How the Composition of Wavelengths in Light Affects the Coherency

Analysis of coherency in a Michelson Interferometer using a sodium lamp, white light with an orange filter and white light.

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1 Abstract

Michelson Interferometry can be used to analyze a lights emission spectrum or to measure small displacements. An interferometer uses the superposition of two beams to create an interference pattern which can then be analyzed. The coherence of the two superimposed waves is crucial for many optical applications. The length of coherence is important as it corresponds to the amount of displacement over which the interference patterns remain discernible. Coherence length is dependent on the spectrum of wavelengths that the light consists of. In this report, we analyzed the coherence length of several different sources to better understand the effect of wavelength composition on its coherence length. Specifically, we compared a sodium lamp, white light, and light passed through an orange narrow band filter. It was found that the coherence of the sodium lamp was the largest of the three, while the white light had the smallest. All values agreed with our expectations and with other sources.

2 Discussion

2.1 Background

The main purpose of an interferometer is to accurately measure small distances by analyzing the interference pattern. Figure 1 is an outline of a Michelson interferometer. It works by splitting the incoming light down two arms and recombining the returning beams. A simple model for this type interference is two monochromatic light sources a distance d apart. An equation for the output intensity is given below.

$$I = \frac{I_0}{2}\left[1 + cos\left(\frac{2\pi dn}{\lambda}\right)\right] \tag{1}$$

Using this simple model, as we adjust d, fringes will appear and disappear. This type of interferometer was used in LIGO which detected the first gravity waves. In order to make reasonable measurements, the light in the interferometer must be coherent. Coherence is a measure of how close two light sources have the same wave form and phase. Two light waves are fully coherent when their intensity, phase and frequency are the same. In a Michelson interferometer, when the two arms have the same optical length (i.e. each takes the same amount of time for light to traverse, this can be different from the physical distance due to refraction) the entire spectrum of light will be in phase. At this point there will be a stable interference pattern, which is extremely important for making measurements. When the light in both arms is fully coherent, this arrangement is known as the ZPL or Zero Path Length difference. As the length is varied from the ZPL, the fringes from the different wavelengths overlap and difference between the light and dark portions of the fringes become less perceptible.

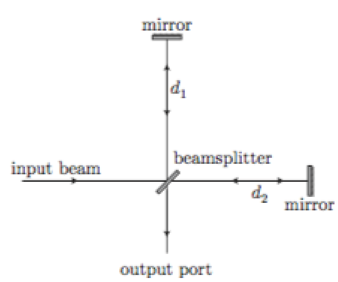


Figure 1: Simple example of a Michelson Interferometer where the initial light wave is split and down two paths. Many configurations of this interferometer also contain a compensator plate to account for the portion of light which travelled through the glass.

In this lab we analyzed the interference pattern created by a sodium lamp, white light with an orange filter and just white light. An important factor of coherency is the amount of frequencies (or wavelengths) in a given source of light. A sodium lamp has two main peaks in it's wavelength spectrum. As a result, as the path length is varied the two peaks will move in and out of phase, producing a beat pattern. If each peak had zero line-width, this pattern would continue to infinity. In reality, the two lines have a non-zero width and so the relative intensity between the dark and light fringes decreases exponentially. White light contains a much larger spectrum of different wavelengths (think about the visible light spectrum). As a result, coherency of white light is much harder to achieve then that of a sodium lamp.

2.2 Results

The general procedure for this lab was to adjust the position of one mirror using a translatable stage while recording the output intensity. The analysis was then performed on the time evolution (and hence displacement) of the interference pattern. As shown in Figure 2, (a) is an example of the output of the Matlab script. The stage had to move slow enough so that the interference pattern didn't alias. For the interference pattern in Figure 2, the mirror was set to move a total of $6mm$ at a speed of $3\mu m/s$. To determine the coherence length, the peaks of the envelope in Figure 2 (a) were recorded. The location of maximum intensity is known as the ZPL. In general, we found a systematic error in all of our results where the interference pattern on the right of the plot appeared to have a smaller intensity then the left. We were unable to determine the cause of this error, but suspect it may have been due to some automatic compensation features in the camera. We also found that the upper part of the envelope was being saturated so we only used to the lower portion of the envelope. With the peaks recorded, we then fit the peak intensity to an exponential curve as seen in Figure 2 (b). The reciprocal of the constant in the exponential is equivalent to the coherence length. We repeated the same procedure for the white light with and without the filter. The results are given in table 1.

