining the **Global cost function** problem is given by the following **GCF1** model:

$$\min \sum_{i\in\mathcal{N}_0} \sum_{j\in\mathcal{N}} \sum_{k=1}^{K} \sum_{t=1}^{t_{ki}} \left(\left(c_{ij}^k + p_{ij}^{kt} + q_{ij}^{kt} \right) \cdot x_{ij}^{kt} \right) \quad (8)$$

subject to

$$\sum_{j\in\mathcal{N}} \sum_{k=1}^{K} \sum_{t=1}^{t_{k0}} \left(t \cdot x_{0j}^{kt} \right) = n \quad (9)$$

$$\sum_{i\in\mathcal{N}_0} \sum_{k=1}^{K} \sum_{t=1}^{t_{ki}} \left(x_{ij}^{kt} \right) = 1, \quad j \in \mathcal{N} \quad (10)$$

$$\sum_{i\in\mathcal{N}_0} \sum_{k=1}^{K} \sum_{t=1}^{t_{ki}} \left(t \cdot x_{ij}^{kt} \right) = \sum_{i\in\mathcal{N}} \sum_{k=1}^{K} \sum_{t=1}^{t_{kj}} \left(t \cdot x_{ji}^{kt} \right) + 1, j \in \mathcal{N} \quad (11)$$

$$x_{ij}^{kt} \in \{0,1\}, i \in \mathcal{N}_0, j \in \mathcal{N}, k = 1, \ldots, K, t = 1, \ldots, t_{ki} \quad (12)$$

The objective function (8) aims to minimize the sum of infrastructure cost and both types of losses costs. Constraint (9) guarantees that the network connects all wind turbines. Constraints (10) are the radial structure constraints; they guarantee that each wind turbine $j \in \mathcal{N}$ has one incoming connection (in the example of Fig. 1, considering all connections in the direction from the substation to the wind turbines, the radial structure is guaranteed when each turbine has exactly one incoming connection while the outgoing connections of a turbine can be of any number). Constraints (11) are the flow conservation constraints and guarantee that, on each wind turbine $j \in \mathcal{N}$, if an incoming connection supporting t downstream wind turbines is in the solution (left-hand side of the constraints), then, the outgoing connections must support a total of $t-1$ donwstream wind turbines (right-hand side of the constraints). Finally, constraints (12) are the variable domain constraints.

The corresponding ILP model defining the **Infrastructure cost function** problem is easily obtained from the previous model by changing in a trivial way the objective function (8). The resulting **ICF1** model is as follows:

$$\min \sum_{i\in\mathcal{N}_0} \sum_{j\in\mathcal{N}} \sum_{k=1}^{K} \sum_{t=1}^{t_{ki}} \left(c_{ij}^k \cdot x_{ij}^{kt} \right) \quad (13)$$

$$\text{subject to } (9)-(12)$$

[1] $\lfloor a \rfloor$ denotes the maximum integer not greater than a.

B. ILP Reformulations

The computational results will show that models **GCF1** and **ICF1** cannot solve the larger problem instance due to out of memory reasons. In order to obtain more efficient ILP formulations, a second model for each problem was derived which combines a variable reformulation technique and the addition of valid inequalities.

Before defining the new ILP formulations, note that, if a connection (i, j) is in the solution supporting the current of t downstream wind turbines, than, the optimal cable type for this connection does not depend on the other connections of the solution and is simply the type k that minimizes the sum $c_{ij}^k + p_{ij}^{kt} + q_{ij}^{kt}$ provided that its maximum current intensity $I_{\mathrm{z_k}}$ is not lower than the current intensity generated by t wind turbines, i.e., $I_{\mathrm{z_k}} \geq t \cdot I_{\mathrm{r}}$. Therefore, for each (i, j) and each possible value of t, the optimal cable type, which is defined as k_{ij}^t, and its cost, which is defined as α_{ij}^t, can be computed in advance in the following way. Consider first parameter k_t as the first cable type k satisfying the relation $t \cdot I_{\mathrm{r}} \leq I_{\mathrm{z_k}}$ (since cable types are defined in an increasing order of their $I_{\mathrm{z_k}}$ value, the possible cable types supporting t wind turbines are $k = k_t, \ldots, K$) and parameter $Q(i)$ defined as:

$$Q(i) = \begin{cases} Q & , i = 0 \\ Q-1 & , i \in N \end{cases}.$$

For each $j \in \mathcal{N}$ and $t = 1, \ldots, Q(i)$, the optimal cable type k_{ij}^t is given by:

$$k_{ij}^t = \arg \min_{k=k_t,\ldots,K} \left(c_{ij}^k + p_{ij}^{kt} + q_{ij}^{kt} \right), i \in \mathcal{N}_0, \quad (14)$$

and its cost is $\alpha_{ij}^t = c_{ij}^k + p_{ij}^{kt} + q_{ij}^{kt}$ for cable type $k = k_{ij}^t$.

Then, for each arc (i, j) and each t, the set of variables, x_{ij}^{kt}, with $k = 1, \ldots, K$, can be replaced by a single binary variables y_{ij}^t obtaining in this way a huge reduction on the total number of variables. Variables y_{ij}^t, when equal to 1, indicate that nodes i and j are connected by a cable of type k_{ij}^t such that i is the node on the substation side and this cable supports the total current of t downstream wind turbines.

With these new parameters and variables, a new ILP formulation defining the **Global cost function** problem is given by the following **GCF2** model:

$$\min \sum_{i\in\mathcal{N}_0} \sum_{j\in\mathcal{N}} \sum_{t=1}^{Q(i)} \left(\alpha_{ij}^t \cdot y_{ij}^t \right) \quad (15)$$

subject to

$$\sum_{j\in\mathcal{N}} \sum_{t=1}^{Q} \left(t \cdot y_{0j}^t \right) = n \quad (16)$$

$$\sum_{i\in\mathcal{N}_0}\sum_{t=1}^{Q(i)}\left(y_{ij}^t\right)=1, j\in\mathcal{N} \tag{17}$$

$$\sum_{i\in\mathcal{N}_0}\sum_{t=1}^{Q(i)}\left(t\cdot y_{ij}^t\right)=\sum_{i\in\mathcal{N}}\sum_{t=1}^{Q(i)}\left(t\cdot y_{ji}^t\right)+1, j\in\mathcal{N} \tag{18}$$

$$y_{ij}^t\in\{0,1\}, i\in\mathcal{N}_0, j\in\mathcal{N}, t=1,\ldots,Q(i) \tag{19}$$

Note that **GCF2** model has the same number of constraints as **GCF1** but, as pointed out before, it has a much smaller number of variables than **GCF1**.

