CS7646 Project 1:
Martingale

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Abstract—PDF report for CS7646 project 1

1 EXPERIMENT 1 QUESTIONS

1.1 Question 1

In Experiment 1, based on the experiment results calculate and provide the estimated probability of winning $80 within 1000 sequential bets.

Based on the results of the experiment, we can conclude that we have a 100% chance of winning 80$ within 1000 sequential bets. Utilizing the p1_results.txt which is composed of statistics calculated from Experiment 1 we can see each episodes first bet # that had a winning of 80$. Using the statistics we can see that each episode earned 80$ in winnings by bet # 201. This means out of each episode's 1000 successive bets we can see that each episode won 80$ within bet # 201 which means based on our experiment we have a 100% chance of winning 80$. We can also confirm this statistic by using the binomial probability distribution to estimate the probability of winning 80$ from 1000 successive bets. Using the formula $P_x = \binom{n}{x} p^x q^{n-x}$ we substitute with the corresponding values to have this new equation $P_x = \binom{1000}{80}(.4737^{80})(1-.4737)^{1000-80}$ which gives us 100%. This formula shows us that if we bet on red or black with a probability of 9/19 we will have a 100% chance of winning 80$ within 1000 successive bets. We can also confirm this by looking at figure 1 and see the convergence of 80$ winnings by bet # 201.

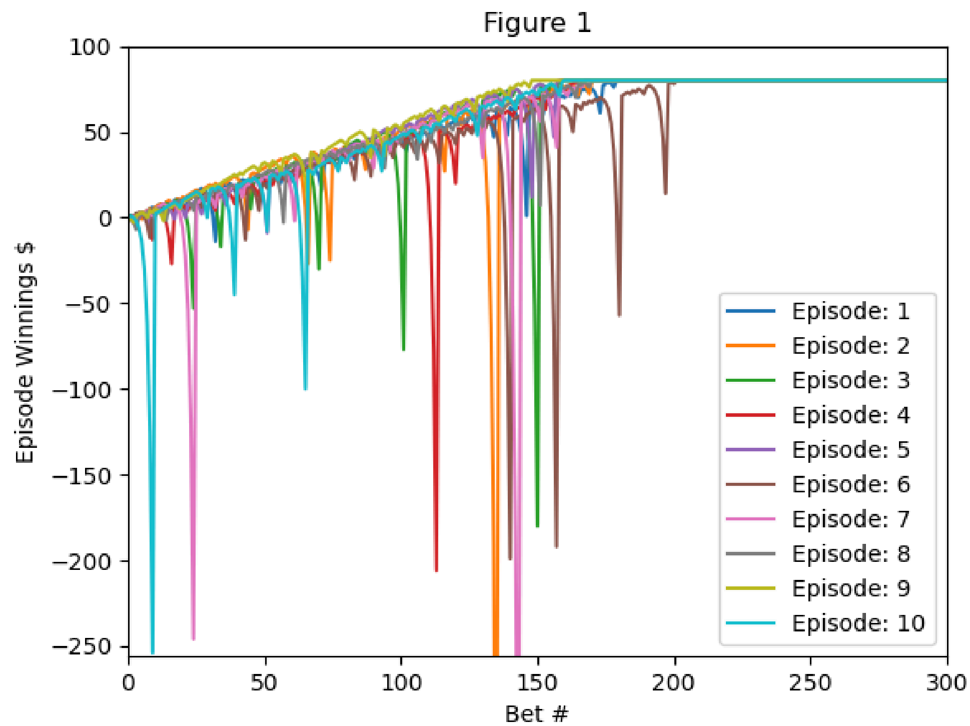

Figure 1—Experiment 1— 10 episodes with 1000 successive bets with an infinite bank roll

1.2 Question 2

In Experiment 1, what is the estimated expected value of winnings after 1000 sequential bets? Thoroughly explain your reasoning for the answer.

Utilizing the formula $E(x) = x_1 p_1$ we can estimate the expected value of winning per bet then multiply that by 1000 to get the estimated value over 1000 sequential bets. Substituting the numbers we get the equation $E(x) = 1(.4737) * 1000$ This gives us an expected value of winning 473.70$ per 1000 bets.

1.3 Question 3

In Experiment 1, do the upper standard deviation line (mean + stdev) and lower standard deviation line (mean – stdev) reach a maximum (or minimum) value and then stabilize?Do the standard deviation lines converge with one another as the number of sequential bets increases? Thoroughly explain why it does or does not.