Table 1: Results for various light sources

	ZPL Location	**Coherence Length**
Sodium Lamp	8.2612 $\pm$ 0.0006mm	2.1 $\pm$ 0.1 mm
Orange Filter	7.7524 $\pm$ 0.0005mm	0.013 $\pm$ 0.004mm
White Light	7.7012 $\pm$ 0.0005mm	0.0021 $\pm$ 0.0005mm

From the table above, we can see our results agree with expectations. Our expectations were that the sodium lamp would have the largest coherence length, then the narrow band or orange filter and finally the white light. The expected value for the white coherence length was around $1\mu m$ where our value is $2.1\mu m$. The results does not agree within uncertainty but within magnitude and since the scale is small we believe this an appropriate result. Furthermore, other online sources have the coherence length of the sodium lamp ranging between 0.6mm to 4mm and our result is within this range. To further illustrate the wavelength compositions and the affect this has on the coherence length we also computed the Fourier transform of the sodium lamp and white light. Figure 3 illustrates that the sodium lamp has a much narrower wavelength spectrum then that of white light.

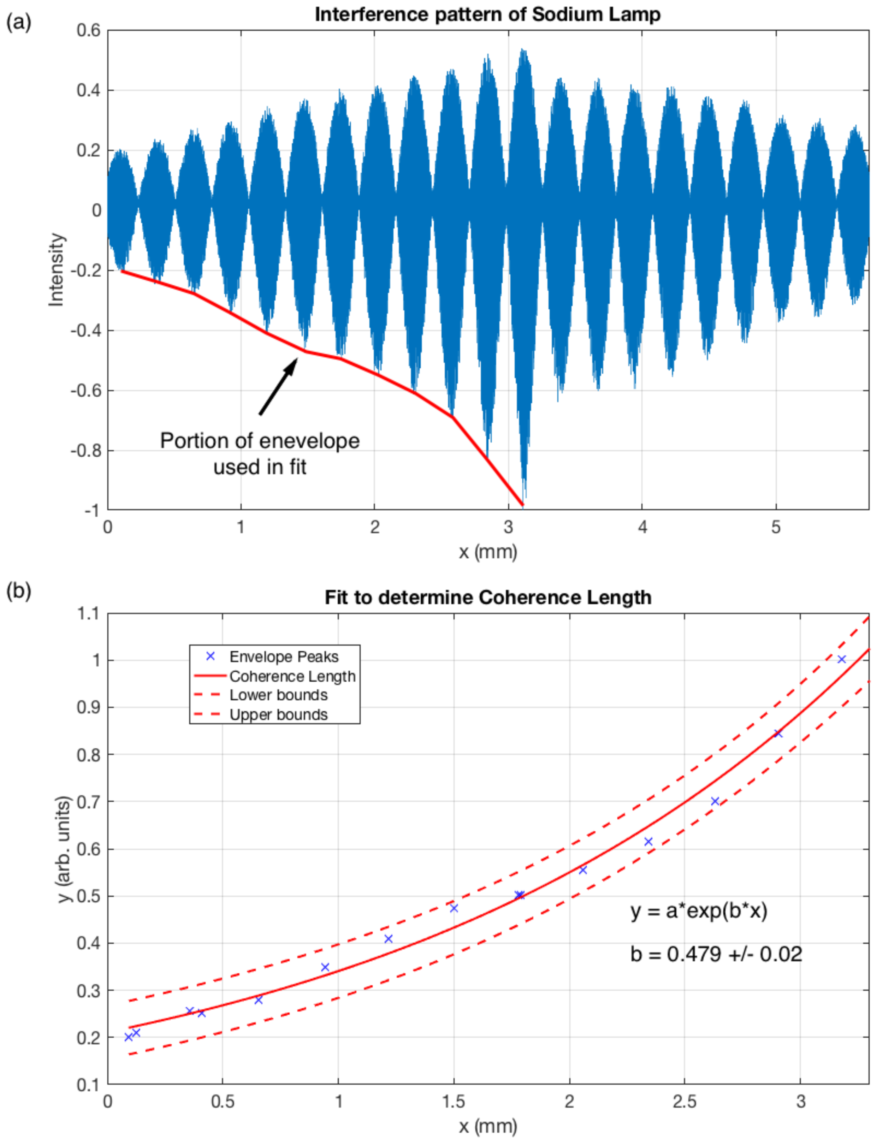
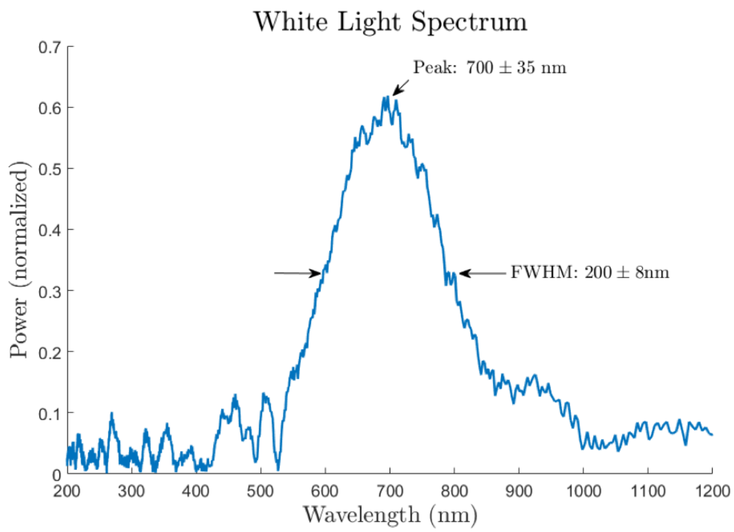