GCF2 model is close to the model proposed in [17] for the Capacitated Minimum Spanning Tree (CMST) problem. In that work, some valid inequalities are proposed to improve the CMST model. Nevertheless, they tested their inequalities on instances with small values of Q, when compared with n, and using fixed costs per connection (here, the coefficients α_{ij}^t in the objective function are different for different values of t while in [17] the coefficients are the same for all values of t). In the wind farm layout problems, the inequalities used in [17] are not effective.

Here, a different set of valid inequalities able to improve the models efficiency is proposed. Remember that constraints (17) guarantee that there is always one connection going into each node $j\in\mathcal{N}$. Note that if this connection supports τ downstream wind turbines, then, the number of connections outgoing from node j supporting t or more downstream wind turbines cannot be higher than $\lfloor\frac{\tau-1}{t}\rfloor$. For example, if the connection going into a particular node $j\in\mathcal{N}$ supports 4 downstream wind turbines, i.e., $\tau=4$, then: (i) the number of connections outgoing from j supporting 1, or more, downstream wind turbines is at most $\lfloor\frac{4-1}{1}\rfloor=3$ (i.e., it can have at most 3 outgoing connections, each one supporting 1 wind turbine); (ii) the number of connections outgoing from j supporting 2, or more, downstream wind turbines is at most $\lfloor\frac{4-1}{2}\rfloor=1$ (i.e., it can have at most 1 outgoing connection supporting 2 or more wind turbines); (iii) the number of connections outgoing from j supporting 3, or more, downstream wind turbines is at most $\lfloor\frac{4-1}{3}\rfloor=1$ (i.e., it can have at most 1 outgoing connection supporting 3 wind turbines) and finally, (iv) the number of connections outgoing from node j supporting t, or more, downstream wind turbines, with $t=4,\ldots,Q-1$ must be zero.

Based on this property, the following valid inequalities are defined, that can be used as additional constraints:

$$\sum_{i\in\mathcal{N}_0}\sum_{\tau=t+1}^{Q(i)}\left(\left\lfloor\frac{\tau-1}{t}\right\rfloor\cdot y_{ij}^\tau\right)\geq\sum_{i\in\mathcal{N}}\sum_{\tau=t}^{Q-1}\left(y_{ji}^\tau\right), \quad j\in\mathcal{N}, t=2,\ldots,Q-2 \tag{20}$$

$$y_{0j}^Q\geq\sum_{i\in\mathcal{N}}\left(y_{ji}^{Q-1}\right), j\in\mathcal{N} \tag{21}$$

For each node $j\in\mathcal{N}$ and each possible value of t, the left-hand side of constraints (20) computes the maximum number of outgoing connection supporting t, or more, downstream wind turbines and the right-hand side of constraints (20) determines the number of outgoing connection supporting t, or more, downstream wind turbines. Note that it is not necessary to define these constraints for $t=1$ because they are guaranteed by the flow conservation constraints (18). Constraints (21) are the equivalent to constraints (20) for the case of $t=Q-1$.

The ILP model that results by adding constraints (20) and (21) to the **GCF2** model is designated as **GCF2+** model.

The corresponding ILP model, designated as **ICF2+**, defining the **Infrastructure cost function** problem is defined in exactly the same way as **GCF2+** and the only difference is in the way the parameters k_{ij}^t and α_{ij}^t are previously computed. In this case, the optimal cable type k_{ij}^t is k_t (the first cable type k satisfying the relation $t\cdot I_{\mathrm{r}}\leq I_{\mathrm{z_k}}$) and its cost α_{ij}^t is equal to c_{ij}^k for cable type k_t.

C. Maximum Allowable Voltage Drop Constraints

This subsection describes the variables and constraints that, when added to the previously proposed formulations, guarantee the maximum allowable voltage drop of a wind farm layout. V denotes the maximum allowable voltage drop. Note that, like in the power loss case, when a cable of type k connects node i to node j and supports the current of t downstream wind turbines, its voltage drop v_{ij}^{kt} is independent of the other connections and depends only on k, t and ℓ_{ij}. Therefore, the parameters v_{ij}^{kt} can be computed in advance for all possible i, j, k and t.

The total voltage drop in all cables forming a path between the substation and each wind turbine cannot be higher than V. Therefore, we need to consider variables identifying the connections of each path. Consider the variables z_{ij}^{dkt} indicating that a cable of type k connecting node i to node j (such that i is the node on the substation side and this cable supports the current of t downstream wind turbines) is in the path from the substation toward turbine d.

With these new parameters and variables, the basic models **GCF1** and **ICF1** augmented with the following constraints (22)–(26) guarantee the maximum allowable voltage drop:

$$\sum_{i\in\mathcal{N}_0}\sum_{k=1}^{K}\sum_{t=1}^{t_{ki}}z_{ij}^{dkt}=\sum_{i\in\mathcal{N}}\sum_{k=1}^{K}\sum_{t=1}^{t_{ki}}z_{ji}^{dkt}, d\in\mathcal{N}, j\in\mathcal{N}\backslash\{d\} \tag{22}$$

$$\sum_{i\in\mathcal{N}_0}\sum_{k=1}^{K}\sum_{t=1}^{t_{ki}}\left(z_{id}^{dkt}\right)=1, d\in\mathcal{N} \tag{23}$$

$$z_{ij}^{dkt}\geq x_{ij}^{kt}, i\in\mathcal{N}_0, j\in\mathcal{N}, k=1,\ldots,K, t=1,\ldots,t_{ki} \tag{24}$$

$$\sum_{i\in\mathcal{N}_0}\sum_{j\in\mathcal{N}}\sum_{k=1}^{K}\sum_{t=1}^{t_{ki}}\left(v_{ij}^{kt}\cdot z_{ij}^{dkt}\right)\leq V, d\in\mathcal{N} \tag{25}$$

$$0\leq z_{ij}^{dkt}\leq 1, i\in\mathcal{N}_0, j\in\mathcal{N}, k=1,\ldots,K, t=1,\ldots,t_{ki} \tag{26}$$

Constraints (22) and (23) are the path conservation constraints. Constraints (24) guarantee that the path variables are one for a cable type k if this is the cable type used in the solution. Constraints (25) guarantee that, for each turbine d, the voltage drop of all cables belonging the path between the substation and turbine d is within the allowable voltage drop value. Constraints (26) are the variable domain constraints of the new variables z_{ij}^{dkt} (they are not required to be integer).

