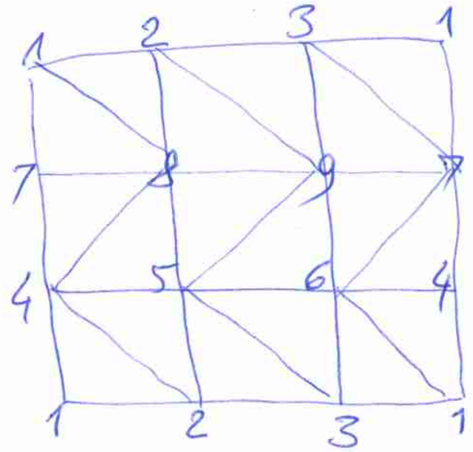
Simplicial torus

#9 0-cells = {9, ..., 1}

#27 1-cells = {9-8, 9-7, 9-6, 9-5, 9-3, 9-2,
8-7, 8-5, 8-4, 8-2, 8-1,
7-6, 7-4, 7-3, 7-1,
6-5, 6-4, 6-3, 6-1,
5-4, 5-3, 5-2,
4-2, 4-1,
3-2, 3-1,
2-1}



#18 2-cells = {9-8-5, 9-8-2, 9-7-6, 9-7-3, 9-6-5, 9-6-3, 9-5-3,
9-5-2, 9-3-2,
8-7-4, 8-7-1, 8-5-4, 8-5-2, 8-4-2, 8-4-1, 8-2-1,
7-6-4, 7-6-3, 7-6-1, 7-4-1, 7-3-1,
6-5-3, 6-4-1, 6-3-1,
5-4-2, 5-3-2,
4-2-1}

+#9 inner 2-cells

= inner 2-cells, #9

#9 3-cells = {9-8-5-2, 9-7-6-3, 9-6-5-3, 9-5-3-2,
8-7-4-1, 8-5-4-2, 8-4-2-1,
7-6-4-1, 7-6-3-1}

Lexiographical order of cells (= Lower Stars) #72

{9-8-5-2, 9-8-5, 9-8-2, 9-8, 9-7-6-3, 9-7-6, 9-7-3, 9-7, 9-6-5-3, 9-6-5,
9-6-3, 9-6, 9-5-3-2, 9-5-3, 9-5-2, 9-5, 9-3-2, 9-3, 9-2, 9
8-7-4-1, 8-7-4, 8-7-1, 8-7, 8-5-4-2, 8-5-4, 8-5-2, 8-5, 8-4-2-1, 8-4-2,
8-4-1, 8-4, 8-2-1, 8-2, 8-1, 8,
7-6-4-1, 7-6-4, 7-6-3-1, 7-6-3, 7-6-1, 7-6, 7-4-1, 7-4, 7-3-1, 7-3, 7-1, 7,
6-5-3, 6-5, 6-4-1, 6-4, 6-3-1, 6-3, 6-1, 6,
5-4-2, 5-4, 5-3-2, 5-3, 5-2, 5,
4-2-1, 4-2, 4-1, 4,
3-2, 3-1, 3
2-1, 2
1}

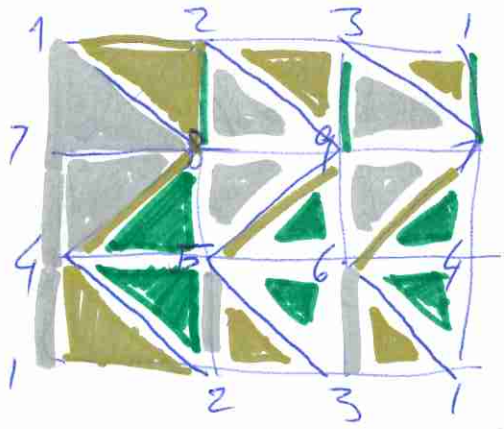
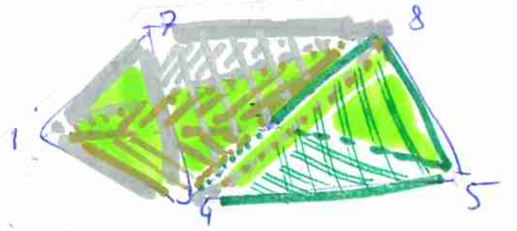


Illustration for building 3-cells



3 3-cells (golden, silver and green) with their respective inner 2-cells

{8-7-4-1} with {8-4-1, 7-4-1}
{8-5-4-2} " {8-5-2, 8-4-2}
{8-4-2-1} " {8-4-2, 8-4-1}

