

Softmax exercise

Complete and hand in this completed worksheet (including its outputs and any supporting code outside of the worksheet) with your assignment submission. For more details see the [assignments page \(https://compsci682-fa18.github.io/assignments2018/assignment1/\)](https://compsci682-fa18.github.io/assignments2018/assignment1/) on the course website.

This exercise is analogous to the SVM exercise. You will:

- implement a fully-vectorized **loss function** for the Softmax classifier
- implement the fully-vectorized expression for its **analytic gradient**
- **check your implementation** with numerical gradient
- use a validation set to **tune the learning rate and regularization strength**
- **optimize** the loss function with **SGD**
- **visualize** the final learned weights

```
In [13]: import random
import numpy as np
from cs682.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

from __future__ import print_function

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading external modules
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2
```

The autoreload extension is already loaded. To reload it, use:

```
%reload_ext autoreload
```

```

In [14]: def get_CIFAR10_data(num_training=49000, num_validation=1000, num_test=1000, num_dev=500):
        """
        Load the CIFAR-10 dataset from disk and perform preprocessing to prepare
        it for the linear classifier. These are the same steps as we used for the
        SVM, but condensed to a single function.
        """
        # Load the raw CIFAR-10 data
        cifar10_dir = 'cs682/datasets/cifar-10-batches-py'

        X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

        # subsample the data
        mask = list(range(num_training, num_training + num_validation))
        X_val = X_train[mask]
        y_val = y_train[mask]
        mask = list(range(num_training))
        X_train = X_train[mask]
        y_train = y_train[mask]
        mask = list(range(num_test))
        X_test = X_test[mask]
        y_test = y_test[mask]
        mask = np.random.choice(num_training, num_dev, replace=False)
        X_dev = X_train[mask]
        y_dev = y_train[mask]

        # Preprocessing: reshape the image data into rows
        X_train = np.reshape(X_train, (X_train.shape[0], -1))
        X_val = np.reshape(X_val, (X_val.shape[0], -1))
        X_test = np.reshape(X_test, (X_test.shape[0], -1))
        X_dev = np.reshape(X_dev, (X_dev.shape[0], -1))

        # Normalize the data: subtract the mean image
        mean_image = np.mean(X_train, axis = 0)
        X_train -= mean_image
        X_val -= mean_image
        X_test -= mean_image
        X_dev -= mean_image

        # add bias dimension and transform into columns
        X_train = np.hstack([X_train, np.ones((X_train.shape[0], 1))])
        X_val = np.hstack([X_val, np.ones((X_val.shape[0], 1))])
        X_test = np.hstack([X_test, np.ones((X_test.shape[0], 1))])
        X_dev = np.hstack([X_dev, np.ones((X_dev.shape[0], 1))])

        return X_train, y_train, X_val, y_val, X_test, y_test, X_dev, y_dev

        # Cleaning up variables to prevent loading data multiple times (which may cause memory issue)
        try:
            del X_train, y_train
            del X_test, y_test
            print('Clear previously loaded data.')
        except:
            pass

        # Invoke the above function to get our data.

```

```
X_train, y_train, X_val, y_val, X_test, y_test, X_dev, y_dev = get_CIFAR10_data()
print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
print('dev data shape: ', X_dev.shape)
print('dev labels shape: ', y_dev.shape)
```

Clear previously loaded data.

Train data shape: (49000, 3073)

Train labels shape: (49000,)

Validation data shape: (1000, 3073)

Validation labels shape: (1000,)

Test data shape: (1000, 3073)

Test labels shape: (1000,)

dev data shape: (500, 3073)

dev labels shape: (500,)

Softmax Classifier

Your code for this section will all be written inside **cs682/classifiers/softmax.py**.

```
In [15]: # First implement the naive softmax loss function with nested loops.
# Open the file cs682/classifiers/softmax.py and implement the
# softmax_loss_naive function.

from cs682.classifiers.softmax import softmax_loss_naive
import time

# Generate a random softmax weight matrix and use it to compute the loss.
W = np.random.randn(3073, 10) * 0.0001
loss, grad = softmax_loss_naive(W, X_dev, y_dev, 0.0)

# As a rough sanity check, our loss should be something close to -log(0.1).
print('loss: %f' % loss)
print('sanity check: %f' % (-np.log(0.1)))
```

loss: 2.397489

sanity check: 2.302585

Inline Question 1:

Why do we expect our loss to be close to $-\log(0.1)$? Explain briefly.**

Your answer:

We expect the loss to be close to $-\log(0.1)$ because we have 10 classes. There is a 1/10th chance of selecting each class.

```
In [16]: # Complete the implementation of softmax_loss_naive and implement a (naive)
# version of the gradient that uses nested loops.
loss, grad = softmax_loss_naive(W, X_dev, y_dev, 0.0)

# As we did for the SVM, use numeric gradient checking as a debugging tool.
# The numeric gradient should be close to the analytic gradient.
from cs682.gradient_check import grad_check_sparse
f = lambda w: softmax_loss_naive(w, X_dev, y_dev, 0.0)[0]
grad_numerical = grad_check_sparse(f, W, grad, 10)

# similar to SVM case, do another gradient check with regularization
loss, grad = softmax_loss_naive(W, X_dev, y_dev, 5e1)
f = lambda w: softmax_loss_naive(w, X_dev, y_dev, 5e1)[0]
grad_numerical = grad_check_sparse(f, W, grad, 10)
```

```
numerical: 6.287602 analytic: 6.287602, relative error: 9.485562e-09
numerical: -1.919614 analytic: -1.919614, relative error: 9.045675e-10
numerical: 4.237143 analytic: 4.237143, relative error: 9.492963e-09
numerical: 0.476701 analytic: 0.476701, relative error: 2.330504e-08
numerical: -2.202478 analytic: -2.202478, relative error: 1.708274e-08
numerical: -0.074193 analytic: -0.074193, relative error: 5.592320e-07
numerical: -1.616027 analytic: -1.616027, relative error: 1.457989e-08
numerical: -0.666957 analytic: -0.666957, relative error: 1.815791e-08
numerical: -2.237066 analytic: -2.237066, relative error: 7.087218e-09
numerical: 1.871338 analytic: 1.871338, relative error: 2.116737e-08
numerical: 2.905001 analytic: 2.909179, relative error: 7.186291e-04
numerical: -2.793639 analytic: -2.793950, relative error: 5.562537e-05
numerical: -0.276179 analytic: -0.282727, relative error: 1.171572e-02
numerical: -2.876874 analytic: -2.869653, relative error: 1.256509e-03
numerical: -0.132321 analytic: -0.134975, relative error: 9.927422e-03
numerical: -0.070010 analytic: -0.068965, relative error: 7.520539e-03
numerical: -3.835544 analytic: -3.832732, relative error: 3.667140e-04
numerical: 0.639878 analytic: 0.638131, relative error: 1.367608e-03
numerical: -1.265790 analytic: -1.259064, relative error: 2.664068e-03
numerical: 0.564685 analytic: 0.568156, relative error: 3.064071e-03
```

