DP Formulations

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Hopscotch

$$
\begin{aligned}
DP(i,j) &:= \text{Max number of pennies attainable by starting at square } (i,j) \\
&:= \max_{(i',j') \text{within distance } k \text{ of } (i,j) \text{ s.t. } pennies(i',j') > pennies(i,j)} DP(i',j') + pennies(i,j)
\end{aligned}
$$

Base case is if no squares within distance k have $pennies(i',j') > pennies(i,j)$,
then value at state is $pennies(i,j)$.

Answer is in $DP(0,0)$.

$$
\begin{aligned}
\text{Time Complexity} &= \text{Number of states * Time to compute each state} \\
&= O(n^2) \cdot O(k) \\
&= O(n^2 k).
\end{aligned}
$$

Forming Quiz Teams

$$
\begin{aligned}
DP(subset) &:= \text{Lowest cost pairing of people in subset} \\
&\quad \text{(let's assume people in subset are numbered } p_0, p_1, \ldots, p_k\text{)} \\
&:= \min_{1 \le j \le k} distance(p_0, p_j) + DP(subset \setminus \{p_0, p_j\}) \\
DP(\emptyset) &:= 0
\end{aligned}
$$

Answer is in $DP(\mathcal{U})$ - the full subset of all $2N$ people.

$$
\begin{aligned}
\text{Time Complexity} &= \text{Number of states * Time to compute each state} \\
&= O(2^{2N}) \cdot O(N) \\
&= O(N \cdot 2^{2N}).
\end{aligned}
$$