Algo 1 process Lower star for simplicial torus

L(1) L(2)
PQ zero = ∅
PQ one = ∅

L(3)
PQ zero = {3-2}
PQ one = ∅

L(4)
PQ zero = {4-2}
PQ one = {4-2-1}

L(5)
PQ zero = {5-3, 5-4}
PQ one = {5-3-2, 5-4-2}

L(6)
PQ zero = {6-3, 6-4, 6-5}
PQ one = {6-3-1, 6-4-1} ∪ {6-5-3}

L(7)
PQ zero = {7-3, 7-4, 7-6}
PQ one = {7-3-1} ∪ {7-6-4}

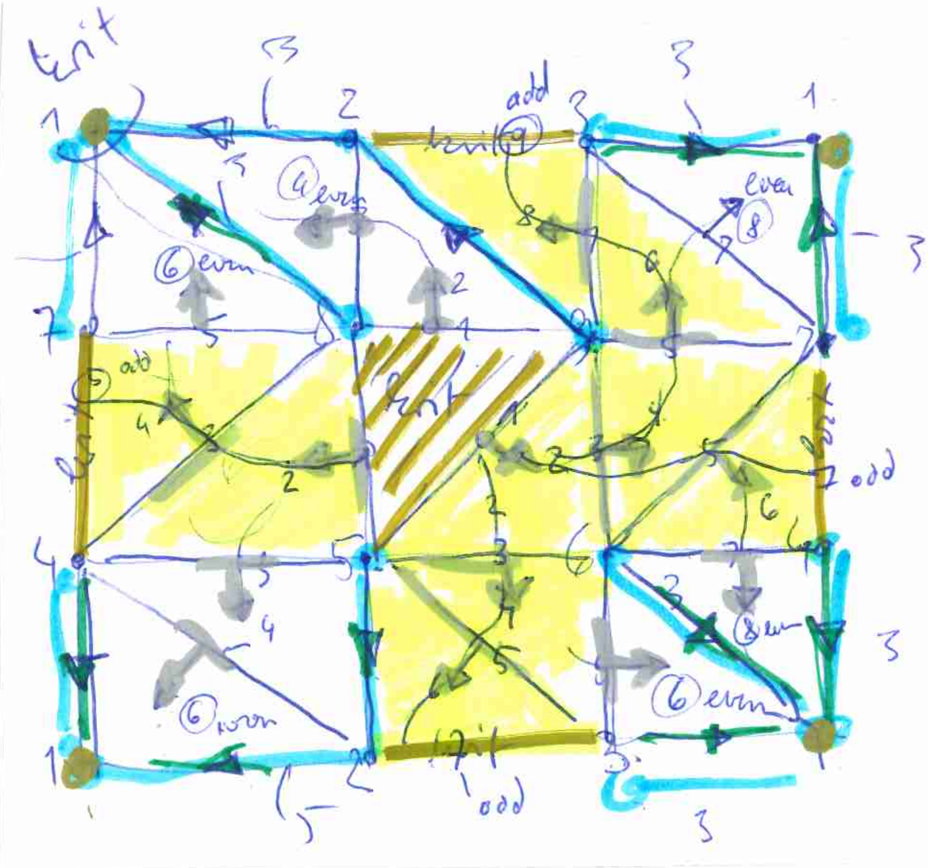
L(8)
PQ zero = {8-2, 8-4, 8-5, 8-7}
PQ one = {8-2-1, 8-7-1} ∪ {8-7-4} ∪ {8-5-4}

L(9)
PQ zero = {9-3, 9-5, 9-6, 9-7, 9-8}
PQ one = {9-3-2, 9-8-2} ∪ {9-7-3} ∪ {9-7-6} ∪ {9-6-5} ∪ {9-8-5}

