2.3.1 Analisis de Fourier

Cualquier función periodica $g(t)$ se puede construir sumando una cantidad posiblemente infinita de senos y cosenos

$$g(t) = \frac{1}{2}c + \sum_{n=1}^{\infty} a_n \operatorname{sen}(2\pi n f t) + \sum_{n=1}^{\infty} b_n \cos(2\pi n f t)$$

$$f = \frac{1}{T}$$ (frecuencia fundamental ó primer armónico)

$$a_n = \frac{2}{T}\int_0^T g(t)\operatorname{sen}(2\pi n f t)\,dt$$

$$b_n = \frac{2}{T}\int_0^T g(t)\cos(2\pi n f t)\,dt$$

$$c = \frac{2}{T}\int_0^T g(t)\,dt$$

- Demostración

• $c = \frac{2}{T}\int_0^T g(t)$

$$g(t) = \frac{c}{2} + \sum_{n=1}^{\infty} a_n \operatorname{sen}(2\pi n f t)_0 + \sum_{n=}^{\infty} b_n \cos(2\pi n f t) \quad \phi$$

$$\int_{-T}^{T} g(t) = \int_{-T}^{T}\frac{c}{2} + \sum_{n=1}^{\infty} a_n \cancel{\int_{-T}^{T} \operatorname{sen}(2\pi n \tfrac{1}{T} t)}^{\phi} + \sum_{n=1}^{\infty} b_n \cancel{\int_{-T}^{T} \cos(2\pi n \tfrac{1}{T} t)}$$

$$\int_{-T}^{T} g(t) = \frac{c}{2}T\Big|_{-T}^{T} = Tc$$

$$\int_{-T}^{T} g(t) = Tc$$

$$c = \frac{1}{T}\int_{-T}^{T} g(t)\,dt$$

$$c = \frac{2}{T}\int_{0}^{T} g(t)\,dt$$

$$b_n = \frac{2}{T}\int_{0}^{T} g(t)\cos(2\pi n f t)\,dt$$

$$g(t)\cos\left(\frac{m\pi t}{T}\right) = \frac{c}{2}\cos\left(\frac{m\pi t}{T}\right) + \sum_{n=1}^{\infty} a_n \operatorname{sen}\left(\frac{2\pi n t}{T}\right)\cos\left(\frac{m\pi t}{T}\right) + \sum b_n \cos\left(\frac{2\pi n t}{T}\right)\cos\left(\frac{m\pi t}{T}\right) =$$

$$\downarrow$$

Integración de $\int_{-T}^{T}$

$$\frac{c}{2}\int_{-T}^{T}\cos\left(\frac{m\pi t}{T}\right)dt = 0$$

$m \neq n = 0$

$m = n$

$$\sum_{n=1}^{\infty} b_n \int_{-T}^{T}\cos\left(\frac{2\pi n t}{T}\right)\cos\left(\frac{m\pi t}{T}\right)dt = b_n\int_{-T}^{T}\cos^2\left(\frac{2\pi t}{T}\right)dt$$

$$\sum_{n=1}^{\infty} a_n \int_{-T}^{T}\operatorname{sen}\left(\frac{2\pi n t}{T}\right)\cos\left(\frac{m\pi t}{T}\right)dt = 0$$

$$\int_{-T}^{T} g(t)\cos\left(\frac{m\pi t}{T}\right)dt = b_n\int_{-T}^{T}\cos^2\left(\frac{2\pi t}{T}\right)dt$$

por propiedades

$$\int_{-T}^{T} g(t)\cos\left(\frac{m\pi t}{T}\right)dt = b_n T$$

$$m = 2n$$

$$b_n = \frac{2}{T}\int_{0}^{T} g(t)\cos(2\pi n f t)\,dt$$

$$a_n = \frac{2}{T}\int_{0}^{T} g(t)\operatorname{sen}(2\pi n f t)\,dt$$

$$\int_{-T}^{T} g(t)\operatorname{sen}\left(\frac{m\pi x}{T}\right) = \ldots\ldots$$

$$\frac{c}{2}\int_{-T}^{T}\operatorname{sen}\left(\frac{m\pi t}{T}\right)dt = 0$$

$$\int_{-T}^{T}\cos\left(\frac{2\pi n}{T}t\right)\operatorname{sen}\left(\frac{m\pi t}{T}\right)dt = 0$$

$$m = 2n$$

$$\int_{-T}^{T}\operatorname{sen}\left(\frac{n\pi x}{T}\right)\operatorname{sen}\left(\frac{m\pi x}{T}\right)dt = \int_{-T}^{T}\operatorname{sen}^2\left(\frac{2\pi n t}{T}\right)dt$$

$$\int_{-T}^{T} g(t)\operatorname{sen}\left(\frac{2\pi n t}{T}\right)dt = a_n\int_{-T}^{T}\operatorname{sen}^2\left(\frac{2\pi n t}{T}\right)dt$$

$$a_n = \frac{2}{T}\int_{0}^{T} g(t)\operatorname{sen}(2\pi n f t)\,dt$$