Based on the results of the experiment, we can conclude that the upper and lower standard deviations reach a maximum and minimum then converges. Utilizing the p1_results.txt which is composed of statistics calculated from Experiment 1 we can see that the upper standard deviation had a maximum of 128429.85 and the lower standard deviation had a minimum of -136708.45. The standard deviations and mean will always converge in this experiment at 80$, because of the unlimited bank roll in this experiment. Due to the betting strategy of Martingale we will always be able to recoup our losses after winning just one game on any losing streak with unlimited bankroll, thus increasing the number of sequential bets just guarantees a better chance of convergence at 80$.

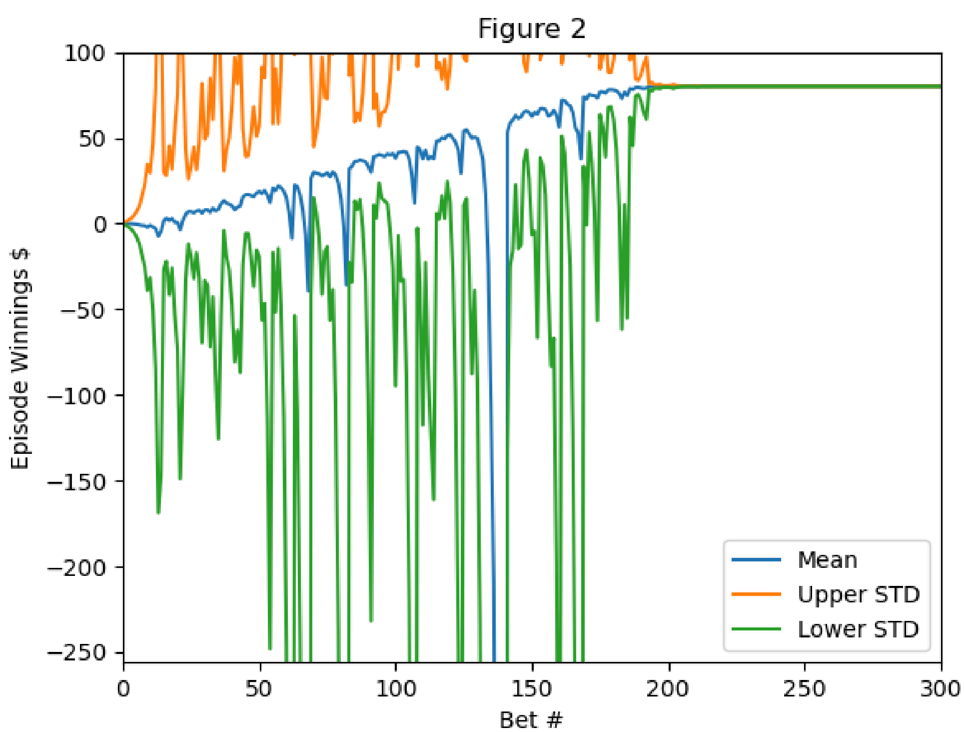

Figure 2—Experiment 1— 1000 episodes with 1000 successive bets with an infinite bank roll