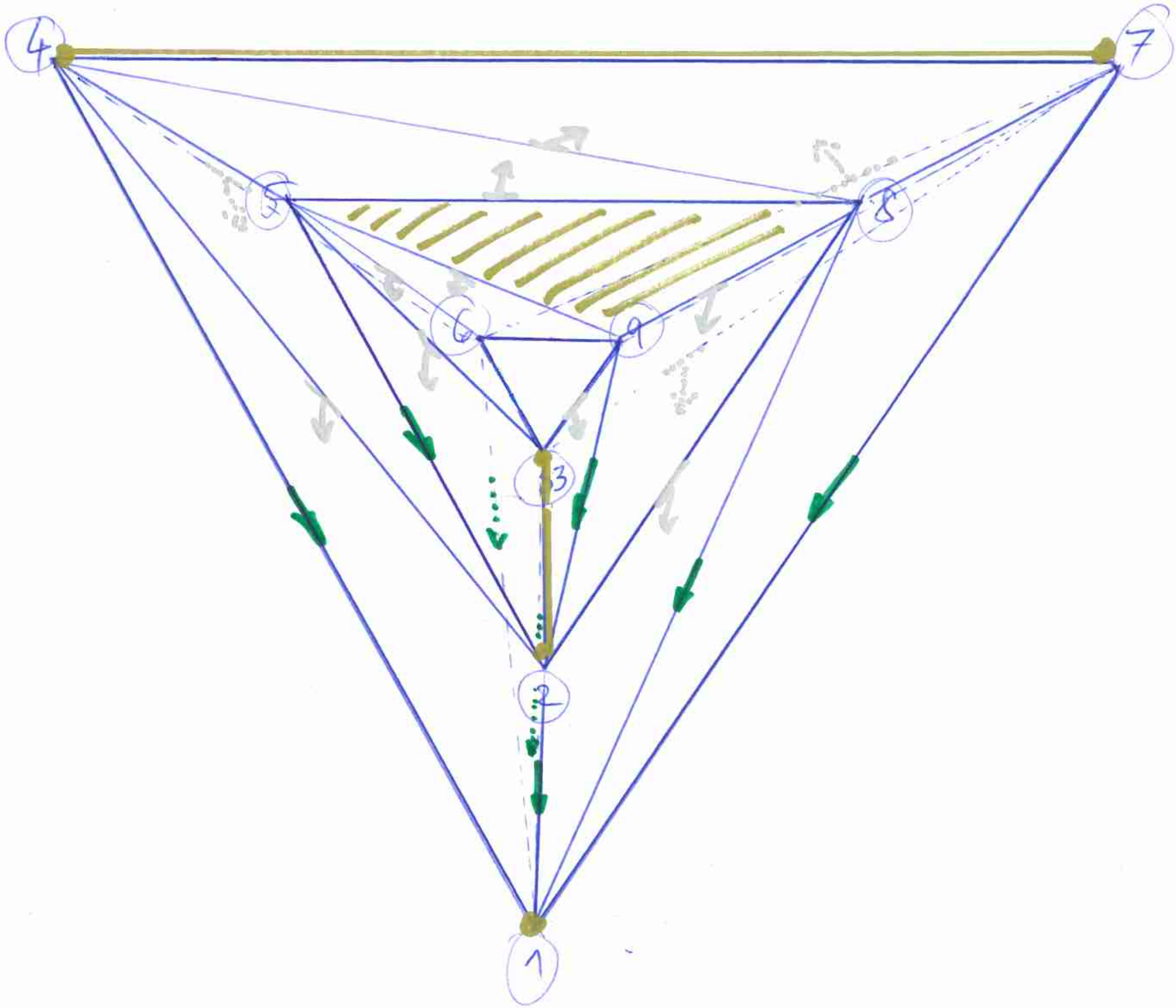
C = {1, 3-2, 7-4, 9-8-5}

V = {2-1|2|3-1|3|4-1|4|4-2-1|4-2|
5-2|5|5-3-2|5-3|5-4-2|5-4|
6-1|6|6-3-1|6-3|6-4-1|6-4|
6-5-3|6-5|7-1|7|7-3-1|7-3|
7-6-4|7-6|8-1|8|8-2-1|8-2|
8-2-1|8-2|8-7-1|8-7|8-7-4|8-4
8-5-4|8-5|9-2|9|9-3-2|9-3|
9-8-2|9-8|9-7-3|9-7|9-7-6|9-6|
9-6-5|9-5}

(no 3-cells considered)



Simplicial torus
(kind of Schlegel's diagram)



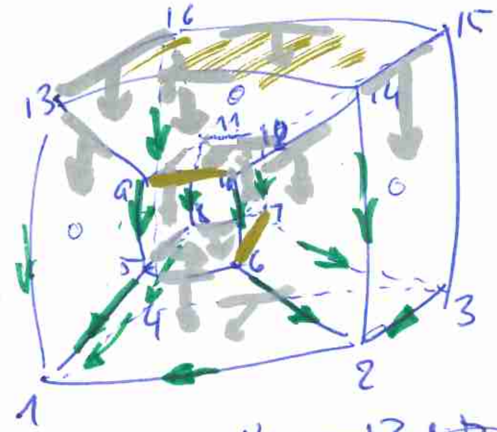
• critical	0-cell	1	} indicated in **gold**
critical	1-cells	3-2, 7-4	
critical	2-cell	9-8-5	

vectorfield pairing a 2-cell with a 1-cell

" " 1-cell with a 0-cell

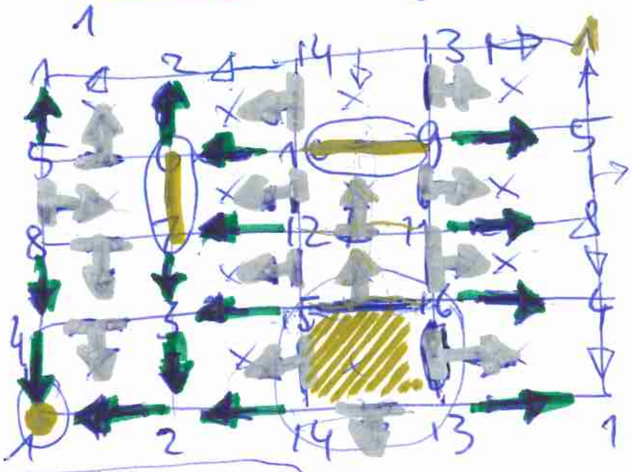
front face   back face

Cubical torus (Cubical Sphere)



#16 0-cells = { 16, ... , 1 }

#32 1-cells = { 16-15, 16-13, 16-11, 16-4,
15-14, 15-12, 15-3,
14-13, 14-10, 14-2,
13-9, 13-1,
12-11, 12-10, 12-7,
11-9, 11-8,
10-9, 10-6,
9-5,
8-7, 8-5, 8-4
7-6, 7-3
6-5, 6-2,
5-1,
4-3, 4-1,
3-2,
2-1 }



16-13-11-9

#16 2-cells = { 16-15-14-13, 16-15-12-11, 16-13-4-1, 16-11-8-4,
+4 4 15-14-12-10
15-14-3-2, 15-12-7-3,
14-13-10-9, 14-10-6-2,
13-9-5-1,
12-11-~~10~~-9, 12-10-7-6,
11-9-8-5,
8-7-6-5, 8-7-4-3, 8-5-4-1
7-6-3-2
6-5-2-1
4-3-2-1 }

