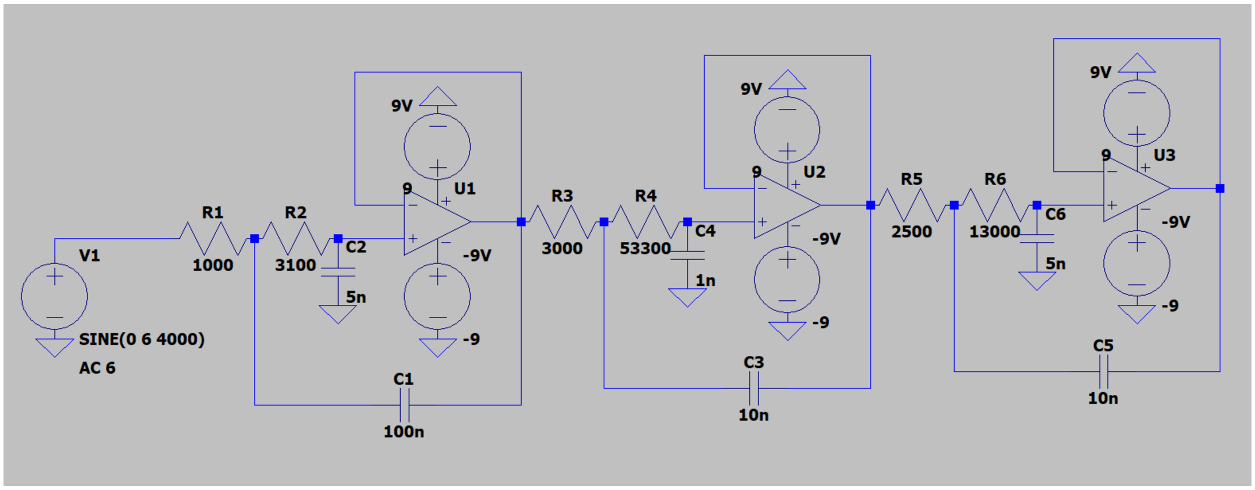
Brycen Craig

4/30/2024

Schematic Diagram: 6^{th} order low pass filter



To come up with component values for the 6^{th} order low pass filter, I used the characteristic equation roots: k=1, 2, 3…6

$$s = 2\pi * 4000 * e^{j(75+k*30°)}$$

Each root **s** has a complex conjugate which I used to get component values. The complex conjugates. The differential equation of each second order low pass filter is given by:

$$\frac{d^2V_o}{dt^2} + \left(\frac{1}{R_1C_1} + \frac{1}{R_1C_2}\right)\frac{dV_o}{dt} + \frac{V_o}{R_1C_1R_2C_2} = \frac{V_{in}}{R_1C_1R_2C_2}$$

The real and imaginary parts of roots s are given by:

$$Re[s] = \frac{-1}{2}\left(\frac{1}{R_1C_1} + \frac{1}{R_1C_2}\right)$$

$$Im[s] = \pm\frac{1}{2}\sqrt{\frac{4}{R_1C_1R_2C_2} - \left(\frac{1}{R_1C_1} + \frac{1}{R_1C_2}\right)^2}$$

Next using each of paired roots by setting the real equation of s to the real part of s and the imaginary equation of s to the imaginary part of s to find component values. To find the component values I set the capacitors 1 and 2 values to a constant and using a system of equations found values for resistors 1 and 2.

I repeated this for each pair of roots until I found component values for 3 2^{nd} order low pass filters. Then to make the 6^{th} order low pass filter, the three-2^{nd} order low pass filters are cascaded where the input voltage is the previous filter.

