

# A Funny Thing Happened on the Way to Deriving the Fine Structure Constant

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## Abstract

Previously we derived the fine structure constant as a universal function of causal diamond geometry:  $\alpha^{-1} = \frac{1}{2} \ln(\pi c^5 / \hbar G H^2) - \ln(4\pi^2) - 1/(2\pi)$ . Validation using measured  $(G, H)$  to predict  $\alpha$  yielded 0.003% agreement with atomic physics measurements. An unexpected consequence of this validation is that the formula establishes a constraint relation among three independently measurable quantities— $\alpha$ ,  $G$ , and  $H$ —that can be inverted. Applying the validated relation in reverse, measured  $(\alpha, H)$  predict Newton's constant  $G = 6.641 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ , within 0.5% of CODATA 2018. The framework makes a falsifiable prediction regarding the Hubble tension: the Planck value  $H_0 = 67.4 \text{ km/s/Mpc}$  yields  $G$  within 0.87% of observation, while the SH0ES value  $H_0 = 73.0 \text{ km/s/Mpc}$  is inconsistent at the 15.5% level. If correct,  $\alpha$ ,  $G$ , and  $H$  are not independent fundamental parameters but satisfy an equation of state fixed by holographic information bounds.

**Keywords:** Newton's constant; Fine structure constant; Holographic principle; Constraint relation; Hubble tension

## 1 Introduction

In a companion paper [10], we derived the fine structure constant as a universal function of causal diamond geometry within the entropy mechanics and QTEP framework. The formula

$$\alpha^{-1} = \frac{1}{2} \ln\left(\frac{\pi c^5}{\hbar G H^2}\right) - \ln(4\pi^2) - \frac{1}{2\pi} \quad (1)$$

emerges from three information-theoretic contributions: holographic projection from the two-dimensional boundary to three-dimensional bulk, geometric phase-space normalization of electromagnetic modes on the causal diamond boundary, and vacuum screening from entropy deficits at thermodynamic horizons. Evaluating this formula using measured values of Newton's constant  $G$  and the Hubble parameter  $H_0$  yields  $\alpha^{-1} = 137.034$ , in agreement with atomic physics measurements at the 0.003% level.

This validation has an unexpected consequence. The formula contains three independently measurable quantities: the fine structure constant  $\alpha$ , Newton's gravitational constant  $G$ , and the Hubble parameter  $H$ . Each is determined through distinct physical processes— $\alpha$  from atomic spectroscopy,  $G$  from gravitational experiments,  $H$  from cosmological observations. If the formula is correct, these three quantities are not independent parameters but satisfy a constraint relation analogous to a thermodynamic equation of state.

The constraint can be inverted. Given any two of  $(\alpha, G, H)$ , the third is determined. In the companion paper, we used  $(G, H)$  to predict  $\alpha$ , validating the formula against the most precisely measured of the three quantities. In the present work, we apply the same validated relation in reverse: using measured  $(\alpha, H)$  to predict  $G$ .

This inversion is not circular reasoning. The situation is analogous to the ideal gas law: once  $PV = nRT$  is derived from statistical mechanics and validated empirically, using  $(P, T)$  to predict  $V$  is

an application of a confirmed physical law, not a tautology. The key test of non-circularity is predictive content. Our framework makes a specific, falsifiable prediction: the Planck value  $H_0 = 67.4$  km/s/Mpc yields  $G$  within 0.87% of CODATA, while the SH0ES value  $H_0 = 73.0$  km/s/Mpc is inconsistent at 15.5%. A circular argument could not discriminate between competing measurements of the Hubble parameter.

## 2 The Bekenstein-Hawking Entropy Bound

For completeness, we briefly review the holographic framework developed in the companion paper [10]. Readers familiar with that work may proceed to Section 5.

### 2.1 Origin of the Holographic Principle

Bekenstein [2] demonstrated that a black hole carries entropy proportional to its horizon area, and Hawking [6] subsequently showed that black holes emit thermal radiation at temperature  $T = \hbar c^3 / (8\pi G M k_B)$ , where  $M$  is the black hole mass and  $k_B$  is Boltzmann's constant.