-#4 3-cells = { 16-15-14-13-12-11-10-9, 16-13-12-9-8-5-4-1,
15-14-11-10-7-6-3-2
8-7-5-6-4-3-2-1 }

Lexicographical order of cells (= Lower Stars)

16-13-11-9

{ 16-15-14-13-12-11-10-9, 16-15-14-13, 16-15-12-11, 16-15, 16-13-12-9-8-5-4-1, 16-13-4-1,
16-13, 16-11-8-4, 16-11, 16-4, 16
15-14-12-10
15-14-11-10-7-6-3-2, 15-14-3-2, 15-14, 15-12-7-3, 15-12, 15-3, 15
14-13-10-9, 14-13, 14-10-6-2, 14-10, 14-2, 14
13-9-5-1, 13-9, 13-1, 13
12-11-10-9, 12-11, 12-10-7-6, 12-10, 12-7, 12
11-9-8-5, 11-9, 11-8, 11
10-9, 10-6, 10
9-5, 9
8-5-4-1
7-6-3-2
8-7-6-5-4-3-2-1, 8-7-6-5, 8-7-4-3, 8-7, 8-5, 8-4, 8
7-6, 7-3, 7
6-5-2-1
6-5, 6-2, 6

continued
5-1, 5
4-3-2-1, 4-3, 4-1, 4
3-2, 3
2-1, 2
1

Algo 1 Process Lower Star for Cubical Torus

L(1), L(2), L(3), L(5), L(9)
PQzero = ∅
PQone = ∅

L(4)
PQzero = {4-3}
PQone = {4-3-2-1}

L(6)
PQzero = {6-5}
PQone = {6-5-2-1}

L(7)
PQzero = {7-6}
PQone = ∅

L(8)
PQzero = {8-5, 8-7}
PQone = {8-7-4-3} ∪ {8-7-6-5}

L(10)
PQzero = {10-9}
PQone = ∅

L(11)
PQzero = {11-9}
PQone = {11-9-8-5}

L(12)
PQzero = {12-10, 12-11}
PQone = {12-10-7-6} ∪ {12-11-10-9}

L(13)
PQzero = {13-9}
PQone = {13-9-5-1}

L(14)
PQzero = {14-10, 14-13}
PQone = {14-10-6-2} ∪ {14-13-10-9}

L(15)
PQzero = {15-12, 15-14}
PQone = {15-14-3-2, 15-12-7-3}

L(16) :
PQzero = {16-11, 16-13, 16-15}
PQone = {16-13-4-1, 16-11-8-4} ∪ {16-15-12-11} ∪ {16-15-14-13}

C = {1, 7-6, 10-9, 16-15-14-13}

V = {2-1 | 2 | 3-2 | 3 | 4-1 | 4 | 4-3-2-1 | 4-3 |
5-1 | 5 | 6-2 | 6 | 6-5-2-1 | 6-5 |
7-3 | 7 | 8-4 | 8 | 8-7-4-3 | 8-7 |
8-7-6-5 | 8-5 | 9-5 | 9 | 10-6 | 10 |
11-8 | 11 | 11-9-8-5 | 11-9 | 12-7 | 12 |
12-10-7-6 | 12-10 | 12-11-10-9 | 12-11 |
13-1 | 13 | 13-9-5-1 | 13-9 | 14-2 | 14 |
14-10-6-2 | 14-10 | 14-13-10-9 | 14-13 |
15-3 | 15 | 15-14-3-2 | 15-14 |
15-12-7-3 | 15-12 | 16-4 | 16 |
16-13-4-1 | 16-13 | 16-11-8-4 | 16-11 |
16-15-12-11 | 16-15}

Algo 1 Process Lower Star for Cubical Torus incl. 3-cells

L(1), L(2), L(3), L(5), L(9)
PQzero = ∅
PQ one = ∅

C = {1, 10-9}

V = { 2-1 | 2 | 3-2 | 3 | 4-1 | 4 | 4-3-2-1 | 4-3 | 5-1 | 5 |
6-2 | 6 | 6-5-2-1 | 6-5 | 7-3 | 7 | 7-6-3-2 | 7-6 |
8-4 | 8 | 8-7-4-3 | 8-7 | 8-5-4-1 | 8-5 |
8-7-6-5-4-3-2-1 | 8-7-6-5 | 9-5 | 9 |
10-6 | 10 | 11-8 | 11 | 11-9-8-5 | 11-9 | 12-7 | 12 |
12-10-7-6 | 12-10 | 12-11-10-9 | 12-11 | 13-1 | 13 |
13-9-5-1 | 13-9 | 14-2 | 14-10-6-2 | 14-10 |
14-13-10-9 | 14-13 | 15-3 | 15 | 15-12-7-3 | 15-12 |
15-14-3-2 | 15-14 | 15-14-12-10-7-6-3-2 |
15-14-12-10 | 16-4 | 16 | 16-11-8-4 | 16-11 |
16-13-4-1 | 16-13 | 16-15-12-11 | 16-15 |
16-13-11-9-8-5-4-1 | 16-13-11-9 |
16-15-14-13-12-11-10-9 | 16-15-14-13 }