Figure 2: (a) The interference pattern of a sodium lamp. Intensity of a small area was recorded as the position of one of the mirrors was changed. The stage moved at a speed of $3\mu m/s$ over a total distance of $6mm$. As the position of the mirror moves the coherence decreases. The bottom envelope was used to determine the coherence length as the top may be slightly saturated. The peaks of the beats in the bottom envelope were saved and used in the fit. (b) Fit used to determine the coherence length of the sodium lamp. Using the peaks of the beats from the interference pattern, an exponential was fit to the data. The upper and lower confidence intervals are given on the plot. The coherence length is given as the distance where the peak intensity has decayed by $1/e$. For the sodium lamp we determined the coherence length to be $1/b = 2.1 \pm 0.1mm$. From examining (a) we can see that this corresponds approximately where the bottom envelope has decayed to about 37% of the maximum intensity as expected.



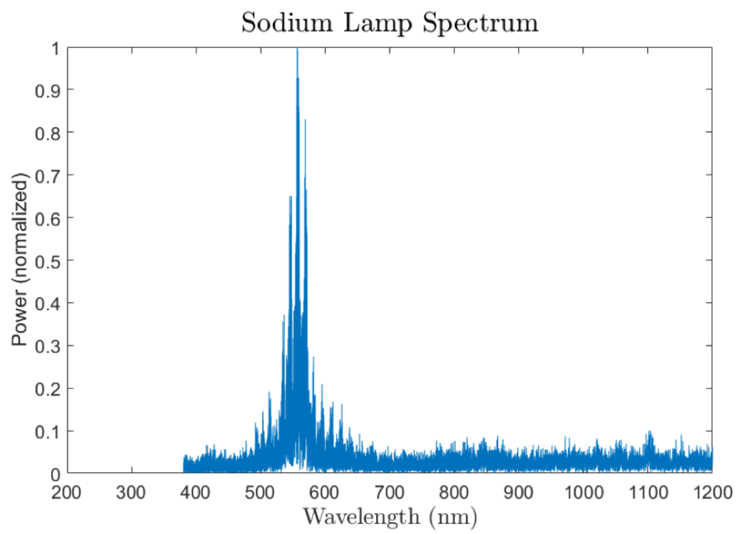


Figure 3: Both spectra were produced from Fourier Transforms of intensity with respect to stage position. As can be seen, the white light has a much wider spectrum, whereas sodium consists of a few relatively narrow bands.

3 Conclusion

The goal of this report was to show that light sources with a combination of multiple wavelengths have a much smaller coherence length. As found from the results, it is clear that sodium, which mainly consists of two wavelengths, has a much larger coherence length than white light. Furthermore, using a narrow band filter also results in a higher coherence length. It is clear to see the difference in wavelength composition when analyzing the Fourier transform as shown in Figure 3. While our results vary slightly from the expected values, they all agree within an order of magnitude of the correct result. Our values may differ due to the limitation of optical components used within the lab or the effectiveness of the CCD camera used to record the output intensity. We also noticed weird errors in our intensity data that could be attributed to issues with the camera. The most challenging portion of this lab was getting the lab equipment to work correctly. We lost an entire lab section due to connectivity issues between the motor and the PC, and lost time on other sections due to the same issue. Regardless of these issues, we were still able to accurately calculate the coherence length values we expected.