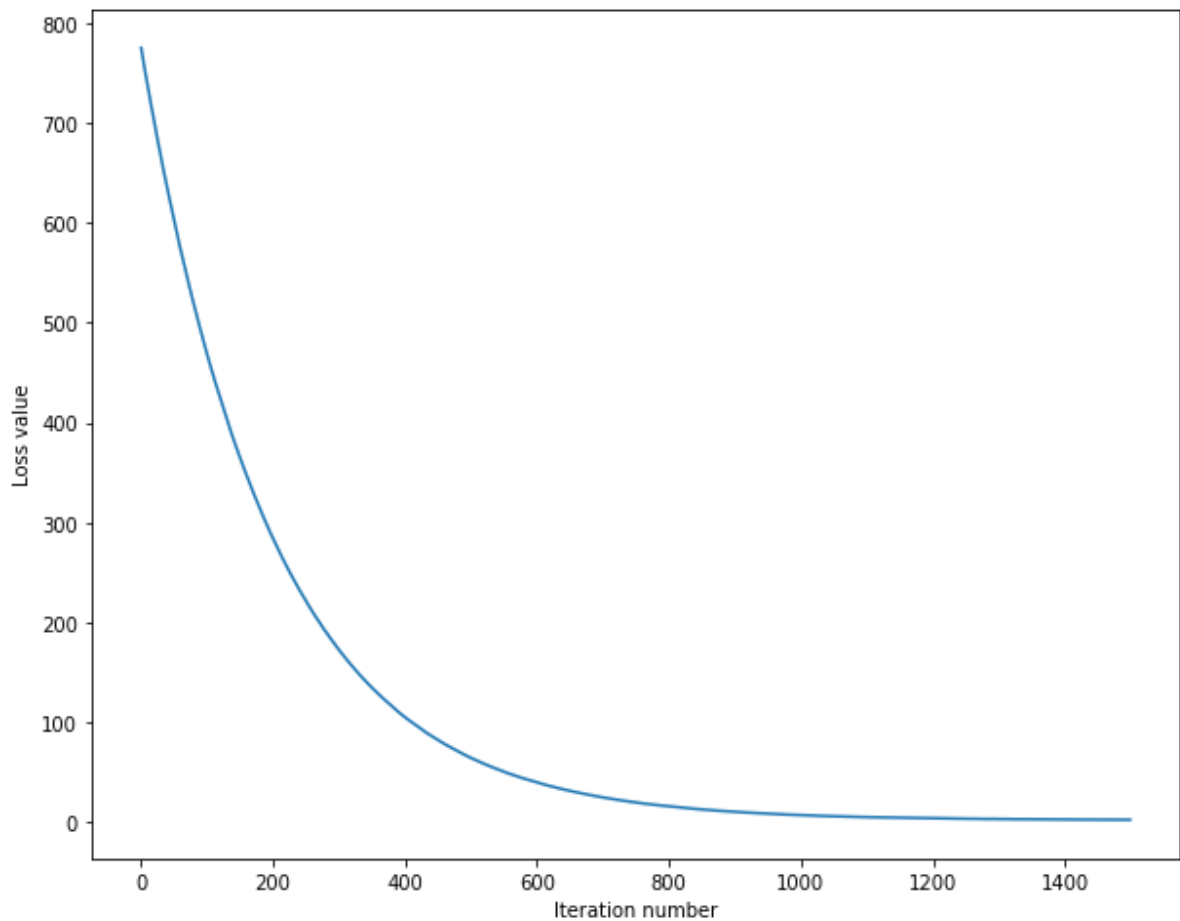
```
In [17]: # Now that we have a naive implementation of the softmax loss function and its gradient,  
# implement a vectorized version in softmax_loss_vectorized.  
# The two versions should compute the same results, but the vectorized version should be  
# much faster.  
tic = time.time()  
loss_naive, grad_naive = softmax_loss_naive(W, X_dev, y_dev, 0.000005)  
toc = time.time()  
print('naive loss: %e computed in %fs' % (loss_naive, toc - tic))  
  
from cs682.classifiers.softmax import softmax_loss_vectorized  
tic = time.time()  
loss_vectorized, grad_vectorized = softmax_loss_vectorized(W, X_dev, y_dev, 0.000005)  
toc = time.time()  
print('vectorized loss: %e computed in %fs' % (loss_vectorized, toc - tic))  
  
# As we did for the SVM, we use the Frobenius norm to compare the two versions  
# of the gradient.  
grad_difference = np.linalg.norm(grad_naive - grad_vectorized, ord='fro')  
print('Loss difference: %f' % np.abs(loss_naive - loss_vectorized))  
print('Gradient difference: %f' % grad_difference)
```

```
naive loss: 2.397489e+00 computed in 0.666219s  
vectorized loss: 2.397489e+00 computed in 0.011968s  
Loss difference: 0.000000  
Gradient difference: 0.000000
```

```
In [18]: #Bryon Kucharski - just for a sanity check like svm
from cs682.classifiers import Softmax
softmax = Softmax()
tic = time.time()
loss_hist = softmax.train(X_train, y_train, learning_rate=1e-7, reg=2.5e4,
                          num_iters=1500, verbose=False)
toc = time.time()
print('That took %fs' % (toc - tic))

plt.plot(loss_hist)
plt.xlabel('Iteration number')
plt.ylabel('Loss value')
plt.show()
```