L(4)
PQzero = {4-3}
PQ one = {4-3-2-1}

L(6)
PQzero = {6-5}
PQone = {6-5-2-1}

L(7)
PQzero = {7-6}
PQ one = {7-6-3-2}

L(8)
PQzero = {8-5, 8-7}
PQ one = {8-7-4-3, 8-5-4-1} ∪ {8-7-6-5} ∪ {8-7-6-5-4-3-2-1}

L(10)
PQ zero = {10-9}
PQ one = ∅

L(11)
PQzero = {11-9}
PQ one = {11-9-8-5}

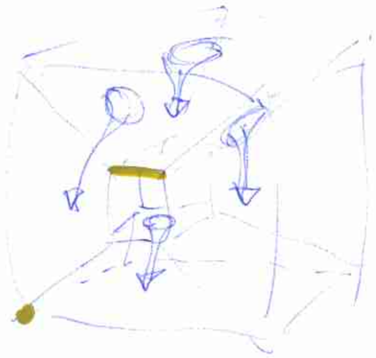
L(12)
PQzero = {12-10, 12-11}
PQone = {12-10-7-6} ∪ {12-11-10-9}

L(13)
PQzero = {13-9}
PQ one = {13-9-5-1}

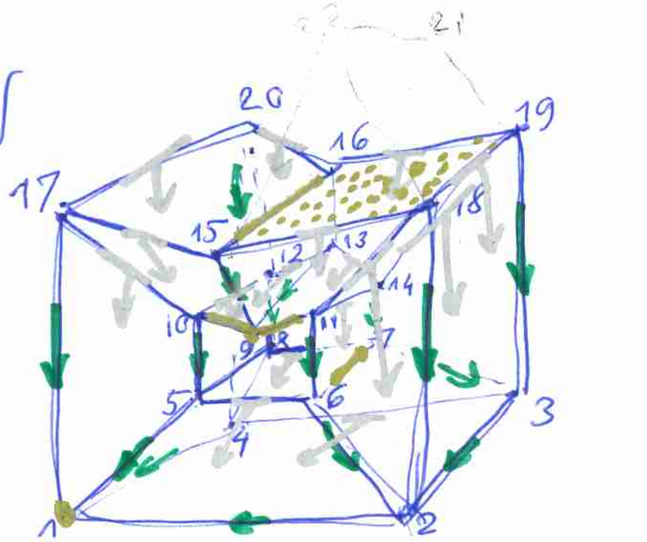
L(14)
PQzero = {14-10, 14-13}
PQ one = {14-10-6-2} ∪ {14-13-10-9}

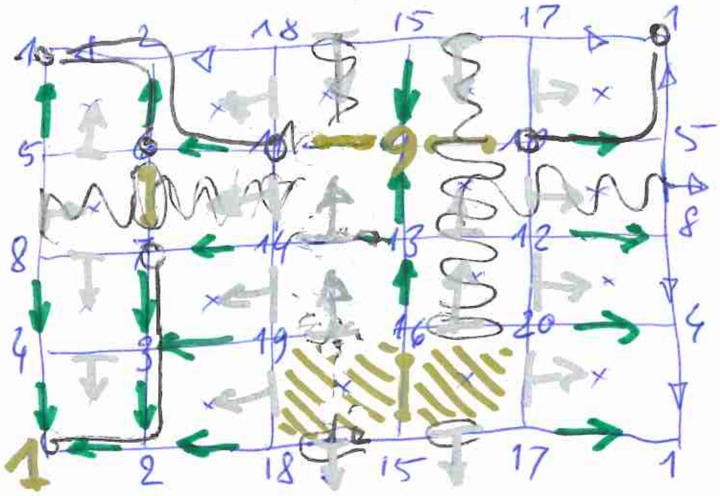
L(15)
PQzero = {15-12, 15-14}
PQ one = {15-12-7-3, 15-14-3-2} ∪ {15-14-12-10} ∪ {15-14-11-10-7-6-3-2}

L(16)
PQ zero = {16-11, 16-13, 16-15}
PQ one = {16-11-8-4, 16-13-4-1} ∪ {16-13-11-9} ∪ {16-15-12-11, 16-15-14-13}
∪ {16-13-11-9-8-5-4-1} ∪ {16-15-14-13-12-11-10-9}



Cubical torus, disturbed





#20 0-cells = {20, … , 1}

#40 1-cells = {20-17, 20-16, 20-12, 20-4,
19-18, 19-16, 19-14, 19-3,
18-15, 18-11, 18-2,
17-15, 17-10, 17-1,
16-15, 16-13,
15-9,
14-13, 14-11, 14-7,
13-12, 13-9,
12-10, 12-8,
11-9, 11-6,
10-9, 10-5,
8-7, 8-5, 8-4,
7-6, 7-3,
6-5, 6-2,
5-1,
4-3, 4-1,
3-2,
2-1 }

#20 2-cells = {20-17-16-15, 20-16-13-12,
20-17-4-1, 20-12-8-4,
19-18-16-15, 19-16-14-13,
19-18-3-2, 19-14-7-3,
18-15-11-9, 18-11-6-2,
17-15-10-9, 17-10-5-1,
14-13-11-9, 14-11-7-6,
13-12-10-9, 12-10-8-5,
8-7-6-5, 8-7-4-3,
6-5-2-1, 4-3-2-1}

Lexicographical order of cells (= Lower Stars)