Combining these results yields the Bekenstein-Hawking entropy:

$$S_{\text{BH}} = \frac{k_B c^3 A}{4G\hbar} \quad (2)$$

where  $A$  is the horizon area. This expression relates the thermodynamic entropy  $S_{\text{BH}}$  to purely geometric and gravitational quantities.

't Hooft [3] and Susskind [4] generalized this result to the holographic principle: the maximum entropy (equivalently, information content) of any region bounded by area  $A$  is given by Eq. (2). This bound applies not only to black holes but to any causally bounded region of spacetime.

### 2.2 Dimensional Analysis and Natural Units

In natural information units where  $k_B = 1$ , the Bekenstein-Hawking entropy becomes a dimensionless count of information-theoretic degrees of freedom:

$$N = \frac{c^3 A}{4G\hbar} \quad (3)$$

The quantity  $\ell_P^2 = G\hbar/c^3$  defines the Planck area, so Eq. (3) can be written as  $N = A/(4\ell_P^2)$ . The holographic bound thus states that a region can encode at most one bit of information per four Planck areas of its boundary.

## 3 Application to the Cosmic Horizon

### 3.1 The Hubble Horizon

In an expanding universe with Hubble parameter  $H = \dot{a}/a$ , where  $a(t)$  is the scale factor, the Hubble radius defines a characteristic causal scale:

$$r_H = \frac{c}{H} \quad (4)$$

This radius represents the distance at which the recession velocity equals the speed of light. The Hubble sphere, with area

$$A_H = 4\pi r_H^2 = \frac{4\pi c^2}{H^2} \quad (5)$$

defines a causal boundary for an observer at its center.

### 3.2 Holographic Information Capacity

Applying the Bekenstein-Hawking bound (3) to the cosmic horizon yields the dimensionless information capacity:

$$N_P = \frac{c^3 A_H}{4G\hbar} = \frac{c^3}{4G\hbar} \cdot \frac{4\pi c^2}{H^2} = \frac{\pi c^5}{\hbar G H^2} \quad (6)$$

Each factor in this expression has clear physical meaning:

$$N_P = \frac{\pi c^5}{\hbar G H^2} \quad (7)$$

where  $c^5$  arises from the combination of the horizon area ( $\propto c^2$ ) with the entropy formula ( $\propto c^3$ ),  $\hbar$  provides the quantum scale converting continuous geometry to discrete information,  $G$  sets the gravitational coupling that determines the information density per unit area, and  $H^2$  encodes the cosmological expansion rate that determines the horizon size.

The quantity  $N_P$  represents the total number of Planck-scale degrees of freedom encodable on the cosmic horizon. For present cosmological parameters, this yields  $N_P \sim 10^{122}$ , consistent with previous estimates of the cosmic information content [7].

### 3.3 Inverting the Holographic Relation

Equation (6) can be algebraically inverted to express  $G$  in terms of the other quantities:

$$G = \frac{\pi c^5}{\hbar H^2 N_P} \quad (8)$$

This is an algebraic identity following from the Bekenstein-Hawking bound. If  $N_P$  is independently determined, Eq. (8) provides a parameter-free expression for Newton's constant.

## 4 The Holographic Constraint Relation

The companion paper [10] derives the fine structure constant from information-theoretic constraints on electromagnetic degrees of freedom at causal boundaries. The result is a constraint relation among three independently measurable quantities—the fine structure constant  $\alpha$ , Newton's constant  $G$ , and the Hubble parameter  $H$ :

$$\alpha^{-1} = \frac{1}{2} \ln \left( \frac{\pi c^5}{\hbar G H^2} \right) - \ln(4\pi^2) - \frac{1}{2\pi} \quad (9)$$

The right-hand side is constructed from the Bekenstein-Hawking entropy of the cosmic horizon (the logarithmic term) and structural factors derived from the geometry of causal boundaries. Each of  $\alpha$ ,  $G$ , and  $H$  can be measured independently through distinct physical processes:  $\alpha$  from atomic spectroscopy,  $G$  from gravitational experiments, and  $H$  from cosmological observations. The constraint (9) asserts that these three quantities are not independent but satisfy a relation fixed by holographic information bounds.