Concerning models **GCF2+** and **ICF2+**, the previous parameters and variables must be redefined in the following way. Consider the parameters v_{ij}^t representing the voltage drop of a cable of type k_{ij}^t connecting node i to node j such that i is the node on the substation side and this cable supports the current of t downstream wind turbines. Consider the variables z_{ij}^{dt} indicating that a cable of type k_{ij}^t connecting node i to node j (such that i is the node on the substation side and this cable supports the current of t downstream wind turbines) is in the path from the substation toward turbine d. With these new parameters and variables, the models **GCF2+** and **ICF2+** augmented with the following constraints (27)–(31) guarantee the maximum allowable voltage drop:

$$\sum_{i\in\mathcal{N}_0}\sum_{t=1}^{Q(i)} z_{ij}^{dt} = \sum_{i\in\mathcal{N}}\sum_{t=1}^{Q-1} z_{ji}^{dt}, d\in\mathcal{N}, j\in\mathcal{N}\backslash\{d\} \quad (27)$$

$$\sum_{i\in\mathcal{N}_0}\sum_{t=1}^{Q(i)} z_{id}^{dt} = 1, d\in\mathcal{N} \quad (28)$$

$$z_{ij}^{dt} \geq y_{ij}^t, i\in\mathcal{N}_0, j\in\mathcal{N}, t=1,\ldots,Q(i) \quad (29)$$

$$\sum_{i\in\mathcal{N}_0}\sum_{j\in\mathcal{N}}\sum_{k=1}^{K}\sum_{t=1}^{Q(i)} \left(v_{ij}^t\cdot z_{ij}^{dt}\right) \leq V, d\in\mathcal{N} \quad (30)$$

$$0\leq z_{ij}^{dt}\leq 1, i\in\mathcal{N}_0, j\in\mathcal{N}, t=1,\ldots,Q(i) \quad (31)$$

The interpretation of these constraints is similar to the previous ones.

IV. Case Studies and Results

The proposed models were applied to 3 Portuguese wind farms, each one with a central substation and a given number of wind turbines: the *Montalegre* wind farm with 25 wind turbines, the *Alto da Coutada* wind farm with 50 wind turbines and the *Gardunha* wind farm with 57 wind turbines. In all cases, the 12 cable types presented in Table I were considered. Additionally, it was considered $c_{\mathrm{e}} = 8\cdot 10^{-6}$ €/Wh as the energy cost, $T = 20$ years as the expected lifetime wind farm, $h = 24\cdot 365\cdot T$ as the number of hours during the expected lifetime, $l_{\mathrm{f}} = 0.5$ as the load factor and $\omega = 100\pi$ rad/s as the angular frequency. The results were obtained solving each model with CPLEX 12.4 running on an Intel Core i7-3610QM CPU @ 2.30 GHz with 6 GBytes of RAM and Windows 7 OS. For the smallest wind farm, the ground solution[2] is compared with the optimal solutions. The results of each wind farm are analyzed separately in the next subsections.

A. Montalegre Wind Farm

The *Montalegre* wind farm, located in the north of Portugal, is formed by $n = 25$ Enercon turbines, model E82/2000 with $P_{\mathrm{r}} = 2$ MW of rated power, interconnected by a $U = 20$ kV grid. With these parameters, the rated current drawn by each turbine is $I_{\mathrm{r}} = 57.735$ A and the maximum number of wind turbines per branch is $Q = 10$. The total nominal power of the wind farm is 50 MW. The coordinates of the substation and the wind turbines, in WGS84, are given in Table IV, in Appendix.

The branches characteristics of the ground solution and the optimal solution obtained using the **Global cost function** and **Infrastructure cost function** are given in Tables VII, VIII and IX, respectively. For a better visualization of these solutions, Figs. 2(a), (b) and (c) present the corresponding layouts. In these figures, the black node is the substation and each main branch is represented by a different color.

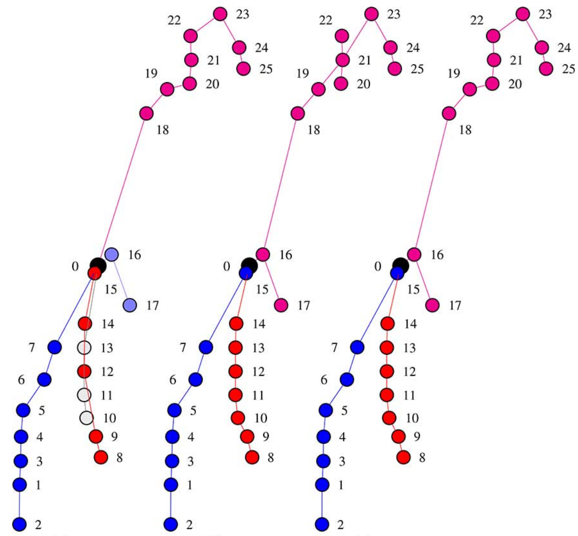

Fig. 2. Montalegre wind farm: (a) Ground layout; (b) Optimum **GCF** layout and (c) Optimum **ICF** layout.

The three solutions differ both in the used cable types and in the layout configuration. The ground solution has four branches (Fig. 2(a)) while the other two solutions (Fig. 2(b)–(c)) have three branches involving the same sets of wind turbines. The two branches 0-15-14-12-9-8 and 0-13-11-10 in the ground solution are almost completely merged into one branch 0-14-13-12-11-10-9-8 (in both optimal solutions) while turbine 15 is included in the branch 0-7-6-5-4-3-1-2. Moreover, in both optimal solutions, the peripheral wind turbines (i.e., the leaves of the spanning tree) are connected by thinner cables than the ones in the ground solution. Finally, the main difference between the two optimal solutions is in the connections of nodes 19, 20, 21, 22 and 23: in the **GCF** solution, nodes 20 and 22 become leaves in the spanning tree (Fig. 2(b)).