To find the actual magnitude response of the circuit, the transfer function for each filter needs to be found. The magnitude response for a single filter is:

$$H(w) = \frac{1}{(\omega - s_1)(\omega - s_6)}$$

$$H(w) = \frac{1}{(\omega - \omega_o(\cos(105) + \sin(105)i)(\omega - \omega_o(\cos(255) + \sin(255)i)}$$

$$H(w) = \frac{1}{\omega^2 - 2\omega\omega_o\cos(105) + \omega_o{}^2(\cos^2(105) + \sin^2(105))}$$

The transfer function of the first filter is:

$$H(w)_1 = \frac{1}{s^2 - 2s\omega_o\cos(165) + \omega_o{}^2}$$

The transfer function of the second filter is:

$$H(w)_2 = \frac{1}{s^2 - 2s\omega_o\cos(135) + \omega_o{}^2}$$

The transfer function of the third filter is:

$$H(w)_3 = \frac{1}{s^2 - 2s\omega_o\cos(105) + \omega_o{}^2}$$

To find the transfer function of the entire circuit each transfer function is multiplied together.

$$H(w)_{tot} = \frac{1}{s^2 - 2s\omega_o\cos(165) + \omega_o{}^2} * \frac{1}{s^2 - 2s\omega_o\cos(135) + \omega_o{}^2} * \frac{1}{s^2 - 2s\omega_o\cos(105) + \omega_o{}^2}$$

$$H(w)_{tot} = \frac{1}{(s - s_1)(s - s_2)(s - s_3)(s - s_4)(s - s_5)(s - s_6)}$$

$$|H(w)_{tot}|^2 = \left(\frac{\omega^6}{|(j\omega - s_1)(j\omega - s_2)(j\omega - s_3)(j\omega - s_4)(j\omega - s_5)(j\omega - s_6)|}\right)^2$$

When plotting the magnitude of the transfer function above and the theoretical response given:

$$|H(w)_{tot}| = \frac{1}{1 + \left(\frac{\omega}{\omega_o}\right)^{12}}$$

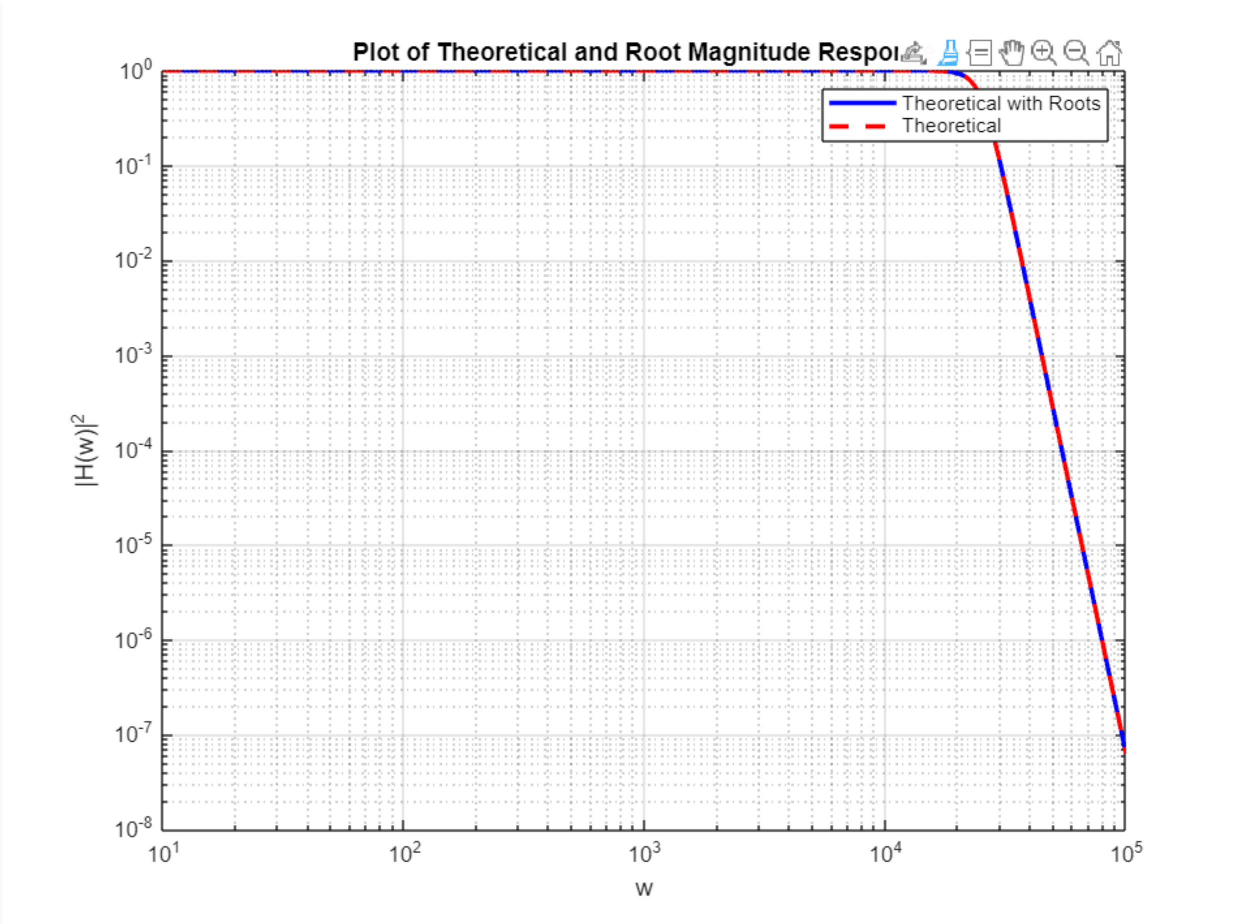


Figure 1: Theoretical and Theoretical Calculated with Roots Graph

The two equations are equal.

To calculate the actual magnitude response of the circuit, we can multiply the transfer function of each filter as proven above.

$$H(\omega) = \frac{1}{1 + j\omega(R_1 + R_2)C_2 - \omega^2 R_1 C_1 R_2 C_2}$$

$$H(\omega)_1 = \frac{1}{1 + j\omega(4.1k)5n - \omega^2(1k * 3.1k * 100n * 5n)}$$

$$H(\omega)_2 = \frac{1}{1 + j\omega(56.3k)1n - \omega^2(3k * 53.3k * 10n * 1n)}$$

$$H(\omega)_3 = \frac{1}{1 + j\omega(15.5k)5n - \omega^2(2.5k * 13k * 10n * 5n)}$$

$$|H(\omega)_{tot}|^2 = |H(\omega)_1 * H(\omega)_2 * H(\omega)_3|^2$$

When plotted together, there is a slight difference because component values won't be ideal.