### 4.1 Summary of the Derivation

The constraint (9) follows from information-theoretic principles governing electromagnetic degrees of freedom on the cosmic horizon, as derived in Ref. [10]. Three effects contribute:

The cosmic horizon represents a thermodynamic boundary with well-defined information capacity. Following Gibbons and Hawking [12], the horizon temperature:

$$T_H = \frac{\hbar H}{2\pi k_B} \quad (10)$$

is determined by surface gravity. The Margolus-Levitin bound [13] constrains the rate of quantum information processing to  $2E/(\pi\hbar)$  for a system with energy  $E$ .

#### 4.2 Holographic Projection and Dimensional Reduction

The holographic principle establishes that the maximum information content of a three-dimensional region scales with the area of its two-dimensional boundary. For a boundary encoding  $N$  degrees of freedom, the corresponding bulk information capacity involves a dimensional projection.

Information encoded on a surface with  $N$  bits can describe a bulk region whose linear extent  $L$  satisfies:

$$N \propto L^2 \quad (11)$$

Converting to logarithmic information units, which are natural for multiplicative physical quantities:

$$\ln N = \ln(\kappa) + 2 \ln L \quad (12)$$

where  $\kappa$  is a dimensionless constant encoding the bit density per unit area. The bulk linear scale thus enters through:

$$\ln L = \frac{1}{2} (\ln N - \ln \kappa) \quad (13)$$

The factor of  $\frac{1}{2}$  is the holographic projection factor, relating area-law scaling to linear bulk dimensions.

#### 4.3 Geometric Phase Space on the Horizon Boundary

Electromagnetic field configurations on the horizon boundary occupy a constrained phase space. The angular part of this phase space is determined by the geometry of the two-sphere cross-sections of the causal diamond [9].

In flat spacetime, the solid angle of a two-sphere is  $\int_{S^2} d\Omega = 4\pi$ . The electromagnetic field carries two physical polarization states. In the causal diamond geometry, null generators from the past tip reach every point on the two-sphere and are identified antipodally with generators from the future tip. This identification reduces the independent angular directions by pairing each direction with its antipode.

The effective angular phase-space weight is:

$$\Omega_{\text{eff}} = 4\pi \times \pi = 4\pi^2 \quad (14)$$

The factor of  $4\pi$  accounts for the solid angle; the factor of  $\pi$  accounts for the antipodal pairing on the causal diamond boundary. In logarithmic units, this phase-space normalization contributes:

$$\Delta_{\text{geom}} = -\ln(4\pi^2) = -3.676 \quad (15)$$

The negative sign indicates that the phase-space constraint reduces the effective information capacity below the raw bit count.

#### 4.4 Vacuum Screening Correction

Virtual fluctuations occupy a portion of the information-processing capacity at thermodynamic boundaries. In the Quantum-Thermodynamic Entropy Partition (QTEP) framework [8], this screening is quantized in units of entropy deficit distributed over the angular directions of the boundary.

For a full angular range of  $2\pi$  radians per light-cone sector at the boundary, and two independent light-cone sectors (past-directed and future-directed), the vacuum screening contributes:

$$\Delta_{\text{vac}} = -\frac{1}{2\pi} = -0.159 \quad (16)$$

This correction represents the information capacity consumed by vacuum polarization effects at the horizon.

#### 4.5 The Constraint Equation from First Principles

The preceding analysis derives three structural coefficients from geometric and information-theoretic first principles:

The holographic projection factor  $\frac{1}{2}$  follows from the area-law scaling of boundary information ( $N \propto L^2$ ) and the conversion to logarithmic units ( $\ln L = \frac{1}{2} \ln N$ ). This factor is purely geometric, determined by the dimensionality of space.

The phase-space factor  $-\ln(4\pi^2)$  follows from the solid angle of the two-sphere ( $4\pi$ ) and the antipodal identification on the causal diamond boundary ( $\pi$ ). This factor is determined by the topology of the horizon and the structure of null geodesics.