The total costs of the wind farm are 1077271.9€, 959248.19€ and 971782€ for ground solution, **GCF** solution and **ICF** solution, respectively. Comparing the ground solution with **GCF** solution, in the latter there is a 19.8% decrease in the infrastructure cost but an increase of 30.2% and 12.9% in the active and reactive losses, respectively, that corresponds to a total decrease of 11.0% in the total costs. On the other hand, comparing the ground solution with **ICF** solution, in the latter there is a 22.9% decrease in the infrastructure cost but an increase of 57.2% and 16.0% in the active and reactive losses, respectively, that corresponds to a total decrease of 9.8% in the total costs.

Comparing the **GCF** solution with **ICF** solution, in the latter the infrastructure cost reduces of 26427.20€, which correspond to a reduction of 3.9%. Nevertheless, there is an increase of 20.8% and 2.8% in the active and reactive losses, respectively. Considering all costs, with **ICF** there is an increment of 12534.00€, which corresponds to an increase of 1.3%.

In the **GCF** solution, the maximum voltage drop occurs in the branch 0-16-18-19-21-23-24-25 where $\Delta V = 240.64$ V.

[2]The ground solution is the installed wind farm which is currently in operation.

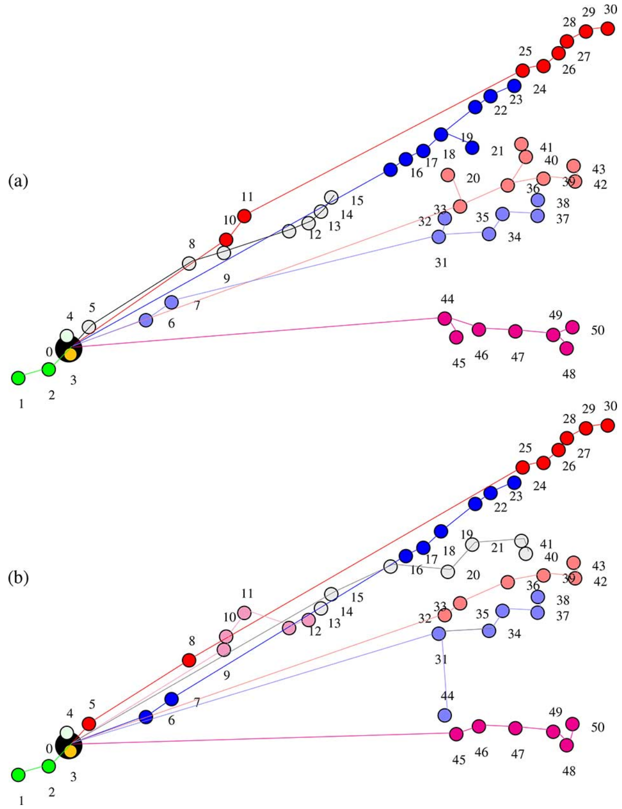

Fig. 3. Alto da Coutada wind farm: (a) Optimum **GCF** layout and (b) Optimum **ICF** layout.

Therefore, $\Delta V_{\mathrm{p}} = \frac{\Delta V}{20000} = 1.2\%$. In the **ICF** solution, the maximum voltage drop occurs in the branch 0-16-18-19-20-21-22-23-24-25 where $\Delta V_{\mathrm{p}} = 1.77\%$. In both cases, the maximum allowable voltage drop of 5% is fulfilled.

B. *Alto da Coutada Wind Farm*

The *Alto da Coutada* wind farm located in the north of Portugal too, is formed by $n = 50$ Enercon turbines model E82/2300 with $P_{\mathrm{r}} = 2.3$ MW of rated power, interconnected by a $U = 20$ kV grid. With these parameters, the rated current drawn by each turbine is $I_{\mathrm{r}} = 66.795$ A and the maximum number of wind turbines per branch is $Q = 8$. The total nominal power of the wind farm is 115 MW. The coordinates of the substation and the wind turbines are given in Table V.

The **GCF** optimal solution, presented in Fig. 3(a), has 9 branches involving 1, 1, 2, 7, 7, 8, 8, 8 and 8 wind turbines, as presented in Table X. On the other hand, the **ICF** optimal solution, presented in Fig. 3(b), has 9 branches involving 1, 3, 6, 6, 6, 6, 6, 8 and 8 wind turbines as can be seen in Table XI.

Comparing these two solutions, there are differences in the section of cables used as well as in the set of turbines included on each branch. With **ICF**, the infrastructure cost reduces 157621.48€ but considering the total cost (including the costs of active and reactive losses) it leads to a higher cost of 78914.15€. In the **GCF** solution, the maximum voltage drop is obtained in the branch 0-10-11-25-26-27-28-29-30 with $\Delta V_{\mathrm{p}} = 1.88\%$, while in the **ICF** solution, the maximum voltage drop is obtained in the branch 0-5-8-25-26-27-28-29-30 with $\Delta V_{\mathrm{p}} = 1.59\%$. Once again, the maximum allowable voltage drop of 5% is fulfilled.