That took 22.561671s



```

In [19]: # Use the validation set to tune hyperparameters (regularization strength and
# learning rate). You should experiment with different ranges for the learning
# rates and regularization strengths; if you are careful you should be able to
# get a classification accuracy of over 0.35 on the validation set.
from cs682.classifiers import Softmax
results = {}
best_val = -1
best_softmax = None
#learning_rates = [1e-7, 5e-7]
#regularization_strengths = [2.5e4, 5e4]
learning_rates = [1e-7, 2e-7, 3e-7, 4e-7, 5e-7]
regularization_strengths = [1.5e4, 1.75e4, 2.0e4]

for lr in learning_rates:
    for reg_str in regularization_strengths:
        #print("Running" + str((lr, reg_str)))
        softmax = Softmax()
        loss_hist = softmax.train( X_train, y_train, learning_rate=lr, reg=reg_str, num_iters=1500, verbose=False)
        y_valid_pred = softmax.predict( X_val)
        y_train_pred = softmax.predict(X_train)
        valid_accuracy = (np.mean(y_val == y_valid_pred) )
        train_accuracy = (np.mean(y_train == y_train_pred) )
        if valid_accuracy > best_val:
            best_val = valid_accuracy
            best_softmax = softmax

        results[(lr, reg_str)] = (train_accuracy, valid_accuracy)

# Print out results.
for lr, reg in sorted(results):
    train_accuracy, val_accuracy = results[(lr, reg)]
    print('lr %e reg %e train accuracy: %f val accuracy: %f' % (
        lr, reg, train_accuracy, val_accuracy))

print('best validation accuracy achieved during cross-validation: %f' % best_val)

```

```

lr 1.000000e-07 reg 1.500000e+04 train accuracy: 0.351286 val accuracy: 0.360000
lr 1.000000e-07 reg 1.750000e+04 train accuracy: 0.349980 val accuracy: 0.358000
lr 1.000000e-07 reg 2.000000e+04 train accuracy: 0.353755 val accuracy: 0.363000
lr 2.000000e-07 reg 1.500000e+04 train accuracy: 0.363041 val accuracy: 0.372000
lr 2.000000e-07 reg 1.750000e+04 train accuracy: 0.355327 val accuracy: 0.370000
lr 2.000000e-07 reg 2.000000e+04 train accuracy: 0.355061 val accuracy: 0.367000
lr 3.000000e-07 reg 1.500000e+04 train accuracy: 0.361980 val accuracy: 0.378000
lr 3.000000e-07 reg 1.750000e+04 train accuracy: 0.355204 val accuracy: 0.375000
lr 3.000000e-07 reg 2.000000e+04 train accuracy: 0.357551 val accuracy: 0.370000
lr 4.000000e-07 reg 1.500000e+04 train accuracy: 0.360898 val accuracy: 0.376000
lr 4.000000e-07 reg 1.750000e+04 train accuracy: 0.362122 val accuracy: 0.374000
lr 4.000000e-07 reg 2.000000e+04 train accuracy: 0.357449 val accuracy: 0.369000
lr 5.000000e-07 reg 1.500000e+04 train accuracy: 0.360367 val accuracy: 0.371000
lr 5.000000e-07 reg 1.750000e+04 train accuracy: 0.355714 val accuracy: 0.367000
lr 5.000000e-07 reg 2.000000e+04 train accuracy: 0.350265 val accuracy: 0.368000
best validation accuracy achieved during cross-validation: 0.378000

```

```
In [20]: # evaluate on test set
# Evaluate the best softmax on test set
y_test_pred = best_softmax.predict(X_test)
test_accuracy = np.mean(y_test == y_test_pred)
print('softmax on raw pixels final test set accuracy: %f % (test_accuracy, ))
```

softmax on raw pixels final test set accuracy: 0.365000

Inline Question - True or False

It's possible to add a new datapoint to a training set that would leave the SVM loss unchanged, but this is not the case with the Softmax classifier loss.

Your answer: True *Your explanation:*

The SVM loss uses the $\max()$ function. If the new datapoint sets this equation to 0, then it does not change this. Softmax takes every datapoint into consideration when computing the loss


```
In [21]: # Visualize the learned weights for each class
w = best_softmax.W[:-1,:] # strip out the bias
w = w.reshape(32, 32, 3, 10)

w_min, w_max = np.min(w), np.max(w)

classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', 'truck']
for i in range(10):
    plt.subplot(2, 5, i + 1)

    # Rescale the weights to be between 0 and 255
    wimg = 255.0 * (w[:, :, :, i].squeeze() - w_min) / (w_max - w_min)
    plt.imshow(wimg.astype('uint8'))
    plt.axis('off')
    plt.title(classes[i])
```