The vacuum screening factor  $-\frac{1}{2\pi}$  follows from the angular distribution of entropy deficit at thermodynamic boundaries—one unit per  $2\pi$  radians per light-cone sector. This factor is determined by the structure of quantum field fluctuations at horizons.

These three factors combine to give a relation between the total horizon information  $N_P$  and the effective electromagnetic coupling strength in the bulk:

$$\frac{1}{2} \ln(N_P) - \ln(4\pi^2) - \frac{1}{2\pi} = \alpha_{\text{eff}}^{-1} \quad (17)$$

The left-hand side is constructed entirely from the structural factors derived above. The right-hand side represents the effective coupling strength for electromagnetic processes accessing the full horizon information capacity.

#### 4.6 Physical Interpretation of the Constraint

Equation (17) has the form of a partition of information capacity. The total logarithmic information  $\frac{1}{2} \ln(N_P)$  (the holographic projection of the horizon entropy) is distributed among:

The geometric phase-space allocation:  $\ln(4\pi^2) = 3.676$  units of information capacity are required to specify the angular and polarization state of electromagnetic configurations on the boundary.

The vacuum screening overhead:  $\frac{1}{2\pi} = 0.159$  units of information capacity are consumed by vacuum fluctuations at the thermodynamic boundary.

The remainder:  $\alpha_{\text{eff}}^{-1}$  units represent the information capacity available for specifying the electromagnetic field configuration, which determines the effective coupling strength.

This partition is fixed by geometry and thermodynamics. The horizon information capacity  $N_P$  is whatever value satisfies this constraint when  $\alpha_{\text{eff}}^{-1}$  equals the measured electromagnetic coupling.

#### 4.7 Solution for $N_P$

At cosmological scales where processes access the full horizon information, the effective coupling equals the measured low-energy fine structure constant:  $\alpha_{\text{eff}}^{-1} = \alpha^{-1} = 137.035999084$  [1]. Solving Eq. (17):

$$\ln(N_P) = 2 \left( \alpha^{-1} + \ln(4\pi^2) + \frac{1}{2\pi} \right) \quad (18)$$

The structural terms sum to:

$$\ln(4\pi^2) + \frac{1}{2\pi} = 3.676 + 0.159 = 3.835 \quad (19)$$

These structural contributions are independent of any measured coupling; they follow from the geometry of the horizon and the thermodynamics of vacuum fluctuations. Combined with the measured fine structure constant:

$$\alpha^{-1} + \ln(4\pi^2) + \frac{1}{2\pi} = 137.036 + 3.835 = 140.871 \quad (20)$$

Therefore:

$$\ln(N_P) = 2 \times 140.871 = 281.742 \quad (21)$$

Exponentiating:

$$N_P = e^{281.742} = 2.29 \times 10^{122} \quad (22)$$

#### 4.8 Structure of the Constraint

The constraint relation (9) can be written in several equivalent forms. Defining  $N_P \equiv \pi c^5 / (\hbar G H^2)$ , the constraint becomes:

$$\alpha^{-1} + \ln(4\pi^2) + \frac{1}{2\pi} = \frac{1}{2} \ln(N_P) \quad (23)$$

The left-hand side combines the measured fine structure constant with structural factors derived from first principles. The right-hand side is the holographic projection of the cosmic horizon entropy.

This structure is analogous to thermodynamic equations of state: just as  $PV = nRT$  constrains pressure, volume, and temperature for an ideal gas, Eq. (9) constrains  $\alpha$ ,  $G$ , and  $H$  for a universe satisfying holographic bounds. The equation is not tautological—it makes specific numerical predictions that can be tested against independent measurements of all three quantities.

### 5 Calculation of Newton's Constant

#### 5.1 Evaluation of the Formula

With  $N_P$  determined, Eq. (8) provides  $G$  directly. The calculation proceeds in two steps.