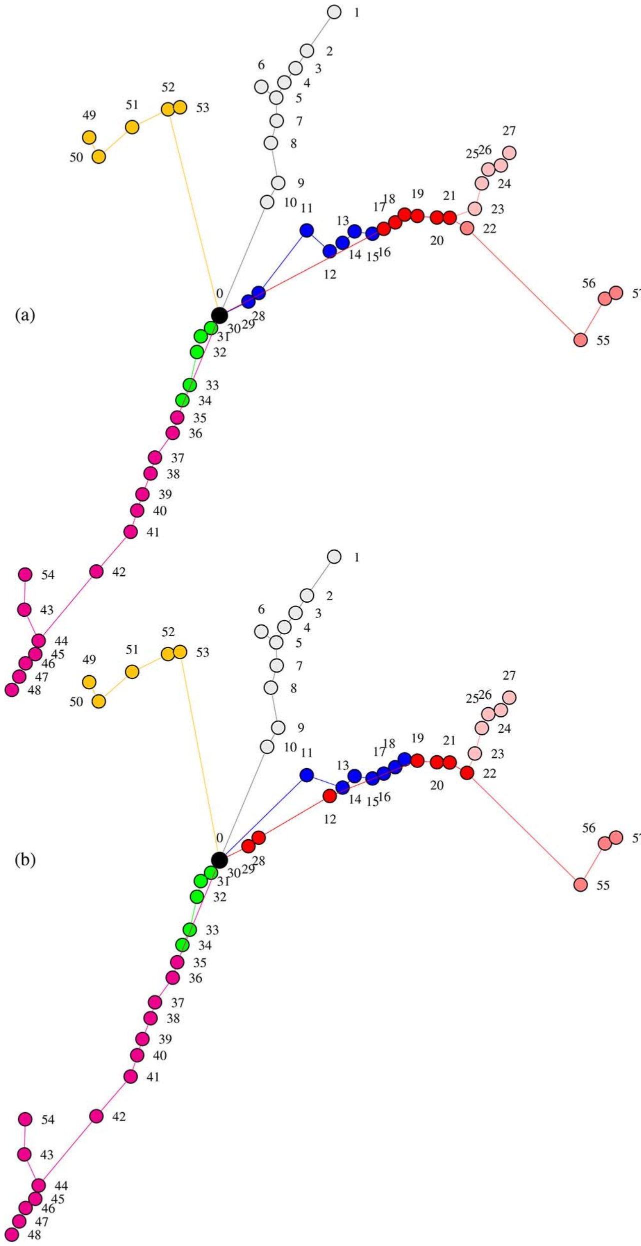

Fig. 4. Gardunha wind farm: (a) Optimum **GCF** layout and (b) Optimum **ICF** layout.

C. *Gardunha Wind Farm*

The *Gardunha* wind farm, located in the center of Portugal, is formed by $n = 57$ Enercon turbines, such that 55 are Enercon E82/2000 and 2 Enercon E70/E4 all with $P_{\mathrm{r}} = 2$ MW of rated power, interconnected by a $U = 30$ kV grid. With these parameters, the rated current drawn by each turbine is $I_{\mathrm{r}} = 38.49$ A and the maximum number of wind turbines per branch is $Q = 15$. The total nominal power of the wind farm is 114 MW. The coordinates of the substation and the wind turbines, in WGS84, are given in Table VI.

TABLE II
MONTALEGRE, ALTO DA COUTADA AND *GARDUNHA* WIND FARMS COSTS

Model	Costs C_c	C_p	C_q	Total
Montalegre				
GCF	686107.00	174302.45	98838.74	**959248.19**
ICF	659679.80	210509.29	101593.10	**971782.19**
Alto da Coutada				
GCF	4498424.56	1057610.79	826913.72	**6382949.07**
ICF	4340803.08	1382032.43	739027.71	**6461863.22**
Gardunha				
GCF	1911323.42	437822.39	267302.05	**2616447.86**
ICF	1823298.84	576643.93	259230.70	**2659173.47**

TABLE III
MONTALEGRE, *ALTO DA COUTADA* AND *GARDUNHA* WIND FARM

Model	Mumber of constr.	var.	Objective values LP	IP	Gaps (%) init.	final	Time (sec.)
Montalegre (Q=10)							
GCF1	52	38152	940775.54	959248.19	1.93	0.00	0.45
GCF2+	252	5651	949637.67	959248.19	1.00	0.00	0.31
ICF1	52	38152	639498.60	659679.80	3.06	0.00	0.25
ICF2+	252	5651	652015.24	659679.80	1.16	0.00	0.23
Alto da Coutada (Q=8)							
GCF1	102	125052	6242909.20	6382949.07	2.19	0.00	81.16
GCF2+	402	17551	6317863.89	6382949.07	1.02	0.00	18.37
ICF1	102	125052	4154978.54	4340803.08	4.28	0.00	317.66
ICF2+	402	17551	4265737.82	4340803.08	1.73	0.00	50.70
Gardunha (Q=15)							
GCF1	116	305465	2398033.55	2629071.01*	8.35	1.94	9562.5
GCF2+	857	45544	2437350.76	2616447.36	6.85	0.00	479.9
ICF1	116	305465	1630954.87	1943051.16*	10.55	8.93	17634.9
ICF2+	857	45544	1687876.05	1823298.84	7.43	0.00	1556.8

* Finish due to out-of-memory.

The **GCF** and **ICF** optimal solutions illustrated in Fig. 4 are presented in Tables XII and XIII, respectively. Both solutions have 6 branches involving 5, 5, 7, 10, 15 and 15 wind turbines. Nevertheless, there are differences in the design of the wind farm as well as in the section of cables.

Comparing the costs of both solutions, the **ICF** solution represents an infrastructure cost reduction of 88024.58€. However, considering the total costs, which includes the costs of active and reactive losses, the **ICF** solutions leads to a cost increase of 42725.61€. In the **GCF** solution, the maximum voltage drop is obtained in the branch 0-16-17-18-19-20-21-23-24-25-26-27 with $\Delta V_{\rm p} = 1.23\%$, while in the **ICF** optimal solution, the maximum voltage drop is obtained in the branch 0-29-28-12-19-20-21-22-55-56-57 with $\Delta V_{\rm p} = 2.40\%$.

D. Summary of Optimization Results

Table II summarizes the costs of the **GCF** and **ICF** optimal solutions. These results show that, the consideration of losses costs in the design objective always produces lower cost solutions. The comparison of the different cost components in the two solutions shows that the cost gains at the end of the wind farm lifetime are obtained at the expense of an higher initial investment cost, i.e., the additional infrastructure cost (which is paid at the beginning) is compensated by the savings in the energy losses registered during the wind farm lifetime.

E. Models Performance

Table III characterizes all models (**GCF1**, **GCF2+**, **ICF1** and **ICF2+**) for all wind farms, in terms of modeling characteristics, integer linear programming parameters and runtimes.

In terms of modeling characteristics, Table III shows the number of constraints (column 'constr.') and the number of variables (column 'var.') of each model. As easily seen, models **GCF2+** and **ICF2+** have much less variables (due to the variable reformulation technique) and much more constraints (due to the additional constraints included in the models) than models **GCF1** and **ICF1**.

Table III also presents the cost value of the linear relaxation (column 'LP'), the cost value or the best solution found (column 'IP'), the LP gap, i.e., the gap between the linear relaxation value LP and the optimal solution value IP, given by $\frac{IP-LP}{IP}$, (column 'init.') and the final gap, (column 'final'). Finally, the last column (column 'Time') gives the total runtime (in seconds) of CPLEX to find the best solution. Note that, when the final gap is 0.0%, it means that the optimal solution has been found.