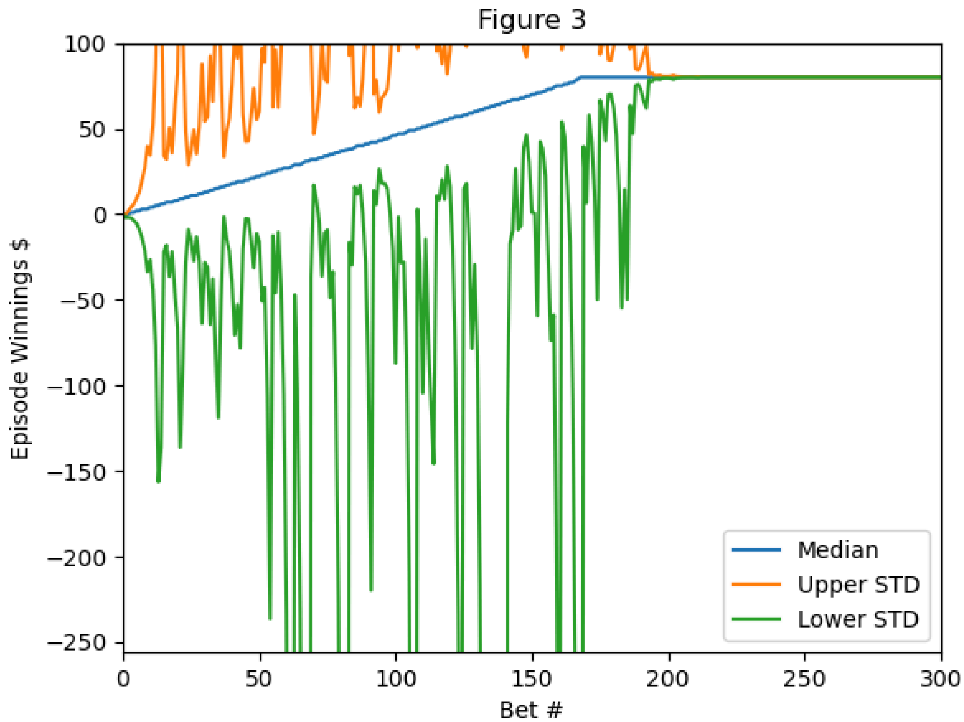

Figure 3—Experiment 1— 1000 episodes with 1000 successive bets with an infinite bank roll

2.1 Question 4

In Experiment 2, based on the experiment results calculate and provide the estimated probability of winning $80 within 1000 sequential bets. Thoroughly explain your reasoning for the answer using the experiment output. Your explanation should NOT be based on estimates from visually inspecting your plots, but from analyzing any output from your simulation.

Based on the results of the experiment, we can conclude that we have a 78.3% chance of winning 80$ within 1000 sequential bets with a bankroll of $256. Utilizing the p1_results.txt which is composed of statistics calculated from Experiment 2 we see that out of the 1000 episodes, 783 of the episodes won 80$ within 1000 sequential bets which gives us the estimated probability of 78.3%.

2.2 Question 5

In Experiment 2, what is the estimated expected value of winnings after 1000 sequential bets? Thoroughly explain your reasoning for the answer.

Utilizing the formula $E(x) = x_1 p_1$ we can estimate the expected value of winning per bet then multiply that by 1000 to get the estimated value over 1000 sequential bets. Substituting the numbers we get the equation $E(x) = 1(.4737) * 1000$ This gives us an expected value of winning 473.70$ per 1000 bets. The expected value doesn't change compared to the first experiment because the probability of landing on red or black remains the same even though the bankroll changed.

2.3 Question 6

In Experiment 2, do the upper standard deviation line (mean + stdev) and lower standard deviation line (mean – stdev) reach a maximum (or minimum) value and then stabilize? Do the standard deviation lines converge with one another as the number of sequential bets increases? Thoroughly explain why it does or does not.

Based on the results of the experiment, we can conclude that the upper and lower standard deviations reach a maximum and minimum then does NOT converge. Utilizing the p1_results.txt which is composed of statistics calculated from Experiment 2 we can see that the upper standard deviation had a maximum of 188.55 and the lower standard deviation had a minimum of -268.76. The standard deviations and mean will not converge in this experiment, because of the limited bank roll in this experiment. Due to the betting strategy of Martingale we will always be able to recoup our losses after winning just one game on any losing streak with an unlimited bankroll, but in this experiment we have a bankroll capped at $256. This means if we lose 9 times in a row then we lose our entire bankroll and cannot keep betting. This will lead to the standard deviation not converging with the mean because we will have episodes that have not hit the $80 winning threshold which leads to the non-convergence. Increasing the number of sequential bets will not help converge the standard deviations with the mean because we are limited by the probability of losing 9 times in a row.

