Leapfrogging Vortices

Bryce Richard

May 2022

1 Introduction

For this project I attempted to simulate and plot a pair of leapfrogging vortices moving through a 2D plane. The model was created using the Julia programming language and based on vector equations for the induced velocity of a fluid from a toroidal vortex. By using a basic time stepping method I was able to achieve a plotted solution that was similar to the provided data and validated by hand calculations.

2 Methods

2.1 Approach

To model leapfrogging vortex rings in two dimensions with two pairs of two vortices, for four vortices total. These modeled vortices are the result of an imaginary cut in the x-y plane of the 3D vortex ring. [htb] The position and velocity of each vortex was treated independently. The movement was simulated using a basic time step approach in which the instantaneous velocity of each

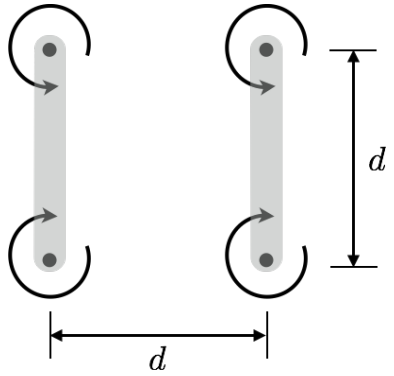

Figure 1: A pair of leapfrogging vortices cut in the x-y plane

vortex was calculated and the new position was then calculated with the resultant velocity and the specified change in time. Each vortex induced a velocity on the other vortices following the equation:

$$\vec{V} = \frac{\vec{\Gamma} \times \vec{r}}{2\pi r^2} \tag{1}$$

Therefore, the resultant instantaneous velocity for each vortex was calculated by summing the resultant velocity vectors induced at that location by the other vortices. With each iteration the x and y position for each vortex was recorded and all these points were then plotted with vortex pairs in matching colors.

2.2 Modeling Parameters

For the plot shown below I used the following initial values: d = 1, $\Gamma = 1$, and time-step = 0.01. The velocity and position of each vortex was calculated over a 40 second time period.

3 Results

The following plot shows the simulated x and y position of each vortex ring throughout time.

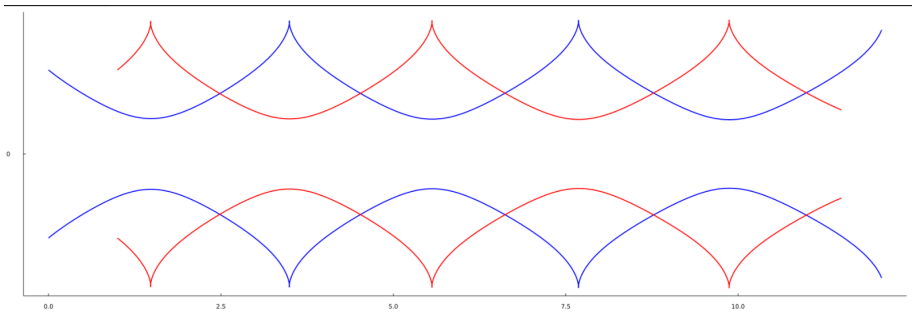

Figure 2: Plotted X and Y position for each vortex over 40 seconds

4 Discussion

The plotted movement of the leapfrogging vortices appears to be correct, as the first several iterations of the movement matched the values calculated by hand. I did some experimentation with changing the values for Γ and d. I found that when Γ increased, the x distance traveled for each 'leapfrog' increased with each successive leapfrog until the eventually the vortex rings decoupled. I also found that as d increased, larger retrograde motion could be observed for the outer ring as the inner vortex ring passed. This makes sense as the backward velocity

induced by the inner ring vortex with the shortest r would be greater than the induced forward velocity from the other two vortices. With the r^2 term on the bottom of equation 1, a shorter r leads to a larger magnitude of induced velocity.