The main conclusion is that models **GCF2+** and **ICF2+** are better than models **GCF1** and **ICF1**. All models of the smallest wind farm (*Montalegre*) were easy to solve. Nevertheless, in the *Alto da Coutada* wind farm, CPLEX took much shorter time to solve the **GCF2+** and **ICF2+** models. Finally, in *Gardunha* wind farm, the more complex case, only models **GCF2+** and **ICF2+** were solved to optimality (runtimes of around 8 minutes for **GCF2+** and 26 minutes for **ICF2+**) while the others ended due to out of memory with non optimal solutions. Note that, in general, when there are alternative models defining the same optimization problem, the best model is usually the one with a better LP value. In all cases shown in Table III, the initial LP values of models **GCF2+** and **ICF2+** are closer to the optimal values (i.e., their initial gap is closer to zero) than the **GCF1** and **ICF1** models, the reason why these models make CPLEX more efficient while solving them.

V. CONCLUSION

Nowadays, the environment preservation is a very hot topic. In this regard, renewable energy has gained popularity due to its low pollution characteristics and the wind energy has become one of the most relevant energy sources. The efficient production of electrical energy from the wind can be improved in several ways. One of them, considered in this work, is the design of the cable network layout of wind farms.

This work has addressed the optimal design of the wind farm cable network aiming to minimize the sum of the infrastructure cost and the cost of the power that is lost in the cables over an expected lifetime of 20 years. In order to assess the gains of considering the losses costs in the design goal, it was also considered the problem of minimizing only the infrastructure cost. For both problems, the integer linear programming models **GCF1** and **ICF1** were initially proposed. Then, new models (**GCF2+** and **ICF2+**) were derived that proved to be much more efficient since they could either solve the problem instances in shorter runtimes or find the optimal solutions in cases such that the previous models failed.

TABLE IV
MONTALEGRE WIND FARM COORDINATES (WGS84)

No.	Latitude	Longitude	No.	Latitude	Longitude
0	41.7422700	−7.9428260	13	41.7345910	−7.9471840
1	41.7217060	−7.9674240	14	41.7368500	−7.9469550
2	41.7179780	−7.9674950	15	41.7415870	−7.9439190
3	41.7239280	−7.9670640	16	41.7433390	−7.9386110
4	41.7262150	−7.9669790	17	41.7385760	−7.9329060
5	41.7287040	−7.9663140	18	41.7566040	−7.9276870
6	41.7315890	−7.9596900	19	41.7588760	−7.9211900
7	41.7346310	−7.9564550	20	41.7594040	−7.9141320
8	41.7242850	−7.9419550	21	41.7616320	−7.9135890
9	41.7262360	−7.9435220	22	41.7638860	−7.9138520
10	41.7279830	−7.9464390	23	41.7659510	−7.9045660
11	41.7301420	−7.9471970	24	41.7627960	−7.8986160
12	41.7323580	−7.9471610	25	41.7608100	−7.8972670

TABLE V
ALTO DA COUTADA WIND FARM COORDINATES (WGS84)

No.	Latitude	Longitude	No.	Latitude	Longitude
0	41.522755	−7.595805			
1	41.517580	−7.604703	26	41.572622	−7.512087
2	41.519109	−7.599337	27	41.574893	−7.509432
3	41.521755	−7.595505	28	41.576982	−7.507969
4	41.524987	−7.596142	29	41.578721	−7.504606
5	41.526588	−7.592248	30	41.579221	−7.500768
6	41.527789	−7.582202	31	41.542497	−7.530569
7	41.530939	−7.577686	32	41.545762	−7.529500
8	41.537791	−7.574587	33	41.547844	−7.526791
9	41.539651	−7.568444	34	41.543005	−7.521698
10	41.541970	−7.568057	35	41.546523	−7.519316
11	41.546164	−7.564848	36	41.551573	−7.518409
12	41.543489	−7.556946	37	41.546209	−7.513137
13	41.544914	−7.553510	38	41.548974	−7.513085
14	41.546958	−7.551330	39	41.552762	−7.512087
15	41.549448	−7.549512	40	41.556594	−7.515213
16	41.554332	−7.539066	41	41.558856	−7.515989
17	41.556190	−7.536363	42	41.552266	−7.506523
18	41.557645	−7.533284	43	41.555008	−7.506796
19	41.560542	−7.530172	44	41.528073	−7.529517
20	41.553402	−7.528967	45	41.524768	−7.527459
21	41.558211	−7.524661	46	41.526140	−7.523484
22	41.565391	−7.524126	47	41.525814	−7.517044
23	41.567302	−7.521420	48	41.522806	−7.508016
24	41.569122	−7.517241	49	41.525187	−7.510382
25	41.571823	−7.515764	50	41.526548	−7.506983

With the proposed models, the optimal solutions for both addressed problems were computed for three wind farms located in Portugal, spanning from 25 up to 57 wind turbines. The optimal solutions that included the power loss costs in the objective function of the problem were always better, showing that the energy lost in the cables is an important component that must be considered in the design of the wind farm layout. Moreover, for one wind farm, the optimal solution and the ground solution were compared showing that significant cost savings can be achieved.

APPENDIX

Tables IV–VI present the geographical coordinates of the three wind farms considered, where node number 0 corresponds to the substation.