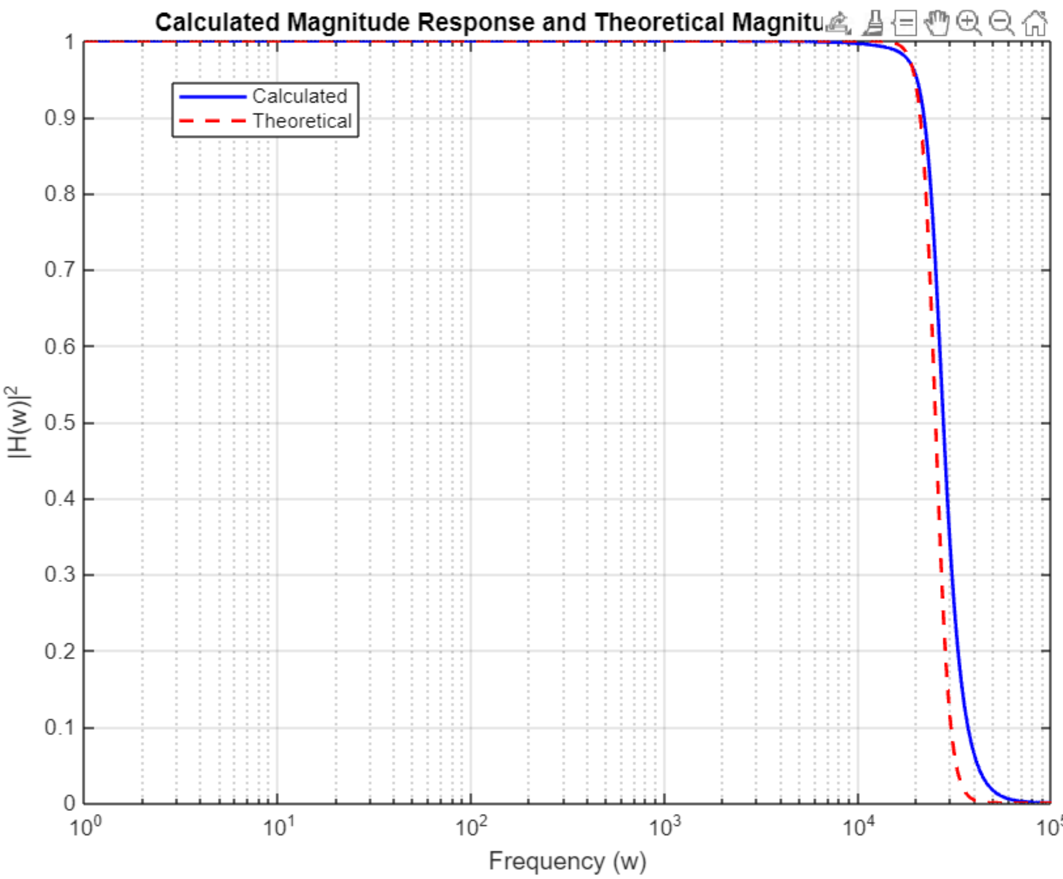


Figure 2: Calculated and Theoretical Magnitude Response Plotted Together

Since the magnitude response of is the product of the magnitude response of 3 Sallen-Key low pass filters, the order doesn't matter for calculations. However, in simulation this is not the case. If component values are not carefully chosen the order of the 3 filters can change the simulated response. This could be due to differences in response of each initial Sallen-Key filter. The Q factor of each could be very different from each other which causes the difference in response. For my circuit there wasn't an issue with difference in response, so the order of the circuits is the same as the circuit design above.

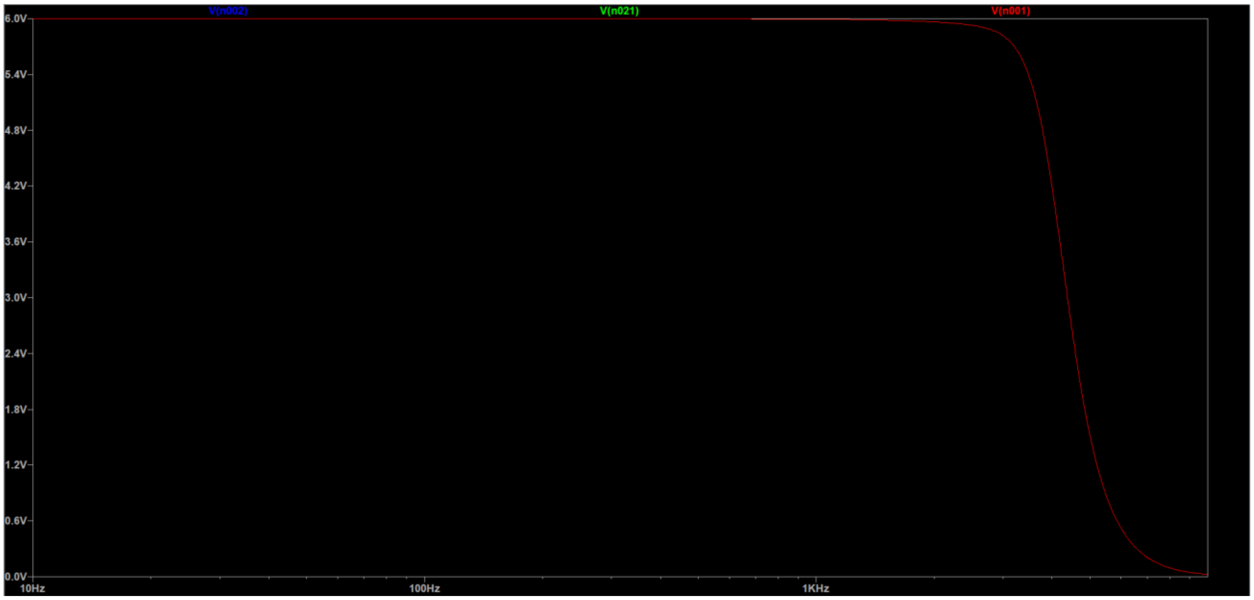


Figure 3: AC Simulation of 3 Different 6th Order Sallen Key Low Pass Filters