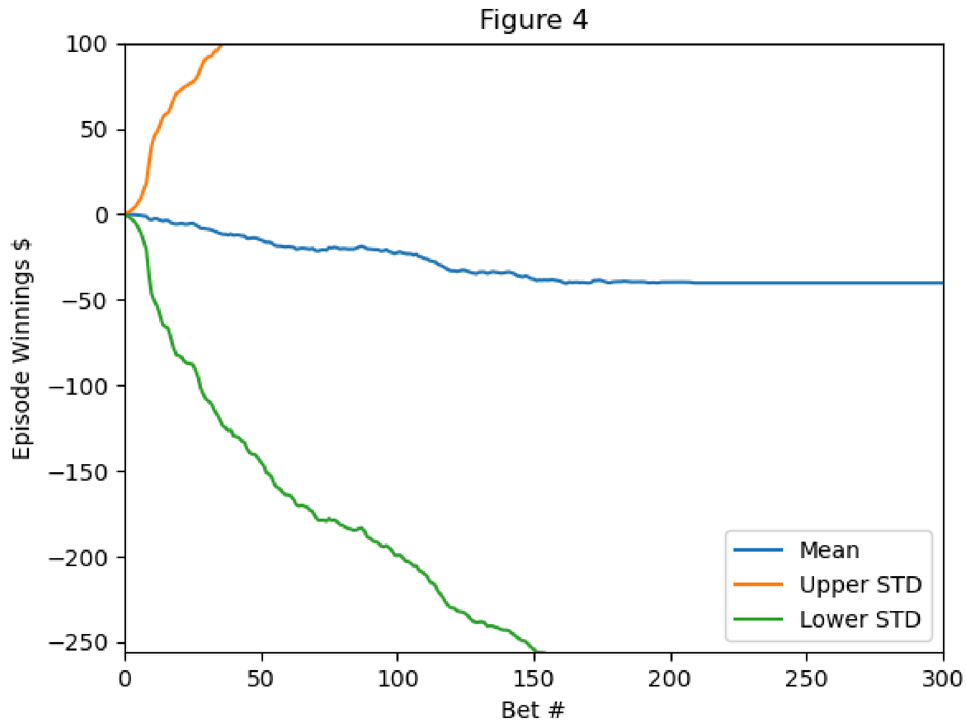

Figure 4—Experiment 2— 1000 episodes with 1000 successive bets

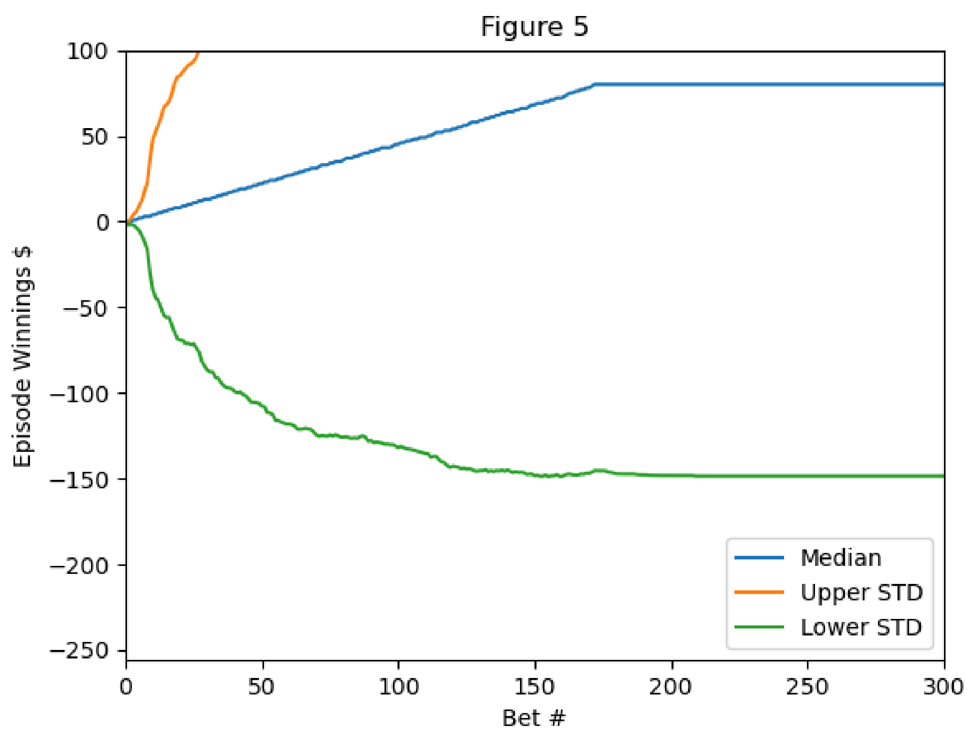

Figure 5—Experiment 2— 1000 episodes with 1000 successive bets with an bankroll set to $256

3.1 Question 7

What are some of the benefits of using expected values when conducting experiments instead of simply using the result of one specific random episode?

We utilize expected values rather than using one specific random episode because the expected values consider the probability of all possible outcomes. Lets create a hypothetical situation on why we want to use expected value, let's say we redid experiment #2 with only one episode. Let's say this episode resulted in 9 consecutive losses from the start, this would mean we lost our entire bankroll of $256. If we used this episode to make predictions on probabilities we would never want to gamble again, but if we use the expected value, the expected value takes into account an episode where you lose 9 times or win consecutively 1000 times. The expected value helps in evening out the probability for all possible scenarios, which makes it better for making predictions without real world data.