TABLE VI
GARDUNHA WIND FARM COORDINATES (WGS84)

No.	Latitude	Longitude	No.	Latitude	Longitude
0	40.071000	−7.6631590	29	40.072761	−7.655931
1	40.109055	−7.634448	30	40.069422	−7.665296
2	40.104194	−7.641263	31	40.068388	−7.667859
3	40.101999	−7.644116	32	40.066428	−7.668822
4	40.100219	−7.646920	33	40.062293	−7.670619
5	40.098309	−7.648963	34	40.060382	−7.672472
6	40.099667	−7.652694	35	40.058216	−7.673765
7	40.095418	−7.648842	36	40.056276	−7.674891
8	40.092641	−7.650316	37	40.053186	−7.679330
9	40.087617	−7.648464	38	40.051191	−7.680456
10	40.085215	−7.651248	39	40.048578	−7.682477
11	40.081661	−7.641331	40	40.046548	−7.683791
12	40.079052	−7.635558	41	40.043879	−7.685438
13	40.080110	−7.632338	42	40.038905	−7.694023
14	40.081538	−7.629338	43	40.034117	−7.712073
15	40.081248	−7.624861	44	40.030223	−7.708477
16	40.081857	−7.622031	45	40.028551	−7.709319
17	40.082673	−7.619152	46	40.027398	−7.711751
18	40.083644	−7.616788	47	40.025719	−7.713332
19	40.083477	−7.613612	48	40.024060	−7.715193
20	40.083284	−7.608701	49	40.093317	−7.695821
21	40.083251	−7.605512	50	40.090912	−7.693421
22	40.081952	−7.601161	51	40.094618	−7.685059
23	40.084375	−7.599219	52	40.096852	−7.676074
24	40.087573	−7.597459	53	40.097101	−7.673058
25	40.089304	−7.595839	54	40.038530	−7.711859
26	40.089812	−7.592704	55	40.067932	−7.572727
27	40.091378	−7.590593	56	40.073107	−7.566636
28	40.073822	−7.653368	57	40.073841	−7.563851

TABLE VII
MONTALEFRE: GROUND SOLUTION

Type	Connections
6	(0,16), (3,1), (1,2), (9,8), (12,9), (13,11), (11,10), (16,17), (21,22), (22,23), (23,24), (24,25)
9	(0,13), (0,15),(4,3), (15,14), (14,12), (5,4), (20,21)
10	(0,7), (0,18), (7,6), (18,19), (6,5), (19,20)

TABLE VIII
MONTALEGRE: **GLOBAL** OPTIMAL CONNECTIONS

Type	Connections
1	(1,2), (9,8), (16,17), (21,20), (21,22), (24,25)
4	(3,1), (10,9), (23,24)
6	(5,4),(4,3),(12,11)(11,10),(21,23)
8	(6,5), (13,12)
9	(0,14), (14,13), (15,7), (7,6), (18,19), (19,21)
10	(0,15), (16,18)

TABLE IX
MONTALEGRE: **INFRASTRUCTURE** OPTIMAL CONNECTIONS

Type	Connections
1	(1,2), (10,9), (9,8), (16,17), (23,24), (24,25)
4	(4,3), (3,1), (11,10), (22,23)
5	(5,4), (12,11), (21,22)
7	(6,5), (5,4), (13,12), (20,21)
8	(7,6), (14,13), (19,20)
9	(0,14), (15,7), (18,19)
10	(0,15), (16,18)
12	(0,16)

Tables VII–XIII present the connections of the solutions as well as its cable type.

TABLE X
ALTO DA COUTADA: **GLOBAL** OPTIMAL CONNECTIONS

Type	Connections
1	(0,3), (0,4), (2,1), (14,15), (19,21), (23,24), (29,30), (31,32), (33,20), (37,38), (40,41), (42,43), (44,45), (49,48), (49,50)
4	(0,2), (13,14), (22,23), (28,29), (35,37), (36,40), (39,42)
6	(9,12), (12,13), (19,22), (26,27), (27,28), (31,34), (34,35), (36,39), (46,47), (47,49)
9	(5,8), (7,31), (8,9), (11,25), (17,18), (18,19), (25,26), (33,36), (44,46)
11	(0,5), (0,44), (6,7), (10,11), (16,17)
12	(0,6), (0,10), (0,16), (0,33)

TABLE XI
ALTO DA COUTADA: **INFRASTRUCTURE** OPTIMAL CONNECTIONS

Type	Connections
1	(0,4), (2,1), (13,14), (23,24), (29,30), (31,44), (37,38), (41,40), (42,43), (49,48), (49,50)
2	(3,2), (12,13), (21,41), (22,23), (28,29), (35,37), (39,42)
3	(0,3), (11,12), (19,22), (20,21), (27,28), (34,35), (36,39), (47,49)
4	(10,11), (16,20), (18,19), (26,27), (31,34), (33,36), (46,47)
5	(9,10), (15,16), (17,18), (25,26), (32,33), (45,46)
6	(0,9), (0,32), (0,45), (0,15), (0,31), (7,17), (8,25)
7	(5,8), (6,7)
8	(0,6), (0,5)

TABLE XII
GARDUNHA: **GLOBAL** OPTIMAL CONNECTIONS

Type	Connections
1	(2,1), (5,6), (14,15), (26,27), (33,34), (43,54), (47,48), (50,49), (52,53), (56,57)
2	(3,2), (13,14), (25,26), (32,33), (44,43), (46,47), (51,50), (55,56)
3	(4,3), (12,13), (22,55), (24,25), (31,32), (45,46), (52,51)
4	(5,4), (11,12), (21,22), (23,24), (44,45), (30,31)
5	(0,30), (0,52), (21,23), (28,11)
6	(7,5), (29,28)
7	(0,29), (8,7), (42,44)
8	(9,8), (41,42)
9	(10,9), (40,41)
10	(0,10), (20,21), (39,40)
11	(19,20), (38,39)
12	(18,19), (37,38)
13	(17,18), (36,37)
14	(16,17), (35,36)
15	(0,16), (0,35)

TABLE XIII
GARDUNHA: **INFRASTRUCTURE** OPTIMAL CONNECTIONS

Type	Connections
1	(2,1), (5,6), (17,18), (26,27), (33,34), (43,54), (47,48), (50,49), (56,57)
2	(3,2), (16,17), (25,26), (32,33), (44,43), (46,47), (51,50), (55,56)
3	(4,3), (15,16), (22,55), (24,25), (31,32), (45,46), (52,51)
4	(5,4), (14,15), (23,24), (30,31), (44,45), (53,52)
5	(0,30), (0,53), (13,14), (22,23)
6	(7,5), (11,13)
7	(0,11), (8,7), (42,44)
8	(9,8), (41,42)
9	(10,9), (21,22), (40,41)
10	(0,10), (20,21), (39,40)
11	(19,20), (38,39)
12	(12,19), (37,38)
13	(28,12), (36,37)
14	(29,28), (35,36)
15	(0,29), (0,35)

REFERENCES

[1] E. M. Carreno, N. Moreira, and R. Romero, "Distribution network reconfiguration using an efficient evolutionary algorithm," in *Proc. IEEE Power Eng. Soc. General Meeting*, Tampa, FL, USA, Jun. 2007, vol. 24–28.