{$\boxed{20-17-16-15}$, $\boxed{20-17-4-1}$, $\boxed{20-17}$, $\boxed{20-16-13-12}$, $\boxed{20-16}$, $\boxed{20-12-8-4}$, $\boxed{20-12}$, $\boxed{20-4}$, 20,

$\boxed{19-18-16-15}$, $\boxed{19-18-3-2}$, $\boxed{19-18}$, $\boxed{19-16-14-13}$, $\boxed{19-16}$, $\boxed{19-14-7-3}$, $\boxed{19-14}$, $\boxed{19-3}$, 19

$\boxed{18-15-11-9}$, $\boxed{18-15}$, $\boxed{18-11-6-2}$, $\boxed{18-11}$, $\boxed{18-2}$, 18

$\boxed{17-15-10-9}$, $\boxed{17-15}$, $\boxed{17-10-5-1}$, $\boxed{17-10}$, $\boxed{17-1}$, 17

$\boxed{16-15}$, $\boxed{16-13}$, 16

$\boxed{15-9}$, 15

$\boxed{14-13-11,9}$, $\boxed{14-13}$, $\boxed{14-11-7-6}$, $\boxed{14-11}$, $\boxed{14-7}$, 14

$\boxed{13-12-10-9}$, $\boxed{13-12}$, $\boxed{13-9}$, 13

$\boxed{12-10-8-5}$, $\boxed{12-10}$, $\boxed{12-8}$, 12

$\boxed{11-9}$, $\boxed{11-6}$, 11

$\boxed{10-9}$, $\boxed{10-5}$, 10

9

$\boxed{8-7-6-5}$, $\boxed{8-7-4-3}$, $\boxed{8-7}$, $\boxed{8-5}$, $\boxed{8-4}$, 8

$\boxed{7-6}$, $\boxed{7-3}$, 7

$\boxed{6-5-2-1}$, $\boxed{6-5}$, $\boxed{6-2}$, 6

$\boxed{5-1}$, 5

$\boxed{4-3-2-1}$, $\boxed{4-3}$, $\boxed{4-1}$, 6

$\boxed{3-2}$, 3

$\boxed{2-1}$, 2

1

Algo1 Process Lower Star for Cubical Torus Standard

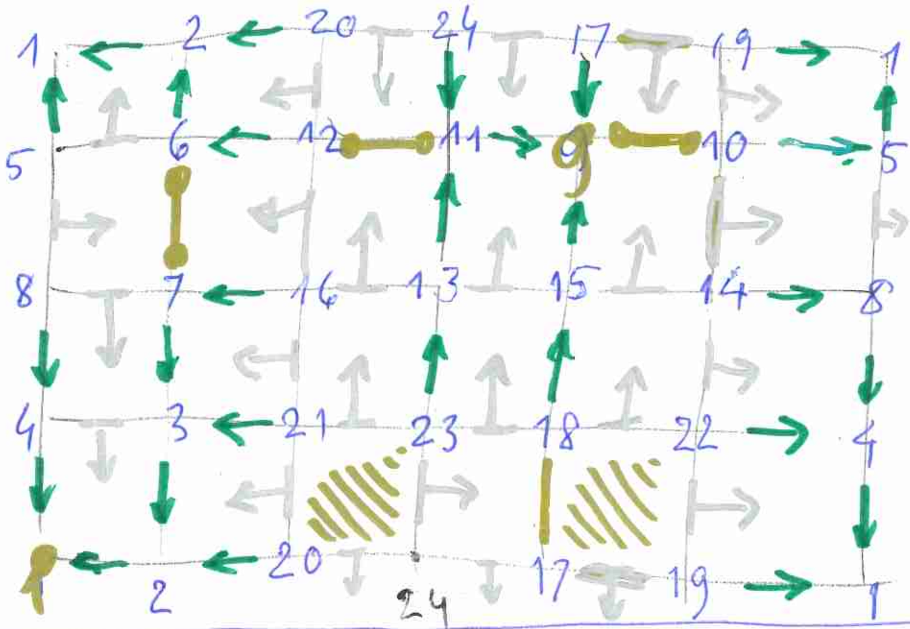
L(1), L(2), L(3), L(5), L(9), L(15)
PQ zero = ∅
PQ one = ∅
L(4)
PQ zero = {4-3}
PQ one = {4-3-2-1}
L(6)
PQ zero = {6-5}
PQ one = {6-5-2-1}
L(7)
PQ zero = {7-6}
PQ one = ∅
L(8)
PQ zero = {8-5, 8-7}
PQ one = {8-7-4-3}{8-7-6-5}
L(10)
PQ zero = {10-9}
PQ one = ∅
L(11)
PQ zero = {11-9}
PQ one = ∅
L(12)
PQ zero = {12-10}
PQ one = {12-10-8-5}
L(13)
PQ zero = {13-12}
PQ one = {13-12-10-9}
L(14)
PQ zero = {14-11, 14-13}
PQ one = {14-11-7-6}{14-13-11-9}
L(16)
PQ zero = {16-15}
PQ one = ∅
L(17)
PQ zero = {17-10, 17-15}
PQ one = {17-10-5-1} {17-15-10-9}
L(18)
PQ zero = {18-11, 18-15}
PQ one = {18-11-6-2} {18-15-11-9}
L(19)
PQ zero = {19-14, 19-16, 19-18}
PQ one = {19-14-7-3, 19-16-14-13} {19-18-3-2}, {19-18-16-15}
L(20)
PQ zero = {20-12, 20-16, 20-17}
PQ one = {20-12-8-4, 20-17-4-1} {20-16-13-12} {20-17-18-15}