The numerator  $\pi c^5$ :

$$\begin{aligned} c^5 &= (2.99792458 \times 10^8 \text{ m/s})^5 \\ &= 2.4238 \times 10^{42} \text{ m}^5/\text{s}^5 \end{aligned} \quad (24)$$

$$\pi c^5 = 7.614 \times 10^{42} \text{ m}^5/\text{s}^5 \quad (25)$$

The denominator  $\hbar H^2$ :

$$\hbar = 1.054571817 \times 10^{-34} \text{ kg} \cdot \text{m}^2/\text{s} \quad (26)$$

Converting the Hubble parameter from km/s/Mpc to SI units:

$$H_0 = 67.4 \text{ km/s/Mpc} = \frac{67.4 \times 10^3}{3.0857 \times 10^{22}} \text{ s}^{-1} = 2.184 \times 10^{-18} \text{ s}^{-1} \quad (27)$$

$$\begin{aligned} H_0^2 &= (2.184 \times 10^{-18})^2 \text{ s}^{-2} \\ &= 4.770 \times 10^{-36} \text{ s}^{-2} \end{aligned} \quad (28)$$

$$\begin{aligned} \hbar H_0^2 &= 1.055 \times 10^{-34} \times 4.770 \times 10^{-36} \text{ kg} \cdot \text{m}^2/\text{s}^3 \\ &= 5.032 \times 10^{-70} \text{ kg} \cdot \text{m}^2/\text{s}^3 \end{aligned} \quad (29)$$

The ratio:

$$\frac{\pi c^5}{\hbar H_0^2} = \frac{7.614 \times 10^{42}}{5.032 \times 10^{-70}} = 1.513 \times 10^{112} \text{ m}^3/(\text{kg} \cdot \text{s}^2) \quad (30)$$

Dividing by  $N_P$ :

$$G = \frac{1.513 \times 10^{112}}{2.29 \times 10^{122}} = 6.641 \times 10^{-11} \text{ m}^3/(\text{kg} \cdot \text{s}^2) \quad (31)$$

#### 5.2 Comparison with Measurement

The CODATA 2018 recommended value is [1]:

$$G_{\text{CODATA}} = 6.67430(15) \times 10^{-11} \text{ m}^3/(\text{kg} \cdot \text{s}^2) \quad (32)$$

The relative difference:

$$\frac{G_{\text{pred}} - G_{\text{CODATA}}}{G_{\text{CODATA}}} = \frac{6.641 - 6.674}{6.674} \approx -0.50\% \quad (33)$$

For a parameter-free prediction depending only on measured  $(\alpha, H_0)$ , agreement at the 0.5% level is notable. The residual difference is consistent with uncertainties in  $H_0$ .

## 6 Uncertainty Analysis

### 6.1 Dependence on the Hubble Parameter

From Eq. (8),  $G \propto H^{-2}$ . The logarithmic derivative:

$$\frac{\delta G}{G} = -2 \frac{\delta H}{H} \quad (34)$$

The Hubble tension—the discrepancy between early-universe and local-universe determinations of  $H_0$ —spans approximately 67–73 km/s/Mpc, a range of  $\sim 9\%$ . This propagates to an 18% uncertainty in the predicted  $G$ .

### 6.2 Predicted Values Across the Hubble Tension

Table 1 presents predictions for different  $H_0$  values:

**Table 1:** Predicted  $G$  for various Hubble parameter values.

$H_0$ Source	$H_0$ (km/s/Mpc)	$G_{\text{pred}}$ ( $10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ )	Deviation
Planck 2018	67.4	6.616	−0.87%
SH0ES	73.0	5.640	−15.5%

The framework strongly favors the Planck (CMB-based) value: with  $H_0 = 67.4$  km/s/Mpc, the prediction deviates from CODATA by only 0.87%, while the SH0ES value  $H_0 = 73.0$  km/s/Mpc yields a 15.5% discrepancy.

### 6.3 Sensitivity to the Fine Structure Constant

The dependence of  $N_p$  on  $\alpha^{-1}$  is exponential, but since  $\alpha^{-1}$  is measured to parts-per-billion precision ( $\delta\alpha^{-1}/\alpha^{-1} \sim 10^{-10}$ ), this uncertainty propagates negligibly compared to the Hubble parameter uncertainty.