[2] A. Jenkins, M. Scutariu, and K. Smith, "Offshore wind farm inter-array cable layout," in *Proc. 2013 IEEE Grenoble PowerTech (POWERTECH)*, June 2013, pp. 1–6.

[3] K. Veeramachaneni, M. Wagner, U.-M. O'Reilly, and F. Neumann, "Optimizing energy output and layout costs for large wind farms using particle swarm optimization," in *Proc. 2012 IEEE Congr. Evolutionary Computation (CEC)*, Jun. 2012, pp. 1–7.

[4] S. Dutta and T. Overbye, "Optimal wind farm collector system topology design considering total trenching length," *IEEE Trans. Sustain. Energy*, vol. 3, no. 3, pp. 339–348, Jul. 2012.

[5] W. Yuan-Kang, L. Ching-Yin, C. Chao-Rong, H. Kun-Wei, and T. Huang-Tien, "Optimization of the wind turbine layout and transmission system planning for a large-scale offshore wind farm by AI technology," in *Proc. 2012 IEEE Ind. Applicat. Soc. Annu. Meeting (IAS)*, Oct. 2012, pp. 1–9.

[6] J. Serrano Gonzalez, M. Burgos Payan, and J. R. Santos, "An improved evolutive algorithm for large offshore wind farm optimum turbines layout," in *Proc. 2011 IEEE Trondheim PowerTech*, Jun. 2011, pp. 1–6.

[7] G. Dobric, M. Zarkovic, and Z. Djurisic, "Fuzzy based computational efficiency for optimal wind farm layout design," in *Proc. 2013 Int. Conf. Renewable Energy Research and Applications (ICRERA)*, Oct. 2013, pp. 274–279.

[8] M. Nandigam and S. Dhali, "Optimal design of an offshore wind farm layout," in *Proc. Int. Symp. Power Electronics, Electrical Drives, Automation and Motion, 2008 (SPEEDAM 2008)*, Jun. 2008, pp. 1470–1474.

[9] P. Zhang, D. Romero, J. Beck, and C. Amon, "Solving wind farm layout optimization with mixed integer programming and constraint programming," in *Integration of AI and OR Techniques in Constraint Programming for Combinatorial Optimization Problems*, ser. Lecture Notes in Computer Science, C. Gomes and M. Sellmann, Eds. Berlin, Germany: Springer, 2013, vol. 7874, pp. 284–299.

[10] J. Bauer and J. Lysgaard, "The offshore wind farm array cable layout problem—A planar open vehicle routing problem," *J. Oper. Res. Soc.*, vol. 66, pp. 360–368, 2015.

[11] S. Lumbreras and A. Ramos, "Optimal design of the electrical layout of an offshore wind farm applying decomposition strategies," *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 1434–1441, May 2013.

[12] W. Fangdong, L. Han, W. Fangdong, and W. Buying, "Substation optimization planning based on the improved orientation strategy of Voronoi diagram," in *Proc. 2010 2nd Int. Conf. Inf. Sci. Eng. (ICISE)*, 2010, pp. 1563–1566.

[13] F. Llorens-Iborra, J. Riquelme-Santos, and E. Romero-Ramos, "Mixed-integer linear programming model for solving reconfiguration problems in large-scale distribution systems," *Electr. Power Syst. Res.*, vol. 88, no. 0, pp. 137–145, 2012.

[14] S. Kalambe and G. Agnihotri, "Loss minimization techniques used in distribution network: Bibliographical survey," *Renew. Sustain. Energy Rev.*, vol. 29, no. 0, pp. 184–200, 2014.

[15] A. Cerveira, J. Baptista, and E. Pires, Á. Herrero, Ed. *et al.*, "Optimization design in wind farm distribution network," in *International Joint Conference SOCO'13-CISIS'13-ICEUTE'13*, 2014, vol. 239, pp. 109–119, ser. Advances in Intelligent Systems and Computing, Springer International Publishing.

[16] B. Moradzadeh and K. Tomsovic, "Mixed integer programming-based reconfiguration of a distribution system with battery storage, 2012," in *Proc. North Amer. Power Symp. (NAPS)*, Sep. 2012, pp. 1–6.

[17] L. Gouveia, "A 2n constraint formulation for the capacitated minimal spanning tree problem," *Oper. Res.*, vol. 43, pp. 130–141, 1995.

Adelaide Cerveira received the B.Sc. degree in mathematics–operations research branch from the University of Coimbra, Coimbra, Portugal, in 1993, the M.Sc. degree in operations research from the University of Lisbon, Lisbon, Portugal, in 1998, and the Ph.D. degree in statistics and operations research from the University of Lisbon, Lisbon, Portugal, in 2006. She is currently an Auxiliary Professor with UTAD. Her current research interests include combinatorial optimization and applications.

Amaro de Sousa received a Ph.D. degree in telecommunications engineering from the University of Aveiro, Portugal, in 2001. He is a professor at the Department of Electronics, Telecommunications and Informatics of the University of Aveiro, Portugal, and also a senior researcher at the Applied Mathematics Group of Institute of Telecommunications-pole of Aveiro. His research interests include network design with a special focus on operations research and computer science techniques.

E. J. Solteiro Pires received the B.Sc. degree in electrical engineering from the University of Coimbra, Coimbra, Portugal, in 1994, the M.Sc. degree in electrical and computer engineering from the University of Porto, Porto, Portugal, in 1999, and the Ph.D. degree from the University of Trás-os-Montes and Alto Douro (UTAD), Vila Real, Portugal, in 2006. He is currently an Auxiliary Professor with UTAD. His current research interests include evolutionary computation, multiobjective problems, and fractional calculus.

José Baptista graduated in electrical engineering from the University of Trás-os-Montes e Alto Douro (UTAD), Portugal in 1991. He obtained the M.Sc. degree in power electronics in 1997 from UTAD and the Ph.D. degree in electrical engineering in 2007 from UTAD. Presently, he is an Auxiliary Professor in the Department of Electrical Engineering, UTAD. He is also a researcher in power quality, electrical machines and renewables. His main interest areas are power quality, electrical machines and renewables.