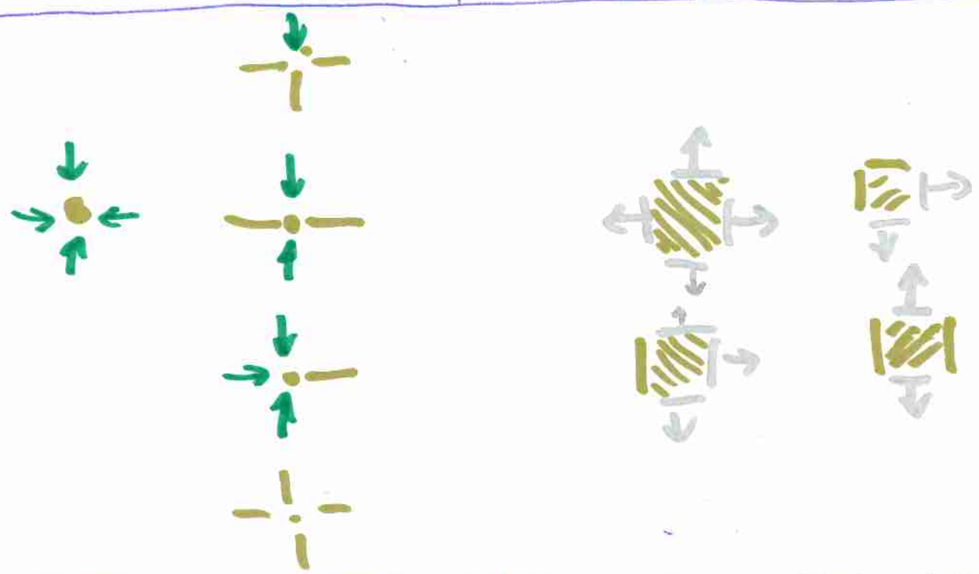
C = (see below)

V = {2-1|2|3-2|3|4-1|4|4-3-2-1|4-3|
5-1|5|5-2|6|6-5-2-1|6-5|7-3|7|8-4|8|
8-7-4-3|8-7|8-7-6-5|8-5|10-5|10|
11-6|11|12-8|12|12-10-8-5|12-10|
13-9|13|13-12-10-9|13-12|14-7|14|
14-11-7-6|14-11|14-13|-11-9|14-13|
15-9|15|16-13|16|17-1|17|
17-10-5-1|17-10|17-15-10-9|17-15|
18-2|18|18-11-6-2|18-11|18-15-11-9|18-15
19-3|19) 19-14-7-3|19-14|
19-16-14-13|19-16|19-18-3-2|19-18|
20-4|20|20-12-8-4|20-12|
20-17 4-1|20-17|20-16-13-12|20-16}

C = {1, 7-6, 9, 10-9, 16-15,
19-18-16-15, 20-17-16-15}

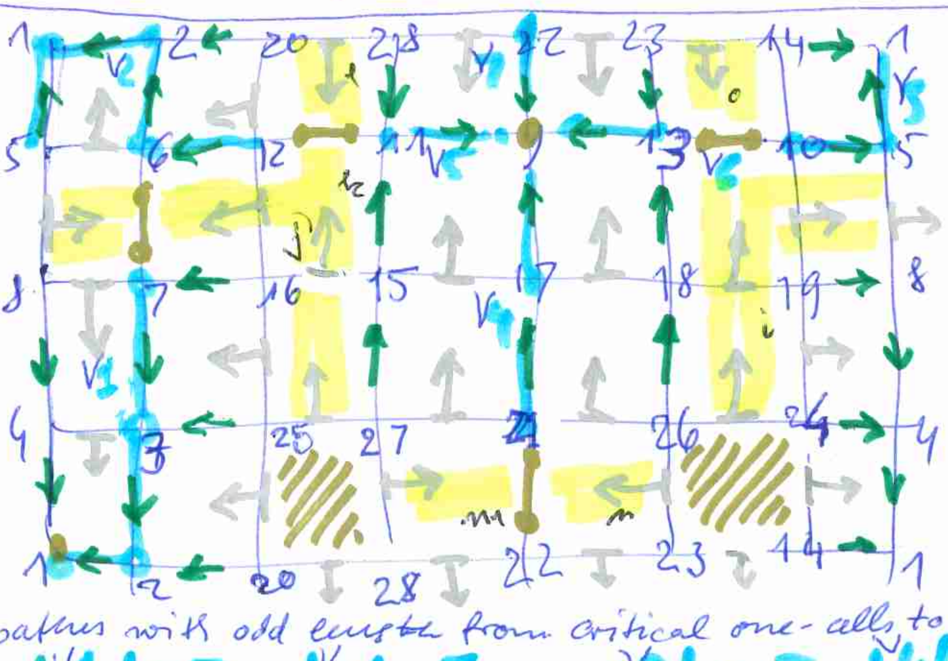
Cubical torus, distorted twice, # 0-cells = 6·4 = 24





