Solutions to Linear Representations of Finite Groups

Bryn Elesedy*
Department of Computer Science
University of Oxford

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*bryn@robots.ox.ac.uk

2 Character Theory

2.1

Let V and V' be the corresponding representations. Then $\chi + \chi'$ is the character of the direct sum $V \oplus V'$. The character of the alternating square is then, for any s in the group,

$$\begin{aligned}(\chi + \chi')_\sigma &= \frac{1}{2}((\chi(s) + \chi'(s))^2 + \chi(s^2) + \chi'(s^2)) \\ &= \frac{1}{2}(\chi(s)^2 + \chi(s^2)) + \frac{1}{2}(\chi'(s)^2 + \chi'(s^2)) + \chi(s)\chi'(s) \\ &= \chi_\sigma^2(s) + {\chi'_\sigma}^2(s) + \chi(s)\chi'(s).\end{aligned}$$

We can do the same thing for the symmetric square

$$\begin{aligned}(\chi + \chi')_\alpha &= \frac{1}{2}((\chi(s) + \chi'(s))^2 - \chi(s^2) + \chi'(s^2)) \\ &= \frac{1}{2}(\chi(s)^2 - \chi(s^2)) + \frac{1}{2}(\chi'(s)^2 - \chi'(s^2)) + \chi(s)\chi'(s) \\ &= \chi_\alpha^2(s) + {\chi'_\alpha}^2(s) + \chi(s)\chi'(s).\end{aligned}$$

2.2

References

[Serre,] Serre, J.-P. *Linear representations of finite groups*, volume 42. Springer.