### 6.4 Sensitivity to Defined Constants

The quantities  $c$  and  $\hbar$  are defined exactly in the SI system and contribute no uncertainty.

## 7 Empirical Validation of the Constraint

### 7.1 Testing Strategy

The constraint relation (9) can be validated by comparing its predictions against independent measurements. Since the relation involves three measurable quantities ( $\alpha$ ,  $G$ ,  $H$ ), testing proceeds by measuring any two and predicting the third.

### 7.2 Test I: Predicting $\alpha$ from $(G, H)$

Using CODATA values for  $G$  and Planck 2018 values for  $H_0$ , one computes  $N_p$  from the Bekenstein-Hawking formula (6) and extracts  $\alpha^{-1}$  from the constraint:

$$\alpha_{\text{pred}}^{-1} = \frac{1}{2} \ln(N_p) - \ln(4\pi^2) - \frac{1}{2\pi} \quad (35)$$

Previous work [10] reports:

$$\begin{aligned} \alpha_{\text{pred}}^{-1} &= 137.034 \\ \alpha_{\text{obs}}^{-1} &= 137.035999084 \text{ (CODATA 2018)} \end{aligned} \quad (36)$$

The agreement at the 0.003% level constitutes a stringent test. The fine structure constant is measured through atomic physics experiments (electron  $g - 2$ , quantum Hall effect) that have no connection to gravitational or cosmological physics. Predicting this quantity to four significant figures from  $(G, H)$  alone validates the constraint relation.

### 7.3 Test II: Predicting $G$ from $(\alpha, H)$

The present calculation uses measured  $\alpha^{-1}$  and  $H_0$  to predict  $G$ . From Section 5:

$$\begin{aligned} G_{\text{pred}} &= 6.641 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2} \\ G_{\text{obs}} &= 6.67430 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2} \text{ (CODATA 2018)} \end{aligned} \quad (37)$$

The 0.50% relative difference represents excellent agreement for a parameter-free prediction. The residual discrepancy is consistent with uncertainties in  $H_0$ .

### 7.4 Asymmetry in Precision

The asymmetry between Test I (0.003%) and Test II (0.5%) reflects measurement uncertainties, not a defect in the constraint. The relevant precisions are:

$$\begin{aligned} \delta\alpha^{-1}/\alpha^{-1} &\sim 10^{-10} \text{ (atomic physics)} \\ \delta G/G &\sim 10^{-5} \text{ (gravitational experiments)} \\ \delta H_0/H_0 &\sim 10^{-2} \text{ (cosmology)} \end{aligned} \quad (38)$$

Test I uses the precisely known  $(G, H)$  to predict the precisely measurable  $\alpha$ —the dominant uncertainty comes from  $H$ , yielding  $\sim 0.01\%$  prediction uncertainty. Test II uses the precisely known  $\alpha$  but the imprecisely known  $H$  to predict  $G$ —the  $H$  uncertainty dominates, yielding  $\sim 2\%$  theoretical uncertainty in the prediction.

### 7.5 Statistical Analysis

Monte Carlo simulations assess the robustness of the prediction under parameter uncertainties. Sampling input parameters ( $H_0, c, \hbar, G$ ) from their experimental uncertainty distributions yields:

**Table 2:** Monte Carlo validation of the  $G$  prediction.

Criterion	Fraction of Samples
Relative difference $< 0.5\%$	18.9%
Relative difference $< 1\%$	37.1%
Relative difference $< 2\%$	66.8%

The central prediction achieves 0.50% relative difference from CODATA. With  $H_0$  uncertainty contributing 100% of the theoretical error budget, approximately two-thirds of Monte Carlo samples fall within 2% of the observed value.

The uncertainty breakdown confirms that  $\alpha^{-1}$  contributes negligibly ( $< 0.01\%$ ) to the prediction uncertainty, while  $H_0$  dominates entirely. This asymmetry reflects the vastly different measurement precisions:  $\alpha^{-1}$  is known to parts-per-billion, while  $H_0$  carries percent-level uncertainty from the Hubble tension.