Classification of critical
0-cells
1-cells
2-cells
in a discrete vectorfield
(gold = critical
silver } arrows
green } for vectorfield-pairings)

Cubical torus, distorted 3-times, # 0-cells = 7·4 = 28



v-paths (odd length) from crit 2-cells to crit 1-cells

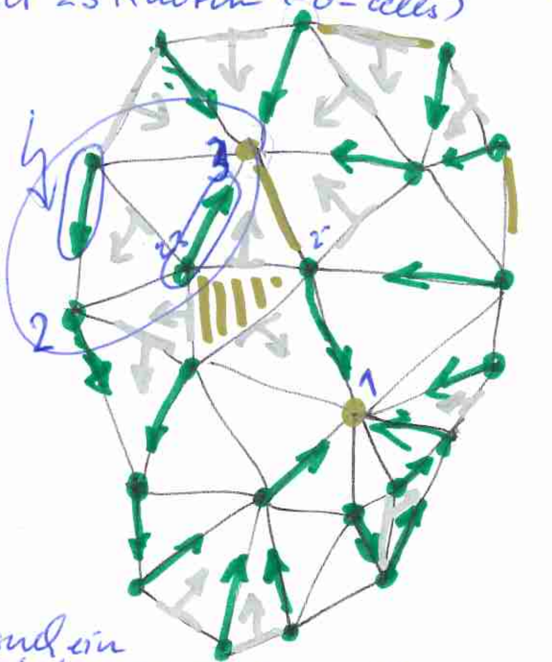
|i| = 9
|j| = 7
|k| = 5
|l| = 3
|m| = 3
|n| = 3
|o| = 3

|v7| = 3
|v5| = 3
|v6| = 3

V-paths with odd length from critical one-cells to critical zero-cells (in blue)

|v1| = 7 |v2| = 7 |v3| = 5 |v4| = 5

Zusammenhängender berandeter simplizialer Komplex #23 mit 23 Knoten (0-cells)



Versuch ein diskretes Vektorfeld zu zeichnen.

Ränder der dreikritischen 2-cells:

$\beta_1 = 23-21-10$, $\beta_1 \in M_p$ $p=2$

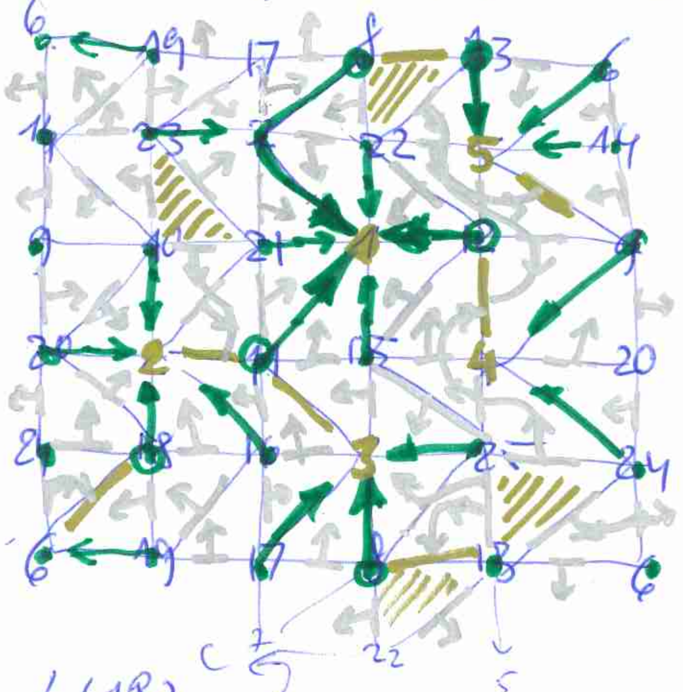
$\partial\beta_1 = \sum_{\alpha \in M_{p-1}} c_{\alpha\beta}\,\alpha = \{11-2\}$

$\partial\beta_2 = \partial\{22-13-8\} = \{9-5, 12-4\} \cup \{13-8\}$

$\partial\beta_3 = \partial\{25-24-13\} = \{12-4, 11-3\} \cup \{18-3\} \cup \{18-6\}$

Werte der #25 Knoten (0-cells) zufällig verteilt.

Kubischer Komplex mit periodischem Rand



L(12)
PQzero = {12-4, 12-5, 12-9}
PQone = ∅ ∪ {12-9-4} ∪ {12-9-5}

L(9)
PQzero = {9-5}
PQone = ∅

L(15) =
PQzero = {15-3, 15-4, 15-11, 15-12}
PQone = {15-11-1, 15-12-1} ∪ {15-11-3} ∪ {15-12-4}

L(13)
PQzero = {13-6, 13-8}
PQone = {13-6-5}

L(11)
PQzero = {11-2, 11-3}
PQone = ∅

L(16)
PQzero = {16-3, 16-11}
PQone = {16-17-2, 16-11-3}

L(17)
PQzero = {17-7, 17-8, 17-16},
PQone = {17-8-3, 17-16-3} ∪ {17-8-7}

L(18)
PQzero = {18-6, 18-16}
PQone = {18-16-2}

L(19)
PQzero = {19-14, 19-16, 19-17, 19-18}
PQone = {19-14-16, 19-18-6} ∪ {19-18-16} ∪ {19-17-16}