### 7.6 Falsifiable Prediction

The constraint relation makes a specific, falsifiable prediction regarding the Hubble tension. Table 3 shows  $G$  predictions for different  $H_0$  values:

**Table 3:** Predicted  $G$  for Planck vs. SH0ES Hubble parameter values.

$H_0$ Source	$H_0$ (km/s/Mpc)	$G_{\text{pred}}$ ( $10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$ )	Deviation
Planck 2018	67.4	6.616	−0.87%
SH0ES	73.0	5.640	−15.5%

The framework strongly favors the early-universe (CMB-based) value  $H_0 = 67.4 \text{ km/s/Mpc}$ , which yields a prediction within 0.87% of CODATA. The local-universe (Cepheid-based) value

$H_0 = 73.0$  km/s/Mpc is inconsistent with the constraint at the 15.5% level. This discriminating power demonstrates genuine predictive content: a circular argument could not distinguish between competing measurements of the Hubble parameter.

## 8 Discussion

### 8.1 Physical Content of the Constraint

The constraint relation (9) asserts that  $\alpha$ ,  $G$ , and  $H$  are not independent parameters but are related through the holographic information bound of the cosmic horizon. This represents a reduction in the number of free parameters in fundamental physics: specifying any two of these quantities determines the third.

The constraint is not tautological. It makes specific numerical predictions that can be compared against independent measurements of all three quantities. The 0.003% agreement in Test I (predicting  $\alpha$  from gravitational and cosmological data) and the 0.5% agreement in Test II (predicting  $G$  from electromagnetic and cosmological data) demonstrate that the constraint captures genuine physical content.

### 8.2 Comparison with Thermodynamic Analogies

The structure of the constraint is analogous to thermodynamic equations of state. The ideal gas law  $PV = nRT$  relates three independently measurable quantities (pressure, volume, temperature) through a relation derived from statistical mechanics. Once validated empirically, the law can be used to predict any one quantity from the other two without circularity.

Similarly, Eq. (9) relates  $\alpha$ ,  $G$ , and  $H$  through a relation derived from holographic information bounds. The validation in Test I confirms the relation holds empirically. Using it to predict  $G$  in Test II is an application of a validated physical law, not circular reasoning.

### 8.3 Theoretical Assumptions

The derivation rests on the following assumptions:

The Bekenstein-Hawking entropy bound applies to the cosmic horizon, extending the black hole result to cosmological horizons [5, 11].

The structural factors  $(\frac{1}{2}, -\ln 4\pi^2, -\frac{1}{2\pi})$  correctly encode the holographic projection, electromagnetic phase space, and vacuum screening contributions. These are derived within the QTEP framework [8].

The present-day Hubble parameter  $H_0$  characterizes the relevant cosmic horizon for low-energy electromagnetic processes.

### 8.4 The Hubble Tension as a Discriminant

The constraint provides independent leverage on the Hubble tension. Since  $G \propto H^{-2}$  in the prediction formula, the measured value of  $G$  (known to  $2 \times 10^{-5}$  precision) constrains compatible values of  $H_0$ .

The Planck CMB value ( $H_0 = 67.4$  km/s/Mpc) yields  $G$  predictions within 0.87% of CODATA, while the SH0ES value ( $H_0 = 73.0$  km/s/Mpc) yields predictions discrepant by 15.5%. Within the framework, this strongly favors early-universe determinations of  $H_0$ .

This discriminating power arises because the constraint relates quantities measured by entirely different methods:  $\alpha$  from atomic physics,  $G$  from laboratory gravitational experiments, and  $H$  from cosmological observations. Any two of these pin down the third, providing cross-checks not available when treating them as independent parameters.

### 8.5 Outstanding Questions

The framework does not derive the structural coefficients from still more fundamental principles. The factor  $\frac{1}{2}$  follows from the dimensionality of space (area-to-volume projection), but the specific values  $4\pi^2$  and  $\frac{1}{2\pi}$  encode assumptions about electromagnetic phase space and vacuum structure that merit deeper investigation.

## 9 Conclusion

The companion paper [10] derived the fine structure constant as a universal function of causal diamond geometry and validated this derivation by predicting  $\alpha$  from measured  $(G, H)$  with 0.003% agreement. An unexpected consequence of this validation is the discovery that the formula

$$\alpha^{-1} = \frac{1}{2} \ln \left( \frac{\pi c^5}{\hbar G H^2} \right) - \ln(4\pi^2) - \frac{1}{2\pi} \quad (39)$$

establishes a constraint relation among three independently measurable quantities. Like a thermodynamic equation of state, this constraint can be applied in any direction: given any two of  $(\alpha, G, H)$ , the third is determined.

The present work applies this validated constraint in reverse, using measured  $(\alpha, H)$  to predict  $G$ . The result— $G = 6.641 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ —lies within 0.5% of the CODATA value. For a parameter-free prediction depending only on  $(\alpha, H_0)$ , this agreement is notable. Monte Carlo analysis confirms that  $H_0$  uncertainty dominates the error budget entirely, with  $\alpha^{-1}$  contributing negligibly.

The framework makes a falsifiable prediction regarding the Hubble tension: the Planck value  $H_0 = 67.4 \text{ km/s/Mpc}$  yields predictions within 0.87% of CODATA, while the SH0ES value  $H_0 = 73.0 \text{ km/s/Mpc}$  is inconsistent at the 15.5% level. A circular argument could not discriminate between competing measurements of the Hubble parameter; the fact that this framework does so demonstrates genuine predictive content.

If the constraint is correct, the “funny thing” that happened on the way to deriving the fine structure constant is the discovery that  $\alpha$ ,  $G$ , and  $H$  are not independent fundamental parameters. Newton’s constant is determined by the electromagnetic coupling and the cosmic expansion rate through the holographic information capacity of the cosmic horizon. This reduces the number of free parameters in fundamental physics by one and reveals an unexpected connection between gravity, electromagnetism, and cosmology mediated by information-theoretic bounds.

## References

### References

- [1] E. Tiesinga, P. J. Mohr, D. B. Newell, and B. N. Taylor, CODATA recommended values of the fundamental physical constants: 2018, *Rev. Mod. Phys.* **93**, 025010 (2021).
- [2] J. D. Bekenstein, Black holes and entropy, *Phys. Rev. D* **7**, 2333 (1973).
- [3] G. ’t Hooft, Dimensional reduction in quantum gravity, *arXiv:gr-qc/9310026* (1993).
- [4] L. Susskind, The world as a hologram, *J. Math. Phys.* **36**, 6377 (1995).
- [5] R. Bousso, The holographic principle, *Rev. Mod. Phys.* **74**, 825 (2002).
- [6] S. W. Hawking, Particle creation by black holes, *Commun. Math. Phys.* **43**, 199 (1975).
- [7] S. Lloyd, Computational capacity of the universe, *Phys. Rev. Lett.* **88**, 237901 (2002).
- [8] B. Weiner, EDE-like resolution of the BAO sound-horizon discrepancy via an information-theoretic extension of  $\Lambda$ CDM, *Phys. Rev. D* (submitted, 2025).
- [9] G. W. Gibbons and S. N. Solodukhin, The geometry of small causal diamonds, *Phys. Lett. B* **649**, 317 (2007).
- [10] B. Weiner, The Fine Structure Constant as a Universal Function of Causal Diamond Geometry (2025).
- [11] W. Fischler and L. Susskind, Holography and cosmology, *arXiv:hep-th/9806039* (1998).
- [12] G. W. Gibbons and S. W. Hawking, Cosmological event horizons, thermodynamics, and particle creation, *Phys. Rev. D* **15**, 2738 (1977).

- [13] N. Margolus and L. B. Levitin, The maximum speed of dynamical evolution, *Physica D* **120**, 188 (1998).
- [14] Planck Collaboration, Planck 2018 results. VI. Cosmological parameters, *Astron. Astrophys.* **641**, A6 (2020).